GRASP: AN ANNOTATED BIBLIOGRAPHY

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Abstract. A greedy randomized adaptive search procedure (GRASP) is a metaheuristic for combinatorial optimization. It is a multi-start or iterative process, in which each GRASP iteration consists of two phases, a construction phase, in which a feasible solution is produced, and a local search phase, in which a local optimum in the neighborhood of the constructed solution is sought. Since 1989, numerous papers on the basic aspects of GRASP, as well as enhancements to the basic metaheuristic have appeared in the literature. GRASP has been applied to a wide range of combinatorial optimization problems, ranging from scheduling and routing to drawing and turbine balancing. This paper is an annotated bibliography of the GRASP literature from 1989 to 2001.

1. Introduction

Optimization problems that involve a large finite number of alternatives often arise in the private and public sectors of the economy. In these problems, given a finite solution set X and a real-valued function $f : X \to \mathbb{R}$, one seeks a solution $x^* \in X$ with $f(x^*) \leq f(x), \forall\, x \in X$. Common examples include designing efficient telecommunication networks, constructing cost effective airline crew schedules, and producing efficient routes for waste management pickup. To find the optimal solution in a combinatorial optimization problem it is theoretically possible to enumerate the solutions and evaluate each with respect to the stated objective. However, in practice, it is often infeasible to follow such a strategy of complete enumeration because the number of combinations often grows exponentially with the size of problem.

Much work has been done over the last five decades to develop optimal seeking methods that do not explicitly require an examination of each alternative. This research has given rise to the field of *combinatorial optimization* (see Papadimitriou and Steiglitz (1982)), and an increasing capability to solve ever larger real-world problems. Nevertheless, most problems found in industry and government are either computationally intractable by their nature, or sufficiently large so as to preclude the use of exact algorithms. In such cases, heuristic methods are usually employed to find good, but not necessarily guaranteed optimal solutions. The effectiveness of these methods depends upon their ability to adapt to a particular realization, avoid entrapment at local optima, and exploit the basic structure of the problem. Building on these notions, various heuristic search techniques have been developed that have demonstrably improved our ability to obtain good solutions to difficult combinatorial optimization problems. The most promising of such techniques include simulated annealing (Kirkpatrick, 1984), tabu search (Glover, 1989, 1990;

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Glover and Laguna, 1997), genetic algorithms (Goldberg, 1989), variable neighborhood search (Hansen and Mladenović, 1998), and GRASP (Greedy Randomized Adaptive Search Procedures) (Feo and Resende, 1989, 1995).

A GRASP is a multi-start or iterative process (Lin and Kernighan, 1973), in which each GRASP iteration consists of two phases, a construction phase, in which a feasible solution is produced, and a local search phase, in which a local optimum in the neighborhood of the constructed solution is sought. The best overall solution is kept as the result.

In the construction phase, a feasible solution is iteratively constructed, one element at a time. The basic GRASP construction phase is similar to the semi-greedy heuristic proposed independently by Hart and Shogan (1987). At each construction iteration, the choice of the next element to be added is determined by ordering all candidate elements (i.e. those that can be added to the solution) in a candidate list C with respect to a greedy function $g : C \rightarrow \mathbb{R}$. This function measures the (myopic) benefit of selecting each element. The heuristic is adaptive because the benefits associated with every element are updated at each iteration of the construction phase to reflect the changes brought on by the selection of the previous element. The probabilistic component of a GRASP is characterized by randomly choosing one of the best candidates in the list, but not necessarily the top candidate. The list of best candidates is called the *restricted candidate list* (RCL). This choice technique allows for different solutions to be obtained at each GRASP iteration, but does not necessarily compromise the power of the adaptive greedy component of the method.

As is the case for many deterministic methods, the solutions generated by a GRASP construction are not guaranteed to be locally optimal with respect to simple neighborhood definitions. Hence, it is almost always beneficial to apply a local search to attempt to improve each constructed solution. A local search algorithm works in an iterative fashion by successively replacing the current solution by a better solution in the neighborhood of the current solution. It terminates when no better solution is found in the neighborhood. The *neighborhood structure* N for a problem P relates a solution s of the problem to a subset of solutions $N(s)$. A solution s is said to be *locally optimal* if there is no better solution in $N(s)$. The key to success for a local search algorithm consists of the suitable choice of a neighborhood structure, efficient neighborhood search techniques, and the starting solution.

While such local optimization procedures can require exponential time (Johnson et al., 1988) from an arbitrary starting point, empirically their efficiency significantly improves as the initial solution improves. The result is that often many GRASP solutions are generated in the same amount of time required for the local optimization procedure to converge from a single random start. Furthermore, the best of these GRASP solutions is generally significantly better than the single solution obtained from a random starting point.

It is difficult to formally analyze the quality of solution values found by using the GRASP methodology. However, there is an intuitive justification that views GRASP as a repetitive sampling technique. Each GRASP iteration produces a sample solution from an unknown distribution of all obtainable results. The mean and variance of the distribution are functions of the restrictive nature of the candidate list. For example, if the cardinality of the restricted candidate list is limited to one, then only one solution will be produced and the variance of the distribution will be zero. Given an effective greedy function, the mean solution value in this case should be good, but probably suboptimal. If a less restrictive cardinality limit is imposed, many different solutions will be produced implying a larger variance. Since the greedy function is more compromised in this case, the mean solution value should degrade. Intuitively, however, by order statistics and the fact that the samples are randomly produced, the best value found should outperform the mean value. Indeed, often the best solutions sampled are optimal.

An especially appealing characteristic of GRASP is the ease with which it can be implemented. Few parameters need to be set and tuned, and therefore development can focus on implementing efficient data structures to assure quick GRASP iterations. Finally, GRASP can be trivially implemented in parallel. Each processor can be initialized with its own copy of the procedure, the instance data, and an independent random number sequence. The GRASP iterations are then performed in parallel with only a single global variable required to store the best solution found over all processors.

In this article, we provide an annotated bibliography of the GRASP literature up to early 2001. This document contains references related to GRASP that have either appeared in the literature or as technical reports. We first look at tutorials and surveys. Papers that propose enhancements to the basic heuristic are considered next. Following that, we examine GRASP as a component of a hybrid metaheuristic. Parallelization of GRASP and GRASP source code follow. The paper concludes with a literature review of operations research and computer science applications of GRASP as well as industrial applications.

If the reader is aware of any uncited reference, incorrectly cited reference, or update to a cited reference, please contact the authors.

C.H. Papadimitriou and K. Steiglitz. *Combinatorial optimization: Algorithms and complexity*. Prentice-Hall, 1982.

A classic book on combinatorial optimization.

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S. Kirkpatrick. Optimization by simulated annealing: Quantitative studies. *J. of Statistical Physics*, 34:975–986, 1984.

Description of the simulated annealing metaheuristic.

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F. Glover. Tabu search – Part I. *ORSA J. on Computing*, 1:190–206, 1989.

Description of the tabu search metaheuristic.

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F. Glover. Tabu search – Part II. *ORSA J. on Computing*, 2:4–32, 1990.

Description of the tabu search metaheuristic.

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F. Glover and M. Laguna. *Tabu Search*. Kluwer Academic Publishers, 1997.

Book on metaheuristics and in particular tabu search.

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D.E Goldberg. *Genetic algorithms in search, optimization and machine learning*. Addison-Wesley, 1989.

Book on genetic algorithms.

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P. Hansen and N. Mladenović. An introduction to variable neighborhood search.

In S. Voss, S. Martello, I. H. Osman, and C. Roucairol, editors, *Meta-heuristics, Advances and trends in local search paradigms for optimization*, pages 433–458. Kluwer Academic Publishers, 1998.
Description of the variable neighborhood search metaheuristic.
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T.A. Feo and M.G.C. Resende. A probabilistic heuristic for a computationally difficult set covering problem. *Operations Research Letters*, 8:67–71, 1989.
This is the first paper to explicitly describe a GRASP. See page 38.
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T.A. Feo and M.G.C. Resende. Greedy randomized adaptive search procedures. *J. of Global Optimization*, 6:109–133, 1995.
An early survey of GRASP. See page 4.
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S. Lin and B.W. Kernighan. An effective heuristic algorithm for the traveling-salesman problem. *Operations Research*, 21:498–516, 1973.
An early random multistart local search technique.
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J.P. Hart and A.W. Shogan. Semi-greedy heuristics: An empirical study. *Operations Research Letters*, 6:107–114, 1987.
This paper presents a randomized greedy heuristic, called semi-greedy heuristic.
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D.S. Johnson, C.H. Papadimitriou, and M. Yannakakis. How easy is local search? *Journal of Computer and System Sciences*, 17:79–100, 1988.
Study of the computational complexity of local search.
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2. Tutorials and surveys

Since GRASP is a relatively recent optimization method, tutorials and surveys on this subject are limited. Below are a few examples of introductory material on GRASP.

T.A. Feo and M.G.C. Resende. Greedy randomized adaptive search procedures. *J. of Global Optimization*, 6:109–133, 1995.
In this tutorial paper, the authors define the various components comprising a GRASP. A general trivial implementation of GRASP on a parallel computer is also discussed. The GRASP literature until 1994 is surveyed.
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J.L. González. GRASP. In A. Díaz, editor, *Heuristic optimization and neural networks in operations management and engineering*, pages 143–161. Editorial Paraninfo, Madrid, 1996.
This is a chapter on GRASP in a book on heuristic procedures for optimization. In Spanish.
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M.G.C. Resende. Greedy randomized adaptive search procedures (GRASP). Technical report, AT&T Labs Research, Florham Park, NJ 07932 USA, 1998.

This paper surveys greedy randomized adaptive search procedures. The basic GRASP is explained in detail and enhancements to the basic procedure are described. Several applications of GRASP are reported, showing how this method can find good approximate solutions to operations research problems and industrial applications. To appear in *Encyclopedia of Optimization*, Kluwer Academic Publishers, 2001.

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C.M.D. Silveira. GRASP – A heuristic for solving combinatorial optimization problems. Technical report, Institute of Informatics, Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil, 1999.

The aim of this report is to provide an exhaustive description of the features of GRASP as a metaheuristic method for solving hard combinatorial optimization problems. The various components comprising a generic GRASP are defined and it is also shown through examples how to develop GRASPs for combinatorial optimization problems. In Portuguese.

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L.S. Pitsoulis and M.G.C. Resende. Greedy randomized adaptive search procedures. In P.M. Pardalos and M.G.C. Resende, editors, *Handbook of Applied Optimization*. Oxford University Press, 2001.

This chapter surveys GRASP. Multi-start heuristics are seen as a way to apply local search to solve combinatorial optimization problems. GRASP is shown to, in some ways, improve upon greedy or random multi-start procedures. Enhancements to GRASP, such as reactive GRASP, hybrid GRASP, and use of long-term memory are discussed. The parallelization of GRASP is also considered. The chapter ends with a survey of GRASP for solving problems in logic, assignment, and location.

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3. Enhancements to the basic GRASP

The standard (or basic) GRASP consists of repeated applications of GRASP construction (using a fixed candidate list strategy), followed by hill climbing local search. Enhancements to this basic GRASP are found in the following papers.

J.L. Bresina. Heuristic-biased stochastic sampling. In *Proceedings of the AAAI-96*, pages 271–278, 1996.

This paper presents a generalization of iterative sampling called Heuristic-Biased Stochastic Sampling (HBSS). The two search techniques have the same overall control structure. The difference lies in how a choice is made at each decision point. HBSS uses a given search heuristic to focus its exploration. The degree of focusing is determined by a chosen *bias function* that reflects the confidence one has in the heuristic's accuracy. This methodology can be directly applied in a GRASP construction phase, by biasing the selection of RCL elements to favor those with higher greedy function values.

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J. Mockus, E. Eddy, A. Mockus, L. Mockus, and G.V. Reklaitis. *Bayesian discrete and global optimization*. Kluwer Academic Publishers, 1997.

This book describes the Bayesian approach to discrete optimization. A Bayesian heuristic algorithm version of GRASP is described.

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J.B. Atkinson. A greedy randomised search heuristic for time-constrained vehicle scheduling and the incorporation of a learning strategy. *J. of the Operational Research Society*, 49:700–708, 1998.

Two forms of adaptive search called *local* and *global* adaptation are identified. In both search techniques, the greedy function takes into account a quantity that measures heuristically the quality of the partial solution. While in local adaptation the decisions made within a particular run influence only the subsequent performance of the heuristic, global adaptation involves making decisions that affect the performance of the heuristic in subsequent runs. See page 16.

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C. Fleurent and F. Glover. Improved constructive multistart strategies for the quadratic assignment problem using adaptive memory. *INFORMS J. on Computing*, 11:198–204, 1999.

Study of alternatives to memoryless multistart approaches like GRASP is conducted for the quadratic assignment problem. Adaptive memory strategies retain and analyze features of selected solutions in order to provide a basis for improving future executions of the constructive process. The authors show that the most effective strategies are tabu search methods, which use memory in both improving and constructive phases. See page 28.

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M. Laguna and R. Martí. GRASP and path relinking for 2-layer straight line crossing minimization. *INFORMS J. on Computing*, 11:44–52, 1999.

This paper incorporates to GRASP a path relinking strategy to search for improved outcomes. Path relinking generates new solutions by exploring trajectories connecting high quality solutions. Starting from an *initiating solution*, path relinking generates a path in the neighborhood space that leads toward the other solutions, called *guiding solutions*. This is accomplished by selecting moves that introduce attributes contained in the guiding solutions. See page 36.

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M. Prais. *Parameter variation in GRASP procedures*. PhD thesis, Department of Computer Sciences, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil, 2000.

This thesis describes a GRASP for a matrix decomposition problem in TDMA traffic assignment. It proposes Reactive GRASP, a refinement of GRASP where the RCL parameter is adjusted dynamically to favor values that produce good solutions. Reactive GRASP is compared with other RCL strategies on matrix decomposition, set covering, maximum satisfiability and graph planarization.

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M. Prais and C.C. Ribeiro. Reactive GRASP: An application to a matrix decomposition problem in TDMA traffic assignment. *INFORMS J. on Computing*, 12: 164–176, 2000a.

A refinement of GRASP, called Reactive GRASP, is proposed. Instead of

using a fixed value for the basic parameter that defines the restrictiveness of the candidate list during the construction phase, Reactive GRASP self-adjusts the parameter value according to the quality of the solutions previously found. See pages 29 and 35.

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M. Prais and C.C. Ribeiro. Parameter variation in GRASP procedures. *Investigación Operativa*, 9:1–20, 2000b.

The GRASP RCL parameter α that determines the size of the restricted candidate list can be adjusted, leading to different behavior of the GRASP implementation. This paper investigates four strategies for the variation of the parameter α: 1) reactive – α is self-adjusted along the iterations; 2) uniform – α is randomly chosen from a discrete uniform probability distribution; 3) hybrid – α is randomly chosen from a fixed probability distribution concentrated around the value corresponding to the purely greedy choice; 4) fixed – α has a fixed value close to the purely greedy choice. The reactive strategy is the most flexible and adherent to the characteristics of the specific problem to be solved. In Portuguese.

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C.C. Ribeiro, E. Uchoa, and R.F. Werneck. A hybrid GRASP with perturbations and adaptive path-relinking for the Steiner problem in graphs. Technical report, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, RJ 22453-900 Brazil, 2000.

This paper describes a hybrid GRASP with weight perturbations and adaptive path-relinking heuristic (HGP-PR) for the Steiner problem in graphs. In this multi-start approach, the greedy randomized construction phase of a GRASP is replaced by the combination of several construction heuristics with a weight perturbation strategy. A strategic oscillation scheme combining intensification and diversification elements is used for the perturbation of the original weights. The improvement phase circularly explores two different local search strategies. An adaptive path-relinking technique is applied to a set of elite solutions as an intensification strategy. Computational experiments on a large set of benchmark problems of three different classes are reported. The new heuristic HGP-PR outperformed other algorithms, obtaining consistently better or comparably good solutions for all classes of test problems.

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4. GRASP in hybrid metaheuristics

GRASP has been used in conjunction with other metaheuristics. The following papers illustrate these hybrid techniques.

M. Laguna and J.L. González-Velarde. A search heuristic for just-in-time scheduling in parallel machines. *J. of Intelligent Manufacturing*, 2:253–260, 1991.

The proposed hybrid metaheuristic combines elements of GRASP with elements of tabu search. See page 13.

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M. Yagiura and T. Ibakari. Genetic and local search algorithms as robust and simple optimization tools. In *Meta-heuristics: Theory and applications*, pages 63–82. Kluwer Academic Publishers, 1996.

Various metaheuristics such as random multi-start local search (MLS) and genetic algorithm (GA) are implemented in this paper and their performance compared. The objective of the authors is not to propose the most powerful technique, but to compare general tendencies of various algorithms. From their analysis it results that a GRASP type modification of MLS improves its performance and that GA combined with local search is quite effective if long computational time is allowed.

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R. Colomé and D. Serra. Consumer choice in competitive location models: Formulations and heuristics. Technical report, Department of Economics and Management, Universitat Pompeu Fabra, Barcelona, Spain, May 1998.

The authors propose a hybrid GRASP which uses tabu search in the local search phase. To appear in *Papers in Regional Science*. See page 38.

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H. Ramalhinho Lourenço, J.P. Paixão, and R. Portugal. Metaheuristics for the bus-driver scheduling problem. Technical report, Department of Economics and Management, Universitat Pompeu Fabra, Barcelona, Spain, 1998.

GRASP is used in a genetic algorithm to implement a type of crossover called *perfect offspring*. See page 15.

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H. Delmaire, J.A. Díaz, E. Fernández, and M. Ortega. Reactive GRASP and Tabu Search based heuristics for the single source capacitated plant location problem. *INFOR*, 37:194–225, 1999.

A hybrid heuristic is proposed. It embeds a reactive GRASP in a tabu search algorithm as a diversification method that provides elite candidate sets. Intensification is also done. See page 20.

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R.K. Ahuja, J.B. Orlin, and A. Tiwari. A greedy genetic algorithm for the quadratic assignment problem. *Computers and Operations Research*, 27:917–934, 2000.

In this paper a hybrid genetic algorithm for the QAP is presented. GRASP is used to generate the initial population. See page 28.

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M. Armony, J.G. Klincewicz, H. Luss, and M.B. Rosenwein. Design of stacked self-healing rings using a genetic algorithm. *J. of Heuristics*, 6:85–105, 2000.

A genetic algorithm for design of stacked self-healing rings is proposed. The objective is to optimize the trade-off between the cost of connecting nodes to the ring and the cost of routing demand on multiple rings. The initial population of the genetic algorithm is made up of randomly generated solutions as well as solutions generated by a GRASP. Computational comparisons are made with a commercial integer programming package.

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S. Abdinnour-Helm and S.W. Hadley. Tabu search based heuristics for multi-floor facility layout. *International J. of Production Research*, 38:365–383, 2000.

Two two-stage heuristics are proposed for solving the multi-floor facility layout problem. GRASP/TS applies a GRASP to find the initial layout and tabu search to refine the initial layout, based on total inter/intra-floor costs. Computational tests indicate that GRASP/TS compares favorably with heuristics that do not rely on exact algorithms.

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C.C. Ribeiro, E. Uchoa, and R.F. Werneck. A hybrid GRASP with perturbations and adaptive path-relinking for the Steiner problem in graphs. Technical report, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, RJ 22453-900 Brazil, 2000.

This paper propose and describes a hybrid GRASP with weight perturbations and adaptive path-relinking heuristic (HGP-PR) for the Steiner problem in graphs. See page 7.

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5. Parallel GRASP

GRASP can be easily implemented in parallel by dividing iterations among several processors. Other parallelization schemes have also been used to implement parallel GRASPs. The following papers exemplify parallel GRASP.

T.A. Feo, M.G.C. Resende, and S.H. Smith. A greedy randomized adaptive search procedure for maximum independent set. *Operations Research*, 42:860–878, 1994.

A GRASP for approximately solving the maximum independent set problem is described. The proposed heuristic can be easily implemented in parallel by decomposing the problem into smaller subproblems, each defined by conditioning on vertices being in the solution. An implementation of this algorithm was tested on a MIMD computer with up to eight processors. Average linear speedup is observed. See page 21.

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P.M. Pardalos, L. Pitsoulis, T. Mavridou, and M.G.C. Resende. Parallel search for combinatorial optimization: Genetic algorithms, simulated annealing and GRASP. In A. Ferreira and J. Rolim, editors, *Parallel Algorithms for Irregularly Structured Problems, Proceedings of the Second International Workshop –Irregular'95*, volume 980 of *Lecture Notes in Computer Science*, pages 317–331. Springer-Verlag, 1995a.

This paper summarizes some parallel search techniques for approximating the global optimal solution of combinatorial optimization problems. For large-scale problems, one of the main limitations of heuristic search is its computational complexity. Therefore, efficient parallel implementation of search algorithms can significantly increase the size of the problems that can be solved.

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P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. A parallel GRASP implementation for the quadratic assignment problem. In A. Ferreira and J. Rolim, editors, *Parallel Algorithms for Irregularly Structured Problems – Irregular'94*, pages 115–130. Kluwer Academic Publishers, 1995b.

Efficient parallel techniques for large-scale sparse quadratic assignment problems are discussed. The paper provides a detailed description of a parallel implementation on an MIMD computer of the sequential GRASP proposed by Li, Pardalos, and Resende (1994) for solving the

QAP. The GRASP iterations are distributed among the processors. Each processor is given its own input data and random number sequence and are run independently. A shared global variable stores the value of the incumbent solution.

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P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. A parallel GRASP for MAX-SAT problems. *Lecture Notes in Computer Science*, 1184:575–585, 1996.

A parallel GRASP for weighted maximum satisfiability (MAX-SAT) problem is proposed. The GRASP is based on the serial GRASP presented by Resende, Pitsoulis, and Pardalos (1997). The parallel implementation distributes the GRASP iterations among several processors operating in parallel, avoiding that two processors have as input the same random number generator seed. The best solution found among all processors is identified and used as solution of the problem.

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A.C.F. Alvim. Parallelization strategies for the metaheuristic GRASP. Master's thesis, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, RJ 22453-900 Brazil, April 1998.

Two parallelization strategies for GRASP are discussed and compared: parallelization by distributing GRASP iterations and parallelization by varying the GRASP random parameter α. Both strategies are adapted to several parallel computation models, such as MPI (Message Passing Interface) and PVM (Parallel Virtual Machine). In Portuguese.

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A.C.F. Alvim and C.C. Ribeiro. Load balancing in the parallelization of the metaheuristic GRASP. Technical report, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, RJ 22453-900 Brazil, 1998.

Two parallelization strategies for GRASP are compared. The difference between the two strategies concerns the way in which data is partitioned: pre-scheduled (static load balancing) or self-scheduled (dynamic load balancing). The strategies have been tested considering an application to optimal traffic assignment in TDMA satellite system. Best results have been obtained by using the self-scheduling strategy. In Portuguese.

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S.L. Martins, C.C. Ribeiro, and M.C. Souza. A parallel GRASP for the Steiner problem in graphs. In A. Ferreira and J. Rolim, editors, *Proceedings of IRREGULAR'98 – 5th International Symposium on Solving Irregularly Structured Problems in Parallel*, volume 1457 of *Lecture Notes in Computer Science*, pages 285–297. Springer-Verlag, 1998.

A parallelization of a sequential GRASP for the Steiner minimal tree problem is proposed. The procedure implemented is one of the procedures described in Martins, Pardalos, Resende, and Ribeiro (1999). The parallelization is accomplished by distributing the GRASP iterations among the processors on a demand-driven basis.

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R.A. Murphey, P.M. Pardalos, and L.S. Pitsoulis. A parallel GRASP for the data association multidimensional assignment problem. In P.M. Pardalos, editor, *Parallel processing of discrete problems*, volume 106 of *The IMA Volumes in Mathematics and Its Applications*, pages 159–180. Springer-Verlag, 1998.

A parallel GRASP for finding good solutions for the data association problem is described. See page 27.

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P.M. Pardalos, M.G.C. Resende, and J. Rappe. An exact parallel algorithm for the maximum clique problem. In R. De Leone et al., editor, *High performance algorithms and software in nonlinear optimization*, pages 279–300. Kluwer Academic Publishers, 1998.

A parallelized version of the exact algorithm of Carraghan and Pardalos (1990) for the unweighted maximum clique problem is described. A variant of the GRASP for the maximum independent set problem of Feo, Resende, and Smith (1994) is used for computing feasible solutions.

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L.I.D. Rivera. Evaluation of parallel implementations of heuristics for the course scheduling problem. Master's thesis, Instituto Tecnologico y de Estudios Superiores de Monterrey, Monterrey, Mexico, 1998.

This thesis presents several parallel implementations of heuristics for the course scheduling problem. One of the heuristics is a GRASP. In Spanish.

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S.L. Martins. *Parallelization strategies for metaheuristics in distributed memory environments*. PhD thesis, Department of Computer Sciences, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil, 1999.

This thesis considers parallelization strategies for metaheuristics in distributed memory environments. GRASPs for the Steiner tree problem in graphs are described and implemented in parallel. In Portuguese.

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R.M. Aiex, M.G.C. Resende, and C.C. Ribeiro. Probability distribution of solution time in GRASP: An experimental investigation. Technical report, AT&T Labs Research, Florham Park, NJ 07733, 2000a.

The authors study the probability distributions of solution time to a sub-optimal target value in five GRASPs that have appeared in the literature and for which source code is available. The distributions are estimated by running 12,000 independent runs of the heuristic. Standard methodology for graphical analysis is used to compare the empirical and theoretical distributions and estimate the parameters of the distributions. They conclude that the solution time to a sub-optimal target value fits a two-parameter exponential distribution. Hence, it is possible to approximately achieve linear speed-up by implementing GRASP in parallel. To appear in *J. of Heuristics*.

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R.M. Aiex, M.G.C. Resende, P.M. Pardalos, and G. Toraldo. GRASP with path relinking for the three-index assignment problem. Technical report, AT&T Labs Research, Florham Park, NJ 07733, 2000b.

This paper describes variants of GRASP with path relinking for the three index assignment problem (AP3). Computational results show clearly that this GRASP for AP3 benefits from path relinking and that the variants considered in this paper compare well with previously proposed heuristics for this problem. The authors also show that the random variable "time to target solution," for all proposed GRASP with path relinking variants, fits a two-parameter exponential distribution. To illustrate the consequence of this, one of the variants of GRASP with path relinking is shown to benefit from parallelization.

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S.L. Martins, M.G.C. Resende, C.C. Ribeiro, and P.M. Pardalos. A parallel GRASP for the Steiner tree problem in graphs using a hybrid local search strategy. *J. of Global Optimization*, 17:267–283, 2000.

A parallel GRASP for the Steiner problem in graphs is described. See page 25.

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6. Source code

Several papers describe computer source code implementing GRASP. The following papers all fall into this category.

M.G.C. Resende, P.M. Pardalos, and Y. Li. Algorithm 754: Fortran subroutines for approximate solution of dense quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software*, 22:104–118, 1996.

This paper describes a set of ANSI standard Fortran 77 subroutines to find approximate solutions to dense quadratic assignment problems having at least one symmetric flow or distance matrix. It is an optimized implementation of the algorithm described in Li, Pardalos, and Resende (1994).

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P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. Algorithm 769: Fortran subroutines for approximate solution of sparse quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software*, 23:196–208, 1997.

A version of the GRASP for the quadratic assignment problem of Li, Pardalos, and Resende (1994), tailored for sparse instances is proposed. A set of ANSI standard Fortran 77 subroutines are described.

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M.G.C. Resende, T.A. Feo, and S.H. Smith. Algorithm 787: Fortran subroutines for approximate solution of maximum independent set problems using GRASP. *ACM Transactions on Mathematical Software*, 24:386–394, 1998.

This article describes a set of ANSI standard Fortran 77 subroutines to find an approximate solution of a maximum independent set problem. The GRASP used to produce the solutions is described in Feo, Resende, and Smith (1994).

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P. Festa, P.M. Pardalos, and M.G.C. Resende. FORTRAN subroutines for computing approximate solution to feedback set problems using GRASP. Technical report, AT&T Labs Research, Florham Park, NJ 07932 USA, 1999.

A set of ANSI standard Fortran 77 subroutines for approximately solving the feedback vertex and arc set problems is described. To appear in *ACM Transactions on Mathematical Software*. See page 24.

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C.C. Ribeiro and M.G.C. Resende. Algorithm 797: Fortran subroutines for approximate solution of graph planarization problems using GRASP. *ACM Transactions on Mathematical Software*, 25:341–352, 1999.

This paper describes a set of Fortran subroutines that implements the GRASP for graph planarization of Resende and Ribeiro (1997).

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M.G.C. Resende, L.S. Pitsoulis, and P.M. Pardalos. Fortran subroutines for computing approximate solutions of MAX-SAT problems using GRASP. *Discrete Applied Mathematics*, 100:95–113, 2000.

A set of Fortran subroutines for computing approximate solutions of MAX-SAT problems is described. See page 18.

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7. Scheduling

GRASP has been applied to a wide variety of scheduling problems. The following papers illustrate GRASPs for scheduling.

J.F. Bard and T.A. Feo. Operations sequencing in discrete parts manufacturing. *Management Science*, 35:249–255, 1989.

This paper presents a method for efficiently sequencing cutting operations associated with the manufacture of discrete parts. The problem is modeled as an integer program. This is relaxed via Lagrangian relaxation into a min-cut problem on a bipartite network. To obtain lower bounds, a max-flow algorithm is applied and the corresponding solution is input to a GRASP.

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T.A. Feo and J.F. Bard. Flight scheduling and maintenance base planning. *Management Science*, 35:1415–1432, 1989.

See page 31.

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T.A. Feo, K. Venkatraman, and J.F. Bard. A GRASP for a difficult single machine scheduling problem. *Computers & Operations Research*, 18:635–643, 1991.

GRASP is applied in this paper to an unusually difficult scheduling problem with flow time and earliness penalties. Two greedy functions are developed and tested. The first is the difference between the flow time and earliness penalties, normalized by the processing time. The second function evaluates the cost of scheduling a job next by estimating the cost of the remaining schedule. The local search uses 2-exchange and insertion exchange.

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M. Laguna and J.L. González-Velarde. A search heuristic for just-in-time scheduling in parallel machines. *J. of Intelligent Manufacturing*, 2:253–260, 1991.

This paper presents a hybrid GRASP/tabu search metaheuristic for the weighted earliness penalty problem with deadlines in identical parallel machines.

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P. De, J.B. Ghosj, and C.E. Wells. Solving a generalized model for CON due date assignment and sequencing. *International J. of Production Economics*, 34: 179–185, 1994.

> This paper deasl with a generalized model for assigning a constant flow allowance (CON) due date to a set of jobs and sequencing them on a single machine. The problem is viewed as a 0-1 quadratic problem and a GRASP is proposed to solve the quadratic problem. The randomization strategy used was inspired by a gradient-based variable forcing methodology proposed by Pardalos and Rodgers (1990) for a branch & bound algorithm. The local search procedure is based on a definition of neighborhood in which two solutions are neighbors if they differ in the value of exactly one variable.

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T.A. Feo, J.F. Bard, and S. Holland. Facility-wide planning and scheduling of printed wiring board assembly. *Operations Research*, 43:219–230, 1995.

> A GRASP is proposed to solve a multiple machine scheduling problem. See page 30.

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J.F. Bard, T.A. Feo, and S. Holland. A GRASP for scheduling printed wiring board assembly. *I.I.E. Transactions*, 28:155–165, 1996.

> GRASP is used for solving the daily scheduling problem that arises in printed wiring board assembly. See page 31.

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T.A. Feo, K. Sarathy, and J. McGahan. A GRASP for single machine scheduling with sequence dependent setup costs and linear delay penalties. *Computers & Operations Research*, 23:881–895, 1996.

> A GRASP for single machine scheduling with sequence dependent setup costs and linear delay penalties is presented. The greedy function of the GRASP construction phase proposed is made up of two components: the switch over cost and the opportunity cost associated with not inserting a specific job in the next position and instead, inserting it after half of the unscheduled jobs have been scheduled. This greedy function tends to lead to a balance between the natural order and nearest neighbor approaches. The local search uses 2-exchange, insertion exchange, and a combination of the two.

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J. Xu and S. Chiu. Solving a real-world field technician scheduling problem. In *Proceedings of the International Conference on Management Science and the Economic Development of China*, pages 240–248, July 1996.

> The objective of the field technician scheduling problem is to assign a set of jobs at different locations with time windows to technicians with different job skills. The greedy choice of the proposed GRASP is to select jobs with the highest unit weight. The local search implements four different moves, among them the 2-exchange and a swap that exchanges an assigned job with another job unassigned under the candidate schedule.

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H. Ramalhinho Lourenço, J.P. Paixão, and R. Portugal. Metaheuristics for the bus-driver scheduling problem. Technical report, Department of Economics and Management, Universitat Pompeu Fabra, Barcelona, Spain, 1998.

The authors present several metaheuristics for solving real crew scheduling problems that can be used in a transportation planning system, in real-time, and in a user-friendly environment. The metaheuristics proposed are based on the following approaches: GRASP, tabu search, and genetic algorithms. See page 8.

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R.Z. Ríos-Mercado and J.F. Bard. Heuristics for the flow line problem with setup costs. *European J. of Operational Research*, pages 76–98, 1998.

This paper presents two new heuristics for the flowshop scheduling problem with sequence-dependent setup times and makespan minimization objective, one of which is a GRASP. Both heuristics are compared to a previously proposed algorithm, based on the traveling salesman problem (TSP), on randomly generated instances. When setup times are an order of magnitude smaller than processing times, the new approaches prove superior to the TSP-based heuristic. When both processing and setup times are identically distributed, the TSP-based heuristic outperforms the proposed procedures.

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L.I.D. Rivera. Evaluation of parallel implementations of heuristics for the course scheduling problem. Master's thesis, Instituto Tecnologico y de Estudios Superiores de Monterrey, Monterrey, Mexico, 1998.

See page 11.

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J. Xu and S. Chiu. Effective heuristic procedure for a field technician scheduling problem. Technical report, US WEST Advanced Technologies, Boulder, CO 80303 USA, 1998.

The objective of the field technician scheduling problem is to assign a set of jobs at different locations with time windows to technicians with different job skills. Several heuristics are designed and tested for solving the problem: a pure greedy heuristic, a GRASP, and a local search algorithm. The greedy choice of the GRASP proposed is to select jobs with the highest unit weight. The local search implements four different moves, among them the 2-exchange and a swap that exchanges an assigned job with another job unassigned under the candidate schedule. To appear in *J. of Heuristics*.

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R.Z. Ríos-Mercado and J.F. Bard. An enhanced TSP-based heuristic for makespan minimization in a flow shop with setup costs. *J. of Heuristics*, 5:57–74, 1999.

An enhanced heuristic for minimizing the makespan of the flow shop scheduling problem with sequence-dependent setup times is presented. To tune the parameters, each component of the heuristic is tested over a wide range of problem instances. An experimental comparison with a GRASP reveals the conditions and data attributes where the proposed procedure works best.

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S. Binato, W.J. Hery, D. Loewenstern, and M.G.C. Resende. A greedy randomized adaptive search procedure for job shop scheduling. In P. Hansen and C.C. Ribeiro, editors, *Essays and surveys on metaheuristics*. Kluwer Academic Publishers, 2001.

This paper proposes a GRASP for the job shop scheduling problem. The standard GRASP is enhanced with a memory based intensification scheme, that improves the local search phase around good solutions. The adaptive greedy function is the makespan resulting from the addition of an unscheduled operation to those already scheduled. The greedy choice favors the operation with the minimum value of the greedy function. The local search uses the 2-exchange based on the disjunctive graph model proposed by Roy and Sussmann (1964).

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8. Routing

GRASP has been applied to vehicle, aircraft, telecommunications, and inventory routing problems. The following papers illustrate the application of GRASP to routing problems.

C.A. Hjorring. *The vehicle routing problem and local search metaheuristics.* PhD thesis, University of Auckland, Auckland, New Zealand, 1995.

Three metaheuristics for effectively searching through the space of cyclic orders are developed. They are based on GRASP, tabu search, and genetic algorithms. For tabu search, different schemes are investigated to control the tabu list length, including a reactive tabu search method. To obtain good solutions when using the genetic algorithm, specialized crossovers are developed, and a local search component is added. GRASP is used to construct an initial good solution.

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G. Kontoravdis and J.F. Bard. A GRASP for the vehicle routing problem with time windows. *ORSA J. on Computing*, 7:10–23, 1995.

A GRASP is proposed for minimizing the fleet size of temporarily constrained vehicle routing problems with two types of service. The greedy function of the construction phase takes into account both the overall minimum insertion cost and the penalty cost. Local search is applied to the best solution found every five iterations of the first phase, rather than to each feasible solution.

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M.F. Argüello, J.F. Bard, and G. Yu. A GRASP for aircraft routing in response to groundings and delays. *J. of Combinatorial Optimization*, 1:211–228, 1997.

This paper presents a GRASP to reconstruct aircraft routings in response to groundings and delays experienced over the course of the day. The objective is to minimize the cost of reassigning aircraft to flights taking into account available resources and other system constraints. See page 32.

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J.B. Atkinson. A greedy randomised search heuristic for time-constrained vehicle scheduling and the incorporation of a learning strategy. *J. of the Operational Research Society*, 49:700–708, 1998.

A GRASP is proposed for solving a complex vehicle-scheduling problem with tight time windows and additional constraints. See page 6.

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J.F. Bard, L. Huang, P. Jaillet, and M. Dror. A decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science*, 32: 189–203, 1998.

A methodology is presented that decomposes the inventory routing problem with satellite facilities over the planning horizon, and then solves daily rather than multi-day vehicle routing problems. Three heuristics are proposed for solving the vehicle routing problem with satellite facilities: randomized Clarke-Wright, GRASP, and modified sweep. The GRASP proposed is a modified version of the GRASP of Kontoravdis and Bard (1995).

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L.I.P. Resende and M.G.C. Resende. A GRASP for frame relay PVC routing. In *Proc. of the Third Metaheuristics International Conference*, pages 397–402, July 1999.

See page 33.

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A. Corberán, R. Martí, and J.M. Sanchís. A GRASP for the mixed postman problem. Technical report, Department of Statistics and Operations Research, University of Valencia, 46100 Burjassot, Valencia, Spain, 2000.

The mixed postman problem, a generalization of the Chinese postman problem, is that of finding the shortest tour that traverses each edge of a given mixed graph (a graph containing both undirected and directed edges) at least once. This paper proposes a GRASP for the mixed postman problem.

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C. Carreto and B. Baker. A GRASP interactive approach to the vehicle routing problem with backhauls. In P. Hansen and C.C. Ribeiro, editors, *Essays and surveys on metaheuristics*. Kluwer Academic Publishers, 2001.

The incorporation of interactive tools into heuristic algorithms is investigated. A GRASP is used in the routes construction and improvement phase. The construction phase is implemented in a clustering heuristic that constructs the routes by clustering the remaining customers according to the vehicles defined by seeds while applying the 3-opt heuristic to reduce the total distance traveled by each vehicle. The greedy function takes into account routes with smallest insertion cost and costumers with biggest difference between the smallest and the second smallest insertion costs and smallest number of routes they can traverse. As the local search phase, 3-opt is used.

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9. Logic

GRASP has been applied to problems in logic, including SAT, MAX-SAT, and logical clause inference, as shown by the following papers.

M.G.C. Resende and T.A. Feo. A GRASP for satisfiability. In D.S. Johnson and M.A. Trick, editors, *Cliques, Coloring, and Satisfiability: The Second DIMACS*

Implementation Challenge, volume 26 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 499–520. American Mathematical Society, 1996.

This paper describes a GRASP for the satisfiability problem that can be also directly applied to both the weighted and unweighted versions of the maximum satisfiability problem. The adaptive greedy function is a hybrid combination of two functions. One function seeks to maximize the number of yet-unsatisfied clauses that become satisfied after the assignment of each construction iteration, while the other maximizes the number of yet-unassigned literals in yet-unsatisfied clauses that become satisfied if opposite assignments were to be made. The local search flips the assignment of each variable, one at a time, checking if the new truth assignment increases the number of satisfied clauses.

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P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. A parallel GRASP for MAX-SAT problems. *Lecture Notes in Computer Science*, 1184:575–585, 1996.

A parallel GRASP is proposed for finding approximate solutions to the weighted maximum satisfiability problem. See page 10.

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M.G.C. Resende, L.S. Pitsoulis, and P.M. Pardalos. Approximate solution of weighted MAX-SAT problems using GRASP. In J. Gu and P.M. Pardalos, editors, *Satisfiability problems*, volume 35 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 393–405. American Mathematical Society, 1997.

This article proposes a GRASP for finding approximate solutions of weighted MAX-SAT problems. The greedy adaptive function is to maximize the total weight of yet-unsatisfied clauses that become satisfied after the assignment of each construction phase iteration. The local search uses the 1-flip neighborhood of a vector x, defined as the set of all binary vectors that differ from x in exactly one literal.

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A.S. Deshpande and E. Triantaphyllou. A greedy randomized adaptive search procedure (GRASP) for inferring logical clauses from examples in polynomial time and some extensions. *Mathematical and Computer Modelling*, 27:75–99, 1998.

Two heuristics are presented in this article for inferring a small size Boolean function from complete and incomplete examples in polynomial time. Each example can be positive or negative depending on whether it must be accepted or rejected, respectively, by the target function. Both of the proposed heuristics are randomized in the sense that instead of choosing the best candidate element, a candidate list is built whose elements are assigned with evaluative function values close to the highest one.

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M.G.C. Resende, L.S. Pitsoulis, and P.M. Pardalos. Fortran subroutines for computing approximate solutions of MAX-SAT problems using GRASP. *Discrete Applied Mathematics*, 100:95–113, 2000.

A set of Fortran subroutines for computing approximate solutions of MAX-SAT problems is described. The algorithm implemented was proposed by Resende, Pitsoulis, and Pardalos (1997). Two versions of the

subroutines are distributed. One version uses a neighborhood data structure in order to speed up the local search phase, while the second version, since it does not make use of this data structure, is more memory efficient but less time efficient. Computational results improve upon those in Resende, Pitsoulis, and Pardalos (1997) using an RCL parameter α randomly chosen each GRASP iteration from the interval $[0, 1]$.

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10. Partitioning

Two types of partitioning problems have been treated with GRASP: number partitioning and VLSI circuit partitioning. The following papers describe these applications.

M.F. Argüello, T.A. Feo, and O. Goldschmidt. Randomized methods for the number partitioning problem. *Computers & Operations Research*, 23(2):103–111, 1996.

This paper presents randomized methodologies for solving the number partitioning problem (the partitioning of a finite set of integers into two disjoint subsets such that the difference of the sums of the elements in the subsets is minimized). The greedy criterion consists in considering only large elements for differencing. Specific selection of the elements to be differenced is made at random. Differences are placed back into the list of remaining elements, and the process of selecting the next element is repeated. The proposed methods are greedy, randomized, and adaptive construction heuristics, but local search is omitted.

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S. Areibi and A. Vannelli. A GRASP clustering technique for circuit partitioning. In J. Gu and P.M. Pardalos, editors, *Satisfiability problems*, volume 35 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 711–724. American Mathematical Society, 1997.

This paper adapts a basic node interchange method for solving the circuit partitioning problem and develop a clustering technique that uses GRASP to generate clusters of moderate sizes. See page 38.

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S.M. Areibi. GRASP: An effective constructive technique for VLSI circuit partitioning. In *Proc. IEEE Canadian Conference on Electrical & Computer Engineering (CCECE'99)*, May 1999.

This article proposes a GRASP for obtaining good initial solutions for an iterative improvement technique. At each iteration of the randomized approach, the gains associated with moving modules to the current block being filled are examined, and a restricted candidate list is built using the modules with the highest gains.

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11. Location and layout

Location and layout are another class of problems successfully handled by GRASP. The following papers show how GRASP is used in this context.

J.G. Klincewicz. Avoiding local optima in the p-hub location problem using tabu search and GRASP. *Annals of Operations Research*, 40:283–302, 1992.

This paper proposes two heuristics based on tabu search and GRASP for the p-hub location problem. The objective is to overcome the difficulty that local search algorithms encounter. The local search procedure of the GRASP algorithm is based on the 2-exchange.

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H. Delmaire, J.A. Díaz, E. Fernández, and M. Ortega. Comparing new heuristics for the pure integer capacitated plant location problem. Technical Report DR97/10, Department of Statistics and Operations Research, Universitat Politecnica de Catalunya, Barcelona, Spain, 1997.

To solve the pure integer capacitated plant location problem, several heuristics are proposed: evolutionary algorithms, GRASP, simulated annealing, and tabu search. All the algorithms share the same neighborhood definition, which can be one of the following: client plant, client reassignment, client interchange, open plans interchange, and open-closed plants interchange.

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K. Holmqvist, A. Migdalas, and P.M. Pardalos. Greedy randomized adaptive search for a location problem with economies of scale. In I.M. Bomze et al., editor, *Developments in Global Optimization*, pages 301–313. Kluwer Academic Publishers, 1997.

This paper proposes a GRASP for finding approximate solutions to a facility location problem with concave costs. The greedy function of the construction phase favors the facilities that give lower cost for a costumer, regarding the effect that already connected costumers have on the solution. The neighborhood function is defined as changing facility connection for one costumer. Instead of a time consuming computation of the objective function value for each neighborhood solution, the difference in cost for changing supplier is examined.

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T.L. Urban. Solution procedures for the dynamic facility layout problem. *Annals of Operations Research*, pages 323–342, 1998.

The concept of incomplete dynamic programming is applied to the dynamic facility layout problem and a lower bound for the general problem is developed. A GRASP and an initialized multi-greedy algorithm are described to provide a solution methodology for large problems. The GRASP is the algorithm proposed by Li, Pardalos, and Resende (1994) for dense quadratic assignment problems.

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H. Delmaire, J.A. Díaz, E. Fernández, and M. Ortega. Reactive GRASP and tabu search based heuristics for the single source capacitated plant location problem. *INFOR*, 37:194–225, 1999.

The single source capacitated plant location problem is a discrete location problem that takes into account capacities in the plants to be opened and imposes that clients be served from a single open plant. The authors propose a hybrid heuristic that embeds reactive GRASP in a tabu search algorithm. See page 8.

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M.J.N. Gomes and J.B.C. da Silva. An experimental evaluation of the GRASP metaheuristic applied to the uncapacitated location problem. Technical Report 004/99, Department of Statistics and Computation, State University of Ceará, Fortaleza, Ceará, Brazil, 1999.

> Two GRASPs, one using the ADD heuristic and the other using the DROP heuristic, are proposed for the uncapacitated location problem. Computational experiments with instances from Beasley's OR-Library show that GRASP-DROP dominates GRASP-ADD, while both GRASPs dominate ADD and DROP. In Portuguese.

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S. Abdinnour-Helm and S.W. Hadley. Tabu search based heuristics for multi-floor facility layout. *International J. of Production Research*, 38:365–383, 2000.

> Two two-stage heuristics are proposed for solving the multi-floor facility layout problem. See page 8.

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T.L. Urban, W.-C. Chiang, and R.A. Russel. The integrated machine allocation and layout problem. *International J. of Production Research*, pages 2913–2930, 2000.

> GRASP is used to solve quadratic assignment sub-problems in a model that aggregates quadratic assignment problems with several network flow problems with side constraints. This model is used to produce machine layouts where machines are not required to be placed in a functional or cellular layout.

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12. Graph theoretic applications

Perhaps the largest class of problems for which GRASP has been applied is graph theory. The following papers illustrate the wide applicability of GRASP to these problems.

T.A. Feo, M.G.C. Resende, and S.H. Smith. A greedy randomized adaptive search procedure for maximum independent set. *Operations Research*, 42:860–878, 1994.

> A GRASP for approximately solving the maximum independent set problem is described. The greedy function chosen orders admissible vertices with respect to the minimum admissible vertex degree. The adjective admissible is referred to a vertex that is not adjacent to any vertex in the current independent set. The neighborhood definition used in the local search is $(2,1)$-exchange, where two nonadjacent vertices can be added to the current solution if a single vertex from the solution is removed. See page 9.

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M. Laguna, T.A. Feo, and H.C. Elrod. A greedy randomized adaptive search procedure for the two-partition problem. *Operations Research*, 42:677–687, 1994.

> A GRASP for the network 2-partition problem is proposed. The greedy function of the construction phase minimizes the augmented weight of the partition. For the local improvement phase, four alternative procedures are considered: best swap, first swap, slight swap, and slightest

swap. The best strategies are slight and slightest swaps. Slight swap selects a near-minimum gain exchange at each iteration, while slightest swap chooses the absolute minimum gain.

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E.M. Macambira and C.C. de Souza. A GRASP for the maximum clique problem with weighted edges. In *Proceedings of the XXIX Brazilian Symposium on Operations Research*, page 70, October 1997.

The authors propose a branch-and-cut algorithm for the maximum clique problem with weighted edges. The initialization phase of the algorithm uses a GRASP to generate good starting solutions. The greedy function minimizes the sum of weights of the edges outgoing from vertices in the partition. The local search uses the exchange of one element in the current partition with one not in it.

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R. Martí and M. Laguna. Heuristics and meta-heuristics for 2-layer straight line crossing minimization. Technical report, Department of Statistics and Operations Research, University of Valencia, 46100 Burjassot, Valencia, Spain, 1997.

See page 36.

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M.G.C. Resende and C.C. Ribeiro. A GRASP for graph planarization. *Networks*, 29:173–189, 1997.

A GRASP for the graph planarization problem is described, extending the two-phase heuristic of Goldschmidt and Takvorian (*Networks*, v. 24, pp. 69–73, 1994). The implementation of the GRASP is described in detail. Computational experience on a large set of standard test problems is presented. On almost all test problems considered, the heuristic either matches or finds a better solution than previously described graph planarization heuristics. In several cases, previously unknown optimal solutions are found.

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R.K. Ahuja, J.B. Orlin, and D. Sharma. New neighborhood search structures for the capacitated minimum spanning tree problem. Technical report, Department of ISE, University of Florida, Gainesville, FL 32611-6595, 1998.

This report presents two generalizations of the best known neighborhood structures for the capacitated minimum spanning tree problem. The new neighborhood structures defined allow cyclic exchanges of nodes among multiple subtrees simultaneously. The first structure proposed allows exchanges of single nodes among several subtrees. The second structure allows moves exchanging subsets of nodes spanning several trees. As the size of both these structures grows exponentially with the problem size, for searching the neighborhood efficiently the authors propose a heuristic search technique based on the concept of *improvement graph* which converts each possible cyclic exchange into a subset-disjoint cycle in the improvement graph and a profitable cyclic exchange into a negative cost cycle, heuristically identified using a modification of the shortest-path label-correcting algorithm. To judge the efficacy of the neighborhoods local improvement and tabu search algorithms have been developed. Local improvement uses a GRASP construction mechanism to generate repeated starting solutions for local improvement.

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M. Laguna and R. Martí. A GRASP for coloring sparse graphs. Technical report, Graduate School of Business, University of Colorado, Boulder, CO 80309-0419 USA, May 1998.

A GRASP for graph coloring is presented. The GRASP construction phase constructs the next coloring, one color at time. The greedy function chooses the vertex having the maximum degree among the uncolored vertices adjacent to at least one colored vertex. At each step, the local search combines the two smallest cardinality color classes into one and tries to find a valid color for each violating vertex. To appear in *Computational Optimization and Applications*.

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E.M. Macambira and C.N. Meneses. A GRASP algorithm for the maximum weighted edge subgraph problem. Technical report, Department of Statistics and Computation, University of Ceará, Fortaleza, CE 60740-000 Brazil, 1998.

A GRASP for the maximum weighted edge subgraph problem is proposed to overcome the difficulties encountered by local search methods. The greedy function of the construction phase favors the vertices corresponding to the maximum sum of the weights associated with its outgoing edges. The local search tries to improve the actual solution by simply swapping one element in the solution set with one not belonging to the solution. In Portuguese.

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S.L. Martins, C.C. Ribeiro, and M.C. Souza. A parallel GRASP for the Steiner problem in graphs. In A. Ferreira and J. Rolim, editors, *Proceedings of IRREGULAR'98 – 5th International Symposium on Solving Irregularly Structured Problems in Parallel*, volume 1457 of *Lecture Notes in Computer Science*, pages 285–297. Springer-Verlag, 1998.

See page 10.

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P.M. Pardalos, M.G.C. Resende, and J. Rappe. An exact parallel algorithm for the maximum clique problem. In R. De Leone et al., editor, *High performance algorithms and software in nonlinear optimization*, pages 279–300. Kluwer Academic Publishers, 1998.

See page 11.

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M.G.C. Resende. Computing approximate solutions of the maximum covering problem using GRASP. *J. of Heuristics*, 4:161–171, 1998.

A GRASP for the maximum covering problem is described. The greedy function is the total weight of the yet-uncovered demand points that become covered after the selection. The local search procedures uses a 2-exchange neighborhood structure. The GRASP is shown to find near optimal solutions. See page 34.

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M.G.C. Resende, T.A. Feo, and S.H. Smith. Algorithm 787: Fortran subroutines for approximate solution of maximum independent set problems using GRASP. *ACM Transactions on Mathematical Software*, 24:386–394, 1998.

See page 12.

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E. Rolland, R.A. Patterson, and H. Pirkul. Memory adaptive reasoning & greedy assignment techniques for the capacitated minimum spanning tree problem. Technical report, Department of Accounting & Management Information Systems, Fisher College of Business, The Ohio State University, Columbus, OH 43210 USA, 1998.

A GRASP for the capacitated minimum spanning tree problem is proposed. Testing, however, is limited to only a semi-greedy version of the GRASP.

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J. Abello, P.M. Pardalos, and M.G.C. Resende. On maximum clique problems in very large graphs. In J. Abello and J. Vitter, editors, *External memory algorithms and visualization*, volume 50 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 199–130. American Mathematical Society, 1999.

An approach for clique and quasi-clique computations in very large multi-digraphs is presented. The authors discuss graph decomposition schemes that break up the original problem into several pieces of manageable dimensions. A semi-external memory GRASP is presented to approximately solve the maximum clique problem and maximum quasi-clique problem. The construction phase uses vertex degrees as a guide for construction. The local search uses a simple $(2,1)$-exchange. See page 34.

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P. Festa, P.M. Pardalos, and M.G.C. Resende. FORTRAN subroutines for computing approximate solution to feedback set problems using GRASP. Technical report, AT&T Labs Research, Florham Park, NJ 07932 USA, 1999.

This article describes a set of ANSI standard Fortran 77 subroutines to find approximate solutions of both the feedback vertex set problem and the feedback arc set problem. The GRASP of Pardalos, Qian, and Resende (1999) is used to produce the approximate solutions of the feedback set problem. Feedback arc set problems are converted into feedback vertex set problems and solved. To appear in *ACM Transactions on Mathematical Software*.

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S.L. Martins, P.M. Pardalos, M.G.C. Resende, and C.C. Ribeiro. Greedy randomized adaptive search procedures for the Steiner problem in graphs. In P.M. Pardalos, S. Rajasekaran, and J. Rolim, editors, *Randomization methods in algorithmic design*, volume 43 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 133–145. American Mathematical Society, 1999.

Four versions of a GRASP for approximately solving general instances of the Steiner problem in graphs are proposed. One is implemented and tested. The construction phase is based on the distance network heuristic. A distance network corresponding to the original graph is built. Associated with each edge of the distance network is a weight that takes into account the shortest distances in the original graph. With respect to the new weight distribution, Kruskal's algorithm is used to solve the minimum spanning tree (MST) problem and the edges in the MST so computed are replaced by the edges in the corresponding shortest paths

in the original graph. The local search is based on insertions and eliminations of nodes to and from the current solution.

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P.M. Pardalos, T. Qian, and M.G.C. Resende. A greedy randomized adaptive search procedure for the feedback vertex set problem. *J. of Combinatorial Optimization*, 2:399–412, 1999.

This article describes a GRASP for finding approximate solutions to the feedback vertex set problem on a digraph. Several greedy functions are tested, all of them taking into account vertices with high degree. The local search procedure tries at each iteration to eliminate redundant vertices. Some efficient problem reduction techniques are also described. They are useful both to simplify the problem instance and to determine whether a digraph is acyclic or not.

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C.C. Ribeiro and M.G.C. Resende. Fortran subroutines for approximate solution of graph planarization problems using GRASP. *ACM Transactions on Mathematical Software*, 25:341–352, 1999.

See page 13.

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S.L. Martins, M.G.C. Resende, C.C. Ribeiro, and P.M. Pardalos. A parallel GRASP for the Steiner tree problem in graphs using a hybrid local search strategy. *J. of Global Optimization*, 17:267–283, 2000.

This paper presents a GRASP for the Steiner problem in graphs. The construction phase is based on the Mehlhorn distance network heuristic, which consists of computing the modified distance network graph and using Kruskal's algorithm to solve the minimum spanning tree problem for the resulting graph. The local search is done by using a combination of key-path based local search and node based local search. See page 12.

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I.H. Osman, B. Al-Ayoubi, M. Barake, and M. Hasan. A greedy random adaptive search procedure for the weighted maximal planar graph problem. Technical report, School of Business and Center for Advanced Mathematical Sciences, American University of Beirut, Beirut, Lebanon, 2000.

A GRASP is proposed and tested for the weighted maximal planar graph problem. The construction is a randomized version of the Green & Al-Hakim algorithm (1985). A new data structure is introduced, reducing the complexity of the construction from $O(n^3)$ to $O(n^2)$. Local search uses four types of moves proposed by Pesch, Glover, Bartsch, Salewski, and Osman (1995).

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C.C. Ribeiro, E. Uchoa, and R.F. Werneck. A hybrid GRASP with perturbations and adaptive path-relinking for the Steiner problem in graphs. Technical report, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, RJ 22453-900 Brazil, 2000.

This paper propose and describes a hybrid GRASP with path-relinking for the Steiner problem in graphs. See page 7.

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R. Martí. Arc crossing minimization in graphs with GRASP. *IEE Transactions*, 2001.

This paper presents a GRASP for minimizing straight-line crossings in hierarchical graphs. See page 36.

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R. Martí and V. Estruch. Incremental bipartite drawing problem. *Computers and Operations Research*, 2001.

A GRASP is proposed for the incremental arc crossing minimization problem for bipartite graphs. To appear in *Computers & Operations Research*. See page 37.

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13. Quadratic and other assignment problems

GRASP has been applied to several hard assignment problems, including quadratic, biquadratic, multidimensional, and frequency assignment. The following articles cover these problems.

Y. Li, P.M. Pardalos, and M.G.C. Resende. A greedy randomized adaptive search procedure for the quadratic assignment problem. In P.M. Pardalos and H. Wolkowicz, editors, *Quadratic assignment and related problems*, volume 16 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 237–261. American Mathematical Society, 1994.

A GRASP for the quadratic assignment problem is described. Construction first makes two assignments, and then completes the solution by making assignments, one at a time. The greedy function is assignment interaction cost. The local search procedure is a 2 assignment exchange.

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T.A. Feo and J.L. González-Velarde. The intermodal trailer assignment problem: Models, algorithms, and heuristics. *Transportation Science*, 29:330–341, 1995.

A GRASP is proposed for solving the problem of assigning highway trailers to railcar hitches in intermodal transportation. See page 32.

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P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. A parallel GRASP implementation for the quadratic assignment problem. In A. Ferreira and J. Rolim, editors, *Parallel Algorithms for Irregularly Structured Problems – Irregular'94*, pages 115–130. Kluwer Academic Publishers, 1995.

This paper discusses an efficient parallel implementation of GRASP for sparse quadratic assignment problems. See page 9.

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M.G.C. Resende, P.M. Pardalos, and Y. Li. Algorithm 754: Fortran subroutines for approximate solution of dense quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software*, 22:104–118, 1996.

See page 12.

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P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. Algorithm 769: Fortran subroutines for approximate solution of sparse quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software*, 23:196–208, 1997.

See page 12.

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R.A. Murphey, P.M. Pardalos, and L.S. Pitsoulis. A parallel GRASP for the data association multidimensional assignment problem. In P.M. Pardalos, editor, *Parallel processing of discrete problems*, volume 106 of *The IMA Volumes in Mathematics and Its Applications*, pages 159–180. Springer-Verlag, 1998.

> A GRASP for finding good solutions for the data association multidimensional assignment problem is described. At each discrete time interval, the data set is formulated as a multidimensional assignment problem (MAP) with a maximum likelihood cost function. A near-optimal solution to each MAP is obtained with a GRASP. The proposed method can be easily parallelized to substantially decrease the running time.

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T. Mavridou, P.M. Pardalos, L.S. Pitsoulis, and M.G.C. Resende. A GRASP for the biquadratic assignment problem. *European J. of Operational Research*, 105: 613–621, 1998.

> This paper proposes a GRASP for finding approximate solutions of the biquadratic assignment problem. As in the case of GRASP for the quadratic assignment problem, the construction phase has two stages. The first stage simultaneously makes four assignments, selecting the pairs corresponding to the smallest interaction costs, while the second stage makes the remaining assignments, one at time. The greedy function in the second stage selects the assignment corresponding to the minimum interaction cost with respect to the already-made assignments. In the local search phase, 2-exchange local search is applied to the permutation constructed in the first phase.

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E.L. Pasiliao. A greedy randomized adaptive search procedure for the multi-criteria radio link frequency assignment problem. Technical report, Department of ISE, University of Florida, Gainesville, FL 32611-6595, 1998.

> A GRASP for computing approximate solutions to a radio link frequency assignment problem is proposed. The objective is to minimize the order and the span of the solution set. See page 34.

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M.C. Rangel, N.M.M. de Abreu, P.O. Boaventura Netto, and M.C.S. Boeres. A modified local search for GRASP in the quadratic assignment problem. Technical report, Production Engineering Program, COPPE, Federal University of Rio de Janeiro, Rio de Janeiro, RJ Brazil, 1998.

> An improvement of the local search phase of the GRASP proposed by Li, Pardalos, and Resende (1994) for solving the quadratic assignment problem is proposed. The new strategy amplifies the local search range and improves the local search's efficiency.

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A.J. Robertson. A set of greedy randomized adaptive local search procedure (GRASP) implementations for the multidimensional assignment problem, 1998.

> This report introduces four GRASP implementations for the multidimensional assignment problem by combining two constructive methods (randomized greedy and randomized max regret) and two local search methods (2-exchange and variable depth exchange). At each iteration of the randomized greedy construction phase, a set of best assignments is constructed from which a random element is selected and added to the

solution set. The greedy function of the randomized max regret construction method is a measure of the competition between the two leading cost candidates. The maximum regret value corresponds to the candidate assignment that has the largest winning margin between itself and its next highest competitor. The variable depth exchange is an extension of the 2-exchange method that allows a more extensive search of the surrounding neighborhood space. To appear in *Computational Optimization and Applications*.

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C. Fleurent and F. Glover. Improved constructive multistart strategies for the quadratic assignment problem using adaptive memory. *INFORMS J. on Computing*, 11:198–204, 1999.

Adaptive memory strategies that are the heart of tabu search methods are shown to be a foundation for alternative, enhanced, multistart approaches. This paper illustrates that constructive multistart methods, such as Random Restart and GRASP, can be improved by the addition of memory and associated heuristic search principles. The improved results indicate that these principles (learning, intensification, candidate list strategies, POP) are not limited to applications with transition neighborhoods, as in local search, but can also be useful for applications characterized by constructive (and destructive) neighborhoods. The paper shows that the GRASP for QAP of Li, Pardalos, and Resende (1994) can be improved upon by using these memory strategies. See page 6.

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C.A.S. Oliveira and F.C. Gomes. Two metaheuristics for channel allocation in mobile telephony. Technical report, Artificial Intelligence Laboratory, Universidade Federal do Ceará, Fortaleza, Brazil, August 1999.

The cellular frequency assignment problem described in this paper consists of minimizing the total system interference in mobile phone covered areas, with respect to co-channel and adjacent-channel interference. Two metaheuristics are proposed: GRASP and Asynchronous Team (A-Teams). See page 34.

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L.S. Pitsoulis. *Algorithms for nonlinear assignment problems*. PhD thesis, Department of Industrial and Systems Engineering, University of Florida, 1999.

This dissertation presents GRASPs for solving the following NP-hard nonlinear assignment problems (NAPs): quadratic assignment problem (QAP), biquadratic assignment problem (BiQAP), turbine balancing problem (TBP), and multidimensional assignment problem (MAP). Computational results indicate that all of the suggested algorithms are among the best in the literature in terms of solution quality and computational time.

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R.K. Ahuja, J.B. Orlin, and A. Tiwari. A greedy genetic algorithm for the quadratic assignment problem. *Computers and Operations Research*, 27:917–934, 2000.

This report suggests a genetic algorithm for the QAP that incorporates the construction phase of the GRASP for QAP of Li, Pardalos, and Resende (1994) to generate the initial population.

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R.M. Aiex, M.G.C. Resende, P.M. Pardalos, and G. Toraldo. GRASP with path relinking for the three-index assignment problem. Technical report, AT&T Labs Research, Florham Park, NJ 07733, 2000.

This paper describes variants of GRASP with path relinking for the three index assignment problem (AP3). See page 11.

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X. Liu, P.M. Pardalos, S. Rajasekaran, and M.G.C. Resende. A GRASP for frequency assignment in mobile radio networks. In S. Rajasekaran, P.M. Pardalos, and F. Hsu, editors, *Mobile Networks and Computing*, volume 52 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 195–201. American Mathematical Society, 2000.

A GRASP for frequency assignment is described in this paper. See page 35.

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M. Prais and C.C. Ribeiro. Reactive GRASP: An application to a matrix decomposition problem in TDMA traffic assignment. *INFORMS J. on Computing*, 12: 164–176, 2000.

This paper describes a GRASP for matrix decomposition problem arising in the context of traffic assignment in communication satellites. The local search phase of the GRASP proposed is based on a new neighborhood, defined by constructive and destructive moves. See pages 6 and 35.

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M.C. Rangel, N.M.M. Abreu, and P.O. Boaventura Netto. GRASP in the QAP: An acceptance bound for initial solutions. *Pesquisa Operacional*, 20:45–58, 2000.

This paper presents a modified version of the GRASP algorithm proposed by Li, Pardalos, and Resende (1994) for the quadratic assignment problem. The new GRASP uses a criterion to accept or reject a given initial solution, thus trying to avoid searches that eventually can be fruitless. It computes a normalized limit cost, defined with the aid of QAP upper and lower bounds easily obtained and discards all solutions with cost less than the computed limit. In Portuguese.

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A. Srinivasan, K.G. Ramakrishnan, K. Kumaram, M. Aravamudam, and S. Naqvi. Optimal design of signaling networks for Internet telephony. In *IEEE INFOCOM 2000*, March 2000.

GRASP is used in an approach for efficient design of a signaling network for a network of software switches supporting Internet telephony. See page 35.

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L.S. Pitsoulis, P.M. Pardalos, and D.W. Hearn. Approximate solutions to the turbine balancing problem. *European J. of Operational Research*, 130:147–155, 2001.

In this paper, the turbine balancing problem is formulated as a standard quadratic assignment problem, and a GRASP implementation for solving the resulting problem is presented.

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14. Manufacturing

GRASP has been used to address several optimization problems in manufacturing. The following papers are examples of this.

J.F. Bard and T.A. Feo. Operations sequencing in discrete parts manufacturing. *Management Science*, 35:249–255, 1989.

> This paper presents a method for efficiently sequencing the cutting operations associated with the manufacture of discrete parts. See page 13.

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T.A. Feo and J.F. Bard. The cutting path and tool selection problem in computer-aided process planning. *J. of Manufacturing Systems*, 8:17–26, 1989.

> The purpose of this paper is to provide a method for minimizing the sum of tool setup and volume removal times associated with metal cutting operations on a flexible machine. The problem is modeled as an integer program, then relaxed into a min-cut problem on a simple network. After obtaining a tentative solution, the authors use a GRASP to identify good feasible points corresponding to alternative process plans. These are seen to speed convergence during branch & bound.

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J.F. Bard and T.A. Feo. An algorithm for the manufacturing equipment selection problem. *IIE Transactions*, 23:83–92, 1991.

> This paper provides a unified framework in which product and process demands can be related to manufacturing system requirements. The authors develop a nonlinear cost minimization model. The objective is to determine how many of each machine type to purchase and what fraction of the time each piece of equipment will be configured for a particular type of operation. Once the original problem is converted into a MILP, a depth-first branch & bound algorithm is used, employing the greedy randomized set covering heuristic of Feo and Resende (1989), to implicitly search for optimality. Viewing the contribution that any machine makes to satisfy the demand of any process as the unit benefit associated with that machine, a benefit-to-cost ratio is computed for each machine. To derive a feasible solution, the heuristic iteratively selects machines with largest ratio and updates benefits to take into account the remaining demand.

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J.G. Klincewicz and A. Rajan. Using GRASP to solve the component grouping problem. *Naval Research Logistics*, 41:893–912, 1994.

> Two new heuristics are proposed for solving a particular set partitioning problem that arises in robotics assembly, as well as in a number of other manufacturing and material logistics application areas. The heuristics are GRASPs involving two alternate procedures for determining starting points: component-based and code-based.

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T.A. Feo, J.F. Bard, and S. Holland. Facility-wide planning and scheduling of printed wiring board assembly. *Operations Research*, 43:219–230, 1995.

> This paper describes a decision support system known as INSITES, designed to assist Texas Instruments in the day-to-day assembly operations

of their printed wiring board (PWB) facilities. A GRASP is used to solve the underlying multiple machine scheduling problem. The schedule produced at each GRASP iteration is evaluated based on one of five different optimization criteria. The choice of the criterion to be followed is made by the user to rank order the schedules provided by multiple GRASP iterations.

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J.F. Bard, T.A. Feo, and S. Holland. A GRASP for scheduling printed wiring board assembly. *I.I.E. Trans.*, 28:155–165, 1996.

The assembly of printed wiring boards (PWBs) typically involves the coordination of thousands of components and hundreds of part numbers in a job shop environment with more than 50 different processes and workstations. The authors propose a GRASP for solving the daily scheduling problem that arises in such environment. The first phase of GRASP obtains a user-specified number of schedules. The greedy function is the product between the weighted processing time and and the slack time window.

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J. Yen, M. Carlsson, M. Chang, J.M. Garcia, and H. Nguyen. Constraint solving for inkjet print mask design. *J. of Imaging Science and Technology*, 44:391–397, 2000.

Print masks are used to control the firing of the nozzles, that is, to determine which nozzles on an inkjet printer cartridge are to spit an ink droplet at each particular instant in a multiple-pass print mode. Masks are generated by minimizing the total costs. A GRASP is proposed for for automatic generation of print masks. It has been used to design the print masks for Hewlett Packard's wide format printers (DeskJet 2500C and 2500CM). This approach can shorten the turn-around time for print mask design.

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15. Transportation

GRASP has been used to find approximate solutions of problems in air, rail, and intermodal transportation. The following papers illustrate these applications.

T.A. Feo and J.F. Bard. Flight scheduling and maintenance base planning. *Management Science*, 35:1415–1432, 1989.

This paper presents a model that can be used by planners to both locate maintenance stations and develop flight schedules that better meet the cyclical demand for maintenance. The problem is formulated as large-scale mixed integer program, i.e. a minimum cost, multicommodity flow network with integral constraints, where each airplane represents a separate commodity and each arc has an upper and lower capacity of flow. Since obtaining feasible solutions from the relative LP relaxation is difficult, the authors propose a GRASP.

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T.A. Feo and J.L. González-Velarde. The intermodal trailer assignment problem: Models, algorithms, and heuristics. *Transportation Science*, 29:330–341, 1995.

This paper deals with the problem of optimally assigning highway trailers to railcar hitches in intermodal transportation. Using a set covering formulation, the problem is modeled as an integer linear program, whose linear programming relaxation yields a tight lower bound. This formulation also provides the basis for developing a branch-and-bound algorithm and a GRASP for solving the problem. The greedy strategy of the construction phase of GRASP consists in selecting at each step a feasible assignment of the most difficult to use available railcar together with the most difficult to assign trailer. To improve the constructed solution, a 2-exchange local search is applied, carrying out a complete enumeration of the solutions in the neighborhood.

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M.F. Argüello, J.F. Bard, and G. Yu. A GRASP for aircraft routing in response to groundings and delays. *J. of Combinatorial Optimization*, 1:211–228, 1997.

This paper presents a GRASP to reconstruct aircraft routings in response to groundings and delays experienced over the course of the day. The objective is to minimize the cost of reassigning aircraft to flights taking into account available resources and other system constraints. The proposed heuristic is a neighborhood search technique that takes as input an initial feasible solution, so that the construction phase is omitted. Two types of partial route exchange operations are described. The first type is the exchange of flight sequences with identical endpoints. In the second type, the sequence of flights being exchanged must have the same origination airport, but the termination airports are swapped.

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J.F. Bard. An analysis of a rail car unloading area for a consumer products manufacturer. *J. of the Operational Research Society*, 48:873–883, 1997.

This paper discusses how to design and analyze the railcar unloading area of Procter & Gamble's principal laundry detergent plant. The related combinatorial problem of assigning railcars to positions on the platform and unloading equipment to railcars is modeled as a mixed-integer nonlinear program. To approximately solve the problem, four alternatives are proposed and evaluated with the help of a GRASP.

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D. Sosnowska. Optimization of a simplified fleet assignment problem with metaheuristics: Simulated annealing and GRASP. In P.M. Pardalos, editor, *Approximation and complexity in numerical optimization*. Kluwer Academic Publishers, 2000.

Two heuristics based on simulated annealing and GRASP are presented for finding approximate solutions for a simplified fleet assignment problem. Both methods are based on swapping parts of sequence of flight legs assigned to an aircraft (rotation cycle) between two randomly chosen aircrafts. In simulated annealing, the exchange is such that a solution is accepted according to a probability distribution, while in GRASP only exchanges leading to a better solution are permitted and the potentially best part of the assignment is conserved and the rest is randomly reattributed. The construction phase does not use a restricted candidate list explicitly, but a solution is built by simply trying to make the time interval between two flights as small as possible.

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16. Telecommunications

Telecommunications, including network design, is a field in which much work with GRASP has been done. The papers below illustrate this.

J.G. Klincewicz. Avoiding local optima in the p-hub location problem using tabu search and GRASP. *Annals of Operations Research*, 40:283–302, 1992.

See page 20.

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J. Xu and S. Chiu. Solving a real-world field technician scheduling problem. In *Proceedings of the International Conference on Management Science and the Economic Development of China*, pages 240–248, July 1996.

See page 14.

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F. Poppe, M. Pickavet, P. Arijs, and P. Demeester. Design techniques for SDH mesh-restorable networks. In *Proceedings of the European Conference on Networks and Optical Communications (NOC'97), Volume 2: Core and ATM Networks*, pages 94–101, 1997.

To design low cost reliable telecommunication networks, the authors propose three algorithms: an integer linear programming algorithm (branch-and-cut-and-price), a GRASP, and a zoom-in approach that combines a genetic algorithm with deterministic optimization routines. The greedy choice of the proposed GRASP is to favor paths having lowest additional cost. The local search iteratively tries to reroute some paths in order to further decrease the overall network cost.

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L.I.P. Resende and M.G.C. Resende. A GRASP for frame relay PVC routing. In *Proc. of the Third Metaheuristics International Conference*, pages 397–402, July 1999.

This paper describes a GRASP for routing permanent virtual circuits (PVC) for frame relay in telecommunications systems. The objective is to minimize PVC delays while balancing trunk loads. The greedy choice selects from the set of not yet routed PVCs the one that minimizes the delay while balancing the trunk loads. The local search procedure reroutes each PVC, one at a time, checking each time if the new route taken together with the remaining fixed routes improves the objective function.

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M.G.C. Resende and O. Ulular. SMART: A tool for AT&T Worldnet access design – Location of Cascade 9000 concentrators. Technical report, AT&T Labs Research, Florham Park, NJ 07932 USA, 1997.

This report describes SMART, a software tool for finding low cost configurations of Cascade 9000 concentrators in the AT&T Worldnet backbone access network. The concentrator location problem is stated and cost model is presented for concentrator configurations. This cost model is used in a GRASP, proposed for finding approximate solutions to the

concentrator location problem. The greedy choice favors the points-of-presence (POPs) with smallest incremental cost. The local search implements a simple 2-exchange.

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E.L. Pasiliao. A greedy randomized adaptive search procedure for the multi-criteria radio link frequency assignment problem. Technical report, Department of ISE, University of Florida, Gainesville, FL 32611-6595, 1998.

A GRASP is presented for computing approximate solutions to the radio link frequency assignment problem. If a feasible solution exists, the objective is to minimize both the order and the span of the solution set. The local search procedure attempts to eliminate each channel from the communication network. This process is repeated until an attempt to eliminate every frequency in the solution set has been made without success.

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M.G.C. Resende. Computing approximate solutions of the maximum covering problem using GRASP. *J. of Heuristics*, 4:161–171, 1998.

A GRASP for facility location on a network with the objective of maximizing service coverage is proposed. See page 23.

.

J. Xu and S. Chiu. Effective heuristic procedure for a field technician scheduling problem. Technical report, US WEST Advanced Technologies, Boulder, CO 80303 USA, 1998.

See page 15.

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J. Abello, P.M. Pardalos, and M.G.C. Resende. On maximum clique problems in very large graphs. In J. Abello and J. Vitter, editors, *External memory algorithms and visualization*, volume 50 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 119–130. American Mathematical Society, 1999.

GRASP is used to identify cliques and quasi-cliques in very large multi-digraphs that arise from a telephone call detail database. See page 24.

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C.A.S. Oliveira and F.C. Gomes. Two metaheuristics for channel allocation in mobile telephony. Technical report, Artificial Intelligence Laboratory, Universidade Federal do Ceará, Fortaleza, Brazil, August 1999.

The cellular frequency assignment problem described consists in minimizing the total system interference in mobile phone covered areas, with respect to co-channel and adjacent-channel interference. Two metaheuristics are proposed: GRASP and Asynchronous Team (A-Team). The construction phase of the proposed GRASP is realized by a procedure that at each step chooses the next antenna to which a frequency will be assigned. In the RCL construction, priority is given to transmitters with fewer options of frequency assignment. To implement the local search phase, the authors use a down hill algorithm, that performs random perturbations in the solution, exchanging the frequency of one antenna by another randomly chosen. The stopping criterion used for the down hill algorithm is execution time.

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M. Armony, J.G. Klincewicz, H. Luss, and M.B. Rosenwein. Design of stacked self-healing rings using a genetic algorithm. *J. of Heuristics*, 6:85–105, 2000.

A hybrid genetic algorithm for design of stacked self-healing rings is proposed and tested. The initial population is made up of randomly generated solutions as well as solutions generated by a GRASP. See page 8.

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X. Liu, P.M. Pardalos, S. Rajasekaran, and M.G.C. Resende. A GRASP for frequency assignment in mobile radio networks. In S. Rajasekaran, P.M. Pardalos, and F. Hsu, editors, *Mobile Networks and Computing*, volume 52 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 195–201. American Mathematical Society, 2000.

A GRASP for frequency assignment is described. Local search uses simulated annealing. The construction phase uses two greedy functions. The first chooses a vertex from the set of unselected vertices with high saturation degrees. The second function is used to assign a frequency to the selected vertex. A frequency is selected from a set of permissible frequencies that contribute little additional cost to the objective function.

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J.G. Klincewicz. Enumeration and search procedures for a hub location problem with economies of scale. Technical report, AT&T Labs, Middletown, NJ 07748, 2000.

An optimal enumeration scheme, as well as other heuristics based on tabu search and GRASP are proposed for locating hubs in a communications or transportation network.

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M. Prais and C.C. Ribeiro. Reactive GRASP: An application to a matrix decomposition problem in TDMA traffic assignment. *INFORMS J. on Computing*, 12: 164–176, 2000.

This paper describes a GRASP for matrix decomposition problem arising in the context of traffic assignment in communication satellites. A geostationary communication satellite has a number of spot beam antennas covering geographically distributed areas. According to the slot switching configuration on the on-board switch, the uplink traffic received at the satellite has to be immediately sent to ground areas through a set of transponders. The slot switching configurations are determined through the solution of a time slot assignment problem, which is equivalent to the decomposition of a nonnegative traffic matrix into the sum of a family of switching mode matrices. See pages 6 and 29.

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A. Srinivasan, K.G. Ramakrishnan, K. Kumaram, M. Aravamudam, and S. Naqvi. Optimal design of signaling networks for Internet telephony. In *IEEE INFOCOM 2000*, March 2000.

This paper presents an approach for efficient design of a signaling network for a network of software switches supporting Internet telephony. Optimal load balancing for given demand forecast is formulated as a quadratic assignment problem, which is solved with a GRASP.

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17. Automatic drawing

GRASP has been used to find approximate solutions to problems that arise in automatic graph drawing. The following papers illustrate this.

R. Martí and M. Laguna. Heuristics and meta-heuristics for 2-layer straight line crossing minimization. Technical report, Department of Statistics and Operations Research, University of Valencia, 46100 Burjassot, Valencia, Spain, 1997.

Extensive computational results are presented using 12 heuristics and two meta-heuristics for the 2-layer straight line crossing minimization problem. On dense graphs, a tabu search meta-heuristic does best with GRASP a close second. On low-density graphs, GRASP outperforms all other approaches.

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M.G.C. Resende and C.C. Ribeiro. A GRASP for graph planarization. *Networks*, 29:173–189, 1997.

GRASP is applied to the graph planarization problem. See page 22.

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E. Fernández and R. Martí. GRASP for seam drawing in mosaicking of aerial photographic maps. *J. of Heuristics*, 5:181–197, 1999.

Commercial aerial photographic maps are often so large that it is necessary to produce one map from two or even more photographs. These are combined, two at a time, in a process called *mosaicking*. The objective is to make the final map appear to be the product of a single photograph. The most difficult step in the mosaicking process is *seam-drawing*. This paper proposes a GRASP for solving the seam-drawing process.

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M. Laguna and R. Martí. GRASP and path relinking for 2-layer straight line crossing minimization. *INFORMS J. on Computing*, 11:44–52, 1999.

This paper develops a GRASP for the problem of minimizing straight line crossings in a 2-layer graph. The method proposed can be coupled with a path relinking strategy to search for improved outcomes. The greedy criterion of the construction phase is based on the degree of the vertices and a value based restricted candidate list is used. Each step of the improvement phase consists in selecting each vertex to be considered for a move. A probabilistic selection rule is used such that vertices with high degree are more likely to be selected first at each step of this process. See page 6.

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C.C. Ribeiro and M.G.C. Resende. Algorithm 797: Fortran subroutines for approximate solution of graph planarization problems using GRASP. *ACM Transactions on Mathematical Software*, 25:341–352, 1999.

See page 13.

.

R. Martí. Arc crossing minimization in graphs with GRASP. *IEE Transactions*, 2001.

A GRASP for minimizing straight-line crossings in hierarchical graphs is presented. GRASP is shown to be faster than more complex heuristics but produces lower-quality solutions. It is not as fast as simple heuristics, but finds better-quality solutions. Hence, it is a candidate for practical implementation in graph drawing systems.

.

R. Martí and V. Estruch. Incremental bipartite drawing problem. *Computers and Operations Research*, 2001.

The goal of limiting the number of arc crossings is a well accepted criterion of how well a graph is drawn. Incremental graph drawing supports interactive updates by users. A GRASP is proposed for the incremental arc crossing minimization problem for bipartite graphs. Computational experiments are done on 450 instances and results are compared with a branch and bound algorithm. To appear in *Computers & Operations Research*.

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18. Electrical power systems

GRASP has been applied to a transmission expansion problem in electrical power systems. The two papers below describe this application.

S. Binato, G.C. Oliveira, and J.L. Araújo. A greedy randomized adaptive search procedure for transmission expansion planning. Technical report, Research Centre for Electrical Energy – CEPEL, Rio de Janeiro, 1998.

This paper presents a GRASP for a long term transmission expansion planning problem. The greedy function is to minimize the load curtailment required to eliminate all operational violations. The local search phase is based on circuit exchanges, i.e. the procedure exchanges selected additions with unselected additions.

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S. Binato and G.C. Oliveira. A Reactive GRASP for transmission network expansion planning. In P. Hansen and C.C. Ribeiro, editors, *Essays and surveys on metaheuristics*. Kluwer Academic Publishers, 2001.

The GRASP previously proposed by Binato, Oliveira, and Araújo (1998) for solving a transmission network expansion problem is enhanced with the reactive scheme of Prais and Ribeiro (2000) to self-adjust the GRASP RCL parameter α. They also propose to apply a bias distribution function of Bresina (1996) to bias the random greedy construction phase towards the most promising variables.

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19. VLSI design

GRASP has been applied to circuit partitioning problems in VLSI design, as indicated by the papers below.

S. Areibi and A. Vannelli. A GRASP clustering technique for circuit partitioning. In J. Gu and P.M. Pardalos, editors, *Satisfiability problems*, volume 35 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 711–724. American Mathematical Society, 1997.

This paper adapts a basic node interchange scheme for solving the circuit partitioning problem and develops a clustering technique that uses GRASP to generate clusters of moderate sizes. The number of clusters is predetermined as a function of the number of partitions required. Initially, the heuristic reads the circuit description and resizes the blocks to be used by GRASP, which utilizes only the construction phase to generate the number of required clusters. The GRASP construction phase is followed by a post-processing stage, in which a simple dynamic hill climbing algorithm is used as local search to improve the initial solution generated.

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S.M. Areibi. GRASP: An effective constructive technique for VLSI circuit partitioning. In *Proc. IEEE Canadian Conference on Electrical & Computer Engineering (CCECE'99)*, May 1999.

A GRASP for VLSI circuit partitioning is proposed. See page 19.

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20. Miscellaneous applications

A number of papers on GRASP have appeared in fields as diverse as consumer choice theory and multitarget multisensor tracking. The following papers could not be classified into one of the previous categories and are grouped as miscellaneous applications.

T.A. Feo and M.G.C. Resende. A probabilistic heuristic for a computationally difficult set covering problem. *Operations Research Letters*, 8:67–71, 1989.

GRASP is proposed for set covering. A value based restricted candidate list is used to construct solutions of difficult set covering problems that arise in computing the 1-width of the incidence matrix of Steiner triple systems. The local search is based on the elimination of redundant elements in the cover.

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J.B. Ghosh. Computational aspects of the maximum diversity problem. *Operations Research Letters*, 19:175–181, 1996.

This paper addresses two variations of the maximum diversity problem. This problem arises when m elements are to be selected from an n-element population based on inter-element distances. Using a reduction from the vertex cover problem, the authors prove that the problem is NP-hard and propose a GRASP for approximately solving it. The neighborhood of a solution is the set of all solutions that can be obtained by replacing an element in the incumbent solution with one element that is not in it.

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R. Colomé and D. Serra. Consumer choice in competitive location models: Formulations and heuristics. Technical report, Department of Economics and Management, Universitat Pompeu Fabra, Barcelona, Spain, May 1998.

This paper studies the importance of customer behavior with respect to distance or transportation costs in the optimality of locations obtained by traditional state-of-art competitive location models. The authors propose four models to represent the problem and propose a hybrid metaheuristic for solving it. The proposed method consists of two phases. In the first phase, a good initial solution is found by applying a GRASP procedure, while in the second phase the previous solution found is improved by applying a tabu search heuristic. To appear in *Papers in Regional Science*.

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X. Gandibleux, D. Vancoppenolle, and D. Tuyttens. A first making use of GRASP for solving MOCO problems. Technical report, University of Valenciennes, France, 1998.

An extension of GRASP, to solve multi-objective combinatorial optimization (MOCO) problems, is considered. In particular, classical covering, assignment, knapsack, and scheduling problems with multiple objectives are used as benchmarks. Computational results compare GRASP solutions for a benchmark set of test problems and results are discussed in comparison with an exact method, when available. In French.

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K. Holmqvist, A. Migdalas, and P.M. Pardalos. A GRASP algorithm for the single source uncapacitated minimum concave-cost network flow problem. In P.M. Pardalos and D.-Z. Du, editors, *Network design: Connectivity and facilities location*, volume 40 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 131–142. American Mathematical Society, 1998.

This paper is concerned with the single source uncapacitated version of the minimum concave-cost network flow problem, that requires establishing a minimum cost flow through a given network from a single source to a set of sinks. The authors propose a GRASP that can be trivially implemented on parallel processors. The construction phase iteratively builds a tree starting from the source node. The elements of the restricted candidate list are end nodes of arcs with a cost close to the best one. The local search phase applies either of the two local search variants proposed by Guisewite and Pardalos (1990).

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N. Krasnogor, D.A. Pelta, W. Russo, and G. Terrazas. A GRASP approach to the protein structure prediction problem. Technical report, LIFIA Lab, University of La Plata, La Plata, Argentina, 1998.

This paper discusses the applicability of a GRASP for solving a special protein folding problem, an important problem in computational biology. The goal is to predict from the molecular sequence of a given protein its particular 3D structure.

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R.A. Murphey, P.M. Pardalos, and L.S. Pitsoulis. A greedy randomized adaptive search procedure for the multitarget multisensor tracking problem. In P.M. Pardalos and D.-Z. Du, editors, *Network design: Connectivity and facilities location*, volume 40 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pages 277–301. American Mathematical Society, 1998.

A GRASP is presented for approximately solving the multitarget multisensor tracking problem, which can be interpreted as a collection of multidimensional assignment problems. Since the objective is to select a target hypothesis and partition of the measurements that is most likely to occur, a likelihood cost function and partitioning constraint set are developed. The GRASP construction phase creates the restricted candidate list containing the most likely to occur (lower cost) tuples. The local search explores all 2-exchange permutations.

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P.L. Hammer and D.J. Rader, Jr. Maximally disjoint solutions of the set covering problem. Technical report, RUTCOR, Rutgers University, 1999.

This paper describes the problem of finding two solutions of a set covering problem that have a minimum number of common variables. It is proved that this problem is NP-complete and three heuristics are proposed for solving it. Two of these algorithms find the solutions sequentially. One of them is a GRASP. The third algorithm finds solutions simultaneously. Each proposed heuristic is a variant of the standard greedy method for set covering problems, whose greedy choice favors unselected variables that maximize the number of uncovered rows that become covered. To reduce the overlap of any pair of solutions, a local search algorithm is used that swaps at each iteration parts of the solution found with a set of variables not in it.

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M.C. Medeiros, M.G.C. Resende, and A. Veiga. Piecewise linear time series estimation with GRASP. Technical report, AT&T Labs Research, Florham Park, NJ 07932 USA, 1999a.

This paper describes a GRASP to build piecewise linear statistical models with multivariate thresholds. The construction phase consists of sequentially choosing hyperplanes until the maximum number of hyperplanes is reached. The greedy function orders the possible hyperplanes with respect to the sum of squared errors of the fitted data. The local search is a 2-exchange heuristic. To appear in *Computational Optimization and Applications*.

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M.C. Medeiros, A. Veiga, and M.G.C. Resende. A combinatorial approach to piecewise linear time series analysis. Technical report, AT&T Labs Research, Florham Park, NJ 07932 USA, 1999b.

This paper presents a new approach to modeling threshold processes, based on a linear model with time-varying parameters. The authors show that this formulation is closely related to the self-exciting threshold autoregressive models (SETAR) with the advantage that it incorporates linear multivariate thresholds. A GRASP is proposed to estimate the parameters of the model. The greedy choice takes into account the sum of squared errors of the fitted data. The local search is a 2-exchange heuristic. To appear in *J. of Graphical and Computational Statistics*.

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J.R. Cano, O. Cordón, F. Herrera, and L. Sánchez. A greedy randomized adaptive search procedure for the clustering problem. Technical report, Department of Computer Science and Artificial Intelligence, University of Granada, 18071

Granada, Spain, 2000.

A GRASP for cluster analysis is described. Construction is done using a randomized greedy Kaufman procedure and local search uses the K-means algorithm. High quality solutions are found for benchmark problems. The best solutions are found with a hybid GRASP/K-means with Kaufman inicialization.

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L.S. Pitsoulis, P.M. Pardalos, and D.W. Hearn. Approximate solutions to the turbine balancing problem. *European J. of Operational Research*, 130:147–155, 2001.

See page 29.

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