A Quadratic Programming Bibliography

Nicholas I. M. Gould

Computational Science and Engineering Department
Rutherford Appleton Laboratory, Chilton
Oxfordshire, OX11 0QX, England, EU
Email : n.gould@rl.ac.uk

and

Philippe L. Toint

Department of Mathematics, University of Namur
61, rue de Bruxelles, B-5000 Namur, Belgium, EU.
Email : pht@math.fundp.ac.be

RAL Numerical Analysis Group Internal Report 2000–1

This version: February 27, 2001

The following is a list of all of the published and unpublished works on quadratic programming that we are aware of. Some are general references to background material, while others are central to the development of the quadratic programming methods and to the applications we intend to cover in our evolving book on the subject. We have deliberately not included any but the most relevant of the hundreds, if not thousands, of citations to sequential/successive/recursive quadratic programming methods for nonlinear programming, nor to those on linear programming or quadratic programming with quadratic constraints. The complete LaTeX bibliography, together with up-to-date additions, is available online at

`ftp://ftp.numerical.rl.ac.uk/pub/qpbook/qpbook.bib`

and

`ftp://thales.math.fundp.ac.be/pub/qpbook/qpbook.bib` .

We would be delighted to receive any corrections or updates to this list.

A Quadratic Programming Bibliography

N. N. Abdelmalek. Restoration of images with missing high-frequency components using quadratic programming. *Applied Optics*, **22**(14), 2182–2188, 1983.

> **Abstract.** A method for restoring an optical image which is subjected to low-pass frequency filtering is presented. It is assumed that the object whose image is restored is of finite spatial extent. The problem is treated as an algebraic image-restoration problem which is then solved as a quadratic programming problem with bounded variables. The regularization technique for the ill-posed system is to replace the consistent system of the quadratic programming problem by an approximate system of smaller rank. The rank which gives a best or near-best solution is estimated. This method is a novel one, and it compares favorably with other known methods. Computer-simulated examples are presented. Comments and conclusions are given.

N. N. Abdelmalek and T. Kasvand. Digital image restoration using quadratic programming. *Applied Optics*, **19**(19), 3407–3415, 1980.

> **Abstract.** The problem of digital image restoration is considered by obtaining an approximate solution to the Fredholm integral equation of the first kind in two variables. The system of linear equations resulting from the discretization of the integral equation is converted to a consistent system of linear equations. The problem is then solved as a quadratic programming problem with bounded variables where the unknown solution is minimized in the L_2 norm. In this method minimum computer storage is needed, and the repeated solutions are obtained in an efficient way. Also the rank of the consistent system which gives a best or near best solution is estimated. Computer simulated examples using spatially separable pointspread functions are presented. Comments and conclusions are given.

R. A. Abrams and A. Ben Israel. A duality theorem for complex quadratic programming. *Journal of Optimization Theory and Applications*, **4**(4), 245–252, 1969.

> **Abstract.** A duality theory for complex quadratic programming over polyhedral cones is developed, following Dorn, by using linear duality theory.

J. W. Adams. Quadratic programming approaches to new optimal windows and antenna arrays. *Conference Record. Twenty Fourth Asilomar Conference on Signals, Systems and Computers Maple Press, San Jose, CA, USA*, **1**, 69–72, 1990a.

> **Abstract.** New window design problems are formulated in terms of quadratic programming. The new windows permit the designer to control the tradeoff between the peak sidelobe level and the total sidelobe energy. In addition, linear constraints can be imposed on the design problem. The proposed methods are applicable to applications in the fields of signal processing and antenna arrays.

J. W. Adams. Quadratic programming approaches to new problems in digital filter design. *Conference Record. Twenty Fourth Asilomar Conference on Signals, Systems and Computers Maple Press, San Jose, CA, USA*, **1**, 307–310, 1990b.

> **Abstract.** New digital filter design problems are formulated in terms of quadratic programming. The new filters permit the designer to control the tradeoff between the peak errors and the total squared errors. In

particular, the mean-squared error can be minimized subject to peak errors constraints, as required in many practical design problems. In addition, equality constraints can be imposed for special applications.

J. W. Adams, P. Kruethong, R. Hashemi, J. L. Sullivan, and D. R. Gleeson. New quadratic programming algorithms for designing FIR digital filters. *Conference Record of The Twenty Seventh Asilomar Conference on Signals, Systems and Computers. IEEE Comput. Soc Press, Los Alamitos, CA, USA*, **2**, 1206–1210, 1993.

Abstract. The Parks-McClellan algorithm (1973) is very popular for designing FIR digital filters. It is based on a linear programming algorithm called the Remez exchange. Our new algorithm is based on quadratic programming, which includes linear programming as a special case. The filters in this paper permit the designer to control the tradeoff between the peak error and the total squared error. These filters are designed according to the peak-constrained least-squares (PCLS) optimality criterion.

J. W. Adams, J. L. Sullivan, D. R. Gleeson, P. H. Chang, and R. Hashemi. Application of quadratic programming to FIR digital filter design problems. *Conference Record of the Twenty Eighth Asilomar Conference on Signals, Systems and Computers. IEEE Comput. Soc Press, Los Alamitos, CA, USA*, **1**, 314–318, 1994.

Abstract. Quadratic programming problems have long been of interest in the business community. Quadratic programming is often used as the basis for ”program trading” where stocks are automatically bought and sold by mutual funds to optimize profits. Quadratic programming algorithms can also be used to optimize digital filters, as discussed in this paper. We present the generalized multiple exchange (GME), simplified generalized multiple exchange (SGME) and modified generalized multiple exchange (MGME) algorithms for designing constrained least-squares (CLS) filters. The CLS filters are generalizations of the popular minimax and least-squares filters. The CLS filters are important not only because of their generality, but also because they are needed for many practical applications.

W. P. Adams and P. M. Dearing. On the equivalence between roof duality and Lagrangian duality for unconstrained 0–1 quadratic programming problems. *Discrete Applied Mathematics*, **48**(1), 1–20, 1994.

Abstract. Considers techniques for computing upper bounds on the optimal objective function value to any unconstrained 0–1 quadratic programming problem (maximization). In particular, the authors study three methods for obtaining upper bounds as presented in a recent paper by Hammer, Hansen, and Simeone (1984) (1) generating two classes of upper-bounding linear functions referred to as paved upper planes and roofs, (2) solving the continuous relaxation of a mixed-integer linear problem by Rhys (1970), and (3) the quadratic complementation of variables which results in a bound called the height. The authors show that all three methods directly result from standard properties of a reformulation of the quadratic problem as a mixed-integer linear program, with methods (1) and (3) resulting from a Lagrangian dual of this reformulation. Based on this reformulation, they expand upon the published results.

W. P. Adams and H. D. Sherali. A tight linearization and an algorithm for 0–1 quadratic programming problems. *Management Science*, **32**(10), 1274–1290, 1986.

Abstract. The paper is concerned with the solution of linearly constrained 0–1 quadratic programming problems. Problems of this kind arise in numerous economic, location decision, and strategic planning situations, including capital budgeting, facility location, quadratic assignment, media selection, and dynamic set covering. A new linearization technique is presented for this problem which is demonstrated to yield a tighter continuous or linear programming relaxation than is available through other methods. An implicit enumeration algorithm which uses Lagrangian relaxation, Benders' cutting planes, and local explorations is designed to exploit the strength of this linearization. Computational experience is provided to demonstrate the usefulness of the proposed linearization and algorithm.

S. N. Afriat. The quadratic form definite on a manifold. *Proceedings of the Cambridge Philosophical Society*, **47**, 1–6, 1951.

A. Aggarwal and C. A. Floudas. A decomposition approach for global optimum search in QP, NLP and MINLP problems. *Annals of Operations Research*, **25**(1–4), 119–145, 1990.

Abstract. A new approach for global optimum search is presented which involves a decomposition of the variable set into two sets-complicating and noncomplicating variables. This results in a decomposition of the constraint set leading to two subproblems. The decomposition of the original problem induces special structure in the resulting subproblems and a series of these subproblems are then solved, using the generalised Benders' decomposition technique, to determine the optimal solution. Mathematical properties of the proposed approach are presented. Even though the proposed approach cannot guarantee the determination of the global optimum, computational experience on a number of nonconvex QP, NLP and MINLP example problems indicates that a global optimum solution can be obtained from various starting points.

F. G. Akhmadov. Computational method for solving the quadratic programming problem in $L_2^n\,(0,T)$ space. *Izvestiya Akademii Nauk Azerbaidzhanskoi SSR, Seriya Fiziko–Tekhnicheskikh i Matematicheskikh Nauk*, **4**(4), 102–106, 1983.

Abstract. A method for solving the quadratic programming problem in the space of all vector-functions, each component square of which is integrable, is given. It is supposed that the matrix of the quadratic form is positively defined.

F. A. Al-Khayyal. Linear, quadratic and bilinear programming approaches to the linear complementarity problems. *European Journal of Operations Research*, **24**, 216–227, 1987.

F. A. Al-Khayyal. Jointly constrained bilinear programs and related problems: An overview. *Computers in Mathematical Applications*, **19**, 53–62, 1990.

F. A. Al-Khayyal and J. E. Falk. Jointly constrained biconvex programming. *Mathematics of Operations Research*, **8**, 273–286, 1983.

C. Alessandri and A. Tralli. Frictionless contact with BEM using quadratic programming—discussion. *Journal of Engineering Mechanics-ASCE*, **119**(12), 2538–2540, 1993.

B. Alidaee, G. A. Kochenberger, and A. Ahmadian. 0–1 quadratic programming approach for optimum solutions of two scheduling problems. *International Journal of Systems Science*, **25**(2), 401–408, 1994.

Abstract. Two scheduling problems are considered: (1) scheduling n jobs non-preemptively on a single machine to minimize total weighted earliness and tardiness (WET); (2) scheduling n jobs non-preemptively on two parallel identical processors to minimize weighted mean flow time. In the second problem, a pre-ordering of the jobs is assumed that must be satisfied for any set of jobs scheduled on each specific machine. Both problems are known to be NP-complete. A 0–1 quadratic assignment formulation of the problems is presented. An equivalent 0–1 mixed integer linear programming approach for the problems are considered and a numerical example is given. The formulations presented enable one to use optimal and heuristic available algorithms of 0–1 quadratic assignment for the problems considered here.

K. S. Alsultan and K. G. Murty. Exterior point algorithms for nearest points and convex quadratic programs. *Mathematical Programming*, **57**(2), 145–161, 1992.

A. Altman. QHOPDM—a higher-order primal-dual method for large-scale convex quadratic programming. *European Journal of Operational Research*, **87**(1), 200–202, 1995.

A. Altman. Higher order primal-dual interior point method for separable convex quadratic optimization. *Control and Cybernetics*, **25**(4), 761–772, 1996.

A. Altman and J. Gondzio. Regularized symmetric indefinite systems in interior point methods for linear and quadratic optimization. Logilab Technical Report 1998.6, Department of Management Sciences, University of Geneva, Geneva, Switzerland, 1998.

Abstract. This paper presents linear algebra techniques used in the implementation of an interior point method for solving linear programs and convex quadratic programs with linear constraints. New regularization techniques for Newton systems applicable to both symmetric positive definite and symmetric indefinite systems are described. They transform the latter to quasidefinite systems known to be strongly factorizable to a form of Cholesky-like factorization. Two different regularization techniques, primal and dual, are very well suited to the (infeasible) primal-dual interior point algorithm. This particular algorithm, with an extension of multiple centrality correctors, is implemented in our solver HOPDM. Computational results are given to illustrate the potential advantages of the approach when applied to the solution of very large linear and convex quadratic programs.

H. Amato and G. Mensch. Rank restrictions on the quadratic form in indefinite quadratic programming. *Unternehmensforschung*, **15**(3), 214–216, 1971.

Abstract. A quadratic programming problem, where $q(x) = a^T x + x^T Qx$ is an indefinite objective function, can be solved with Swarup's approach to optimizing $(c^T x + \alpha)(d^T x + \beta)$ only if the rank of Q is two; if Q is definite, the rank of Q must be one.

D. E. Amos and M. L. Slater. Polynomial and spline approximation by quadratic programming. *Communications of the ACM*, **12**(7), 379–381, 1969. See also, *Collected Algorithms from ACM*, 1985.

Abstract. The problem of approximation to a given function, or of fitting a given set of data, where the approximating function is required to have certain of its derivatives of specified sign over the whole range of approximation, is studied. Two approaches are presented, in each of which quadratic programming is used to provide both the constraints on the derivatives and the selection of the function which yields the best fit. The first is a modified Bernstein polynomial scheme, and the second is a spline fit.

P. Anand. Decomposition principle for indefinite quadratic programe. *Trabajos de Estadistica y de Investigacion*, **23**, 61–71, 1972.

S. C. Anand, F. E. Weisgerber, and H. S. Hwei. Direct solution vs quadratic programming technique in elastic-plastic finite element analysis. *Computers and Structures*, **7**(2), 221–228, 1977.

Abstract. Elastic-plastic plane stress finite element analysis of a disk rolling on a rigid track is performed by the direct method as well as the quadratic programming technique. Tresca and von Mises' yield conditions are used in the former whereas an approximate piecewise linear Tresca yield condition is used in the later case. It is concluded from a comparison of the computer times needed in the two cases that the direct method is far superior to the quadratic programming technique.

A. Anckonie. A quadratic program for determining efficient frontier portfolio compositions using the SAS language. *In* 'SUGI 10—Proceedings of the Tenth Annual SAS Users Group International Conference', Vol. 15, pp. 55–60, 1985.

W. Anfiloff. Gravity interpretation with the aid of quadratic programming. *Geophysics*, **46**(3), 340–341, 1981.

P. L. De Angelis, P. M. Pardalos, and G. Toraldo. Quadratic programming with box constraints. *In* I. M. Bomze, ed., 'Developments in Global optimization', pp. 73–93, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1997.

K. M. Anstreicher. On long step path following and SUMT for linear and quadratic programming. *SIAM Journal on Optimization*, **6**(1), 33–46, 1996.

Abstract. We consider a long step barrier algorithm for the minimization of a convex quadratic objective subject to linear inequality constraints. The algorithm is a dual version of a method developed by K. M. Anstreicher et al. (1993), and requires $O(nL)$ or $O(sqrtnL)$ iterations to solve a problem with n constraints, depending on how the barrier parameter is reduced. As a corollary of our analysis we demonstrate that the classical SUMT algorithm, exactly as implemented in 1968, solves linear and quadratic programs in $O(\sqrt{n}LlogL)$ iterations, with proper initialization and choice of parameters.

K. M. Anstreicher. On the equivalence of convex programming bounds for boolean quadratic programming. Working paper, Department of Management Science, University of Iowa, Iowa City, USA, 1997.

Abstract. Recent papers have shown the equivalence of several tractable bounds for Boolean quadratic programming. In this note we give simplified proofs for these results, and also show that all of the bounds considered are simultaneously attained by one diagonal perturbation of the quadratic form.

K. M. Anstreicher and N. W. Brixius. Solving quadratic assignment problems using convex quadratic programming relaxations. Working paper, Department of Management Science, University of Iowa, Iowa City, USA, 2000.

Abstract. We describe a branch-and-bound algorithm for the quadratic assignment problem (QAP) that uses a convex quadratic programming (QP) relaxation to obtain a bound at each node. The QP subproblems are approximately solved using the Frank-Wolfe algorithm, which in this case requires the solution of a linear assignment problem on each iteration. Our branching strategy makes extensive use of dual information associated with the QP subproblems. We obtain state-of-the-art computational results on large benchmark QAPs.

K. M. Anstreicher and N. W. Brixius. A new bound for the quadratic assignment problem based on convex quadratic programming. *Mathematical Programming*, **89**(3), 341–357, 2001.

Abstract. We describe a new convex quadratic programming bound for the quadratic assignment problem (QAP). The construction of the bound uses a semidefinite programming representation of a basic eigenvalue bound for QAP. The new bound dominates the well-known projected eigenvalue bound, and appears to be competitive with existing bounds in the trade-off between bound quality and computational effort.

K. M. Anstreicher, D. den Hertog, C. Roos, and T. Terlaky. A long-step barrier method for convex quadratic programming. *Algorithmica*, **10**(5), 365–382, 1993.

Abstract. In this paper we propose a long-step logarithmic barrier function method for convex quadratic programming with linear equality constraints. After a reduction of the barrier parameter, a series of long steps along projected Newton directions are taken until the iterate is in the vicinity of the center associated with the current value of the barrier parameter. We prove that the total number of iterations is $O(\sqrt{n}L)$ or $O(nL)$, depending on how the barrier parameter is updated.

K. Aoki and T. Fujikawa. VAR planning and nonconvex quadratic programming. *Transactions of the Institute of Electrical Engineers of Japan*, **100**(3), 78–88, 1980.

Abstract. So far, many methods based on linear programming, nonlinear programming and integer programming have been proposed for var planning. In the linear programming method, the relationship between voltage, active power and reactive power is represented by linear equations. The nonlinear programming method, in which this relationship is expressed exactly, is not suited for a large-scale power system. Both these methods neglect discrete variables representing the number of the condenser or reactor units and the transformer tap positions. The integer programming method, which enables handling these discrete variables, is of course not suited for a large-scale power system. The authors show that the condenser planning problem is formulated into a parametric convex quadratic programming and the reactor planning problem is formulated into a parametric nonconvex quadratic programming.

K. Aoki and M. Kanezashi. A decomposition algorithm for a dual angular type quadratic programming. *In* A. Lew, ed., 'Proceedings of the 6th Hawaii International Conference on Systems Sciences. Western Periodicals, North Hollywood, CA, USA', pp. 358–360, 1973.

Abstract. This paper deals with a decomposition procedure for the problem whose objective function is linear for a coupling variable. Moreover it is described by taking advantage of such characteristics that one can easily obtain the variation of a coupling variable by computing linear forms.

K. Aoki and T. Satoh. Economic dispatch with network security constraints using parametric quadratic programming. *IEEE Transactions on Power Apparatus and Systems*, **PAS-101**(12), 4548–4556, 1982.

Abstract. This paper presents an efficient method to solve an economic load dispatch problem with DC load flow type network security constraints. The conventional linear programming and quadratic programming methods cannot deal with transmission losses as a quadratic form of generator outputs. In order to overcome this defect, the extension of the quadratic programming method is proposed, which is designated as the parametric quadratic programming method. The upper bounding technique and the relaxation method are coupled with the proposed method for the purpose of computational efficiency. The test results show that the proposed method is practical for real-time applications.

M. Arioli. The use of QR factorization in sparse quadratic programming and backward error issues. *SIAM Journal on Matrix Analysis and Applications*, **21**(3), 825–839, 2000.

Abstract. We present a roundoff error analysis of a null space method for solving quadratic programming minimization problems. This method combines the use of a direct QR factorization of the constraints with an iterative solver on the corresponding null space. Numerical experiments are presented which give evidence of the good performances of the algorithm on sparse matrices.

B. Armstrong and B. A. Wade. Nonlinear pid control with partial state knowledge: design by quadratic programming. *In* 'Proceedings of the 2000 American Control Conference, Danvers, MA, USA', Vol. 2, pp. 774–778, 2000.

Abstract. Nonlinear PID (NPID) control is implemented by allowing the controller gains to vary as a function of system state. NPID controllers will in general depend on knowledge of the full state vector. In this work, NPID controllers which operate without knowledge of some state variables are demonstrated. A general but conservative design method is presented with an experimental demonstration. For a special case, complete necessary and sufficient conditions are established.

D. A. Arsamastsev, P. I. Bartolomey, and S. K. Okulovski. New improved quadratic-programming methods for super-large power-systems analysis. *In* 'Proceedings of the Eighth Power Systems Computation Conference', Vol. 178, pp. 710–716, 1984.

L. Arseneau and M. J. Best. Resolution of degenerate critical parameter values in parametric quadratic programming. Technical Report CORR 99-47, Department of Combinatorics and Optimization, University of Waterloo, Ontario, Canada, 1999.

J. Atkociunas. Quadratic programming for degenerate shakedown problems of bar structures. *Mechanics Research Communications*, **23**(2), 195–203, 1996.

A. M. Aurela and J. J. Torsti. A quadratic programming method for stabilized solution of unstable linear systems. *Annales Universitatis Turkuensis, Ser AI (Astronomica Chemica Physica Mathematica)*, **123**, 1968.

Abstract. An improved version is presented of the quadratic programming method introduced in 1967 for stabilized solution of unstable systems of linear equations. The most probable solution and its confidence limits are discussed. In the present work, the method proper was examined more systematically by using the second artificial example of Phillips (1962), in which the correct result was known. The dependence of the computing time t on the number of variables adjusted simultaneously, l, was studied, taking a total of $m = 15$ variables, subject to the non negativity constraint. The optimum was achieved with $l = 3$ to 6 (final precision $\varepsilon = 1.10^5$, $t \simeq 3$ min in the IBM 1130). The different solutions exhibited marked consistency with each other, indicating the accuracy and reliability of the method

G. Auxenfants, L. Barthe, and P. Gibert. Architecture for scientific software. II. Analysis of a quadratic programming algorithm. *Recherche Aerospatiale*, **4**, 247–255, 1982.

Abstract. Presents a quadratic programming algorithm with linear constraints, working in the case of large-scale optimization problems. The number of variables is reduced by a partial dualization of constraint relations. It enables one to determine whether or not the admissible set is empty. The programming has been implemented on a CYBER 170/750 using a method of architecture based on (1) data centralization and (2) management of information exchange between processors by database management system. This algorithm represents one of the elements of a more optimization code.

T. Aykin. On a quadratic integer-program for the location of interacting hub facilities. *European Journal of Operational Research*, **46**(3), 409–411, 1990.

A. Bachem and B. Korte. An algorithm for quadratic optimization over transportation polytopes. *Zeitschrift für Angewandte Mathematik und Mechanik*, **58**, 459–461, 1978.

R. Bacher and H. P. van Meeteren. Security dispatch based on coupling of linear and quadratic programming techniques. *Power Systems, Modelling and Control Applications. IFAC Symposium Pergamon, Oxford, England*, pp. 211–217, 1989.

Abstract. Security dispatch can be defined as the real-time closed loop cost-optimal allocation of active generator output while considering branch flow limits of the intact network and lower and upper generation limits. Most OPF algorithms fail to guarantee accuracy. reliability and speed at the same time and can thus not be used in real-time closed loop application. Accuracy, reliability and speed can be obtained by executing a LP based OPF and a QP based constrained economic dispatch at different execution frequencies. The QP based algorithm uses the critical constraint set as determined by the LP based algorithm. Constrained economic dispatch can substitute the classical economic dispatch and will provide a secure dispatch.

W. E. Baethgen, D. B. Taylor, and M. M. Alley. Quadratic-programming method for determining optimum nitrogen rates for winter-wheat during tillering. *Agronomy Journal*, **81**(4), 557–559, 1989.

A. Bagchi and B. Kalantari. A method for computing approximate solution of the trust region problem with application to projective methods for quadratic programming. Working paper, Department of Computer Science, Rutgers University, New Brunswick, New Jersey, USA, 1988.

J. R. Baker. Determination of an optimal forecast model for ambulance demand using goal and quadratic programming. *In* 'Proceedings—Southeastern Chapter of the Institute of Management Sciences Twentieth Annual Meeting', Vol. 6, pp. 154–157, 1984.

E. Balas. Duality in discrete programming: II. The quadratic case. *Management Science*, **16**, 14–32, 1969.

E. Balas. Nonconvex quadratic programming via generalized polars. *SIAM Journal on Applied Mathematics*, **28**(2), 335–349, 1975.

Abstract. A new approach is proposed to linearly constrained nonconvex quadratic programming. The approach is based on generalized polar sets, and is akin to the convex analysis approach to integer programming. The author constructs a generalized polar of the Kuhn-Tucker polyhedron associated with a quadratic program. This generalized polar is a convex polyhedral cone whose interior contains no complementary feasible solution better than the best known one. An algorithm is then proposed, which does not use cutting planes, but constructs a polytope containing the feasible set and contained in the polar of the latter. The best complementary solution found in the process is optimal, or none exists.

C. C. Baniotopoulos, K. M. Abdalla, and P. D. Panagiotopoulos. A variational inequality and quadratic programming approach to the separation problem of steel bolted brackets. *Computers and Structures*, **53**(4), 983–991, 1994.

Abstract. A variational inequality and quadratic programming approach is proposed for the investigation of the separation problem of steel bolted brackets. By applying the classic unilateral contact law to describe the separation process along the contact surfaces between the bracket and the column flange, the continuous problem is formulated as a variational inequality or as a quadratic programming problem. By applying an appropriate finite element discretization scheme, the discrete problem is formulated as a quadratic optimization problem with inequality constraints which, in turn, can be effectively treated numerically by means of an appropriate quadratic optimization algorithm. The applicability and the effectiveness of the method is illustrated by means of a numerical application.

C. C. Baniotopoulos and K. M. Abdalla. Steel column-to-column connections under combined load—a quadratic-programming approach. *Computers and Structures*, **46**(1), 13–20, 1993.

Abstract. The aim of this paper is the investigation of the mechanical behaviour of bolted steel column-to-column connections under moment and axial loads by means of a method that takes into account the possibility of the appearance of detachment phenomena between the splice plates. As is well known, regions of detachment (called nonactive contact regions below), due to the appearance of the prying-action phenomenon, do appear on the adjacent fronts of such steel splice plates, greatly affecting the mechanical response of steel connections of this type. The significance of the problem under investigation arises from the fact that column-to-column splices are extensively applied in any possible combination to the design and construction of steel structures. It is therefore obvious that, since local failure phenomena on such connections due to undesirable-and not a priori defined-detachment between the splice plates (as consequence of the development of the prying- action phenomenon) may cause a total destruction of the whole steel structure. For this reason, it is important for such a behaviour to be accurately predicted and the previously mentioned nonactive contact regions on the splice plates to be defined. In this sense, such an investigation leads to an amelioration of the design principles for bolted steel column- to-column splices and to a refinement of the respective steel construction standards.

B. Bank and R. Hansel. Stability of mixed-integer quadratic-programming problems. *Mathematical Programming Studies*, **21**, 1–17, 1982.

F. Barahona, M. Junger, and G. Reinelt. Experiments in quadratic 0–1 programming. *Mathematical Programming*, **44**(2), 127–137, 1989.

E. W. Barankin and R. Dorfman. Toward quadratic programming. Report to the logistics branch, Office of Naval Research, 1955.

E. W. Barankin and R. Dorfman. A method for quadratic programming. *Econometrica*, **24**, 1956.

E. W. Barankin and R. Dorfman. On quadratic programming. *University of California Publications in Statistics*, **2**(13), 285–318, 1958.

H. J. C. Barbosa, F. M. P. Raupp, and C. C. H. Borges. Numerical experiments with algorithms for bound constrained quadratic programming in mechanics. *Computers and Structures*, **64**(1–4), 579–594, 1997.

Abstract. In this work, the computational performance of some algorithms for solving bound constrained quadratic programming problems is compared by means of numerical experiments. The model problems used to test the behaviour of the algorithms considered were the obstacle problem for a membrane and the contact problem in infinitesimal elasticity. Both problems involved different load conditions and parameters. The finite element method was used for the spatial discretization process.

J. L. Barlow and G. Toraldo. The effect of diagonal scaling on projected gradient methods for bound constrained quadratic programming problems. *Optimization Methods and Software*, **5**(3), 235–245, 1995.

R. O. Barr. An efficient computational procedure for a generalized quadratic programming problem. *SIAM Journal on Control*, **7**(3), 415–429, 1999.

R. H. Bartels, G. H. Golub, and M. A. Saunders. Numerical techniques in mathematical programming. *In* J. B. Rosen, O. L. Mangasarian and K. Ritter, eds, 'Nonlinear Programming', pp. 123–176. Academic Press, London, England, 1970.

G. Bashein and M. Enns. Computation of optimal controls by a method combining quasi-linearization and quadratic programming. *International Journal of Control*, **16**(1), 177–187, 1972.

Abstract. Quadratic programming (QP) has previously been applied to the computation of the optimal controls for linear systems with quadratic cost criteria. This paper extends the application of QP to nonlinear problems through quasi-linearization and the solution of a sequence of linear-quadratic sub-problems whose solutions converge to the solution of the original non-linear problem. The method is called quasi-linearization- quadratic programming or Q-QP.

E. M. L. Beale. On quadratic programming. *Naval Research Logistics Quarterly*, **6**(3), 227–243, 1959.

E. M. L. Beale. The use of quadratic programming in stochastic linear programming. Technical Report P-2404-1, The RAND Corporation, Santa Monica, CA, USA, 1961.

E. M. L. Beale. Note on 'a comparison of two methods in quadratic programmng. *Operations Research*, **14**, 442–443, 1966.

E. M. L. Beale. An introduction to Beale's method of quadratic programming. *In* J. Abadie, ed., 'Nonlinear programming', pp. 143–153, North Holland, Amsterdam, the Netherlands, 1967.

E. M. L. Beale and R. Benveniste. Quadratic programming. *In* 'Design and Implementation of Optimization Software', pp. 249–258. Sijthoff and Noordhoff, Alphen aan den Rijn, Netherlands, 1978.

Abstract. Following a general introduction to the theory of quadratic programming, the paper describes computational aspects of a new algorithm for convex quadratic programming. An essential feature is that the only information needed about the objective function is the gradient direction at successive trial solutions (and the value of the objective function at the final solution). The constraints are handled as in the Reduced Gradient Method. The method is essentially a generalization of the method of Conjugate Gradients. But pure Conjugate Gradients, although finite, require a complete restart whenever the set of active constraints changes. If storage space is available, the algorithm stores additional directions in a way that avoids the need for a complete restart.

J. E. Beasley. Heuristic algorithms for the unconstrained binary quadratic programming problem. Technical report, Department of Mathematics, Imperial College of Science and Technology, London, England, 1998.

Abstract. In this paper we consider the unconstrained binary quadratic programming problem. This is the problem of maximising a quadratic objective by suitable choice of binary (zero-one) variables. We present two heuristic algorithms based upon tabu search and simulated annealing for this problem. Computational results are presented for a number of publically available data sets involving up to 2500 variables. An interesting feature of our results is that whilst for most problems tabu search dominates simulated annealing for the very largest problems we consider the converse is true. This paper typifies a ”multiple solution technique, single paper” approach, i.e. an approach that within the same paper presents results for a number of different heuristics applied to the same problem. Issues relating to algorithmic design for such papers are discussed.

C. R. Bector. Indefinite quadratic programming with standard errors in objective. *Cahiers du Centre d'Etudes de Recherche Opérationalle*, **10**, 247–253, 1968.

C. R. Bector and M. Dahl. Simplex type finite iterative technique and duality for a special type of pseudo-concave quadratic program,. *Cahiers du Centre d'Etudes de Recherche Opérationalle*, **16**, 207–222, 1974.

L. Behjat and A. Vannelli. VLSI concentric partitioning using interior point quadratic programming. *In* 'ISCAS'99. Proceedings of the 1999 IEEE International Symposium on Circuits and Systems VLSI. IEEE, Piscataway, NJ, USA', Vol. 6, pp. 93–96, 1999.

Abstract. This paper presents a novel approach for solving the standard cell placement problem. A relaxed quadratic formulation of the problem is solved iteratively incorporating techniques to increase the spreading of cells, including introducing attractors and dynamic first moment constraints. At each iteration, a percentage of the cells that are close to the boundary of the chip are fixed. This procedure is done recursively until at least eighty percent of the cells are fixed. Numerical simulation of the proposed approach is presented for test systems.

L. Y. Belousov. Quadratic programming in problems of optimal planning of trajectory measurements. *Cosmic Research*, **9**(6), 750–759, 1971.

Abstract. The problem of optimal planning of trajectory measurements of two different measured parameters is investigated for the case of a limited dispersion, an arbitrary correlation coupling of the measurement errors of each parameter separately, and absence of correlation between measurements of different parameters. It is shown that the problem posed can be reduced to solving a problem in quadratic programming based on the linear-programming method generalized for the continuous case. In conclusion, the problem is stated by induction for an arbitrary number of independent measured parameters.

T. Belytschko. Discussion of elastic-plastic analysis by quadratic programming. *American Society of Civil Engineering, Journal of the Engineering Mechanics Division*, **100**, 130–131, 1974.

A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos. The explicit solution of model predictive control via multiparametric quadratic programming. *In* 'Proceedings of the 2000 American Control Conference, Danvers, MA, USA', Vol. 2, pp. 872–876, 2000.

Abstract. The control based on online optimization, popularly known as model predictive control (MPC), has long been recognized as the winning alternative for constrained systems. The main limitation of MPC is, however, its online computational complexity. For discrete-time linear time-invariant systems with constraints on inputs and states, we develop an algorithm to determine explicitly the state feedback control law associated with MPC, and show that it is piecewise linear and continuous. The controller inherits all the stability and performance properties of MPC, but the online computation is reduced to a simple linear function evaluation instead of the expensive quadratic program. The new technique is expected to enlarge the scope of applicability of MPC to small-size/fast-sampling applications which cannot be covered satisfactorily with anti-windup schemes.

M. Ben Daya. Line search techniques for the logarithmic barrier function in quadratic programming. *Journal of the Operational Research Society*, **46**(3), 332–338, 1995.

Abstract. In this paper, we propose a line-search procedure for the logarithmic barrier function in the context of an interior point algorithm for convex quadratic programming. Preliminary testing shows that the proposed procedure is superior to some other line-search methods developed specifically for the logarithmic barrier function in the literature.

M. Ben Daya and K. S. Al Sultan. A new penalty function algorithm for convex quadratic programming. *European Journal of Operational Research*, **101**(1), 155–163, 1997.

Abstract. We develop an exterior point algorithm for convex quadratic programming using a penalty function approach. Each iteration in the algorithm consists of a single Newton step followed by a reduction in the value of the penalty parameter. The points generated by the algorithm follow an exterior path that we define. Convergence of the algorithm is established. The proposed algorithm was motivated by the work of Al-Sultan and Murty (1991) on nearest point problems, a special quadratic program. A preliminary implementation of the algorithm produced encouraging results. In particular, the algorithm requires a small and almost constant number of iterations to solve the small to medium size problems tested.

M. Ben Daya and C. M. Shetty. Polynomial barrier function algorithms for convex quadratic programming. *Arabian Journal for Science and Engineering*, **15**(4B), 656–670, 1990.

J. M. Bennett. Quadratic programming and piecewise linear networks with structural engineering applications,. *In* A. Prekopa, ed., 'Survey of Mathematical Programming, Vol. 3', pp. 95–105, North Holland, Amsterdam, the Netherlands, 1979.

P. Benson, R. L. Smith, I. E. Schochetman, and J. C. Bean. Optimal solution approximation for infinite positive-definite quadratic programming. *Journal of Optimization Theory and Applications*, **85**(2), 235–248, 1995.

Abstract. We consider a general doubly-infinite, positive-definite, quadratic programming problem. We show that the sequence of unique optimal solutions to the natural finite-dimensional subproblems strongly converges to the unique optimal solution. This offers the opportunity to arbitrarily well approximate the infinite-dimensional optimal solution by numerically solving a sufficiently large finite-dimensional version of the problem. We then apply our results to a general time-varying, infinite- horizon, positive-definite, LQ control problem.

R. Benveniste. A quadratic programming algorithm using conjugate search directions. *Mathematical Programming*, **16**(1), 63–80, 1979.

Abstract. A quadratic programming algorithm is presented, resembling Beale's (1955) quadratic programming algorithm and Wolfe's reduced gradient method. It uses conjugate search directions. The algorithm is conceived as being particularly appropriate for problems with a large Hessian matrix. An outline of the solution to the quadratic capacity-constrained transportation problem using the above method is also presented.

R. Benveniste. One way to solve the parametric quadratic programming problem. *Mathematical Programming*, **21**(2), 224–228, 1981.

Abstract. A method is presented for the solution of the parametric quadratic programming problem by the use of conjugate directions.

C. Bergthaller. A quadratic equivalent of the minimum risk problem. *Revue Roumaine do Mathematiques Pure et Appliquees*, **15**, 17–23, 1970.

C. Bergthaller. Parametric quadratic programming. *In* '4th conference on probability theory. Abstracts. Acad Socialist Republic of Rumania, Bucharest, Romania', pp. 23–24, 1971a.

Abstract. This paper deals with the parametric programming problem $\min c^T x + 1/2 x^T D x, A(\lambda)x = b, x \geq 0$, where: $c, x \in R^n$, $b \in R^m$, D is a symmetric nxn positive definite matrix, $A(\lambda) = A_o + \lambda A_l$, A_o and A_l are fixed $m \times n$ matrices, such that the rank of A_l is l and $\lambda \geq 0$ is real parameter. Some particular cases are: 1) One element of the matrix A is a linear function of lambda and all others are constant. 2) A column of mod A is a linear (vectorial) function of $\lambda(a_j = a_j^o + \lambda a_j^l)$ and the others are constant. 3) A row of A is a linear (vectorial) function of $\lambda(\alpha_j = \alpha_j^o + \lambda \alpha_i^l)$ and the others are constant.

C. Bergthaller. Quasi-convex quadratic programming. *Comptes Rendus Hebdomadaires des Seances de l'Academie des Sciences, Serie A (Science Mathematiques)*, **273**(15), 685–686, 1971b.

Abstract. A simplicial algorithm is given for the program $\min q^T x + 1/2 x^T Q x, Ax \leq b, x \geq 0$ where q is an n-dimensional vector and Q is a symmetrical matrix such that the objective function $f(x)$ identical to $q^T x + 1/2 x^T Q x$ is quasi-convex for $x \geq 0$ without being convex.

A. B. Berkelaar. Sensitivity analysis in (degenerate) quadratic programming. Econometric Institute Report 30, Econometric Institute, Erasmus University, Rotterdam, The Netherlands, 1997.

Abstract. In this paper we deal with sensitivity analysis in convex quadratic programming, without making assumptions on nondegeneracy, strict convexity of the objective function, and the existence of a strictly complementary solution. We show that the optimal value as a function of a right–hand side element (or an element of the linear part of the objective) is piecewise quadratic, where the pieces can be characterized by maximal complementary solutions and tripartitions. Further, we investigate differentiability of this function. A new algorithm to compute the optimal value function is proposed. Finally, we discuss the advantages of this approach when applied to mean–variance portfolio models.

A. B. Berkelaar, B. Jansen, C. Roos, and T. Terlaky. Sensitivity analysis in quadratic programming. Report 96-11, Econometric Institute, Erasmus University, Rotterdam, The Netherlands, 1996.

Abstract. In this paper we deal with sensitivity analysis in convex quadratic programming, without making assumptions on nondegeneracy, strict convexity of the objective function, and the existence of a strictly complementary solution. We show that the optimal value as a function of a right–hand side element (or an element of the linear part of the objective) is piecewise quadratic, where the pieces can be characterized by maximal complementary solutions and tripartitions. Further, we investigate differentiability of this function. A new algorithm to compute the optimal value function is proposed. Finally, we discuss the advantages of this approach when applied to mean–variance portfolio models.

A. B. Berkelaar, B. Jansen, K. Roos, and T. Terlaky. Basis- and partition identification for quadratic programming and linear complementarity problems. *Mathematical Programming*, **86**(2), 261–282, 1999.

Abstract. Optimal solutions of interior point algorithms for linear and quadratic programming and linear complementarity problems provide maximally complementary solutions. Maximally complementary solutions can be characterized by optimal partitions. On the other hand, the solutions provided by simplex-based pivot algorithms are given in terms of complementary bases. A basis identification algorithm is an algorithm which generates a complementary basis, starting from any complementary solution. A partition identification algorithm is an algorithm which generates a maximally complementary solution (and its corresponding partition), starting from any complementary solution. In linear programming such algorithms were respectively proposed by Megiddo in 1991 and Balinski and Tucker in 1969. In this paper we will present identification algorithms for quadratic programming and linear complementarity problems with sufficient matrices. The presented algorithms are based on the principal pivot transform and the orthogonality property of basis tableaus.

A. B. Berkelaar, K. Roos, and T. Terlaky. The optimal partition and optimal set approach to linear and quadratic programming. Econometric Institute Report 51, Econometric Institute, Erasmus University, Rotterdam, The Netherlands, 1997.

Abstract. In this chapter we describe the optimal set approach for sensitivity analysis for LP. We show that optimal partitions and optimal sets remain constant between two consecutive transition-points of the optimal value function. The advantage of using this approach instead of the classical approach (using optimal bases) is shown. Moreover, we present an algorithm to compute the partitions, optimal sets and the optimal value function. This is a new algorithm and uses primal and dual optimal solutions. We also extend some of the results to parametric quadratic programming, and discuss differences and resemblances with the linear programming case.

H. Bernau. Upper bound techniques for quadratic programming. *Alkalmazott Matematikai Lapok*, **3**(1–2), 161–170, 1977. See also, A. Prekopa, ed. *Survey of Mathematical Programming, Vol.1*, North-Holland, Amsterdam, pp. 347–356, 1979.

Abstract. An extension of the methods of Wolfe, Jagannathan and Beale is presented for quadratic programming problems with upper bounds for the variables. It is shown that the upper bounds technique for linear programming problems can be very easily incorporated in these methods.

H. Bernau. Quadratic programming problems and related linear complementarity problems. *Journal of Optimization Theory and Applications*, **65**(2), 209–222, 1990.

Abstract. Investigates the general quadratic programming problem, i.e. the problem of finding the minimum of a quadratic function subject to linear constraints. In the case where, over the set of feasible points, the objective function is bounded from below, this problem can be solved by the minimization of a linear function, subject to the solution set of a linear complementarity problem, representing the Kuhn-Tucker conditions of the quadratic problem. To detect in the quadratic problem the unboundedness from below of the objective function, necessary and sufficient conditions are derived. It is shown that, when these conditions are applied, the general quadratic programming problem becomes equivalent to the investigation of an appropriately formulated linear complementary problem.

O. Bertoldi, M. V. Cazzol, A. Garzillo, and M. Innorta. A dual quadratic programming algorithm oriented to the probabilistic analysis of large interconnected networks. *In* ‘PSCC. Proceedings of the Twelfth Power Systems Computation Conference. Power Syst. Comput. Conference, Zurich, Switzerland’, Vol. 2, pp. 1249–1255, 1996.

Abstract. A fast and robust innovative computing procedure has been developed aimed at allowing the use of optimal power flow techniques in the framework of the probabilistic adequacy assessment of large interconnected power systems. The paper describes the methodological approach and the relevant implemented algorithm. Several numerical results are supplied which demonstrate the high computing efficiency of the procedure so that it is suitable in the probabilistic simulation domain.

M. J. Best. Equivalence of some quadratic programming algorithms. *Mathematical Programming*, **30**(1), 71–87, 1984.

Abstract. The author formulates a general algorithm for the solution of a convex (but not strictly convex) quadratic programming problem. Conditions are given under which the iterates of the algorithm are uniquely determined. The quadratic programming algorithms of Fletcher (1971), Gill and Murray (1978), Best and Ritter (1976), and van de Panne and Whinston/Dantzig (1969) are shown to be special cases and consequently are equivalent in the sense that they construct identical sequences of points. The various methods are shown to differ only in the manner in which they solve the linear equations expressing the Kuhn-Tucker system for the associated equality constrained subproblems. Equivalence results have been established by Goldfarb (1972) and Djang (1979) for the positive definite Hessian case. The analysis extends these results to the positive semi-definite case.

M. J. Best. An algorithm for parametric quadratic programming. *In* H. Fischer, B. Riedüller and S. Schäffler, eds, ‘Applied Mathematics and Parallel Computing—Festschrift for Klaus Ritter’, pp. 57–76. Physica-Verlag, Heidelburg, 1996.

M. J. Best and R. J. Caron. A method to increase the computational efficiency of certain quadratic programming algorithms. *Mathematical Programming*, **25**(3), 354–358, 1983.

Abstract. Presents a method for computing the Kuhn-Tucker multipliers associated with equality constraints in quadratic programming problems. When applied to a certain class of algorithms a significant reduction in computation time and in storage is achieved.

M. J. Best and R. J. Caron. A parameterized Hessian quadratic programming problem. *Annals of Operations Research*, **5**(1–4), 373–394, 1986.

Abstract. Presents a general active set algorithm for the solution of a convex quadratic programming problem having a parametrized Hessian matrix. The parametric Hessian matrix is a positive semidefinite Hessian matrix plus a real parameter multiplying a symmetric matrix of rank one or two. The algorithm solves the problem for all parameter values in the open interval upon which the parametric Hessian is positive semidefinite. The algorithm is general in that any of several existing quadratic programming algorithms can be extended in a straightforward manner for the solution of the parametric Hessian problem.

M. J. Best and N. Chakravarti. Stability of linearly constrained convex quadratic programs. *Journal of Optimization Theory and Applications*, **64**(1), 43–53, 1990.

M. J. Best and N. Chakravarti. An $O(n^2)$ active set method for solving a certain parametric quadratic program. *Journal of Optimization Theory and Applications*, **72**(2), 213–224, 1992.

Abstract. The paper presents an $O(n^2)$ method for solving the parametric quadratic program $\min(1/2)x^T Dx - a^T x + (\lambda/2)(\sum_{j=1}^{n} \gamma_j x_j - c)^2$, having lower and upper bounds on the variables, for all non-negative values of the parameter lambda . Here, D is a positive diagonal matrix, a an arbitrary n-vector, each γ_j, $j = 1,...,n$, and c are arbitrary scalars. An application to economics is also presented.

M. J. Best and K. Ritter. An effective algorithm for quadratic minimization problems. Technical report 1691, University of Wisconsin, Madison, Wisconsin, USA, 1976.

M. J. Best and K. Ritter. A quadratic programming algorithm. *ZOR, Methods and Models of Operations Research*, **32**(5), 271–297, 1988.

Abstract. By using conjugate directions a method for solving convex quadratic programming problems is developed. The algorithm generates a sequence of feasible solutions and terminates after a finite number of iterations. Extensions of the algorithm for nonconvex and large structured quadratic programming problems are discussed.

D. Bhatia. Duality for quadratic programming in complex space. *Zeitschrift für Angewandte Mathematik und Mechanik*, **54**(1), 55–57, 1974.

Abstract. The duality theorems of Rami (1972) for symmetric dual quadratic programs are extended in complex space over arbitrary polyhedral cones.

S. Bhowmik, S. K. Goswami, and P. K. Bhattacherjee. Distribution system planning through combined heuristic and quadratic programming approach. *Electric Machines and Power Systems*, **28**(1), 87–103, 2000.

Abstract. The present paper reports a new technique for the planning of a radial distribution system. Distribution system planning has been formulated as a problem of quadratic mixed integer programming (QMIP). A two-stage iterative solution technique has been proposed where the first stage determines the optimum substation sites and the second stage determines the optimum network configurations. To reduce the dimensionality problem, the integer constraints are first related, thus converting the quadratic mixed integer programming problem into a quadratic programming (QP) problem. After the solution of the QP problem, integer constraints are imposed using heuristic techniques.

D. Bienstock. Computational study of a family of mixed-integer quadratic programming problems. *Mathematical Programming*, **74**(2), 121–140, 1996. See also, Integer Programming and Combinatorial Optimization. 4th International IPOC Conference, Proceedings (Balas, E. and Clausen, J., eds.), Springer-Verlag, Berlin, Germany, pages 90–94, 1995.

Abstract. We present computational experience with a branch-and-cut algorithm to solve quadratic programming problems where there is an upper bound on the number of positive variables. Such problems arise in financial applications. The algorithm solves the largest real-life problems in a few minutes of run-time.

A. Billionnet and A. Sutter. Persistency in quadratic 0–1 optimization. *Mathematical Programming*, **54**(1), 115–119, 1992.

Å. Björck. Constrained least-squares problems. *In* 'Numerical Methods for Least Squares Problems', chapter 5, pp. 187–213. SIAM, Philadelphia, USA, 1996.

E. Blum and W. Oettli. Direct proof of the existence theorem for quadratic programming. *Operations Research*, **20**(1), 165–167, 1972.

Abstract. A direct analytical proof is given for the following theorem: If the infimum of a quadratic function on a nonempty (possibly unbounded) polyhedral set R contained in $\mathbb{R}^n$ is finite, then the infimum is assumed somewhere on R, thus being a minimum.

P. T. Boggs, P. D. Domich, and J. E. Rogers. An interior point method for general large-scale quadratic programming problems. *Annals of Operations Research*, **62**, 419–437, 1996a.

Abstract. Presents an interior point algorithm for solving both convex and nonconvex quadratic programs. The method, which is an extension of the authors' interior point work on linear programming problems, efficiently solves a wide class of large-scale problems and forms the basis for a sequential quadratic programming (SQP) solver for general large scale nonlinear programs. The key to the algorithm is a three-dimensional cost improvement subproblem, which is solved at every iteration. The authors have developed an approximate recentering procedure and a novel, adaptive big-M Phase I procedure that are essential to the success of the algorithm. The authors describe the basic method along with the recentering and big-M Phase I procedures. Details of the implementation and computational results are also presented.

P. T. Boggs, P. D. Domich, J. E. Rogers, and C. Witzgall. An interior-point method for linear and quadratic programming problems. *COAL Newsletter*, **19**, 32–40, 1991.

P. T. Boggs, P. D. Domich, J. E. Rogers, and C. Witzgall. An interior point method for general large scale quadratic programming problems. *Annals of Operations Research*, **62**, 419–437, 1996b.

Abstract. In this paper we present an interior point algorithm for solving both convex and nonconvex quadratic programs. The method, which is an extension of our interior point work on linear programming problems, efficiently solves a wide class of large scale problems and forms the basis for a sequential quadratic programming (SQP) solver for general large scale nonlinear programs. The key to the algorithm is a 3-dimensional cost-improvement subproblem, which is solved at every iteration. We have developed an approximate recentering procedure and a novel, adaptive big-*M* Phase I procedure that are essential to the success. We describe the basic method along with the recentering and big-M Phase I procedures. Details of the implementation and computational results are also presented.

N. L. Boland. A dual-active-set algorithm for positive semidefinite quadratic programming. *In* ‘Optimization : Techniques and Applications’, pp. 80–89, 1992.

N. L. Boland. A dual-active-set algorithm for positive semi-definite quadratic programming. *Mathematical Programming*, **78**(1), 1–27, 1997.

Abstract. Because of the many important applications of quadratic programming, fast and efficient methods for solving quadratic programming problems are valued. Goldfarb and Idnani (1983) describe one such method, Well known to be efficient and numerically stable, the Goldfarb and Idnani method suffers only from the restriction that in its original form it cannot be applied to problems which are positive semi-definite rather than positive definite. In this paper, we present a generalization of the Goldfarb and Idnani method to the positive semi-definite case and prove finite termination of the generalized algorithm. In our generalization, we preserve the spirit of the Goldfarb and Idnani method, and extend their numerically stable implementation in a natural way.

I. M. Bomze and G. Danninger. A global optimization algorithm for concave quadratic programming problems. *SIAM Journal on Optimization*, **3**(4), 826–842, 1993.

J. F. Bonnans and M. Bouhtov. The trust region affine interior point algorithm for convex and nonconvex quadratic programming. *RAIRO-Recherche Opérationnelle—Operations Research*, **29**(2), 195–217, 1995.

Abstract. We study from a theoretical and numerical point of view an interior point algorithm for quadratic QP using a trust region idea, formulated by Ye and Tse (1989). We show that, under a nondegeneracy hypothesis, the algorithm converges globally in the convex case. For a nonconvex problem, under a mild additional hypothesis, the sequence of points converges to a stationary point. We obtain also an asymptotic linear convergence rate for the cost that depends only on the dimension of the problem. When we show that, provided some modifications are added to the basic algorithm, the method has a good numerical behaviour.

J. C. G. Boot. Notes on quadratic programming: The Kuhn-Tucker and Thiel-Van de Panne conditions, degeneracy and equality constraints. *Management Science*, **8**, 85–98, 1961.

J. C. G. Boot. Binding constraint procedures of quadratic programming. *Econometrica*, **31**, 464–498, 1963a.

J. C. G. Boot. On sensitivity analysis in convex quadratic programming problems. *Operations Research*, **11**, 771–786, 1963b.

J. C. G. Boot. *Quadratic Programming. Algorithms—Anomalies—Algorithms*. North Holland, Amsterdam, the Netherlands, 1964.

J. C. G. Boot and H. Theil. A procedure for integer maximization of a definite quadratic form. *In* G. Kreweras and G. Morlat, eds, 'Proceedings of the 3rd International Conference on Operations Research'. English Universities Press, 1964.

V. L. Borre and S. G. Kapoor. A multi-stage quadratic-programming optimization technique for optimal management of large hydro-power operations. *Engineering Optimization*, **8**(2), 103–118, 1985.

J. M. Borwein. Necessary and sufficient conditions for quadratic minimality. *Numerical Functional Analysis and Optimization*, **5**, 127–140, 1982.

R. J. Bosch, Y. Ye, and G. G. Woodworth. A convergent algorithm for quantile regression with smoothing splines. *Computational Statistics and Data Analysis*, **19**, 613–630, 1995.

Abstract. An important practical problem is that of determining a nonparametric estimate of the conditional quantile of y given x. If we balance fidelity to the data with a smoothness requirement, the resulting quantile function is a cubic smoothing spline. We reformulate this estimation procedure as a quadratic programming problem, with associated optimality conditions. A recently developed interior point algorithm with proven convergence is extended to solve the quadratic program. This solution characterizes the desired nonparametric conditional quantile function. These methods are illustrated in a study of audiologic performance following cochlear implants.

A. Bouzaher. Symmetric QP and linear programming under primal-dual uncertainty. *Operations Research Letters*, **6**(5), 221–225, 1987.

Abstract. A saddle-point formulation of linear programming problems with random objective function and RHS coefficients is proposed. Under a certainty equivalent criterion, a pair of primal-dual deterministic equivalents is derived. These problems are symmetric dual quadratic programs, and can be interpreted as generalizations of the classical mean-variance model.

J. Bouzitat. On Wolfe's method and Dantzig's method for convex quadratic programming. *RAIRO Recherche Operationelle*, **13**(2), 151–184, 1979.

Abstract. Both Wolfe's and Dantzig's methods solve linear-constrained convex quadratic programming problems by simplex-like algorithms. They use the Kuhn-Tucker conditions, which are necessary and sufficient for such problems. The author presents those two methods, with complete theoretical proofs the greater part of which, to the author's knowledge, is new. Wolfe's method receives here a complement and is then found to be more efficient than it previously appeared, on account of 'blocking' phenomena which are proved not to stop the convergent process of the completed algorithm. The 'shear form' of the method is consequently applicable to solve any nonparametric problem, and the 'long form' may be reserved for parametric problems only. The present proof of Dantzig's algorithm convergence is not based on the direct study of computation schemata, but uses the convexity of the quadratic function to be minimized, which leads to quite simple proof. The general presentation of Wolfe's and Dantzig's methods is illustrated by a numerical problem which is solved by both of them, so as to permit a comparison.

R. J. Braitsch. A computer comparison of four quadratic programming algorithms. *Management Science*, **18**(11), 632–643, 1972.

Abstract. This paper compares the computational performance of four quadratic programming algorithms. Problems are generated and solved on the computer with iteration count serving as the principal method of comparison. The effect of certain problem parameters on rate of convergence is considered and computer time and storage requirements of the four algorithms are discussed.

N. J. Breytenbach. A structured quadratic program. *In* J. A. Snyman, ed., 'Proceedings of the Tenth South African Symposium on Numerical Mathematics', p. 128, 1984.

J. F. Brotchie. A generalized design approach to solution of the nonconvex quadratic-programming problem. *Applied Mathematical Modelling*, **11**(4), 291–295, 1987.

B. M. Brown and C. R. Goodall. Applications of quadratic programming in statistics. Technical Report 93-09, Department of Statistsics, The Pennsylvania State University, USA, 1993.

K. S. Brown and K. A. Reiman. An ALGOL program for quadratic programming using the method of Wolfe and Frank. *Bulletin of the Operations Research Society of America*, **23**, B374, 1975.

Abstract. To meet the needs of making available computer programs, this paper presents an ALGOL program for applying the method of Wolfe and Frank to properly structured quadratic programming problems. The procedure is then demonstrated by solving a representative sample of structured problems. Where feasible, problems are formulated in such a way that relevance to real problems is readily noted.

C. G. Broyden and N. F. Attia. Penalty functions, Newton's method, and quadratic programming. *Journal of Optimization Theory And Applications*, **58**(3), 377–385, 1988.

J. R. Bunch and L. Kaufman. A computational method for the indefinite quadratic programming problem. *Linear Algebra and Its Applications*, **34**, 341–370, 1980.

Abstract. Presents an algorithm for the quadratic programming problem of determining a local minimum of $f(x) = 1/2x^T Qx + c^T x$ such that $A^T x \geq b$ where Q is a symmetric matrix which may not be positive definite.

C. A. Burdet. General quadratic programming. Technical report, Carnegie Mellon Univ , Pittsburgh, PA, USA, 1971.

Abstract. An algorithm is presented for the general (not necessarily convex or concave) quadratic programming problem over a linearly constrained set. The algorithm is finitely convergent and makes use of a convex quadratic programming method as a subroutine (like the quadratic simplex for instance). The basic tool for this method is a facial decomposition for polyhedral sets.

S. J. Byrne. Solution of quadratic programming problems. *New Zealand Operational Research*, **12**(2), 73–89, 1984.

Abstract. Quadratic programming problems arise in a number of situations. Typical examples, e.g. portfolio selection, economic modelling, regression analysis, and solution of non-linear programming problems, are briefly described. The methods that have been proposed for solving this problem are reviewed. The approach using a linear complementarity program is selected, and an efficient, numerically well-behaved procedure is developed, based on Lemke's algorithm and using orthogonal factorizations. A principal pivot version is also presented, which parallels Dantzig and Cottle's solution procedure, but is applicable to the same very wide class of matrices processed by Lemke's algorithm. A restart facility is developed for this version, which accelerates the solution of a sequence of related quadratic programs, by proceeding from the optimum of the last problem. The methods have been coded in Fortran IV and perform well. The process is easily extended to handle both upper and lower bounds on constraints and has proved acceptable for use in a general non-linear programming algorithm.

R. Caballero and A. Santos. A new dual method for solving strictly convex quadratic programs. Technical report, Department of Applied Economics (Mathematics), University of Malaga, Spain, 1998.

A. V. Cabot and R. L. Francis. Solving nonconvex quadratic minimization problems by ranking the extreme points. *Operations Research*, **18**, 82–86, 1970.

L. M. Cabral. An efficient algorithm for the quadratic programming problem with inequality constraints. *In* ‘Proceedings of the 1982 American Control Conference. IEEE, New York, NY, USA’, Vol. 3, pp. 1016–1017, 1982.

Abstract. The solution to the general linear problem $Ax = y$ with side conditions can be interpreted as a quadratic optimization problem. Such a requirement arises in the solution of illposed problems where the method of regularization is exploited. A compact quadratic optimization procedure is presented which obviates the requirement to invert large matrices, encountered in typical applications, and thereby reduce computation time and errors due to numerical round-off.

P. H. Calamai and L. N. Vicente. ALGORITHM 728: Fortran subroutines for generating quadratic bilevel programming test problems. *ACM Transactions on Mathematical Software*, **20**(1), 120–123, 1994a.

P. H. Calamai and L. N. Vicente. Generating quadratic bilevel programming test problems. *ACM Transactions on Mathematical Software*, **20**(1), 103–119, 1994b.

P. H. Calamai, J. J. Júdice, and L. N. Vicente. Generation of disjointly constrained bilinear programming test problems. *Computational Optimization and Applications*, **1**, 299–306, 1992.

P. H. Calamai, L. N. Vicente, and J. J. Júdice. A new technique for generating quadratic-programming test problems. *Mathematical Programming*, **61**(2), 215–231, 1993.

Abstract. This paper describes a new technique for generating convex, strictly concave and indefinite (bilinear or not) quadratic programming problems. These problems have a number of properties that make them useful for test purposes. For example, strictly concave quadratic problems with their global maximum in the interior of the feasible domain and with an exponential number of local minima with distinct function values and indefinite and jointly constrained bilinear problems with nonextreme global minima, can be generated. Unlike most existing methods our construction technique does not require the solution of any subproblems or systems of equations. In addition, the authors know of no other technique for generating jointly constrained bilinear programming problems.

A. J. Calise. Statistical design of sampled-data control systems via quadratic programming. Technical report, Univ Pennsylvania, PA, USA, 1968.

Abstract. Quadratic programming is applied to the statistical design of Linear sampled-data control systems. Given a fixed plant, a compensator is chosen which minimizes the mean-square value of an error sequence subject to a set of constraints. The system inputs are described in terms of sampled values from autocorrelation and cross-correlation functions, eliminating the need for the analytical expressions required in the Wiener-Hopf equation. Constraints which characterize the closed-loop response to deterministic inputs and constraints which limit the complexity of the compensating net-work can be simultaneously employed. The solution generates the parameters of the optimum compensator. The case where the error signal is not sampled and where the mean-square value of the error is the index of the performance is also considered.

E. K. Can. A quadratic-programming solution to cost-time trade-off for CPM. *In* ‘Applied Simulation and Modelling—ASM ’85’, Vol. 3, pp. 253–256, 1985.

W. Candler and R. J. Townsley. The maximization of a quadratic function of variables subject to linear inequalities. *Management Science*, **10**, 515–523, 1964.

E. Canestrelli, S. Giove, and R. Fuller. Stability in possibilistic quadratic programming. *Fuzzy Sets and Systems*, **82**(1), 51–56, 1996.

Abstract. We show that possibilistic quadratic programs with crisp decision variables and continuous fuzzy number coefficients are well-posed, i.e. small changes in the membership function of the coefficients may cause only a small deviation in the possibility distribution of the objective function.

J. Canisius and J. L. van Hemmen. A polynomial time algorithm in general quadratic programming and ground-state properties of spin glasses. *Europhysics Letters*, **1**(7), 319–326, 1986.

Abstract. An algorithm is presented which finds a surprisingly good approximation to the global minimum of a not necessarily convex, quadratic function in N variables, restricted to a N-dimensional cube. For instance, all Ising Hamiltonians with pair interactions belong to this class. In general, the time complexity of the algorithm is $O(N^4)$. For nearest-neighbour interactions, it reduces to $O(N^3)$. The algorithm is used to find the ground state of various Ising spin glass models and to study the zero-temperature behavior of the magnetization as a function of the external field h in both and three dimensions. It is found that the two-dimensional +or-J model has a nonzero magnetization as $h \leftarrow 0$.

M. D. Canon and J. H. Eaton. A new algorithm for a class of quadratic programming problems with application to control. *SIAM Journal on Control*, **4**, 34–45, 1996.

J. M. Cao. Necessary and sufficient condition for local minima of a class of nonconvex quadratic programs. *Mathematical Programming*, **69**(3), 403–411, 1995.

M. Capurso. A quadratic programming approach to the impulsive loading analysis of rigid plastic structures. *Meccanica*, **7**(1), 45–57, 1972.

Abstract. This paper discusses the dynamic problem of rigid plastic structures subjected to impulsive loading. A couple of 'dual' extremum theorems reduces the problem to the optimization of convex quadratic functions subject to linear equalities and equations: the first theorem takes as variables stress and accelerations, the second accelerations and plastic multiplier rates. The problem is discussed in matrix notation on the basis of finite element discretization of the structure and piecewise linear approximation of the yield surfaces, using some quadratic programming concepts. The procedure is illustrated by a simple numerical example.

R. J. Caron. Parametric quadratic programming. Windsor Mathematics Report 86-01, Department of Mathematics, University of Windsor, Ontario, Canada, 1986.

T. J. Carpenter and D. F. Shanno. An interior point method for quadratic programs based on conjugate projected gradients. *Computational Optimization and Applications*, **2**(1), 65–28, 1993.

Abstract. We propose an interior point method for large-scale convex quadratic programming where no assumptions are made about the sparsity structure of the quadratic coefficient matrix Q. The interior point method described is a doubly iterative algorithm that invokes a conjugate projected gradient procedure to obtain the search direction. The effect is that Q appears in a conjugate direction routine rather than in a matrix factorization. By doing this, the matrices to be factored have the same nonzero structure as those in linear programming. Further, one variant of this method is theoretically convergent with only one matrix factorization throughout the procedure.

T. J. Carpenter, I. J. Lustig, J. M. Mulvey, and D. F. Shanno. Higher-order predictor-corrector interior point methods with application to quadratic objectives. *SIAM Journal on Optimization*, **3**(4), 696–725, 1993a.

T. J. Carpenter, I. J. Lustig, J. M. Mulvey, and D. F. Shanno. Separable quadratic programming via a primal-dual interior point method and its use in a sequential procedure. *ORSA Journal on Computing*, **5**(2), 182–191, 1993b.

Abstract. Extends a primal-dual interior point procedure for linear programs to the case of convex separable quadratic objectives. Included are efficient procedures for: attaining primal and dual feasibility, variable upper bounding, and free variables. A sequential procedure that invokes the quadratic solver is proposed and implemented for solving linearly constrained convex separable nonlinear programs. Computational results are provided for several large test cases from stochastic programming. The proposed methods compare favorably with MINOS, especially for the larger examples. The nonlinear programs range in size up to 8700 constraints and 22000 variables.

J. L. Carpentier, G. Cotto, and P. L. Niederlander. New concepts for automatic generation control in electric power systems using parametric quadratic programming. *In* A. Alonso-Concheiro, ed., 'Real Time Digital Control Applications. Proceedings of the IFAC/IFIP Symposium. Pergamon, Oxford, England', pp. 595–600, 1984.

Abstract. New concepts for automatic generation control in electric power systems are presented, where the two components of automatic generation control, load frequency control and economic dispatch are performed at the same rate, i.e. a few seconds, and where economic dispatch takes network security into account. This gives network security and good transients, avoiding contradictory actions of load frequency control and economic dispatch on the generating units. The corner stone of the solution is the use of a new fast on-line optimal power flow, using a new parametric quadratic programming method, which is presented in details.

P. Carraresi, F. Farinaccio, and F. Malucelli. Testing optimality for quadratic 0-1 problems. Technical Report TR-95-11, Dipartimento di Informatica, Università di Pisa, Italy, 1995.

Abstract. The issue tackled is testing whether a given solution of a quadratic 0-1 problem is optimal. The paper presents an algorithm based on the necessary and sufficient optimality condition introduced by Hirriart-Urruty for general convex problems. A measure of the quality of the solution is provided. Computational results show the applicability of the method. The method is extended to constrained quadratic 0-1 problems such as quadratic assignment and quadratic knapsack.

E. Casas and C. Pola. An algorithm for indefinite quadratic programming based on a partial Cholesky factorization. *RAIRO-Recherche Operationnelle-Operations Research*, **27**(4), 401–426, 1993.

Abstract. A new algorithm is described for quadratic programming that is based on a partial Cholesky factorization that uses a diagonal pivoting strategy and allows computation of the null of negative curvature directions. The algorithm is numerically stable and has shown efficiency in solving positive-definite and indefinite problems. It is specially interesting in indefinite cases because the initial point does not need to be a vertex of the feasible set. The authors thus avoid introducing artificial constraints in the problem, which turns out to be very efficient in parametric programming. At the same time, techniques for updating matrix factorizations are used.

Y. Chabrillac and J.-P. Crouzeix. Definiteness and semidefiniteness of quadratic forms revisited. *Linear Algebra and its Applications*, **63**, 283–292, 1984.

T. F. Chan, J. A. Olkin, and D. W. Cooley. Solving quadratically constrained least squares using black box solvers. *BIT*, **32**, 481–495, 1992.

S. W. Chang. A method for quadratic programming. *Naval Research Logistics Quarterly*, **33**(3), 479–487, 1986.

Abstract. A solution to the quadratic programming is presented with the constraint of the form $Ax \leq b$ using the linear complementary problem approach.

Y. Y. Chang and R. W. Cottle. Least-index resolution of degeneracy in quadratic programming. *Mathematical Programming*, **18**(2), 127–137, 1980.

Abstract. Combines least-index pivot selection rules with Keller's algorithm for quadratic programming to obtain a finite method for processing degenerate problems.

A. Charnes and J. Semple. Practical error bounds for a class of quadratic programming problems. *Informatica*, **2**(3), 352–366, 1991.

Abstract. The absolute error between an approximate feasible solution, generated via a dual formulation, and the true optimal solution is measured. These error bounds involve considerably less computational work than existing estimates.

B. Chen and P. T. Harker. A noninterior continuation method for quadratic and linear programming. *SIAM Journal on Optimization*, **3**(3), 503–515, 1993.

M. Chen and J. A. Filar. Hamiltonian cycles, quadratic programming, and ranking of extreme points. *In* C. Floudas and P. Pardalos, eds, 'Global Optimization', pp. 32–9. Princeton University Press, USA, 1992.

Y.-H. Chen and S. C. Fang. Neurocomputing with time delay analysis for solving convex quadratic programming problems. *IEEE Transactions on Neural Networks*, p. (to appear), 1999.

Y. H. Chen and S. C. Fang. Neurocomputing with time delay analysis for solving convex quadratic programming problems. *IEEE Transactions on Neural Networks*, **11**(1), 230–240, 2000.

Abstract. This paper presents a neural-network computational scheme with time-delay consideration for solving convex quadratic programming problems. Based on some known results, a delay margin is explicitly determined for the stability of the neural dynamics, under which the states of the neural network does not oscillate. The configuration of the proposed neural network is provided. Operational characteristics of the neural network are demonstrated via numerical examples.

Z. Chen and N. Y. Deng. Some algorithms for the convex quadratic programming problem via the ABS approach. *Optimization Methods and Software*, **8**(2), 157–170, 1997.

F. T. Cheng, T. H. Chen, and Y. Y. Sun. Efficient algorithm for resolving manipulator redundancy—the compact QP method. *In* '1992 IEEE International Conference on Robotics and Automation : Proceedings', Vol. 1–3, pp. 508–513, 1992a.

F. T. Cheng, T. H. Chen, Y. S. Wang, and Y. Y. Sun. Efficient algorithm for resolving manipulator redundancy-the compact QP method. *In* 'Proceedings. 1992 IEEE International Conference on Robotics And Automation. IEEE Comput. Soc. Press, Los Alamitos, CA, USA', Vol. 1, pp. 508–513, 1992b.

Abstract. Due to hardware limitations, physical constraints, such as joint rate bounds and joint angle limits, always exist. In the present work, these constraints are included in the general formulation of the redundant inverse kinematic problem. To take into account these physical constraints, the computationally efficient compact QP (quadratic programming) method is derived to resolve the kinematic redundancy problem. In addition, the compact-inverse QP method is developed to remedy the singularity problem. The compact QP

(compact and inverse QP) method makes use of the compact formulation to obtain the general solutions and to eliminate the equality constraints. As such, the variables are decomposed into basic and free variables, and the basic variables are expressed by the free variables. Thus, the problem size is reduced and it only requires an optimization algorithm, such as QP, for the free variables subject to pure inequality constraints. This approach will expedite the optimization process and make real-time implementation possible.

F. T. Cheng, T. H. Chen, Y. S. Wang, and Y. Y. Sun. Obstacle avoidance for redundant manipulators using the compact QP method. *In* 'Proceedings IEEE International Conference on Robotics and Automation. IEEE Comput. Soc. Press, Los Alamitos, CA, USA', Vol. 3, pp. 262–269, 1993.

Abstract. The compact QP (quadratic programming) method is proposed to resolve the obstacle avoidance problem for a redundant manipulator. The drift-free criterion is considered when a redundant manipulator performs a repeated motion. Due to the computational efficiency and versatility of the compact QP method, real-time implementation is able to be achieved, and physical limitations such as joint rate bounds and joint angle limits can be easily taken into account. An example is given to demonstrate that this method is able to avoid the throat of a cavity, and to remedy the drift problem while a primary goal of the manipulators is carried out. Simulation results show that multiple goals can easily be fulfilled by this method.

F. T. Cheng, R. J. Sheu, T. H. Chen, Y. S. Wang, and F. C. Kung. The improved compact QP method for resolving manipulator redundancy. *In* 'IROS '94. Proceedings of the IEEE/RSJ/GI International Conference on Intelligent Robots and Systems. Advanced Robotic Systems and the Real World. IEEE, New York, NY, USA', Vol. 2, pp. 1368–1375, 1994.

Abstract. The compact QP method is an effective and efficient algorithm for resolving the manipulator redundancy under inequality constraints. In this paper, a more computationally efficient scheme which will improve the efficiency of the compact QP method-the improved compact QP method -is developed. With the technique of workspace decomposition, the redundant inverse kinematics problem can be decomposed into two subproblems. Thus, the size of the redundancy problem can be reduced. For an n degree-of-freedom spatial redundant manipulator, instead of a $6n$ matrix, only a $3(n-3)$ matrix is needed to be manipulated by Gaussian elimination with partial pivoting for selecting the free variables. The simulation results on the CESAR manipulator indicate that the speedup of the compact QP method as compared with the original QP method is about 4.3. Furthermore, the speedup of the improved compact QP method is about 5.6. Therefore, it is believed that the improved compact QP method is one of the most efficient and effective optimization algorithm for resolving the manipulator redundancy under inequality constraints.

C. C. N. Chu and D. F. Wong. A quadratic programming approach to simultaneous buffer insertion/sizing and wire sizing. *IEEE Transactions on Computer Aided Design of Integrated Circuits and Systems*, **18**(6), 787–798, 1999.

Abstract. In this paper, we present a completely new approach to the problem of delay minimization by simultaneous buffer insertion and wire sizing for a wire. We show that the problem can be formulated as a convex quadratic program, which is known to be solvable in polynomial time. Nevertheless, we explore some special properties of our problem and derive an optimal and very efficient algorithm, modified active set method (MASM), to solve the resulting program. Given m buffers and a set of m discrete choices of wire width, the running time of our algorithm is $O(mn^2)$ and is independent of the wire length in practice. For example, an instance of 100 buffers and 100 choices of wire width can be solved in 0.92 s. In addition, we extend MASM to consider simultaneous buffer insertion, buffer sizing, and wire sizing. The resulting algorithm MASM-BS is again optimal and very efficient. For example, with six choices of buffer size and 10 choices of wire width, the optimal solution for a 15000 μ m long wire can be found in 0.05 s. Besides, our formulation is so versatile that it is easy to consider other objectives like wire area or power dissipation, or to add constraints to the solution. Also, wire capacitance lookup tables, or very general wire capacitance models which can capture area capacitance, fringing capacitance, coupling capacitance, etc. can be used.

C. S. Chung and D. Gale. A complementarity algorithm for optimal stationary programs in growth models with quadratic utility. Technical Report ORC 81-10, Operations Research Center, University of California, Berkeley, CA, USA, 1981.

S. J. Chung and K. G. Murty. Polynomially bounded ellipsoid algorithm for convex quadratic programming. *In* O. L. Mangasarian, R. R. Meyer and S. M. Robinson, eds, 'Nonlinear Programming, 4', pp. 439–485, Academic Press, London and New York, 1981a.

S. J. Chung and K. G. Murty. Polynomially bounded ellipsoid algorithms for convex quadratic programming. *Methods of Operations Research*, **40**, 63–66, 1981b.

Abstract. Let B, b be respectively a given square nonsingular integer matrix of order n, and an integer column vector in $\mathbb{R}^n$. It is required to find the nearest point to b in the Cone $\text{Pos}(B) = (x : x = Bz, z \geq 0)$. This leads to the linear complementarity problem: find $w = (w_1, \ldots, w_n)$, $z = (z_1, \ldots, z_n)$ satisfying $W - (B^T B)z = -B^T b$, $w \geq 0, z \geq 0$, $w_z^T = 0$.

T. T. Chung. Analysis of plate bending by the quadratic programming approach. Technical report, Washington Univesity, St. Louis, MO, USA, 1974.

Abstract. Finite element analysis of plate bending is interpreted as a quadratic programming problem. The total potential energy, expressed in terms of the coefficients of the approximating polynomials is the objective function the minimum of which is sought subject to linear equality constraints. The constraints require satisfaction of all kinematic boundary conditions and inter-element continuity conditions. Convergence characteristics of this approach with respect to increasing orders of polynomial approximation, as well as with respect to progressively reduced element sizes, are discussed and illustrated with a number of examples. Advantages of the proposed approach are discussed, and topics requiring further investigations are outlined.

L. Churilov, D. Ralph, and M. Sniedovich. A note on composite concave quadratic programming. *Operations Research Letters*, **23**(3–5), 163–169, 1998.

Abstract. We present a pivotal-based algorithm for the global minimization of composite concave quadratic functions subject to linear constraints. It is shown that certain subclasses of this family yield easy-to-solve line search subproblems. Since the proposed algorithm is equivalent in efficiency to a standard parametric complementary pivoting procedure, the implication is that conventional parametric quadratic programming algorithms can now be used as tools for the solution of much wider class of complex global optimization problems.

T. F. Coleman and L. A. Hulbert. A direct active set algorithm for large sparse quadratic programs with simple bounds. *Mathematical Programming, Series B*, **45**(3), 373–406, 1989.

T. F. Coleman and L. A. Hulbert. A globally and superlinearly convergent algorithm for convex quadratic programs with simple bounds. *SIAM Journal on Optimization*, **3**(2), 298–321, 1993.

T. F. Coleman and Y. Li. A reflective Newton method for minimizing a quadratic function subject to bounds on some of the variables. *SIAM Journal on Optimization*, **6**(4), 1040–1058, 1996.

Abstract. We propose a new algorithm, a reflective Newton method, for the minimization of a quadratic function of many variables subject to upper and lower bounds on some of the variables. The method applies to a general (indefinite) quadratic function for which a local minimum subject to bounds is required and is particularly suitable for the large-scale problem. Our new method exhibits strong convergence properties and global and second-order convergence and appears to have significant practical potential. Strictly feasible

points are generated. We provide experimental results on moderately large and sparse problems based on both sparse Cholesky and preconditioned conjugate gradient linear solvers.

T. F. Coleman and J. G. Liu. An interior Newton method for quadratic programming. *Mathematical Programming*, **85**(3), 491–523, 1999.

Abstract. We propose a new (interior) approach for the general quadratic programming problem. We establish that the new method has strong convergence properties: the generated sequence converges globally to a point satisfying the second-order necessary optimality conditions, and the rate of convergence is 2-step quadratic if the limit point is a strong local minimizer. Published alternative interior approaches do not share such strong convergence properties for the nonconvex case. We also report on the results of preliminary numerical experiments: the results indicate that the proposed method has considerable practical potential.

T. F. Coleman and J. G. Liu. An exterior Newton method for strictly convex quadratic programming. *Computational Optimization and Applications*, **15**(1), 5–32, 2000.

Abstract. We propose an exterior Newton method for strictly convex quadratic programming (QP) problems. This method is based on a dual formulation: a sequence of points is generated which monotonically decreases the dual objective function. We show that the generated sequence converges globally and quadratically to the solution (if the QP is feasible and certain nondegeneracy assumptions are satisfied). Measures for detecting infeasibility are provided. The major computation in each iteration is to solve a KKT-like system. Therefore, given an effective symmetric sparse linear solver, the proposed method is suitable for large sparse problems. Preliminary numerical results are reported.

D. C. Collins. Terminal state dynamic programming: quadratic costs, linear differential equations. *Journal of Mathematical Analysis and Applications*, **31**(2), 235–253, 1970.

Abstract. A fairly general class of control problems can be posed in terms of minimizing a cost functional involving the state of the system to be controlled and the control exerted over a fixed interval of time. The state and control variables are related by a state equation, often with additional constraints of various forms upon the control or state. That is, it is desired to find $\min_{y(t_\varepsilon Y)}(p(x(T)) + \int_o^T q(x(t))dt)$, where x and y are the N-dimensional state vector and the M-dimensional control vector, respectively, while p and q are scalar-valued functions of their arguments. The state equation is $dx/dt = h(x,y)$, $x(0) = c$, with y constrained to some class of admissible controls, Y. This paper discusses a specialization of the above problem, a terminal control problem, in which the cost of state is measured only at the terminal time, T.

A. R. Conn and J. W. Sinclair. Quadratic programming via a non-differentiable penalty function. Technical Report CORR 75/15, Faculty of Mathematics, University of Waterloo, Ontario, Canada, 1975.

A. R. Conn, N. I. M. Gould, and Ph. L. Toint. A primal-dual algorithm for minimizing a nonconvex function subject to bound and linear equality constraints. *In* G. Di Pillo and F. Giannessi, eds, 'Nonlinear Optimization and Related Topics', pp. 15–50, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1999.

Abstract. A new primal-dual algorithm is proposed for the minimization of non-convex objective functions subject to simple bounds and linear equality constraints. The method alternates between a classical primal-dual step and a Newton-like modified barrier step in order to ensure descent on a suitable merit function. Convergence of a well-defined subsequence of iterates is proved from arbitrary starting points. Preliminary numerical results for quadratic programming problems are presented.

A. R. Conn, N. I. M. Gould, D. Orban, and Ph. L. Toint. A primal-dual trust-region algorithm for non-convex nonlinear programming. *Mathematical Programming*, **87**(2), 215–249, 2000.

Abstract. A new primal-dual algorithm is proposed for the minimization of non-convex objective functions subject to general inequality and linear equality constraints. The method uses a primal-dual trust-region model to ensure descent on a suitable merit function. Convergence is proved to second-order critical points from arbitrary starting points. Numerical results are presented for general quadratic programs.

G. C. Contaxis, C. Delkis, and G. Korres. Decoupled optimal load flow using linear or quadratic programming. *IEEE Transactions on Power Systems*, **PWRS-1**(2), 1–7, 1986.

Abstract. The authors propose an iterative scheme for the optimal load flow (OLF) problem, a constrained optimization problem of large size and great complexity. For online implementation, fast execution times and minimum computer storage are required. The iterative scheme decomposes the optimal load flow problem into real and reactive subproblems. The real and reactive subproblems are solved alternately until they converge. Quadratic or linear programming is utilized to solve the two subproblems. If the cost curve of each generator is approximated as a quadratic function then the cost function of the OLF problem is in quadratic form and quadratic programming is applied for the solution. If the effect of valve point loading is to be examined, the cost function of the OLF problem becomes linear and linear programming is applied.

G. C. Contaxis, C. Delkis, E. Glytsis, and B. C. Papadias. Economic power dispatch with line security limits using *Z*-matrix techniques and quadratic programming. *In* E. Lauger and J. Moltoft, eds, ‘Reliability in Electrical and Electronic Components and Systems. Fifth European Conference on Electrotechnics EUROCON ’82’, pp. 709–713, North Holland, Amsterdam, the Netherlands, 1982.

Abstract. Presents an efficient algorithm for solving the economic power dispatch problem. The general problem of the economic power dispatch is formulated as a non-linear constrained optimization problem, recognizing system losses, operating limits on the generation units and line security limits. The optimization problem is solved by an iterative scheme. At each iteration the optimization problem is transformed to a quadratic programming problem by utilizing Z-matrix techniques and the generalized generation distribution factors.

G. C. Contaxis, C. Delkis, B. Paraskevopoulos, and B. C. Papadias. Economic power dispatch using quadratic programming and *Z*-matrix. *In* ‘Electrotechnology for Development. Proceedings of MELECON ’81 the First Mediterranean Electrotechnical Conference. IEEE, New York, NY, USA’, pp. 67–70, 1981.

Abstract. Describes an algorithm developed at the National Technical University of Athens for solving the problem of economic power dispatch. This algorithm uses quadratic programming and Z-matrix techniques to minimize the total operational cost of the thermal units. The algorithm recognizes operating limits on the generation units and can easily be modified to account for the line security limits also.

B. L. Contesse. Une caractérisation complète des minima locaux en programmation quadratique. *Numerische Mathematik*, **34**(3), 315–332, 1980.

Abstract. The general second order conditions for the characterization of local minima are shown to be necessary and sufficient in the case of a quadratic objective function subject to linear constraints. Specifically, it is shown that the well known second order hand, that the well known second order sufficient condition for an isolated local minimum is, actually, necessary. These results are proved from scratch and include several existing ones. In the particular case of equality constrained quadratic problems, Orden’s second order conditions for global minima are recovered.

F. Cotiu. On the quadratic programming problem. *In* ‘4th conference on probability theory. Abstracts. Acad Socialist Republic of Rumania, Bucharest, Romania’, p. 42, 1971.

Abstract. Quadratic programming is considered in the case in which conditions are linear and the function to be optimized is of a positively defined quadratic form. A method of solving such a program is discussed and an algorithm for solving quadratic programs with mutually exclusive constraints is also given.

J. S. Cotner and R. R. Levary. A quadratic programming model for determining short-term multiple currency portfolios. *Opsearch*, **24**(4), 218–227, 1987.

Abstract. Based on Eurocurrency deposit interest rates and currency exchange rates, a quadratic programming model is developed for defining optimal short-term multiple currency-denominated portfolios. It is shown that this model can be applied in defining efficient portfolios at various levels of expected return, and that investing in these multicurrency portfolios will provide higher risk-adjusted returns than comparable single-currency holdings.

R. W. Cottle. Symmetric dual quadratic programs. *Quarterly of Applied Mathematics*, **21**, 237–243, 1963.

R. W. Cottle. A fundamental theorem in quadratic programming. *Journal of the Society of Industrial and Applied Mathematics*, **12**, 663–665, 1964.

R. W. Cottle. On the convexity of quadratic forms over convex sets. *Operations Research*, **15**, 170–173, 1967.

R. W. Cottle. The principal pivoting method of quadratic programming. *In* G. B. Dantzig and A. F. Veinott, eds, 'Lectutes in Applied Mathematics II', number 1 *in* 'Mathematics of the Decision Sciences', pp. 144–162, American Mathematical Society, Providence, Rhode Island, USA, 1968.

R. W. Cottle. Three remarks about two papers on quadratic forms. *Zeitschrift fur Operations Research*, **19**, 123–124, 1975.

R. W. Cottle. Fundamentals of quadratic programming and linear complementarity. *In* M. Z. Cohn and G. Maier, eds, 'Engineering Plasticity by Mathematical Programming', pp. 293–323. Pergamon Press, New York, USA, 1979.

R. W. Cottle and A. Djang. Algorithmic equivalence in quadratic programming. I. A least-distance programming problem. *Journal of Optimization Theory and Applications*, **28**(3), 275–301, 1979.

Abstract. It is demonstrated that Wolfe's algorithm for finding the point of smallest Euclidean norm in a given convex polytope generates the same sequence of feasible points as does the van de Panne-Whinston symmetric algorithm applied to the associated quadratic programming problem. Furthermore, it is shown how the latter algorithm may be simplified for application to problems of this type.

R. W. Cottle and M. S. Goheen. A special class of large quadratic programs. *In* O. L. Mangasarian, R. R. Meyer and S. M. Robinson, eds, 'Nonlinear Programming, 3', pp. 361–390, Academic Press, London and New York, 1977.

R. W. Cottle and W. C. Mylander. Ritter's cutting plane method for nonconvex quadratic programming. *In* J. Abadie, ed., 'Integer and nonlinear programming', pp. 257–283, North Holland, Amsterdam, the Netherlands, 1970.

Abstract. Reviews the methods of Ritter (1964, 1965, 1966) for obtaining a global solution to a linearly-constrainted quadratic minimization problem. The problem concerned can be stated without loss of generality as minimize $\phi(x) = c^T x + 1/2 x^T Dx$ subject to $Ax \geq b$, $x \geq 0$, where the matrix A is of order $m \times n$ and $D = D^T$.

R. W. Cottle and S. Schaible. On pseudoconvex quadratic forms. *In* E. F. Beckenbach, ed., 'General Inequalities. II', pp. 81–88. Birkhauser, Basel, Switzerland, 1978.

R. W. Cottle, G. J. Habetler, and C. E. Lemke. Quadratic forms semi-definite over convex cones. *In* H. W. Kuhn, ed., 'Proceedings of the Princeton Symposium on Mathematical Programming', pp. 551–565. Princeton University Press, Princeton, USA, 1970.

R. W. Cottle, J.-S. Pang, and R. E. Stone. *The Linear Complementarity Problem*. Academic Press, London, England, 1992.

CPLEX 6.0. *High-performance linear, integer and quadratic programming software*. ILOG SA, Gentilly, France, 1998.

C. W. Cryer. The solution of a quadratic programming problem using systematic overrelaxation. *SIAM Journal on Control*, **9**(3), 385–392, 1971.

Abstract. Let A be a real symmetric positive definite $n \times n$ and b a real column n-vector, then find real column n-vectors x and y such that $Ax = b + y$, and $x^T y = 0, x \geq 0, y \geq 0$, Problems of this type occur when the method of Christopherson is used to solve free boundary problems for journal bearings. In such cases, A is a 'finite difference' matrix. A method for solving the above problem is presented which is a modification of systematic over- relaxation. This method is particularly suitable when A is a finite difference matrix.

W. Cui and D. I. Blockley. Decision making with fuzzy quadratic programming. *Civil Engineering Systems*, **7**(3), 140–147, 1990.

Abstract. Decision making in civil engineering systems necessarily involves imprecise data. Fuzzy linear programming (FLP) has been used to deal with imprecision in parameter definitions. In this paper, it is argued that present methods of FLP can be improved upon by using a new method of fuzzy quadratic programming (FQP) which enables the modelling of independent fuzzy parameters. Several numerical examples are presented and compared with other techniques.

J. W. Daniel. Stability of the solution of definite quadratic programs. *Mathematical Programming*, **5**(1), 41–53, 1973.

N. P. Danilkin, P. F. Denisenko, and V. V. Sotskiy. Computation of ionospheric $N(h)$ profiles by quadratic programming. *Geomagnetism and Aeronomy*, **15**(2), 292–293, 1975.

Abstract. Proposes a method making it possible to obtain from two components a nonmonotonous $N(h)$ profile under adverse conditions, i.e. when the virtual heights are insufficiently accurately determined, in the presence of horizontal N gradients, and in computations from averaged ionograms.

G. Danninger and I. M. Bomze. Using copositivity for global optimality criteria in concave quadratic-programming problems. *Mathematical Programming*, **62**(3), 575–580, 1993.

Abstract. In this note we specify a necessary and sufficient condition for global optimality in concave quadratic minimization problems. Using this condition, it follows that, from the perspective of worst-case complexity of concave quadratic problems, the difference between local and global optimality conditions is not as large as in general. As an essential ingredient, we here use the ε-subdifferential calculus via an approach of Hiriart-Urruty and Lemarechal (1990).

G. B. Dantzig. Quadratic programming: A variant of the Wolfe-Markowitz algorithms. Research report 2, Operations Research Center, University of California, Berkeley, California, USA, 1961.

G. B. Dantzig. Quadratic programming. *In* 'Linear Programming and Extensions', chapter 12-4, pp. 490–498. Princeton Uiversity Press, Princeton, USA, 1963.

M. D'Apuzzo, V. De Simone, and G. Toraldo. A parallel algorithm for box constrained convex quadratic programming. Technical Report TR-96-8, Center for Research on Parallel Computing and Supercomputers, University of Naples "Federico II", Italy, 1996a.

M. D'Apuzzo, V. De Simone, and G. Toraldo. Parallel software for box constrained convex quadratic programming. Technical Report TR-96-13, Center for Research on Parallel Computing and Supercomputers, University of Naples "Federico II", Italy, 1996b.

M. S. Daskin, J. L. Schofer, and A. E. Haghani. A quadratic programming model for designing and evaluating distance-based and zone fares for urban transit. *Transportation Research, Part B (Methodological)*, **22**(1), 25–44, 1988.

Abstract. A microcomputer based system for designing and evaluating distance-based and zone fares for transit properties is described. At the heart of the system is an optimization model that finds the fixed charge, mileage charge, and transfer charge that maximize gross revenue subject to constraints on ridership and the form of the fare equation. A linear approximation to the demand curve at the base case values results in a quadratic programming problem. Three alternative modes of using the model system are demonstrated using selected data from the Chicago Transit Authority. Model extensions and proposed future work are outlined.

V. A. Daugavet and A. V. Lazarev. Development of Dantzig's method in quadratic programming. *USSR Computational Mathematics and Mathematical Physics*, **26**(2), 67–72, 1986.

V. A. Daugavet and T. M. Salikh. A complementary basis procedure for convex quadratic programming. *Zhurnal Vychislitel'noi Matematiki i Matematicheskoi Fiziki*, **32**(12), 1981–1992, 1992.

Abstract. The complementary basis procedure, used for the solution of quadratic programming problems, is based on ideas of the Lemke, Wolfe and Dantzig methods. It can be applied to an arbitrary case of convex quadratic programming, without reducing it to any kind of a standard form.

A. Dax. The gradient projection method for quadratic programming. Technical report, Insitute of Mathematics, The Hebrew University of Jerusalem, Isreal, 1978.

A. Dax. An active set algorithm for convex quadratic programming. Technical report, Hydrological Service, Jerusalem, Israel, 1982.

G. Dayal, L. L. Grigsby, and L. Hasdorff. Quadratic programming for optimal active and reactive power dispatch using special techniques for reducing storage requirements. *IEEE Power Engineering Society Summer Meeting. (Text of A papers) IEEE, New York, NY, USA*, pp. A76 388–389/1–5, 1976.

Abstract. This paper presents some of the techniques which can be effectively employed for reducing the computational effort and storage requirements, when linear and quadratic programming methods are used for solving large size power system problems. The effectiveness of these procedures has been tested by implementing them in a new computer code, specially developed for solving economic dispatch problem. The approved IEEE test systems up to 118 buses were used for the test purposes. The results show that the improvement in the case of 118 bus system is rather dramatic. Number of nonzero entries in the basis inverse, while using the new code, are found to be only 25% of those reported earlier.

O. de Donato and A. Franchi. A modified gradient method for finite element elastoplastic analysis by quadratic programming. *Computer Methods in Applied Mechanics and Engineering*, **2**(2), 107–131, 1973.

Abstract. The finite analysis problem with piecewise linear constitutive laws is formulated as a linear complementarity and a quadratic programming problem. The solution techniques using the well-known optimization methods of mathematical programming are discussed and a procedure, belonging to the class of gradient methods, is proposed which overcomes the computational difficulties that arise when there is a large number of variables. Through a physical interpretation of the gradient of the objective function, each mathematical step of the proposed optimization technique is translated into a corresponding physical operation on the structure and a mechanical solution procedure with a finite number of steps is derived. Finally, for the incremental analysis problem under non-holonomic constitutive laws the same procedure is adopted combined with the multistage loading technique. Illustrative examples for each of the preceding problems are given.

G. Debreu. Definite and semidefinite forms. *Econometrica*, **20**, 295–300, 1952.

H. Dejonghe. A quadratic programming technique for modeling gravitating systems. *Astrophysical Journal*, **343**(1), 113–124, 1989.

Abstract. The paper presents a general method that produces analytic distribution functions for gravitating systems. A typical model fits photometric and kinematical data and is a superposition of components. The coefficients of the linear combination are determined by a quadratic programming technique. The author gives an explicit example for systems with spherical symmetry, and subsequently applies the method to the Coma Cluster of galaxies. Statistically significant fits are obtained using only a small number of components. This technique is also a dynamical mass estimator. The author obtains masses between $1.7 * 10^{15} h_{50}^{-1}\ M_{(.)}$ and $5 * 10^{15} h_{50}^{-1}\ M_{(.)}$ within $5 h_{50}^{-1}$ Mpc, and finds some evidence for luminosity segregation in the orbital structure.

C. L. DeMarco and D. M. Divan. Synthesis of optimal discrete pulse modulation waveforms through an integer quadratic program. *In* ‘1988 IEEE International Symposium on Circuits and Systems. Proceedings. IEEE, New York, NY, USA’, Vol. 2, pp. 1377–1380, 1988.

Abstract. The authors introduce a physically based objective function for determining optimal discrete pulse modulation waveforms in power electronics applications. Using this objective function, it is demonstrated that the problem of selecting an optimal waveform reduces to a quadratic 0–1 integer program. It is shown that the structure of this 0–1 program lends itself to a bounding algorithm that allows many of the branches in the search tree to be eliminated, considerably reducing the computational cost.

R. S. Dembo and U. Tulowitzki. On the minimization of quadratic functions subject to box constraints. School of Organization and Management Working paper Series B no. 71, Yale University, Yale, USA, 1983.

R. S. Dembo and U. Tulowitzki. Computing equilibria on large multicommodity networks: an application of truncated quadratic programming algorithms. *Networks*, **18**(4), 273–284, 1988.

Abstract. Presents a general scheme for improving the asymptotic behavior of a given nonlinear programming algorithm without incurring a significant increase in storage overhead. To enhance the rate of convergence the authors compute search directions by partially solving a sequence of quadratic programming (QP) problems. The idea is illustrated on a class of extremely large nonlinear programming problems arising from traffic equilibrium calculations using both the Frank-Wolfe and PARTAN algorithms to partially solve the QP subproblems. Computational results indicate that the convergence rate of the underlying algorithm is indeed enhanced significantly when Frank-Wolfe is used to solve the QP subproblems but only marginally so in the case of PARTAN. It is conjectured, and supported by the theory, that with better algorithms for the QP subproblems the improvements due to the proposed framework would be more marked.

N. Demokan and A. H. Land. A parametric quadratic program to solve a class of bicriteria decision problems. *Journal of the Operational Research Society*, **32**(6), 477–488, 1981.

Abstract. Presents an extension of Beale's quadratic programming algorithm to perform parametric analysis on the right hand side element of a single constraint. Specific examples are given to illustrate the utilization of the method for a class of bi-criteria decision problems, where one of the criteria can be formulated as a quadratic function, such as risk, sum of squared deviations or a utility function, and the other one is usually in linear cost or return function which can be treated as a parametric constraint.

D. den Hertog. *Interior point approach to linear, quadratic, and convex programming : algorithms and complexity*. Kluwer Academic Publishers, Dordrecht, The Netherlands, 1994.

D. den Hertog, C. Roos, and T. Terlaky. A polynomial method of weighted centers for convex quadratic programming. *Journal of Information & Optimization Sciences*, **12**(2), 187–205, 1991.

M. A. Diamond. The solution of a quadratic programming problem using fast methods to solve systems of linear equations. *International Journal of Systems Science*, **5**(2), 131–136, 1974.

Abstract. Let A be a real symmetric positive definite $n*n$ matrix and b a real column n-vector. The problem of finding n-vectors x and y such that $Ax = b + y$, $x^T y = 0, x \geq 0 and y \geq 0$, occurs when the method of Christopherson is used to solve free boundary problems for journal bearings. In this case A is a 'finite difference' matrix. The author presents a direct method for solving the above problem by solving a number of linear systems $A_k x = b_k$. Each system can be solved using one of the recently developed fast direct or iterative procedures. He considers the solution by factorization techniques.

C. R. Dietrich. Unit graph estimation and stabilization using quadratic programming and difference norms—reply. *Water Resources Research*, **31**(10), 2635, 1995.

C. R. Dietrich and T. G. Chapman. Unit graph estimation and stabilization using quadratic programming and difference norms. *Water Resources Research*, **29**(8), 2629–2635, 1993.

Abstract. For a rainfall-runoff event, quadratic programming is particularly suited to the estimation of unit graph as linear and positivity constraints on unit graph ordinates can be naturally implemented. In this paper, the quadratic programming framework is invoked in a novel way to stabilize the unit graph estimation procedure via the use of difference norms. The advantage of the latter over standard ridge regression is that penalties are placed on oscillations of the unit graph rather than on the size of its ordinates. Application of the methodology to real rainfall-runoff data is provided and comparisons with existing approaches are made. The latter indicate that our approach generally yields unit graph estimates of more realistic shape and smaller variance resulting in a better fit to the runoff data.

I. I. Dikin. Iterative solution of problems of linear and quadratic programming. *Doklady Akademiia Nauk USSR*, **174**, 747–748, 1967. See also, *Soviet Mathematics Doklady* volume 8, pages 674–675, 1967.

V. V. Dikusar. Extended quadratic programming problem. *Doklady Akademii Nauk*, **349**(1), 29–31, 1996.

M. A. Diniz-Ehrhardt, M. A. Gomes-Ruggiero, and S. A. Santos. Numerical analysis of leaving-face parameters in bound-constrained quadratic minimization. Technical Report 52/98, Department of Applied Mathematics, IMECC-UNICAMP, Campinas, Brasil, 1998.

Abstract. In this work, we focus our attention on the quadratic subproblem of trust-region algorithms for large-scale bound-constrained minimization. An approach that combines a mild active set strategy with gradient projection techniques is employed in the solution of large-scale bound-constrained quadratic problems.

To fill in some gaps that have appeared in previous work, we propose, test and analyze heuristics which dynamically choose the parameters in charge of the decision of leaving or not the current face of the feasible set. The numerical analysis is based on problems from CUTE collection and randomly generated convex problems with controlled conditioning and degeneracy. The practical consequences of an appropriate decision of such parameters are shown to be crucial, particularly when dual degenerate and ill-conditioned problems are solved.

A. Djang. *Algorithmic equivalence in quadratic programming*. PhD thesis, Department of Operations Research, Stanford University, Stanford, California, USA, 1979.

M. Dormany. A parametric method for the solution of indefinite quadratic programming problems. *Szigma*, **13**(4), 285–303, 1980.

Abstract. The solution of indefinite quadratic programming problems raises serious difficulties. Parametric methods yield a way of solution when a part of program variables are treated as parameters and a series of multiparametric convex quadratic or linear programming problems of reduced size can be formulated. The procedure presented in this paper draws attention to a new possibility of parametrization making use of the minor matrices taken from the main diagonal of an indefinite matrix D. Primal-dual multiparametric linear programming is dealt with in detail; furthermore, it is shown how the efficiency of the algorithm can be raised by using Branch and Bound techniques.

W. Dorn. Duality in quadratic programming. *Quarterly of Applied Mathematics*, **18**, 155–162, 1960a.

W. S. Dorn. Self-dual quadratic programs. *Journal of the Society for Industrial and Applied Mathematics*, **9**, 51–54, 1960b.

W. S. Dorn. A symmetric dual theorem for quadratic programming. *Journal of the Operations Research Society of Japan*, **2**, 93–97, 1960c.

Z. Dostál. On the penalty approximation of quadratic programming problem. *Kybernetika*, **27**(2), 151–154, 1991.

Abstract. An upper bound for the difference of the exact solution of the problem of minimization of a quadratic functional on the subspace and its penalty approximation is given. The paper supplies a numerical example.

Z. Dostál. Box constrained quadratic programming with controlled precision of auxiliary problems and applications. *Zeitschrift für Angewandte Mathematik und Mechanik*, **76**(S3), 413–414, 1996.

Abstract. We review our recent results on the solution of quadratic programming problems with simple bounds by means of the conjugate gradient method with inexact solution of auxiliary subproblems and projections. Precision of the solution of auxiliary problems is controlled by the product of a positive constant Gamma with the norm of violation of the Kuhn-Tucker contact conditions. The resulting algorithm converges for any positive Gamma and reaches the solution in a finite number of steps provided the problem is non-degenerate. A lower bound on Gamma is given so that the finite termination property is preserved even for degenerate problems. The algorithm may be implemented with projections so that it can drop and add many constraints whenever the active set is changed. Applications to the solution of inner obstacle problems and contact problems of elasticity are reported.

Z. Dostál. Box constrained quadratic programming with proportioning and projections. *SIAM Journal on Optimization*, **7**(3), 871–887, 1997.

Abstract. Two new closely related concepts are introduced that depend on a positive constant Gamma . An iteration is proportional if the norm of violation of the Kuhn-Tucker conditions at active variables does not excessively exceed the norm of the part of the gradient that corresponds to free variables, while a progressive direction determines a descent direction that enables the released variables to move far enough from the boundary in a step called proportioning. An algorithm that uses the conjugate gradient method to explore the face of the region defined by the current iterate until a disproportional iteration is generated is proposed. It then changes the face by means of the progressive direction. It is proved that for strictly convex problems, the proportioning is a spacer iteration so that the algorithm converges to the solution. If the solution is nondegenerate then the algorithm finds the solution in a finite number of steps. Moreover, a simple lower bound on Gamma is given to ensure finite termination even for problems with degenerate solutions. The theory covers a class of algorithms, allowing many constraints to be added or dropped at a time and accepting approximate solutions of auxiliary problems. Preliminary numerical results are promising.

Z. Dostál, A. Friedlander, and S. A. Santos. Adaptive precision control in quadratic programming with simple bounds and/or equality constraints. *In* R. D. Leone, A. Murli, P. M. Pardalos and G. Toraldo, eds, 'High Performance Algorithms and Software in Nonlinear Optimization', pp. 161–173, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1998.

Z. Dostál, A. Friedlander, and S. A. Santos. Augmented Lagrangians with adaptive precision control for quadratic programming with equality constraints. *Computational Optimization and Applications*, **14**(1), 37–53, 1999.

Abstract. In this paper we introduce an augmented Lagrangian type algorithm for strictly convex quadratic programming problems with equality constraints. The new feature of the proposed algorithm is the adaptive precision control of the solution of auxiliary problems in the inner loop of the basic algorithm. Global convergence and boundedness of the penalty parameter are proved and an error estimate is given that does not have any term that accounts for the inexact solution of the auxiliary problems. Numerical experiments illustrate efficiency of the algorithm presented.

I. Doukhovni and D. Givoli. Quadratic programming algorithms for obstacle problems. *Communications in Numerical Methods in Engineering*, **12**(4), 249–256, 1996.

Abstract. The numerical solution of problems involving frictionless contact between an elastic body and a rigid obstacle is considered. The elastic body may undergo small or large deformation. Finite element discretization and repetitive linearization lead to a sequence of quadratic programming (QP) problems for incremental displacement. The performances of several QP algorithms, including two new versions of a modified steepest descent algorithm, are compared in this context. Numerical examples include a string, a membrane and an Euler-Bernoulli beam, in contact with flat and non-flat rigid obstacles.

J. P. Dussault, J. A. Ferland, and B. Lemaire. Convex quadratic programming with one constraint and bounded variables. *Mathematical Programming*, **36**(1), 90–104, 1986.

Abstract. The authors propose an iterative algorithm for solving a convex quadratic program with one equality constraint and bounded variables. At each iteration, a separable convex quadratic program with the same constraint set is solved. Two variants are analyzed: one that uses an exact line search, and the other a unit step size. Preliminary testing suggests that this approach is efficient for problems with diagonally dominant matrices.

N. Dyn and W. E. Ferguson. The numerical solution of equality-constrained quadratic programming problems. *Mathematics of Computation*, **41**(163), 165–170, 1983.

Abstract. This paper proves that a large class of iterative schemes can be used to solve a certain constrained minimization problem. The constrained minimization problem considered involves the minimization of a quadratic functional subject to linear equality constraints. Among this class of convergent interactive schemes are generalizations of the relaxed Jacobi, Gauss-Seidel, and symmetric Gauss-Seidel schemes.

B. C. Eaves. On quadratic programming. *Management Science*, **17**(11), 698–711, 1971.

Abstract. A procedure based on Lemke's algorithm is developed which either computes stationary points for general quadratic programs or else shows that the program has no optimum. If a general quadratic program has an optimum and satisfies a nondegeneracy condition then it is demonstrated that there are an odd number of stationary points.

B. C. Eaves. A finite procedure for determining if a quadratic form is bounded below on a closed polyhedral convex set. *Mathematical Programming*, **14**(1), 122–124, 1978.

J. G. Ecker. Computational procedures in quadratic programming and ℓ_p- approximation. *Bulletin of the Operations Research Society of America*, **17**(2), B237–238, 1969.

Abstract. Abstract only given, substantially as follows. In the framework of a duality theory of extended geometric programming, computational procedures are given for obtaining optimal solutions to quadratic programs with quadratic constraints and programs which minimize the ℓ_p-norm of the difference between a fixed vector and variable linear combination of other fixed vectors, subject to inequality constraints expressed by means of ℓ_p-norms. To apply the procedures, one basically need only solve a finite 'Duffin sequence' of linearly-constrained, concave, upper semi-continuous programs. The general class of programs to which the computational procedures apply properly contains a variety of special problems.

U. Eckhardt. Quadratic programming by successive overrelaxation. Technical Report Jül-1064-MA, Kernforschungsanlage, Juelich, West Germany, 1974.

Abstract. The idea of solving the definite linear complementarity problem by successive overrelaxation was originally proposed by Cryer. A detailed discussion of Cryer's method applied to quadratic programming problems is given. The convergence behaviour is treated without assumptions on solvability of the problem. Numerical examples indicate the efficiency of the method.

U. Eckhardt. Iterative lösung quadratischer optimierungsaufgaben. *Zeitschrift für Angewandte Mathematik und Mechanik*, **55**, T236–T237, 1975.

U. Eckhardt. Semi-infinite quadratic programming. *OR Spektrum*, **1**(1), 51–55, 1979.

Abstract. A method is presented for minimizing a definite quadratic function under an infinite number of linear inequality restrictions. Special features of the method are that it generates a sequence of feasible solutions and a sequence of basic solutions simultaneously and that it has very favourable properties concerning numerical stability.

U. Eckhardt. Linear inequalities and quadratic programming-some applications. *Methods of Operations Research*, **53;**, 67–81, 1986.

Abstract. Some applications of linear inequality systems and quadratic programming problems are presented. It is not intended to go into theoretical details or to investigate numerical specialities.

M. M. El Metwally and Z. M. Al Hamouz. Transmission networks planning using quadratic programming. *Electric Machines and Power Systems*, **18**(2), 137–148, 1990.

Abstract. The problem of transmission networks expansion has been solved by considering the cost of losses as well as the cost of investment in the objective function. The problem is solved using an exact quadratic programming technique. This new formulation has been applied to a 6-bus system. The final configurations, which are characterized by minimum cost of losses show that as the cost of the kWh increases, the total system cost decreases.

M. M. El Metwally and A. M. Harb. Transmission planning using admittance approach and quadratic programming. *Electric Machines and Power Systems*, **21**(1), 69–83, 1993.

Abstract. A method for transmission networks planning is proposed using quadratic programming and the admittance approach. The cost of investment and cost of losses, load flow and security constraints and the interest and inflation rates are included. The developed method can be used for static and dynamic modes of transmission planning.

M. A. H. El Sayed, T. M. Abdel Rahman, and M. O. Mansour. A fast quadratic programming approach for large-scale reactive power optimization. *Electric Machines and Power Systems*, **20**(1), 17–23, 1992. See also, European Transactions on Electrical Power Engineering, volume 2, number 4, pages 253–257, 1992.

Abstract. A fast quadratic programming approach has been developed to control the reactive power (VAR) generation in lead dispatching centers. The main objective of VAr control is to minimize the transmission losses taking into consideration the voltage constraints at each node of the network. The developed approach utilizes the decomposition of the whole system into smaller subsystems and the coordination of different subsystem solutions to obtain optimal VAr control in the large scale systems. The smallest practical size of the decomposed subsystems is the one-generator-bus subsystem. The fast and non-iterative VAr optimization of the decomposed subsystems is obtained based on considering only one active constraint at a time by solving the quadratic programming sub-problems. The numerical test results of IEEE-118 and 200 buses power system have indicated that the speed and accuracy of the proposed approach are adequate for real-time applications on large-scale systems. This approach is also applicable for large system deviations from its normal conditions and needs small memory size.

M. A. El Shibini and E. S. Ibrahim. Quadratic programming approach to reactive power optimization on the primary feeders. *Archiv fur Elektrotechnik*, **68**(4), 267–271, 1985.

Abstract. Suggests the use of the quadratic programming technique to determine the optimum size and location of shunt capacitors on radial distribution feeders so as to maximize overall savings, including the cost of capacitors. The saving function which is of quadratic form is maximized for a set of linear inequality constraints by using quadratic programming. For quadratic programming, efficient algorithms have been developed which can easily be implemented on digital computers. The approach is illustrated by an application to a typical distribution feeder of 23 kV.

N. Elia and M. A. Dahleh. A quadratic programming approach for solving the ℓ_1 multiblock problem. *IEEE Transactions on Automatic Control*, **43**(9), 1242–1252, 1998. See also, Proceedings of the 35th IEEE Conference on Decision and Control, IEEE, New York, NY, USA, volume 4, pages 4028–4033, 1996.

Abstract. We present a new method to compute solutions to the general multi-block l_1 control problem. The method is based on solving a standard H_2 problem and a finite-dimensional semidefinite quadratic programming problem of appropriate dimension. The new method has most of the properties that separately characterize many existing approaches, in particular, as the dimension of the quadratic programming problem increases, this method provides converging upper and lower bounds on the optimal ℓ_1 norm and, for well posed multi-block problems, ensures the convergence in norm of the suboptimal solutions to an optimal ℓ_1 solution. The new method does not require the computation of the interpolation conditions, and it allows the direct computation of the suboptimal controller.

D. Endres and P. Foldiak. Quadratic programming for learning sparse codes. *In* 'Proceedings of ICANN99, Edinburgh, 1999', p. (to appear), 1999.

S. S. Erenguc and H. P. Benson. An algorithm for indefinite integer quadratic programming. *Computers and Mathematics with Applications*, **21**(6–7), 99–106, 1991.

Abstract. Presents an algorithm for finding the global minimum of an indefinite quadratic function over the integers contained in a compact, convex set. To find this minimum, the algorithm first transforms the problem into an equivalent problem with a separable objective function. It then uses a branch and bound search on the values of the constraints, rather than the variables, of the transformed problem.

S. E. Eriksen and P. D. Berger. A quadratic programming model for product configuration optimization. *Zeitschrift für Operations Research, Serie B (Praxis)*, **31**(6), 143–159, 1987.

Abstract. The various features of any product are differentially appealing to the various portions of the consumer population and have differential costs of production and marketing. This paper considers the problem of 'overall product optimization', or more specifically, 'optimal product configuration'. Product price is included as a part of this configuration.

S. M. Faber. Quadratic programming applied to the problem of galaxy population synthesis. *Astronomy and Astrophysics*, **20**(3), 361–374, 1972.

Abstract. The technique of quadratic programming as applied to the problem of galaxy population synthesis is described. The method offers significant advantages over the trial-and-error approach usually employed. This technique is applied to 38-color data on the nuclei of M 31, M 32 and M 81 and to integrated 10-color photometry for elliptical galaxies. The results indicate that estimates of mean line strengths in external galaxies by means of population synthesis are well determined. Ages based on the main-sequence turnoff point are uncertain by a factor of two. The method is not sensitive to the number of stars on the main sequence between K 0 V and M 7 V. Consequently the slope of the luminosity function below turnoff cannot be determined. The mass-to-light ratio of the computed population for M 31 is uncertain by a factor of 4. The corresponding factors for M 32 and M 81 are 50 and 16 respectively. Models for elliptical galaxies suggest that the mean metal abundance of the stellar population increases with increasing galaxy luminosity.

B. J. Falkowski. Risk analysis using perceptrons and quadratic programming. *In* 'Computational Intelligence. Theory and Applications. International Conference, 6th Fuzzy Days, Springer-Verlag, Berlin, Germany', pp. 543–547, 1999.

Abstract. A heuristic method (computation of weighted averages) is considered for risk analysis. It is improved upon by utilizing the perception learning theorem and quadratic programming. Experimental work shows that both techniques give good results, the former one being somewhat more efficient in terms of CPU-time used. In spite of certain theoretical shortcomings it is argued that the familiar paradigm offers considerable potential for practical applications.

J. Y. Fan and L. Zhang. Real-time economic dispatch with line flow and emission constraints using quadratic programming. *IEEE Transactions on Power Systems*, **13**(2), 320–325, 1998.

Abstract. The presence of multiple constraints due to network line flow limits and emission allowances in the economic dispatch of modern power systems makes the conventional Lambda-Delta iterative approach no longer effective. This paper proposes a practical strategy based on quadratic programming (QP) techniques to solve the real-time economic dispatch problem. It formulates the problem with a quadratic objective function based on the unit's cost curves in quadratic or piecewise-quadratic forms. The operation constraints are modeled as linear equality/inequality equations, resulting in a typical QP problem. Goal programming techniques are also incorporated in the formulation which guarantees the best available solution even under infeasible conditions. In addition, the proposed strategy formulates the problem in the second phase dispatch in real-time by including a set of emergency control variables to provide effective control strategies for properly relieving constraint violations if they exist. The effectiveness of the proposed strategy is demonstrated by an example power dispatch problem.

L. Fan, I. Chatterton, and L. Walker. Quadratic programming for grid supply point mw demand composition analysis. *In* '34-th Universities Power Engineering Conference. University of Leicester, UK', Vol. 1, pp. 245–248, 1999.

Abstract. This paper reports work of analysing the grid supply point (GSP) load composition across the entire National Grid system. A quadratic programming based algorithm has been developed for the nodal load composition analysis. The static load composition at GSP level is the primary information crucial to further load analysis, including load dynamics, reactive load composition, and GSP load modelling and forecasting.

S. C. Fang and S. C. Puthenpura. Affine scaling for convex quadratic programming. *In* 'Linear Optimization and Extensions: Theory and Algorithms', chapter 9. Prentice Hall, Englewood Cliffs, New Jersey, USA, 1993.

S. C. Fang and H. S. J. Tsao. An unconstrained convex programming approach to solving convex quadratic programming problems. *Optimization*, **27**(3), 235–243, 1993.

Abstract. Derives an unconstrained convex programming approach to solving convex quadratic programming problems in standard form. Related duality theory is established by using two simple inequalities. An ε-optimal solution is obtained by solving an unconstrained dual convex program. A dual-to-primal conversion formula is also provided. Some preliminary computational results of using a curved search method is included.

S. C. Fang, T. M. Huang, C. H. Lin, and W. W. Lin. A relaxed interior path following primal–dual algorithm for convex quadratic programming. *Mathematics Today, Special Issue on Mathematical Programming*, **XII-A**, 115–144, 1994.

H. Farhat and S. From. A quadratic programming approach to estimating the testability and random or deterministic coverage of a VLSI circuit. *VLSI Design*, **2**(3), 223–231, 1994.

Abstract. The testability distribution of a VLSI circuit is modeled as a series of step functions over the interval (0, 1). The model generalizes previous related work on testability. Unlike previous work, however, we include estimates of testability by random vectors. Quadratic programming methods are used to estimate the parameters of the testability distribution from fault coverage data (random and deterministic) on a sample of faults. The estimated testability is then used to predict the random and deterministic fault coverage distributions without the need to employ test generation or fault simulations. The prediction of fault coverage distribution can answer important questions about the ”goodness” of a design from a testing point of view. Experimental results are given on the large ISCAS-85 and ISCAS-89 circuits.

H. Farhat, S. From, and A. Lioy. A quadratic programming approach to estimating the testability and coverage distributions of a VLSI circuit. *Microprocessing and Microprogramming*, **35**(1–5), 479–483, 1992.

Abstract. The testability distribution of a VLSI circuit is modeled as a series of step functions over the interval (0,1). The model generalizes previous related work on testability. Quadratic programming methods are used to estimate the parameters of the testability distribution from fault coverage data on a sample of faults. The estimated testability is then used to predict the fault coverage distribution without the need to employ test generation or fault simulations. The prediction of fault coverage distribution can answer important questions about the 'goodness' of a design from testing point of view. Experimental results on three of the large ISCAS-89 benchmark circuits reflect the accuracy of the model.

A. M. Faustino and J. J. Júdice. A slcp algorithm for bilinear and concave quadratic programming. *Investigación Operacional*, **8**(2), 67–87, 1988.

A. M. Faustino and J. J. Júdice. Solution of large-scale convex quadratic programs by Lemke's method. *In* M. A. Turkman and M. L. Carvalho, eds, 'Actas da 1a Conferẽncia em Estatística e Optimizaąão', pp. 681–695, 1992.

A. M. Faustino and J. J. Júdice. Minimization of a concave quadratic function subject to box constraints. *Investigación Operativa*, **4**, 49–68, 1994.

L. Y. Faybusovich. Wolfe's algorithm for infinite-dimensional quadratic programming problems. *Engineering Cybernetics*, **20**(3), 20–30, 1982.

Abstract. Wolfe's algorithm is generalized to infinite-dimensional quadratic programming problems with a finite number of inequality constraints. A coordinate-free approach is developed enabling one to consider infinite-dimensional analogues of computational procedures of the simplex type.

L. Y. Faybusovich and J. B. Moore. Long-step path-following algorithm for convex quadratic programming problems in a Hilbert space. *Journal of Optimization Theory and Applications*, **95**(3), 615–635, 1997. See also, Proceedings of the 34th IEEE Conference on Decision and Control, IEEE, New York, NY, USA, volume 2, pages 1109–1114, 1995.

Abstract. We develop an interior-point technique for solving quadratic programming problems in a Hilbert space. As an example, we consider an application of these results to the linear-quadratic control problem with linear inequality constraints. It is shown that the newton step in this situation is basically reduced to solving the standard linear-quadratic control problem.

K. A. Fegley, S. Blum, J. O. Bergholm, A. J. Calise, J. E. Marowitz, G. Porcelli, and L. P. Sinha. Stochastic and deterministic design and control via linear and quadratic programming. *IEEE Transactions on Automatic Control*, **AC-16**(6), 759–766, 1971.

Abstract. The application of linear and quadratic programming to optimal control problems and to stochastic or deterministic system design problems is discussed and illustrated with examples.

L. Fernandes, A. Fischer, J. J. Júdice, C. Requejo, and J. Soares. A block active set algorithm for large-scale quadratic programming with box constraints. *Annals of Operations Research*, **81**, 75–95, 1998.

Abstract. An algorithm for computing a stationary point of a quadratic program with box constraints (BQP) is proposed. Each iteration of this procedure comprises a guessing strategy which forecasts the active bounds at a stationary point, the determination of a descent direction by means of solving a reduced strictly convex quadratic program with box constraints and an exact line search. Global convergence is established in the sense that every accumulation point is stationary. Moreover, it is shown that the algorithm terminates after a finite number of iterations, if at least one iterate is sufficiently close to a stationary point which satisfies a certain sufficient optimality condition. The algorithm can be easily implemented for sparse large-scale BQPs. Furthermore, it simplifies for concave BQPs, as it is not required to solve strictly convex quadratic programs in this case. Computational experience with large-scale BQPs is included and shows the appropriateness of this type of methodology.

M. C. Ferris. Parallel constraint distribution in convex quadratic programming. *Mathematics of Operations Research*, **19**(3), 645–658, 1994.

Abstract. We consider convex quadratic programs with large numbers of constraints. We distribute these constraints among several parallel processors and modify the objective function for each of these subproblems with Lagrange multiplier information from the other processors. New Lagrange multiplier information is aggregated in a master processor and the whole process is repeated. Linear convergence is established for strongly convex quadratic programs by formulating the algorithm in an appropriate dual space. The algorithm corresponds to a step of an iterative matrix splitting algorithm for a symmetric linear complementarity problem followed by a projection onto a subspace.

J. A. Filar. Quadratic programming and the single-controller stochastic game. *Journal of Mathematical Analysis and Applications*, **113**(1), 136–147, 1986.

J. A. Filar. The Hamiltonian cycle problem, controlled Markov chains and quadratic programming. *In* 'Proceedings of ASOR 12th National Conference', Vol. 15, pp. 263–281, 1993.

J. A. Filar, M. Oberije, and P. M. Pardalos. Hamiltonian cycle problem, controlled Markov chains and quadratic programming. *In* 'Proceedings of the 12th National Conference of the Australian Society for Operations Research', pp. 263–281, 1993.

F. L. Filippelli, M. Forti, and S. Manetti. New linear and quadratic programming neural network. *Electronics Letters*, **30**(20), 1693–1694, 1994.

Abstract. A neural network is proposed for solving linear and quadratic programming problems. The main feature is that the required conditions of symmetry and asymmetry in the interconnections are automatically met in practical implementations, so that stability is guaranteed.

B. Finkbeiner and P. Kall. Direct algorithms in quadratic programming. *Zeitschrift für Operations Research, Serie A (Theorie)*, **17**(1), 45–54, 1973.

Abstract. According to the literature there seem to be some difficulties in the semidefinite case. A numerical example of a semicomplementary solution is presented, where Cottle's algorithm fails. It will be proved that at least in Zangwill's version of Cottle's algorithm this situation cannot occur theoretically. Since, however, in practical computations such cases may occur due to round off errors. a slight modification of the algorithm is proposed, for which monotonicity and finiteness are proved.

F. D. Fischer. To the solution of the contact problem of elastic bodies with extended contact areas by quadratic programming. *Computing*, **13**(3–4), 353–384, 1974.

Abstract. An elastic body in contact with an elastic or rigid subgrade is represented by finite elements. The total potential energy of the system under consideration of linearly elastic material and small deformations is now a quadratic function of the nodal deformations and the nodal values of the contact pressure which is approximated by a polynomial. Only a part of the surface of the body must be proposed which includes the real contact surface. After evaluation of the equilibrium equations and the contact condition in an inequality and a linear transformation of the nodal variables, all relations are now so formulated, the minimisation of total potential energy can be expressed as a quadratic program in the unknown nodal deformations and nodal values of contact pressure. Standard programs can now be used for solution.

J. Fischer. Stackelberg solutions in constrained quadratic programming problems. *In* K. Reinisch and M. Thoma, eds, 'Large Scale Systems: Theory and Applications 1989. Selected Papers from the 5th IFAC/IFORS/IMACS Symposium. Pergamon, Oxford, England', pp. 241–246, 1990.

Abstract. Deals with two-person static Stackelberg games with quadratic objectives and common linear constraints for the leader and the follower. Important properties of the rational reaction function of the follower can be derived from the outlined relations to the theory of parametric optimization. From the result the solution of the specified Stackelberg problem is reduced to the sequential solution of a finite number of standard constrained quadratic programming problems and the iterative selection of solution structures for the follower reaction. The proposed algorithm has been implemented on a large-scale nonlinear programming solver.

N. J. Fisher. Gravity interpretation with the aid of quadratic programming—reply. *Geophysics*, **46**(3), 341–342, 1981.

N. J. Fisher. A quadratic programming algorithm for geophysical gravity inversion and other applications. *Bulletin of the Australian Mathematical Society*, **25**(1), 159–160, 1982.

Abstract. The geophysical gravity method is described and the corresponding inverse problem state as an integral equation. A survey of previous work associated with gravity data inversion is given followed by an account of the inherent difficulties associated with gravity inversion; this is done by a discussion of the integral equation already given, the inversion of which constitutes an ill-posed problem. The integral equation gives rise to an ill-conditioned system of linear equations; existing methods for the solutions of such systems

are reviewed. Then follows an account of the previous use of quadratic programming techniques in the solution of ill-posed problems.

N. J. Fisher and L. E. Howard. Gravity interpretation with the aid of quadratic programming. *Geophysics*, **45**(3), 403–419, 1980.

Abstract. The inverse gravity problem is posed as a linear least-squares problem with the variables being densities of two-dimensional prisms. Upper and lower bounds on the densities are prescribed so that the problem becomes a linearly constrained least-squares problem, which is solved using a quadratic programming algorithm designed for upper and lower bound-type constraints. The solution to any problem is smoothed by damping, using the singular value decomposition of the matrix of gravitational attractions. If the solution is required to be monotonically increasing with depth, then this feature can be incorporated. The method is applied to both field and theoretical data. The results are plotted for (1) undamped, nonmonotonic, (2) damped, nonmonotonic, and (3) damped, monotonic solutions; these conditions illustrate the composite approach of interpretation where both damping techniques and linear constraints are used in refining a solution which at first is unacceptable on geologic grounds while fitting the observed data well.

R. Fletcher. A FORTRAN subroutine for general quadratic programming. Technical report, UKAEA, Harwell, Berks, England, 1970.

Abstract. A FORTRAN subroutine is described and listed for minimizing a quadratic function of n variables subject to m linear equality and inequality constraints. The method is a general one so that there are no restrictions on the types of quadratic function which can be minimized. The solution is usually found in a moderate multiple of n^3 (or at worst n^2m) computer operations. The subroutine is very flexible and allows options which can considerably shorten the time taken to solve any particular problem.

R. Fletcher. A general quadratic programming algorithm. *Journal of the Institute of Mathematics and its Applications*, **7**, 76–91, 1971.

Abstract. An effective algorithm is presented for quadratic programming which is of general applicability, but which is not dependent upon the availability of a linear programming code for its implementation. It is an algorithm of exchange type, the exchanges being chosen so as to avoid the accumulation of error to as large an extent as possible.

R. Fletcher. Quadratic programming. *In* ‘Practical Methods of Optimization’, chapter 10, pp. 229–258. J. Wiley and Sons, Chichester, England, second edn, 1987a.

R. Fletcher. Recent developments in linear and quadratic programming. *In* A. Iserles and M. J. D. Powell, eds, ‘State of the Art in Numerical Analysis. Proceedings of the Joint IMA/SIAM Conference’, pp. 213–243. Oxford University Press, Oxford, England, 1987b.

Abstract. Describes recent developments in linear programming, including the ellipsoid algorithm, the Karmarkar algorithm, new strategies for updating LU factors in the simplex method, and methods with guaranteed termination in the presence of degeneracy and round-off errors. Various new algorithms for quadratic programming are discussed, and the choice of matrix factorizations and their updates is considered. The use of ℓ_1 penalty functions in linear and quadratic programming is also mentioned briefly.

R. Fletcher. Resolving degeneracy in quadratic programming. *Annals of Operations Research*, **46–47**(1–4), 307–334, 1993.

Abstract. A technique for the resolution of degeneracy in an active set method for quadratic programming is described. The approach generalises Fletcher’s method (1988) which applies to the LP case. The method is described in terms of a linear complementarity problem tableau, which is seen to provide useful insights. It is shown that the degeneracy procedure only needs to operate when the degenerate constraints are linearly dependent on those in the active set. No significant overheads are incurred by the degeneracy procedure. It is

readily implemented in a null space format, and no complications in the matrix algebra are introduced. The guarantees of termination provided by Fletcher's method, extending in particular to the case where round-off error is present, are preserved in the QP case. It is argued that the technique gives stronger guarantees than are available with other popular methods such as Wolfe's method (1963) or the method of Goldfarb and Idnani (1983).

R. Fletcher. Stable reduced hessian updates for indefinite quadratic programming. *Mathematical Programming*, **87**(2), 251–264, 2000.

Abstract. Stable techniques are considered for updating the reduced Hessian matrix that arises in a null-space active set method for quadratic programming when the Hessian matrix itself may be indefinite. A scheme for defining and updating the null-space basis matrix is described which is adequately stable and allows advantage to be taken of sparsity in the constraint matrix. A new canonical form for the reduced Hessian matrix is proposed that can be updated in a numerically stable way. Some consequences for the choice of minor iteration search direction are described.

R. Fletcher and M. P. Jackson. Minimization of a quadratic function of many variables subject only to lower and upper bounds. *Journal of the Institute of Mathematics and its Applications*, **14**(2), 159–174, 1974.

O. E. Flippo and A. H. G. Rinnooy Kan. A note on Benders decomposition in mixed-integer quadratic programming. *Operations Research Letters*, **9**(2), 81–83, 1990.

C. A. Floudas and V. Visweswaran. Quadratic optimization. *In* R. Horst and P. M. Pardalos, eds, 'Handbook of Global Optimization', Kluwer Academic Publishers, Dordrecht, The Netherlands, 1995.

Abstract. Quadratic optimization comprises one of the most important areas of nonlinear programming. Numerous problems in real world applications, including problems in planning and scheduling, economies of scale, and engineering design, and control are naturally expressed as quadratic problems. Moreover, the quadratic problem is known to be NP-hard, which makes this one of the most interesting and challenging class of optimization problems. In this chapter, we review various properties of the quadratic problem, and discuss different techniques for solving various classes of quadratic problems. Some of the more successful algorithms for solving the special cases of bound constrained and large scale quadratic problems are considered. Examples of various applications of quadratic programming are presented. A summary of the available computational results for the algorithms to solve the various classes of problems is presented.

F. Forgo. Relationship between mixed zero-one integer linear programming and certain quadratic programming problems. *Studia Scientiarum Mathematicarum Hungarica*, **4**, 37–43, 1969.

A. Forsgren and G. Sporre. On weighted linear least-squares problems related to interior methods for convex quadratic programming. Report TRITA-MAT-2000-OS11, Department of Mathematics, Royal Institute of Technology, Stockholm, Sweden, 2000.

Abstract. It is known that the norm of the solution to a weighted linear least-squares problem is uniformly bounded for the set of diagonally dominant symmetric positive definite weight matrices. This result is extended to weight matrices that are nonnegative linear combinations of symmetric positive semidefinite matrices. Further, results are given concerning the strong connection between the boundedness of weighted projection onto a subspace and the projection onto its complementary subspace using the inverse weight matrix. In particular, explicit bounds are given for the Euclidean norm of the projections. We apply these results to the Newton equations arising in a primal-dual interior method for convex quadratic programming and prove boundedness for the corresponding projection operator.

A. L. Forsgren, P. E. Gill, and W. Murray. On the identification of local minimizers in inertia-controlling methods for quadratic programming. *SIAM Journal on Matrix Analysis and Applications*, **12**(4), 730–746, 1991.

Abstract. The verification of a local minimizer of a general (i.e., nonconvex) quadratic program is in general an NP-hard problem. The difficulty concerns the optimality of certain points (which we call dead points) at which the first-order necessary conditions for optimality are satisfied, but strict complementarity does not hold. Inertia-controlling quadratic programming (ICQP) methods form an important class of methods for solving general quadratic programs. We derive a computational scheme for proceeding at a dead point that is appropriate for a general ICQP method.

M. Forti and A. Tesi. New conditions for global stability of neural networks with application to linear and quadratic programming problems. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, **42**(7), 354–366, 1995.

Abstract. In this paper, we present new conditions ensuring existence, uniqueness, and Global Asymptotic Stability (GAS) of the equilibrium point for a large class of neural networks. The results are applicable to both symmetric and nonsymmetric interconnection matrices and allow for the consideration of all continuous nondecreasing neuron activation functions. Such functions may be unbounded (but not necessarily surjective), may have infinite intervals with zero slope as in a piece-wise-linear model, or both. The conditions on GAS rely on the concept of Lyapunov Diagonally Stable (or Lyapunov Diagonally Semi-Stable) matrices and are proved by employing a class of Lyapunov functions of the generalized Lur'e-Postnikov type. Several classes of interconnection matrices of applicative interest are shown to satisfy our conditions for GAS. In particular, the results are applied to analyze GAS for the class of neural circuits introduced for solving linear and quadratic programming problems. In this application, the principal result here obtained is that these networks are GAS also when the constraint amplifiers are dynamical, as it happens in any practical implementation.

M. Frank and P. Wolfe. An algorithm for quadratic programming. *Naval Research Logistics Quarterly*, **3**, 95–110, 1956.

P. D. Frank, M. J. Healy, and R. A. Mastro. A range-space implementation for large quadratic programs with small active sets. *Journal of Optimization Theory and Applications*, **69**(1), 109–127, 1991.

R. M. Freund. Dual gauge programs, with applications to quadratic programming and the minimum-norm problem. *Mathematical Programming*, **38**(1), 47–67, 1987.

Abstract. A gauge function $f(\cdot)$ is a nonnegative convex function that is positively homogeneous and satisfies f(0)=0. Norms and pseudonorms are specific instances of a gauge function. The paper presents a gauge duality theory for a gauge program, which is the problem of minimizing the value of a gauge function $f(\cdot)$ over a convex set. The gauge dual program is also a gauge program, unlike the standard Lagrange dual. The author presents sufficient conditions on $f(\cdot)$ that ensure the existence of optimal solutions to the gauge program and its dual, with no duality gap. These sufficient conditions are relatively weak and are easy to verify, and are independent of any qualifications on the constraints. The theory is applied to a class of convex quadratic programs, and to the minimum ℓ_p norm problem. The gauge dual program is shown to provide a smaller duality than the standard dual, in a certain sense discussed.

A. Friedlander and J. M. Martínez. On the maximization of a concave quadratic function with box constraints. *SIAM Journal on Optimization*, **4**(1), 177–192, 1994.

A. Friedlander, J. M. Martínez, and M. Raydan. A new method for large-scale box constrained convex quadratic minimization problems. *Optimization Methods and Software*, **5**(1), 57–74, 1995.

K. R. Frisch. Quadratic programming by the multiplex methodin the general case where the quadratic form may be singular. Memorandum, University Institute for Economics, Oslo, 1960.

M. Fu, Z. Q. Luo, and Y. Ye. Approximation algorithms for quadratic programming. *Journal of Combinatorial Optimization*, **2**(1), 29–50, 1998.

Abstract. We consider the problem of approximating the global minimum of a general quadratic program (QP) with n variables subject to m ellipsoidal constraints. For $m = 1$, we rigorously show that an ε-minimizer, where error $\varepsilon \in (0,1)$, can be obtained in polynomial time, meaning that the number of arithmetic operations is a polynomial in n, m, and $\log(1/\varepsilon)$. For $m \geq 2$, we present a polynomial-time $(1-1/m^2)$-approximation algorithm as well as a semidefinite programming relaxation for this problem. In addition, we present approximation algorithms for solving QP under the box constraints and the assignment polytope constraints.

S. T. Fu and R. T. Wang. Power-system security analysis and enhancement using Fletcher's quadratic-programming method. *In* 'Proceedings of the Eighth Power Systems Computation Conference', pp. 439–445, 1984.

S. T. Fu, E. Yu, and X. Zhang. A decoupled optimal power flow approach using Fletcher's quadratic programming method. *Proceedings of the Chinese Society of Electrical Engineering*, **6**(1), 1–11, 1986. See also, Bridge Between Control Science and Technology. Proceedings of the Ninth Triennial World Congress of IFAC, Pergamon Press, Oxford, England, volume 4, pages 2145–2150, 1985.

Abstract. A decoupled model with active and reactive optimization is derived. In a P-model, a very sparse scheme is used as the equality constraints. In a Q-model, transmission losses are expressed as deviation of swing generation. The active power at generator buses (except swing generator) is chosen as independent variable in P-optimization. The voltage amplitude of PV nodes and V θ node, and the ratio of transformation of all tap-changing transformers are chosen as independent variables in Q-optimization. The procedure is carried out only in the subspace determined by independent variables. The equality constraints are eliminated, and the relationships between dependent and independent variables are established by solution of a fast decoupled load flow method. During P-optimization, all bus voltages and reactive injections are assumed constant and the objective function is to minimize the total fuel consumption in the system. During Q-optimization, all active generations (except swing generator) and all voltage phase angles are assumed constant and the objective function is to minimize total transmission losses in the network. This approach has been programmed in FORTRAN language and tested on a CLASSIC 7835 computer. The sparse matrix programming technique being used, the memory requirement of a network with 150 buses is only 100 kB. For comparison purposes, the computation time and results of the same IEEE 30 bus test system using a different method are tabulated. A Chinese 133 bus system with 164 lines was simulated using this method. There were 2 lines overloaded and 3 bus voltages out of limit in the initial state. An OPF result is tabulated. All line overloads are relieved and all bus voltages are within limit.

T. Fujie and M. Kojima. Semidefinite programming relaxation for nonconvex quadratic programs. Technical Report Research Report on Information Sciences B-298, Tokyo Institute of Technology, 1995.

Abstract. Any quadratic inequality in the n-dimensional Euclidean space can be relaxed into a linear matrix inequality in $(1+n)\times(1+n)$ symmetric matrices. Based on this principle, we extend the Lovász-Schrijver SDP (semidefinite programming) relaxation developed for a 0–1 integer program to a general nonconvex QP (quadratic program), and present some fundamental characterization of the SDP relaxation including its equivalence to a relaxation using convex-quadratic valid inequalities for the feasible region of the QP.

Y. Fujimoto and A. Kawanura. Biped walking control with optimal foot force distribution by quadratic programming. *IEEE/ASME International Conference on Advanced Intelligent Mechatronics '97. Final Program and Abstracts IEEE, New York, NY, USA*, p. 108, 1997.

Abstract. Summary form only given. This paper describes a novel biped walking system based on optimal foot force distribution by quadratic programming (QP). The hierarchical control system consists of a robust force controller for the supporting leg, a robust position controller for the non-supporting leg, an attitude controller for the body, and a free-leg motion planner. The attitude of the body of the biped robot is controlled using the reactive force as the input. The main result of this paper is an introduction of QP to distribute the force required by the attitude controller to each foot. The method can stabilize the attitude of the robot and the contact between the foot and the ground, in spite of the existence of the limitation of the frictional force between the foot and the ground. Also a new walking pattern generator based on an inverted pendulum model is proposed, which realizes globally stable tracking of the center of mass of the biped robot.

K. Fukuda and A. Kawanaka. Adaptive processing with quadratic programming and lpf for reducing blocking artifacts in DCT image coding. *Transactions of the Institute of Electronics, Information and Communication Engineers*, **J82-A**(1), 142–150, 1999.

Abstract. This paper presents an adaptive post-processing method to reduce the blocking artifacts in DCT image coding. In this method, the blocks are classified into the smooth group and the variation group according to the received DCT coefficients. For the blocks in the smooth group, we estimate the DCT coefficients by solving a quadratic programming problem considering the continuity across the block boundaries and the quantization matrix used on the coder. Furthermore, the DCT coefficients are sequentially obtained from lower order to higher. For the blocks classified into the variation group, the low pass filters are applied for smoothing the block boundaries. Experimental results for some images coded at low bit rates show the proposed method improves the reconstructed image quality objectively as well as subjectively.

R. Gabasov, F. M. Kirillova, and V. M. Raketskii. Methods for solving the general convex quadratic programming problem. *Doklady Akademii Nauk SSSR*, **258**(6), 1289–1293, 1981.

Abstract. A theoretical and numerical investigation is reported of 4 methods (Bil's, the direct, dual and adaptive methods) for solving the (mxn) convex quadratic programming problem: $(F(x) = c^T x + x^T Dx/2$ to min, $Ax = b$, $d_* \leq x \leq d^*$, where $D = D(J,J) \geq O$, $A = A(I,J)$, rank $A = m$, $I = (1,2,\ldots,m), J = (1,2,\ldots n)$.

E. Galligani and V. Ruggiero. Numerical solution of equality-constrained quadratic programming problems on vector-parallel computers. *Optimization Methods and Software*, **2**(3), 233–247, 1993.

E. Galligani and L. Zanni. Error analysis of elimination methods for equality constrained quadratic-programming problems. *In* ‘Mathematical Research’, Vol. 89, pp. 107–112, 1996.

E. Galligani and L. Zanni. Error analysis of an algorithm for equality-constrained quadratic programming problems. *Computing*, **58**(1), 47–67, 1997.

Abstract. The numerical stability of the orthogonal factorization method (R. Fletcher, 1987) for linear equality constrained quadratic programming problems is studied using a backward error analysis. A perturbation formula for the problem is analyzed; the condition numbers of this formula are examined in order to compare them with the condition numbers of the two matrices of the problem. A class of test problems is also considered in order to show experimentally the behaviour of the method.

E. Galligani, V. Ruggiero, and L. Zanni. Splitting methods for constrained quadratic programs in data analysis. *Computers Math. Applic.*, **32**, 1–9, 1996.

E. Galligani, V. Ruggiero, and L. Zanni. Splitting methods and parallel solution of constrained quadratic programs. *Rend. Circ. Matem. Palermo Series II*, **48**, 121–136, 1997a.

E. Galligani, V. Ruggiero, and L. Zanni. Splitting methods for quadratic optimization in data analysis. *International Journal of Computer Mathematics*, **63**, 289–307, 1997b.

E. Galligani, V. Ruggiero, and L. Zanni. Parallel solution of large-scale quadratic programs. *In* R. D. Leone, A. Murli, P. M. Pardalos and G. Toraldo, eds, 'High Performance Algorithms and Software in Nonlinear Optimization', pp. 189–205, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1998.

G. Gallo, P. L. Hammer, and B. Simeone. Quadratic knapsack problems. *Mathematical Programming Studies*, **12**, 132–149, 1980.

C. E. Garcia and A. M. Morshedi. Quadratic-programming solution of dynamic matrix control (QDMC). *Chemical Engineering Communications*, **46**(1–3), 73–87, 1986.

M. K. Gavurin and V. N. Malozemov. Foundations of quadratic programming theory. *Vestnik Leningradskogo Universiteta, Seriya Matematika Mekhanika i Astronomiya*, **1**, 9–16, 1980.

Abstract. A compact description of quadratic programming theory is given. It includes the existence theorem different forms of optimality criterion and duality theorems. Only compatibility condition for a linear algebraic system and a necessary optimality condition in the linear programming problem are supposed to be known. The general convex programming theory is not used.

J. A. George. Assessment of errors in approximating the objective function of a quadratic programming problem. *Asia Pacific Journal of Operational Research*, **5**(1), 21–36, 1988.

Abstract. Deals with the effectiveness of various decision rules to approximate the objective function of a quadratic program (QP). It is assumed that the constraint set of the QP is known precisely but the objective function is known only from the data of observations made of that function. Decision rules, some linear and some nonlinear, are used to derive functions to approximate the true unknown function from the data. A series of problems are generated to test these decision rules. In addition to comparing them with each other, the study also considers the effects of the type of quadratic function (matrix type), the size of the problem, the number of observations, the scatter pattern of the observations and the curvature (degree of nonlinearity) of the quadratic.

V. Georgescu. Estimation of fuzzy regression models with possibilistic constraints, using quadratic programming. *Economic Computation and Economic Cybernetics Studies and Research*, **31**(1–4), 105–123, 1997.

Abstract. We introduce a formal criterion, the decoupling principle, which allows us to naturally extend the projection theorem into a fuzzy regression framework. This criterion justifies a radical revision of the usual estimation methods. Actually dominant in the fuzzy regression analysis is another approach, which resorts to linear programming and fuzzy arithmetic in order to minimize the fuzziness of the model, subject to some possibilistic constraints. Our criticism relating to this approach is concluded by the proposal of a new strategy, where we give up using fuzzy arithmetic (identified as a source of estimation distortions) and we insist in the reestablishment of the natural framework for solving a minimum norm problem. Since the orthogonality concept is related to the choice of a norm induced by an inner product, our fuzzy estimation procedure inevitably leads to a quadratic programming problem and not to a linear one. The decoupling principle consists in a set of rules, which allow us to express the fuzzy regression model as a system of two classical equations and to choose the corresponding projection subspaces. We explore the algorithmic consequences, which follow from such theoretical considerations for various situations and concrete types of problems, and we propose some MATLAB implementations.

F. Giannessi and G. Tomasin. Nonconvex quadratic programming, linear complementarity problems and integer programming. *In* F. Giannessi and G. Tomasin, eds, 'Linear complementarity problems and integer Programming', pp. 162–201, North-Holland, Amsterdam, 1974.

P. E. Gill and W. Murray. Numerically stable methods for quadratic programming. *Mathematical Programming*, **14**(3), 349–372, 1978.

Abstract. Numerically stable algorithms for quadratic programming are discussed. A new algorithm is described for indefinite quadratic programming which utilizes methods for updating positive-definite factorizations only. Consequently all the updating procedures required are common to algorithms for linearly-constrained optimization. The new algorithm can be used for the positive-definite case without loss of efficiency.

P. E. Gill, N. I. M. Gould, W. Murray, M. A. Saunders, and M. H. Wright. Range-space methods for convex quadratic programming problems. Technical Report SOL 82-9, Department of Operations Research, Stanford University, California, USA, 1982a.

P. E. Gill, N. I. M. Gould, W. Murray, M. A. Saunders, and M. H. Wright. A range-space quadratic programming algorithm for problems with a mixture of bounds and general constraints. Technical Report SOL 82-10, Department of Operations Research, Stanford University, California, USA, 1982b.

P. E. Gill, N. I. M. Gould, W. Murray, M. A. Saunders, and M. H. Wright. A weighted Gramm-Schmidt method for convex quadratic programming. *Mathematical Programming*, **30**(2), 176–195, 1984.

Abstract. Range-space methods for convex quadratic programming improve in efficiency as the number of constraints active at the solution decreases. In this paper the authors describe a range space method based upon updating weighted Gram-Schmidt factorization of the constraints in the active set. The updating methods described are applicable to both primal and dual quadratic programming algorithms that use an active-set strategy. Many quadratic programming problems include simple bounds on all the variables as well as general linear constraints. A feature of the proposed method is that it is able to exploit the structure of simple bound constraints. This allows the method to retain efficiency when the number of general constraints active at the solution is small. Furthermore, the efficiency of the method improves as the number of active bound constraints increases.

P. E. Gill, W. Murray, and M. A. Saunders. User's guide for QPOPT 1.0: A Fortran package for quadratic programming. Technical Report SOL ??, Department of Operations Research, Stanford University, California, USA, 1995.

P. E. Gill, W. Murray, and M. H. Wright. Quadratic programming. *In* 'Practical Optimization', chapter 5.3.2–5.4.1, pp. 177–184. Academic Press, London, England, 1981.

P. E. Gill, W. Murray, D. B. Ponceleón, and M. A. Saunders. Solving reduced KKT–systems in barrier methods for linear and quadratic programming. Technical Report SOL 91-7, Department of Operations Research, Stanford University, California, USA, 1991a.

P. E. Gill, W. Murray, M. A. Saunders, and M. H. Wright. User's guide for SOL/QPSOL: A Fortran package for quadratic programming. Technical Report SOL 86-2, Department of Operations Research, Stanford University, California, USA, 1986.

P. E. Gill, W. Murray, M. A. Saunders, and M. H. Wright. A Schur-complement method for sparse quadratic programming. *In* M. G. Cox and S. J. Hammarling, eds, 'Reliable Scientific Computation', pp. 113–138, Oxford University Press, Oxford, England, 1990.

P. E. Gill, W. Murray, M. A. Saunders, and M. H. Wright. Inertia-controlling methods for general quadratic programming. *SIAM Review*, **33**(1), 1–36, 1991b.

Abstract. Active-set quadratic programming (QP) methods use a working set to define the search direction and multiplier estimates. In the method proposed by Fletcher in 1971, and in several subsequent mathematically equivalent methods, the working set is chosen to control the inertia of the reduced Hessian, which is never permitted to have more than one nonpositive eigenvalue. (Such methods will be called inertia-controlling.) This paper presents an overview of a generic inertia-controlling QP method, including the equations satisfied by the search direction when the reduced Hessian is positive definite, singular and indefinite. Recurrence relations are derived that define the search direction and Lagrange multiplier vector through equations related to the Karush-Kuhn-Tucker system. Discussion is included of connections with inertia-controlling methods that maintain an explicit factorization of the reduced Hessian matrix.

D. Givoli and I. Doukhovni. Finite element-quadratic programming approach for contact problems with geometrical nonlinearity. *Computers and Structures*, **61**(1), 31–41, 1996.

Abstract. The finite element-quadratic programming (FE-QP) approach for problems involving frictionless contact between an elastic body and a rigid obstacle is presented in a general setting. The validity of this approach is first proved in the context of small deformation problems. Then a way to extend it to the case of large deformation problems is proposed, and the validity and implementation of this procedure are discussed. Numerical examples involving Euler-Bernoulli beams are presented to demonstrate these issues, and to show the regimes where the method is successful or failing.

C. R. Glassey. Analysis of the federal energy agency program by a quadratic programming model. *In* 'Proceedings of Lawrence Symposium on Systems and Decision Sciences. Western Periodicals Co, North Hollywood, CA, USA', pp. 146–150, 1977.

Abstract. A 21 sector input-output model of the United States economy, in which exports, imports and consumer demands are determined endogenously by linear price-demand functions, is used to examine the impact of increases in world market oil prices. Equilibrium prices and flows in the model economy are computed by maximizing a quadratic program. Policies of the Federal Energy Agency to mitigate these impacts are analysed by suitable modifications of the model.

C. R. Glassey. Price sensitive consumer demands in energy modeling-a quadratic programming approach to the analysis of some federal energy agency policies. *Management Science*, **24**(9), 877–886, 1978a.

Abstract. A 21 sector input-output model of the 1972 U.S. economy is extended to include consumer demands, imports, and exports as endogeneous variables, and used to analyze some consequences of the price control policy adopted to mitigate the impact of the quadrupling of world crude oil prices in 1973–1974. It is assumed that household consumption of goods is linearly related to prices. An equilibrium of the model economy is then computed by solving a quadratic program. It is shown that the price control policy is equivalent to subsidizing imported oil with revenues from a tax on domestic crude production. Equilibria are computed under this policy and in its absence. The comparison indicates the policy was effective in reducing the price index increase and GNP reduction that would otherwise have occurred, but at the cost of adversely affecting the balance of payments.

C. R. Glassey. A quadratic network optimization model for equilibrium single commodity trade flows. *Mathematical Programming*, **14**(1), 98–107, 1978b.

J. H. Glick, P. M. Pardalos, and J. B. Rosen. Global minimization of indefinite quadratic problems. *Computing*, **39**, 281–291, 1987.

M. S. Goheen. *Large scale bounded variable quadratic programming*. PhD thesis, Department of Operations Research, Stanford University, Stanford, California, USA, 1976.

R. Goldbach. Some randomized algorithms for convex quadratic programming. *Applied Mathematics and Optimization*, **32**, 121–142, 1999.

Abstract. We adapt some randomized algorithms of Clarkson [3] for linear programming to the framework of so-called LP-type problems, which was introduced by Sharir and Welzl [10]. This framework is quite general and allows a unified and elegant presentation and analysis. We also show that LP-type problems include minimization of a convex quadratic function subject to convex quadratic constraints as a special case, for which the algorithms can be implemented efficiently, if only linear constraints are present. We show that the expected running times depend only linearly on the number of constraints, and illustrate this by some numerical results. Even though the framework of LP-type problems may appear rather abstract at first, application of the methods considered in this paper to a given problem of that type is easy and efficient. Moreover, our proofs are in fact rather simple, since many technical details of more explicit problem representations are handled in a uniform manner by our approach. In particular, we do not assume boundedness of the feasible set as required in related methods.

D. Goldfarb. Analogs of Newton's method for quadratic programming. *Notices of the American Mathematics Society*, **15**(2), 400, 1968.

D. Goldfarb. Extensions of Newton's method and Simplex methods for solving quadratic programs. *In* F. A. Lootsma, ed., 'Numerical methods for non linear optimization', pp. 255–263. Academic Press, London, England, 1972.

Abstract. Two closely related methods, that may be viewed either as extensions of Newtons' method to handle linear equalities and inequalities or as quadratic analogues of the gradient projection method, are presented for solving strictly convex quadratic programs. One of these methods and the Simplex method for quadratic programming are shown to follow the same solution path if started at a vertex. A useful relationship between the Lagrange multiplier variables for a nested set of constraint bases is also shown

D. Goldfarb. Efficient primal algorithms for strictly convex quadratic programs. *In* J. P. Hennart, ed., 'Numerical Analysis', Lecture Notes in Mathematics, p. ???, Springer Verlag, Heidelberg, Berlin, New York, 1985.

D. Goldfarb. Strategies for constrint deletion in active set algorithms. *In* D. F. Griffiths and G. A. Watson, eds, 'Numerical Analysis', number 140 *in* 'Pitman Research Notes in Mathematics Series', pp. 66–81, Longman Scientific and Technical, Harlow, Essex, England, 1986.

D. Goldfarb and A. U. Idnani. Dual and primal-dual methods for solving strictly convex quadratic programs. *In* J. P. Hennart, ed., 'Numerical Analysis', number 909 *in* 'Lecture Notes in Mathematics', pp. 226–239, Springer Verlag, Heidelberg, Berlin, New York, 1982.

D. Goldfarb and A. U. Idnani. A numerically stable dual method for solving strictly convex quadratic programs. *Mathematical Programming*, **27**(1), 1–33, 1983.

D. Goldfarb and S. C. Liu. Interior point potential function reduction algorithm for solving convex quadratic programming. Technical report, Department of Industrial Engineering and Operations Research, Columbia University, New York, USA, 1990.

D. Goldfarb and S. C. Liu. An $O(n^3L)$ primal interior point algorithm for convex quadratic programming. *Mathematical Programming*, **49**(3), 325–340, 1991.

Abstract. We present a primal interior point method for convex quadratic programming which is based upon a logarithmic barrier function approach. This approach generates a sequence of problems, each of which is approximately solved by taking a single Newton step. It is shown that the method requires $O(\sqrt{n}L)$ iterations and $O(n^{3.5}L)$ arithmetic operations. By using modified Newton steps the number of arithmetic operations required by the algorithm can be reduced to $O(n^3L)$.

D. Goldfarb and S. C. Liu. An $O(n^3L)$ primal dual potential reduction algorithm for solving convex quadratic programs. *Mathematical Programming*, **61**(2), 161–170, 1993.

D. Golovashkin and I. E. Grossmann. Interior cuts—a new type of cutting planes for indefinite quadratic programming. Technical report, Center for Advanced Process Decision-Making, Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, PA, USA, 1998.

G. H. Golub and M. A. Saunders. Linear least squares and quadratic programming. *In* J. Abadie, ed., ‘Integer and nonlinear programming’, pp. 229–256, North Holland, Amsterdam, the Netherlands, 1970.

Abstract. One of the most common problems in any computation center is that of finding linear least squares solutions. These problems arise in a variety of areas and in a variety of contexts. For instance, the data may be arriving sequentially from a source and there may be some constraint on the solution. Linear least squares problems are particularly difficult to solve because they frequently involve large quantities of data, and they are ill-conditioned by their very nature. This paper presents several numerical algorithms for solving linear least squares problems in a highly accurate manner. In addition, it gives an algorithm for solving linear least squares problems with linear inequality constraints.

M. A. Gomez. An $o(n^2)$ active set algorithm for the solution of a parametric quadratic program. *Numerical Algorithms*, **22**(3–4), 305–316, 1999.

Abstract. In this paper, an $O(n^2)$ active set method is presented for minimizing the parametric quadratic function $frac12x^TDx - a^Tx + \lambda \max(c - \gamma^T x, 0)$ subject to $l \leq x \leq b$, for all nonnegative values of the parameter lambda . Here, D is a positive diagonal nxn matrix, a and γ are arbitrary n-vectors, c is an arbitrary scalar, l and b are arbitrary n-vectors, such that $l \leq b$. An extension of this algorithm is presented for minimizing the parametric function $\frac{1}{2}x^TDx - a^Tx + \lambda|\gamma^T x - c|$ subject to $l \leq x \leq b$. It is also shown that these problems arise naturally in a tax programming problem.

A. S. Gonçalves. A generalized primal-dual technique for quadratic programming in parametric form. *Revista Faculdade Ciéncias Universidade Coimbra*, **47**, 1–23, 1971.

A. S. Gonçalves. A primal-dual method for quadratic programming with bounded variables. *In* F. A. Lootsma, ed., ‘Numerical methods for non linear optimization’, pp. 255–263. Academic Press, London, England, 1972.

Abstract. A primal-dual algorithm for quadratic programming with bounded variables is established here, which has the same characteristics as the method by Eisemann (1963) for the linear programming case, and which may be considered as an extension of that method. However, distinct from the linear programming case, the restricted problems constructed here have no bounds in their variables. The computations may be performed by pivotal operations on a tableau. Programming of the algorithm is facilitated by its efficient use of the product form of the inverse mechanism available in most commercial linear programming systems.

A. S. Gonçalves. A L.P. algorithm for non-convex quadratic programming and its economic interpretation. *Bulletin of the Operations Research Society of America*, **22**, B95, 1974.

Abstract. Considers the problem of maximizing a convex quadratic objective. The algorithm devised first 'normalizes' the given problem, i.e., substitutes it by another equivalent problem whose solutions can be found very easily by a L.P. column generation procedure. Extensions to the general non-convex quadratic programming are also considered.

J. Gondzio. Hopdm version 2.30: Benchmark results: Solving convex quadratic programming problems. Logilab technical note, Department of Management Sciences, University of Geneva, Geneva, Switzerland, 1998.

V. N. Gordeev. The finiteness of methods of solving a problem in quadratic programming. *Cybernetics*, **7**(1), 110–114, 1971.

V. N. Gordeev. Simple method of solving an auxiliary quadratic programming problem. *Cybernetics*, **16**(4), 616–619, 1980.

N. I. M. Gould. *Numerical methods for linear and quadratic programming*. D. Phil. thesis, Oxford University, England, 1982.

N. I. M. Gould. The stability of the solution of general quadratic programs. Technical Report CORR 83-11, Department of Combinatorics and Optimization, University of Waterloo, Ontario, Canada, 1983.

N. I. M. Gould. On practical conditions for the existence and uniqueness of solutions to the general equality quadratic programming problem. *Mathematical Programming*, **32**(1), 90–99, 1985.

Abstract. The author presents practical conditions under which the existence and uniqueness of a finite solution to a given equality quadratic program may be examined. Different sets of conditions allow this examination to take place when null-space, range-space or Lagrangian methods are used to find stationary points for the quadratic program.

N. I. M. Gould. An algorithm for large-scale quadratic programming. *IMA Journal of Numerical Analysis*, **11**(3), 299–324, 1991.

Abstract. The paper describes a method for solving large-scale general quadratic programming problems. The method is based upon a compendium of ideas which have their origins in sparse matrix techniques and methods for solving smaller quadratic programs. A discussion is included on resolving degeneracy, on single phase methods and on solving parametric problems. Some numerical results are included.

N. I. M. Gould and Ph. L. Toint. A quadratic programming bibliography. Numerical Analysis Group Internal Report 2000-1, Rutherford Appleton Laboratory, Chilton, Oxfordshire, England, 2000. See "http://www.numerical.rl.ac.uk/qp/qp.html".

N. I. M. Gould and Ph. L. Toint. Numerical methods for large-scale non-convex quadratic programming. Technical Report RAL-TR-2001–017, Rutherford Appleton Laboratory, Chilton, Oxfordshire, England, 2001.

N. I. M. Gould, M. E. Hribar, and J. Nocedal. On the solution of equality constrained quadratic problems arising in optimization. Technical Report RAL-TR-98–069, Rutherford Appleton Laboratory, Chilton, Oxfordshire, England, 1998.

F. Granot and J. Skorin-Kapov. Some proximity and sensitivity results in quadratic integer programming. *Mathematical Programming*, **47**(2), 259–268, 1990a.

F. Granot and J. Skorin-Kapov. Towards a strongly polynomial algorithm for strictly convex quadratic programs—an extension of Tardos' algorithm. *Mathematical Programming*, **46**(2), 225–236, 1990b.

F. Granot, J. Skorin-Kapov, and A. Tamir. Using quadratic programming to solve high multiplicity scheduling problems on parallel machines. *Algorithmica*, **17**(2), 100–110, 1997.

Abstract. We introduce and analyze several models of scheduling n different types (groups) of jobs on m parallel machines, where in each group all jobs are identical. Our main goal is to exhibit the usefulness of quadratic programming approaches to solve these classes of high multiplicity scheduling problems, with the total weighted completion time as the minimization criterion. We develop polynomial algorithms for some models, and strongly polynomial algorithms for certain special cases. In particular, the model in which the weights are job independent, as well as the generally weighted model in which processing requirements are job independent, can be formulated as an integer convex separable quadratic cost now problem, and therefore solved in polynomial time. When we specialize further, strongly polynomial bounds are achievable. Specifically, for the weighted model with job-independent processing requirements if we restrict the weights to be machine independent (while still assuming different machine speeds), an $O(mn+n\log n)$ algorithm is developed. If it is also assumed that all the machines have the same speed, the complexity of the algorithm can be improved to $O(m\log m+n\log n)$. These results can be extended to related unweighted models with variable processing requirements in which all the machines are available at time zero.

R. L. Graves. A principal pivoting Simplex algorithm for linear and quadratic programming. *Operations Research*, **15**, 482–494, 1967.

R. L. Graves. Quadratic programming in Hilbert space. *Bulletin of the Operations Research Society of America*, **20**, B–78, 1972.

Abstract. Problems are analysed which arise in optimising over infinite horizons with discrete time, while satisfying a finite number of inequality constraints in each time period. These problems are transformed into quadratic programs on a separable Hilbert space and the existence of solutions to the resulting programs is established. Ways to calculated approximate solutions by solving problems over a finite horizon are shown. Some results (not complete) for the existence of dual variables are given.

R. L. Graves and L. G. Telser. An infinite-horizon discrete-time quadratic program as applied to a monopoly problem. *Econometrica*, **35**(2), ??, 1967.

M. Grigoriadis and K. Ritter. A parametric method for semidefinite quadratic programming. *SIAM Journal on Control and Optimization*, **7**(4), 559–577, 1969.

L. Grippo and S. Lucidi. A differentiable exact penalty function for bound constrained quadratic programming problems. *Optimization*, **22**(4), 557–578, 1991.

Abstract. Defines a continuously differentiable exact penalty function for the solution of bound constrained quadratic programming problems. The authors prove that there exists a computable value of the penalty parameter such that global and local minimizers of the penalty function yield global and local solutions to the original problem. This permits the construction of Newton-type algorithms based on consistent approximations of the Newton's direction of the penalty function. Conditions that ensure finite termination are established.

N. Grudinin. Combined quadratic-separable programming OPF algorithm for economic dispatch and security control. *IEEE Transactions on Power Systems*, **12**(4), 1682–1688, 1997.

Abstract. This paper presents a new algorithm for optimal power flow (OPF), which is based on P/Q decomposition of OPF problem and on combined application of quadratic and separable programming methods. Initially the unit cost curves are approximated by the quadratic functions and the quadratic programming algorithm is applied as a starting method. This gives a good initial point for optimization and reduces the total computation time. The modification of a separable programming (SP) algorithm with generation of approximating intervals is considered. A new quadratic-separable algorithm for OPF is proposed, which combines the main advantages of quadratic and separable programming methods. A bi-criterion formulation of security control problem on the base of economic and security objective functions (OF) is proposed. The numerical results of OPF for large-scale power systems are given for different methods.

L. Guan. An optimal neuron evolution algorithm for constrained quadratic programming in image restoration. *IEEE Transactions on Systems Man and Cybernetics Part A—Systems and Humans*, **26**(4), 513–518, 1996.

Abstract. An optimal neuron evolution algorithm for the restoration of linearly distorted images is presented in this paper. The proposed algorithm is motivated by the symmetric positive- definite quadratic programming structure inherent in restoration. Theoretical analysis and experimental results show that the algorithm not only significantly increases the convergence rate of processing, bat also produces good restoration results. In addition, the algorithm provides a genuine parallel processing structure which ensures computationally feasible spatial domain image restoration.

L. Guan and X. L. Zhou. Real-time image filtering: from optimal neuron evolution to vector quadratic programming. *1994 IEEE International Conference on Systems, Man, and Cybernetics. Humans, Information and Technology IEEE, New York, NY, USA*, pp. 694–699, 1994.

Abstract. In this paper, a new scheme is introduced for the partitioning of images in image filtering using neural networks. The proposed scheme takes into account the physical nature of the image formation process, and thus simplifies the ordering scheme associated with the image processing framework based on neural networks with hierarchical cluster architecture. Also presented in the paper is a vector processing algorithm. By utilizing this algorithm, the pixels in the same row/column of an image are processed simultaneously. Compared with the scalar neuron evolution algorithms, the vector algorithm provides good visual quality in image filtering. Visual examples are provided to demonstrate the performance of the new approach.

J. Guddat. Stability in convex quadratic parametric programming. *Mathematische Operationsforschung und Statistik*, **7**(2), 223–245, 1976.

F. Guder. Pairwise reactive SOR algorithm for quadratic programming of net import spatial equilibrium-models. *Mathematical Programming*, **43**(2), 175–186, 1989.

F. Guder and J. G. Morris. Optimal objective function approximation for separable convex quadratic programming. *Mathematical Programming*, **67**(1), 133–142, 1994.

Abstract. We present an optimal piecewise-linear approximation method for the objective function of separable convex quadratic programs. The method provides guidelines on how many grid points to use and how to position them for a piecewise-linear approximation if the error induced by the approximation is to be bounded a priori.

I. Guevski. An analytic form of solution in quadratic programming. *Tekhnicheska Misul*, **9**(2), 7–21, 1972.

Abstract. A parametric method is proposed to find a non-negative vector x*, maximizing (minimizing) the goal function $F(x) = p^T x + x^T Cx$, where A is a diagonal negatively defined matrix having dimensions of $n * n$, provided that $dx \leq b(p > 0, d > 0, b \geq 0)$. The method is used to express the relation $x = f(p, A, d, b)$ in a closed form.

I. Guevski. Parametric method for analytical solution of quadratic programming problems. *Izvestiya na Instituta po Tekhnickeska Kibernetika*, **15**, 15–28, 1973.

Abstract. A parametric method for solution of a quadratic square programming problem, arising in optimization of resource systems, is used. The dependence between the optimum vector and the coefficients in the aim functions and the limitations is obtained in apparent form. The possibility for solution of problems with a large number of variables without significant computing difficulties, is shown. A numerical example is given.

T. D. Guo and S. Q. Wu. Predictor-corrector algorithm for convex quadratic programming with upper bounds. *Journal of Computational Mathematics*, **13**(2), 161–171, 1995.

Abstract. The predictor-corrector algorithm for linear programming, proposed by S. Mizuno et al. (1990), became the most well known of the interior point methods. The purpose of the paper is to extend these results in two directions. First, we modify the algorithm in order to solve convex quadratic programming with upper bounds. Second, we replace the corrector step with an iteration of R. C. Monteiro and Adler's algorithm (1989). With these modifications, the duality gap is reduced by a constant factor after each corrector step for convex quadratic programming. It is shown that the new algorithm has a $O(\sqrt{n}L)$ iteration complexity.

T. D. Guo and S. Q. Wu. Interior ellipsoid method for convex quadratic programming. *Acta Mathematicae Applacatae Sinica*, **19**(1), 46–50, 1996a.

Abstract. In this paper, a new variant of the interior ellipsoid method for convex quadratic programming is developed. Compared to the algorithm proposed by Ye and Tse (1989), the sub-problem of the new algorithm is a linear programming problem which is easy to solve, although both have the same complexity.

T. D. Guo and S. Q. Wu. Properties of primal interior point methods for QP. *Optimization*, **37**(3), 227–238, 1996b.

Abstract. Studies the properties of a primal interior point algorithm for convex quadratic programs. The algorithm can be viewed as a modification of the algorithms presented by Monteiro and Adler (1989), Goldfarb and Liu (1991), etc. In each iteration, it computes an approximately projected Newton direction h_k and needs $O(n^{2.5})$ arithmetic operations in average. Moving along h_k with a fixed stepsize which will make the log-barrier function a significant decrease. The algorithm terminates after $O(\sqrt{n}L)$ iterations. So the total complexity of this algorithm is $O(n^3L)$.

Z. Guo and G. Xu. Calculation of economic dispatch of interconnected system using quadratic programming. *Automation of Electric Power Systems*, **22**(1), 40–44, 1998.

Abstract. Based on the principle of decomposition coordination and sensitivity analysis, a new model of the economic dispatch of interconnected power systems is presented in this paper, in which active powers of tie lines are considered as coordinating variables. The primal problem is decomposed into N individual subproblems of optimization and one simple coordination problem. The algorithm is based on quadratic programming with the idea of parametric programming and relaxation techniques. The features of the proposed algorithm are little computation and reliable convergence for economic dispatch of interconnected power systems concerning safety constraints. Numerical tests have been carried out for a real interconnected power system composed of three individual systems. The test results show that the model and the algorithm are both feasible and effective.

A. K. Gupta and J. K. Sharma. A generalized Simplex technique for solving quadratic programming problem. *Indian Journal of Technology*, **21**(5), 198–201, 1983.

Abstract. A finite iteration procedure is presented for solving the quasi-concave quadratic function subject to linear constraints. In each iteration two basic variables are replaced by two non-basic variables. The problem is solved starting with a basic feasible solution and showing the conditions under which the solution can be improved. An illustrative example is given.

O. K. Gupta. Applications of quadratic programming. *Journal of Information and Optimization Sciences*, **16**(1), 177–194, 1995.

Abstract. Quadratic programming (QP) has long been studied as an important OR technique. Many algorithms have been developed for solving QP problems. QP has also been very successful for modeling many real-life problems. This paper reviews application areas where QP has been effectively applied. Most applications of QP have been in finance, agriculture, economics, production operations, marketing, and public policy. Applications in each of these areas are briefly described.

P. Gupta and D. Bhatia. Multiparametric analysis of the maximum tolerance in quadratic programming problems. *Opsearch*, **37**(1), 36–46, 2000.

Abstract. In this paper, we use an alternative approach to study multiparametric sensitivity analysis in quadratic programming problems by using the concept of maximum volume in the tolerance region. A numerical example is given to illustrate the results of the paper.

R. P. Gupta. Symmetric dual quadratic program in complex space. *Proceedings of the Indian Academy of Sciences, Section A*, **72**(2), 74–87, 1970.

Abstract. Symmetric dual quadratic program in complex space is presented and some duality theorems are proved. Self-dual linear and quadratic programs in complex space are formed and self-duality theorem is extended to these cases.

C. D. Ha. An algorithm for structured, large-scale quadratic programming problems. Technical report, Virginia Commonwealth University, Virginia, USA, 1981.

W. W. Hager and D. W. Hearn. The dual active set method and quadratic networks. Research Report 90-7, Department of Industrial and Systems Engineering, Gainesville, Florida, USA, 1990.

W. W. Hager and Y. Krylyuk. Graph partitioning and continuous quadratic programming. *SIAM Journal on Discrete Mathematics*, **12**(4), 500–523, 1999.

Abstract. A continuous quadratic programming formulation is given for min-cut graph partitioning problems. In these problems, we partition the vertices of a graph into a collection of disjoint sets satisfying specified size constraints, while minimizing the sum of weights of edges connecting vertices in different sets. An optimal solution is related to an eigenvector (Fiedler vector) corresponding to the second smallest eigenvalue of the graph's Laplacian. Necessary and sufficient conditions characterizing local minima of the quadratic program are given. The effect of diagonal perturbations on the number of local minimizers is investigated using a test problem from the literature.

W. W. Hager, T. A. Davis, Y. Krylyuk, and S.-C. Park. Graph partitioning using quadratic programming. Technical report, Computer and Information Science and Engineering Department, Gainesville, Florida, USA, 1997.

Abstract. A continuous quadratic programming formulations are given for min-cut graph partitioning problems. An optimal solution is related to an eigenvector (Fiedler vector) corresponding to the second smallest eigenvalue of the graph's Laplacian. Necessary and sufficient conditions characterizing local minima of the quadratic program are given. A generalization of the Kernighan and Lin exchange algorithm to a block exchange algorithm can be used to escape from a local minimum of the continuous quadratic program. Comparisons to the optimal cut obtained by other approaches to graph partitioning are presented.

W. W. Hager, P. M. Pardalos, I. M. Roussos, and H. D. Sahinoglou. Active constraints, indefinite quadratic programming, and test problems. *Journal of Optimization Theory and Applications*, **68**(3), 499–511, 1991.

H. H. Hall, E. O. Heady, A. Stoecker, and V. A. Sposito. Spatial equilibrium in us agriculture: a quadratic programming analysis. *SIAM Review*, **17**(2), 323–338, 1975.

Abstract. A spatial competitive equilibrium for the crop and livestock sectors of the US agricultural economy is approximated. Agricultural commodities are classified into three mutually exclusive classes: primary, intermediate and desired. Primary commodities represent available resources; intermediate commodities are produced only as inputs for further production; desired commodities are wanted either for consumption or for other uses outside the system (export). Production of crops and livestock and intermarket commodity shipments are represented by linear activities. The objective function maximizes aggregate producer profits. Since the demand functions are linear, total revenue is quadratic in the prices of desired commodities, hence the quadratic programming formulation. In the results, estimated prices for desired commodities are lower than observed prices and, with minor exceptions, estimated quantities exceed observed quantities.

P. L. Hammer and A. A. Rubin. Some remarks on quadratic programming with 0–1 variables. *Revue Francaise d'Informatique et de Recherche Operationnelle*, **4**(3), 67–79, 1970.

Abstract. The aim of this paper is to show that every bivalent (0,1) quadratic programming problem is equivalent to one having a positive (negative) semi-definite matrix in the objective function, to establish conditions for different classes of local optimality, and to show that any problem of bivalent (0,1) programming is equivalent (a) to the problem of minimizing a real valued function, partly in (0,1) and partly in non-negative variables, (b) to the problem of finding the minimax of a real valued function in bivalent (0,1) variables.

P. L. Hammer, P. Hansen, and B. Simeone. Roof duality, complementation and persistency in quadratic 0–1 optimization. *Mathematical Programming*, **28**(2), 121–155, 1984.

C. G. Han, P. M. Pardalos, and Y. Ye. Computational aspects of an interior point algorithm for quadratic-programming problems with box constraints. *In* T. F. Coleman and Y. li, eds, 'Large-Scale Numerical Optimization', pp. 92–112, SIAM, Philadelphia, USA, 1990a.

C. G. Han, P. M. Pardalos, and Y. Ye. Interior point algorithms for quadratic programming problems. *In* 'Proceedings of the Conference on Optimization Methods and their Applications, Nauka, USSR', pp. 194–213, 1990b. (In Russian).

C. G. Han, P. M. Pardalos, and Y. Ye. Algorithms for the solution of quadratic knapsack problems. *Linear Algebra and its Applications*, **152**, 69–91, 1991.

C. G. Han, P. M. Pardalos, and Y. Ye. On the solution of indefinite quadratic problems using an interior point method. *Informatica*, **3**(4), 474–496, 1992.

S. P. Han. Solving quadratic programs with an exact penalty function. *In* O. L. Mangasarian, R. R. Meyer and S. M. Robinson, eds, 'Nonlinear Programming, 4', pp. 25–55, Academic Press, London and New York, 1981.

S. P. Han. On the hessian of the Lagrangian and second-order optimality conditions. *SIAM Journal on Control and Optimization*, **24**, 339–345, 1985.

S. P. Han and O. Fujiwara. An inertia theorem for symmetric matrices and its application to nonlinear programming. *Linear Algebra and its Applications*, **72**, 47–58, 1985.

S. P. Han and O. L. Mangasarian. Characterization of positive definite and semidefinite matrices via quadratic programming duality. *SIAM Journal of Algebraic and Discrete Methods*, **5**(1), 26–32, 1984.

Abstract. Positive definite and semidefinite matrices induce well-known duality results in quadratic programming. The converse is established here. Thus if certain duality results hold for a pair of dual quadratic programs, then the underlying matrix must be positive definite or semidefinite. For example, if a strict local minimum of a quadratic program exceeds or equals a strict global maximum of the dual, then the underlying symmetric matrix Q is positive definite. If a quadratic program has a local minimum, then the underlying matrix Q is positive semidefinite if and only if the primal minimum exceeds or equals the dual global maximum and $x^T Qx = 0$ implies $Qx = 0$. A significant implication of these results is that the Wolfe dual may not be meaningful for nonconvex quadratic programs and for nonlinear programs without locally positive definite or semidefinite Hessians, even if the primal second order sufficient optimality conditions are satisfied.

M. T. Hanna and M. Simaan. A closed form solution to a quadratic programming problem in complex variables. *In* 'Proceedings of the 23rd IEEE Conference on Decision and Control. IEEE, New York, NY, USA', Vol. 2, pp. 1087–1092, 1984.

Abstract. A special quadratic programming problem in complex variables is investigated for a closed form solution. Two different approaches are used. The first is a direct approach that leads to a family of solutions because of a singular matrix encountered in the solution process. The second is an indirect approach based on parameterizing the objective function. It leads to a solution which is a member in the above family and which is shown to be bounded.

P. Hansen. Quadratic 0–1 programming by implicit enumeration. *In* F. A. Lootsma, ed., 'Numerical methods for non linear optimization', pp. 265–278. Academic Press, London, England, 1972.

P. Hansen and B. Jaumard. Reduction of indefinite quadratic programs to bilinear programs. *Journal of Global Optimization*, **2**, 41–60, 1992.

P. Hansen, B. Jaumard, M. Ruiz, and J. Xiong. Global minimization of indefinite quadratic functions subject to box constraints. Report G-91-54, GERAD, Ecole Polytechnique, Université McGill, Montreal, Quebec, Canada, 1991.

F. Harrigan and I. Buchanan. A quadratic-programming approach to input-output estimation and simulation. *Journal of Regional Science*, **24**(3), 339–358, 1984.

K. Hassan and F. Mahmoud. An incremental approach for the solution of quadratic programming problems. *Mathematical Modelling*, **8**, 34–36, 1987.

Abstract. An approach for the solution of quadratic programming problems is introduced. It is based on an incremental method, and the maximum number of increments is equal to the number of constraints plus 1.

E. J. Haug, R. Chand, and K. Pan. Multibody elastic contact analysis by quadratic programming. *Journal of Optimization Theory and Applications*, **21**(2), 189–198, 1977.

Abstract. A quadratic programming method for contact problems is extended to a general problem involving contact of n elastic bodies. Sharp results of quadratic programming theory provide an equivalence between the original n-body contact problem and the simplex algorithm used to solve the quadratic programming problem. Two multibody examples are solved to illustrate the technique.

B. S. He. A projection and contraction method for a class of linear complementarity-problems and its application in convex quadratic programming. *Applied Mathematics and Optimization*, **25**(3), 247–262, 1992.

Abstract. In this paper we propose a new iterative method for solving a class of linear complementarity problems: $u \geq 0$, $Mu + q \geq$, $u^T(Mu + q) = 0$, where M is a given $l \times l$ positive semidefinite matrix (not necessarily symmetric) and q is a given l-vector. The method makes two matrix-vector multiplications and a trivial projection onto the nonnegative orthant at each iteration, and the Euclidean distance of the iterates to the solution set monotonously converges to zero. The main advantages of the method presented are its simplicity, robustness, and ability to handle large problems with any start point. It is pointed out that the method may be used to solve general convex quadratic programming problems. Preliminary numerical experiments indicate that this method may be very efficient for large sparse problems.

G. L. Hefley and M. E. Thomas. A comparison of two quadratic programming algorithms. Technical report, Florida Univ , Gainesville, FL, USA, 1970.

Abstract. The paper compares Wolfe's quadratic programming algorithm with Cottle and Dantzig's principle pivot method. It is shown that Wolfe's algorithm requires more operations.

R. Helgason, J. L. Kennington, and H. Lall. A polynomially bounded algorithm for a singly constrained quadratic program. *Mathematical Programming*, **18**(3), 338–343, 1980.

Abstract. Presents a characterization of the solutions of a singly constrained quadratic program. This characterization is then used in the development of a polynomially bounded algorithm for this class of problems.

C. Helmberg. Fixing variables in semidefinite relaxations. *SIAM Journal on Matrix Analysis and Applications*, **21**(3), 952–969, 2000.

Abstract. The standard technique of reduced cost fixing from linear programming is not trivially extensible to semidefinite relaxations because the corresponding Lagrange multipliers are usually not available. We propose a general technique for computing reasonable Lagrange multipliers for constraints that are not part of the problem description. Its specialization to the semidefinite $\{-1,1\}$ relaxation of quadratic 0–1 programming yields an efficient routine for fixing variables. The routine offers the possibility of exploiting problem structure. We extend the traditional bijective map between $\{0,1\}$ and $\{-1,1\}$ formulations to the constraints so that the dual variables remain the same and structural properties are preserved. Consequently, the fixing routine can be applied efficiently to optimal solutions of the semidefinite $\{0,1\}$ relaxation of constrained quadratic 0–1 programming as well. We provide numerical results showing the efficacy of this approach.

M. P. Helme and T. L. Magnanti. Designing satellite communication networks by 0–1 quadratic programming. *Networks*, **19**(4), 427–450, 1989.

Abstract. In satellite communication networks, distinctive facilities called homing stations perform special transmission functions. Local demand nodes clustered around each homing station communicate with each other via a local switch at the homing station; demand nodes in different clusters communicate with each other via satellite earth stations at the homing stations. Designing such a communication network requires choices on the locations of the earth stations and on the assignments of demand nodes to the local clusters at the earth stations. The authors formulate this problem as a 0–1 quadratic facility location problem and transform it into an equivalent 0–1 integer linear program. Computational experience on real data shows that a branch and bound procedure is effective in solving problems with up to 40 demand nodes (major cities) and that the solutions that this algorithm finds improve considerably upon management generated solutions. It is also shown that a greedy add heuristic, as implemented on an IBM PC, consistently generates optimal or near-optimal solutions.

C. T. Herakovich and P. G. Hodge. Elastic-plastic torsion of hollow bars by quadratic programming. *International Journal of Mechanical Sciences*, **11**(1), 53–63, 1969.

Abstract. A numerical method which provides the complete history of the stress function for elastic-plastic torsion of hollow bars during quasistatic, monotonic twist is presented. Results are given for several cross-sections and are compared to other available results. Plastic unloading is explicitly shown.

G. T. Herman and A. Lent. A family of iterative quadratic optimization algorithms for pairs of inequalities, with application in diagnostic radiology. *Mathematical Programming Studies*, **9**, 15–29, 1978.

C. Hildreth. A quadratic programming procedure. *Naval Research Logistics Quarterly*, **4**, 79–85, 1957. Erratum, ibid. p. 361.

K. Hitomi and H. Nagasawa. Note on "Solution of the aggregate production planning problem in a multi-stage-multi-product manufacturing system using functional space analysis and quadratic programming approaches". *International Journal of Systems Science*, **15**(11), 1257–1262, 1984.

Abstract. The paper of Teny and Kochhar (see ibid., vol. = 14, num. 3, p. 325, 1983) concluded that the solution calculated by using the functional space analysis technique presented by Hitomi and Nakamura (1976) is inferior to that rendered by the quadratic programming approach (Beale, 1968). This paper shows the contradiction included in their discussion and denies the above conclusion. Functional space analysis is also developed to obtain a mixed-integer solution.

D. S. Hochbaum and S. P. Hong. About strongly polynomial-time algorithms for quadratic optimization over submodular constraints. *Mathematical Programming*, **69**(2), 269–309, 1995.

D. S. Hochbaum, R. Shamir, and J. G. Shanthikumar. A polynomial algorithm for an integer quadratic nonseparable transportation problem. *Mathematical Programming*, **55**(3), 359–371, 1992.

P. G. Hodge, T. Belytschko, and C. T. Herakovich. Plasticity and quadratic programming. *Computer Approaches in Applied Mechanics ASME*, pp. 73–84, 1969.

V. R. Hoffner. A quadratic programming formulation of the Markov student flow problem. *TIMS/ORSA-Bulletin*, **1**, 91, 1976.

Abstract. Discusses the problems of forecasting educational enrollments with the use of a Markov flow model. In many cases, the student transition data needed to estimate the transition probabilities by the maximum likelihood technique is not available. The aggregate enrollments are known, the transition probabilities can be estimated by using a quadratic programming formulation of the problem. This method is used to forecast elementary, secondary, and higher education enrollments in a continuous flow process.

W. Hollenstein and H. Glavitsch. Constraints in quadratic programming treated by switching concepts (power systems). *In* 'Proceedings of the Tenth Power Systems Computation Conference', pp. 551–558. Butterworths, London, England, 1990.

Abstract. When applying standard quadratic programming methods to large power systems the size of tableaus is a handicap. Hence, an approach was chosen whereby penalty terms as usually employed for constraining variables are converted to straight inequalities thus yielding linear programming techniques. The method developed works in two steps. The first solves the unconstrained problem which is linear, the second solves superimposed LP problem. For computational reasons the violation of constraints is monitored and the LP tableau is built up as violations are detected. Sparsity, forward-backward substitution and updating techniques are exploited. Economic load dispatching and loss minimization are examples to illustrate the effectiveness of the method.

S. P. Hong and S. Verma. A note on the strong polynomiality of convex quadratic programming. *Mathematical Programming*, **68**(2), 131–139, 1995.

Abstract. We prove that a general convex quadratic program (QP) can be reduced to the problem of finding the nearest point on a simplicial cone in O(n(3) + n log L) steps, where n and L are, respectively, the dimension and the encoding length of QP. The proof is quite simple and uses duality and repeated perturbation, The implication, however, is nontrivial since the problem of finding the nearest point on a simplicial cone has been considered a simpler problem to solve in the practical sense due to its special structure, Also we

show that, theoretically, this reduction implies that (i) if an algorithm solves QP in a polynomial number of elementary arithmetic operations that is independent of the encoding length of data in the objective function then it can be used to solve QP in strongly polynomial time, and (ii) if L is bounded by a 'first order' exponential function of n then (i) can be stated even in stronger terms: to solve QP in strongly polynomial time, it suffices to find an algorithm running in polynomial time that is independent of the encoding length of the quadratic term matrix or constraint matrix. Finally, based on these results, we propose a conjecture.

R. Horst and N. Van Thoai. A new algorithm for solving the general quadratic programming problem. *Computational Optimization and Applications*, **5**(1), 39–48, 1996.

Abstract. For the general quadratic programming problem (including an equivalent form of the linear complementarity problem) a new solution method of branch and bound type is proposed. The branching procedure uses a well-known simplicial subdivision and the bound estimation is performed by solving certain linear programs.

H. S. Houthakker. The Capacity method of quadratic programming. *Econometrica*, **28**, 62–87, 1960.

S. Hoyle. A single-phase method for quadratic programming. Technical Reort SOL 86-9, Department of Operations Research, Stanford University, California, USA, 1986.

W. S. Hsia. A method for strictly convex quadratic programming problems. *Opsearch*, **14**, 118–124, 1977.

G. H. Huang and B. W. Baetz. Grey quadratic programming and its application to municipal solid-waste management planning under uncertainty. *Engineering Optimization*, **23**(3), 201–223, 1995.

Abstract. This paper introduces a grey quadratic programming (GQP) method as a means for decision making under uncertainty. The method improves upon existing grey linear programming (GLP) methods by allowing the consideration of the effects of economies of scale on cost coefficients in the objective function. The approach also has advantages over a grey nonlinear programming method, since a global optimum is obtainable and the model is moderately easy to solve through commercially available quadratic programming packages. The modelling approach is applied to a hypothetical problem of waste flow allocation within a municipal solid waste management system. The results indicate that, compared with the GLP method, GQP provides a more effective means for reflecting system cost variations and may therefore generate more realistic and applicable solutions.

G. H. Huang, B. W. Baetz, and G. G. Patry. Waste flow allocation planning through a grey fuzzy quadratic-programming approach. *Civil Engineering Systems*, **11**(3), 209–243, 1994.

Abstract. This paper proposes a grey fuzzy quadratic programming (GFQP) approach as a means for optimization analysis under uncertainty. The method combines the ideas of grey fuzzy linear programming (GFLP) and fuzzy quadratic programming (FQP) within a general optimization framework. It improves upon the previous GFLP method by using n grey control variables, $x(\lambda(i))(i = 1, 2, ..., n)$, for n constraints instead of one $x(\lambda)$ for n constraints in order to incorporate the independent properties of the stipulation uncertainties; it also improves upon the FQP method by further introducing grey numbers for coefficients in A and C to effectively reflect the lefthand side uncertainties. Compared with the GFLP method, the GFQP approach is helpful for better satisfying model objective/constraints and providing grey solutions with higher system certainty and lower system cost; compared with the FQP method, more information of the independent uncertain features of not only the stipulations but also the lefthand side coefficients are effectively reflected in the GFQP method. The GFQP modelling approach is applied to a hypothetical case study of waste flow allocation planning under uncertainty, with the input model stipulations fluctuating within wide intervals and having independent uncertain characteristics. The results indicated that reasonable solutions have been generated. Comparisons between the GFQP and FQP/GFLP solutions are also provided, which demonstrate

that the GFQP method could better reflect system uncertainties and provide more realistic and applicable solutions with lower system uncertainties and higher system benefits.

M. Huang. Linear and quadratic programming methods for solving security-constrained economic dispatch problems. Master's thesis, Department of Electrical and Computer Engineering, University of Waterloo, Ontario, Canada, 1991.

J. Huber. A cross-over method for quadratic programming. Diplomathesis, Ruprecht-Karls-UniversitÄt Heidelberg, Germany, 1998.

IBM Optimization Solutions and Library. *QP Solutions User Guide*. IBM Corportation, 1998.

A. Ichikawa, T. Yoshida, K. Ichino, and Y. Sakai. Image extrapolation for padding-DCT using quadratic programming. *ITG Fachberichte*, **143**, 701–704, 1997.

Abstract. This paper proposes an extrapolation method for padding-DCT in encoding an arbitrary shaped image. First the extrapolation problem is formulated as an error minimization problem and then it is iteratively solved as a standard quadratic programming problem. Two examples given in this paper show that the proposed method outperforms an average extrapolation method and the POCS-based extrapolation method in an exchange of the calculation time.

A. U. Idnani. *Numerically stable dual projection methods for solving positive definite quadratic programs*. PhD thesis, Department of Computer Science, City College of New York, New Youk, USA, 1980.

M. R. Irving and M. J. H. Sterling. Economic dispatch of active power by quadratic programming using a sparse linear complementary algorithm (transmission networks). *International Journal of Electrical Power and Energy Systems*, **7**(1), 2–6, 1985.

Abstract. A new method for the economic dispatch of active power in transmission networks is presented. Formulation of the problem as a quadratic program allows the inclusion of a linear network model and operational constraints together with an explicit representation of the cost incurred by transmission losses. The problem is solved by a linear complementary pivoting algorithm which takes full advantage of the sparse form of the objective function and constraints. Computational experience indicates that the method is highly efficient and is capable of solving realistic problems in elapsed times that are compatible with online applications using minicomputer hardware.

H. Isoda. Decision of optimum supply voltage of substation transformers by quadratic programming. I. Search of a feasible solution using Crout's method. *Electrical Engineering in Japan*, **95**(2), 33–39, 1975a.

Abstract. Proposes a digital computing method to determine the optimum supply voltage of substation transformers. The supply voltage of substation transformers is regulated so that the supply voltage to consumers is kept within an allowable range without overloading substation transformers and distribution lines. This problem can be formulated into quadratic programming, which can be solved most typically by Beale's method. Crout's method is useful for solving large-scale simultaneous linear equations and for finding a feasible solution.

H. Isoda. Decision of optimum supply voltage of substation transformers by quadratic programming. II. Optimization technique and its program for practical networks by Beale's method. *Electrical Engineering in Japan*, **95**(2), 39–45, 1975b.

Abstract. For pt.I see ibid., vol.95, no.2, p.33 (1975). Beale's method is used to determine the truly optimal solution of the quadratic programming. A practical digital computer program is also developed and applied to a large-scale distribution network containing six transformer banks, 34 feeders and 177 load points. The total computing time is about 4.0 sec. The required memory size is as small as 20 kilowords.

A. N. Iusem and A. R. Depierro. On the convergence properties of Hildreth's quadratic-programming algorithm. *Mathematical Programming*, **47**(1), 37–51, 1990.

M. Jackson and M. D. Staunton. Quadratic programming applications in finance using excel. *Journal of the Operational Research Society*, **50**(12), 1256–1266, 1999.

Abstract. The paper describes two applications of quadratic programming in finance, one from the early years (Markowitz's efficient portfolios with minimum risk) and the other a more recent innovation (Sharpe's style analysis which estimates an implied asset allocation for an investment fund). We show how, in the presence of inequality constraints, Excel's Solver can be used to find the optimal weights in both quadratic programming applications. We also implement a direct analytic solution for generating the efficient frontier when there are no inequality constraints using the matrix functions in Excel. Both applications use only a small number of asset classes and require repeated use of the minimisation task. We show how Visual Basic for Applications (Microsoft's macro language for Excel) can be used to program such tasks, confirming that techniques that were the preserve of dedicated software only a few years ago can now be easily replicated using Excel to solve real problems.

H. Jaddu and E. Shimemura. Computation of optimal control trajectories using Chebyshev polynomials: parametrization, and quadratic programming. *Optimal Control Applications and Methods*, **20**(1), 21–42, 1999.

Abstract. An algorithm is proposed to solve the optimal control problem for linear and nonlinear systems with quadratic performance index. The method is based on parametrizing the state variables by Chebyshev series. The control variables are obtained from the system state equations as a function of the approximated state variables. In this method, there is no need to integrate the system state equations, and the performance index is evaluated by an algorithm which is also proposed in this paper. This converts the optimal control problem into a small size parameter optimization problem which is quadratic in the unknown parameters, therefore the optimal value of these parameters can be obtained by using quadratic programming results. Some numerical examples are presented to show the usefulness of the proposed algorithm.

F. Jarre. On the convergence of the method of analytic centers when applied to convex quadratic programs. *Mathematical Programming*, **49**(3), 341–358, 1991.

F. Jarre, G. Sonnevend, and J. Stoer. On the complexity of a numerical algorithm for solving generalized convex quadratic programs by following a central path. *In* J. C. Lagarias and M. J. Todd, eds, 'Mathematical Developments Arising from Linear Programming : Proceedings of a Joint Summer Research Conference held at Bowdoin College, Brunswick, Maine, USA, June/July 1988', Vol. 114 of *Contemporary Mathematics*, pp. 233–242. American Mathematical Society, Providence, Rhode Island, USA, 1990.

T. R. Jefferson and C. H. Scott. Quadratic geometric-programming with application to machining economics. *Mathematical Programming*, **31**(2), 137–152, 1985.

D. L. Jensen and A. J. King. A decomposition method for quadratic programming. *IBM Systems Journal*, **31**(1), 39–48, 1992.

Abstract. We discuss the algorithms used in the Optimization Subroutine Library for the solution of convex quadratic programming problems. The basic simplex algorithm for convex quadratic programming is described. We then show how the simplex method for linear programming can be used in a decomposition crash procedure to obtain a good initial basic solution for the quadratic programming algorithm. We show how this solution may be used as a starting solution for the simplex-based algorithm. Besides its ability to obtain good starting solutions, this procedure has several additional properties. It can be used directly to find an optimal solution to a quadratic program instead of simply finding a good initial solution; it provides both

upper and lower bounds on the objective function value as the algorithm proceeds; it reduces the complexity of intermediate calculations; it avoids certain numerical difficulties that arise in quadratic, but not linear programming.

S. Jha and P. M. Pardalos. Parallel search algorithms for quadratic 0–1 programming. Preprint, Department of Computer Science, The Pennsylvania State University, USA, 1988.

S. Jha and P. M. Pardalos. Graph separation techniques for quadratic 0–1 programming. *Computers Math. Applic.*, **21**(6/7), 107–113, 1991.

S. Jha and P. M. Pardalos. Complexity of uniqueness and local search in quadratic 0–1 programming. *Operations Research Letters*, **11**, 119–123, 1992.

A. R. Johnson and C. J. Quigley. Frictionless geometrically non-linear contact using quadratic programming. *International Journal for Numerical Methods in Engineering*, **28**(1), 127–144, 1989.

H. Jones and G. Mitra. Solution of the convex quadratic programming problem using the interior point method. Technical Report TR/03/97, Department of Mathematical Sciences, Brunel University, Uxbridge, Middlesex, England, 1997.

J. J. Júdice. The duality theory of general quadratic programs. *Portugaliae Mathematica*, **42**, 113–121, 1984.

J. J. Júdice and A. M. Faustino. A computational analysis of LCP methods for bilinear and concave quadratic programming. *Computers and Operations Research*, **18**(8), 645–654, 1991.

Abstract. The use of a sequential linear complementarity problem (SLCP) algorithm for finding a global minimum of bilinear programming problem (BLP) or a concave quadratic program (CQP) is examined. The algorithm consists of solving a sequence of linear complementarity problems (LCP). A branch-and-bound method is also considered. This algorithm is based on the reformulation of a BLP into an LCP with a linear function to minimize. Computational experience with small and medium scale BLPs and CQPs indicates that the SLCP algorithm is quite efficient in finding a global minimum but it is, in general, unable to establish that such a solution has been found. An algorithm to find a lower-bound for the BLP can overcome this drawback in some cases. Furthermore the SLCP algorithm is shown to be robust and compares favorably with the branch-and-bound method and another alternative technique.

J. J. Júdice and K. G. Murty. On the complexity of finding stationary points of nonconvex quadratic programs. *Opsearch*, **33**, 162–166, 1996.

J. J. Júdice and F. M. Pires. A comparison between direct and iterative methods for solving large-scale convex quadratic programs on the simplex. *Pesquisa Operacional*, **9**, 55–78, 1989a.

J. J. Júdice and F. M. Pires. Direct methods for convex quadratic programs subject to box constraints. *Investigación Operacional*, **9**(1), 23–56, 1989b.

J. J. Júdice and F. M. Pires. Solution of large-scale separable strictly convex quadratic programs on the simplex. *Linear Algebra and its Applications*, **170**, 214–220, 1992.

J. J. Júdice, P. M. Pardalos, and L. N. Vicente. Parametric linear programming techniques for the indefinite quadratic programming problem. *IMA Journal of Mathematics Applied in Business and Industry*, **4**, 343–349, 1993.

J. J. Júdice, G. Savard, and L. N. Vicente. Descent approaches for quadratic bilevel programming. *Journal of Optimization Theory and Applications*, **81**, 379–399, 1994.

M. Junger, A. Martin, G. Reinelt, and R. Weismantel. Quadratic 0–1 optimization and a decomposition approach for the placement of electronic-circuits. *Mathematical Programming, Series B*, **63**(3), 257–279, 1994.

D. G. Kabe. A note on a quadratic programming problem. *Industrial Mathematics*, **23**(1), 61–66, 1973.

Abstract. This paper gives a simpler exposition of a quadratic programming problem considered by Nelson, Lewis and Boullion.

D. G. Kabe. Bimatrix games and symmetric vector quadratic programming problems. *Industrial Mathematics*, **29**(1), 17–26, 1979.

Abstract. Some results on minima of vector quadratic forms given by Kabe (1968) are used to derive direct solutions to two person nonzero sum matrix (i.e., bimatrix) games and to obtain symmetric matrix solutions to vector quadratic programming problems, and to obtain solutions to linear interval programming problems.

D. G. Kabe. A quadratic-programming approach to the construction of simultaneous confidence-intervals. *Communications in Statistics-Theory and Methods*, **12**(17), 2053–2058, 1983.

D. G. Kabe. On discrete linear and quadratic-programming problems. *Utilitas Mathematica*, **27**, 275–286, 1985.

D. G. Kabe. On solving 2 variables continuous parameter linear and quadratic-programming problems. *Utilitas Mathematica*, **30**, 181–189, 1986a.

D. G. Kabe. On solving linear and quadratic programming problems. *Industrial Mathematics*, **36**(2), 189–196, 1986b.

Abstract. With the aid of the generalized inverse of matrices and vanishing Jacobians theory, a straightforward methodology is developed for solving linear and quadratic programming problems. In particular, the methodology solves the transportation problem very efficiently. The theory is illustrated by numerical examples.

D. G. Kabe. On solving hypercomplex linear and quadratic programming problems. *Industrial Mathematics*, **38**(1), 1–15, 1988a.

Abstract. Some results of the minimization theory of hypercomplex vector quadratic forms, which are subjected to linear restrictions are utilized to obtain direct solutions to hypercomplex linear and quadratic programming problems. In some particular cases, the methodology presented yields faster solutions to these problems that the mathematical programming methodologies.

D. G. Kabe. On some multivariate generalizations of linear and quadratic programming problems. *Industrial Mathematics*, **38**(1), 17–27, 1988b.

Abstract. Although the univariate linear and quadratic programming theory is now thoroughly explored in the literature, its multivariate generalizations appear to have been neglected. The paper presents some possible generalizations of the univariate linear complementary programming problem is presented. The generalizations are attempted by the classical calculus methodology.

D. G. Kabe. On solving matrix quadratic-programming problems. *Journal of Mathematical Analysis and Applications*, **161**(1), 212–217, 1991.

P. D. Kaklis and G. D. Koras. A quadratic-programming method for removing shape-failures from tenser-product B-spline surfaces. *In* 'Geometric Modelling', Vol. 13, pp. 177–188, 1998.

R. E. Kalaba and F. E. Udwadia. Lagrangian mechanics, Gauss's principle, quadratic programming, and generalized inverses: new equations for nonholonomically constrained discrete mechanical systems. *Quarterly of Applied Mathematics*, **52**(2), 229–241, 1994.

Abstract. We formulate Lagrangian mechanics as a constrained quadratic minimization problem. This quadratic minimization problem is then solved using the theory of generalized inverses of matrices thereby obtaining the explicit equations of motion of constrained, discrete mechanical systems. The approach extends the boundaries of Lagrangian mechanics in that we provide a general formulation for describing the constrained motion of such systems without either the use of Lagrange multipliers or the use of quasi-coordinates. An important feature of the approach is that we do not require prior knowledge of the specific set of constraints to accomplish this formulation. This makes the equations presented here more generally useful, and perhaps more aesthetic, than the Gibbs-Appell equations which require a felicitous choice of problem-specific quasi-coordinates. The new equations of motion presented here are applicable to both the holonomic and nonholonomic constraints that Lagrangian mechanics deals with. They are obtained in terms of the usual generalized coordinates used to describe the constrained system. Furthermore, they can be integrated by any of the currently available numerical integration methods, thus yielding analytical and/or computational descriptions of the motions of constrained mechanical systems.

B. Kalantari. *Large scale global minimization of linearly constrained concave quadratic functions and related problems*. PhD thesis, Computer Science Department, University of Minnesota, Minneapolis, Minnesota, USA, 1984.

B. Kalantari. Canonical problems for quadratic programming and projective methods for their solution. *In* J. C. Lagarias and M. J. Todd, eds, 'Mathematical Developments Arising from Linear Programming : Proceedings of a Joint Summer Research Conference held at Bowdoin College, Brunswick, Maine, USA, June/July 1988', Vol. 114 of *Contemporary Mathematics*, pp. 243–263. American Mathematical Society, Providence, Rhode Island, USA, 1990.

B. Kalantari and J. B. Rosen. Construction of large-scale global minimum concave quadratic test problems. *Journal of Optimization Theory and Applications*, **48**(2), 303–313, 1986.

Abstract. Construction of problems with known global solutions is important for the computational testing of constrained global minimization algorithms. It is shown how to construct a concave quadratic function which attains its global minimum at a specified vertex of a polytope in $\mathbb{R}^{n+k}$. The constructed function is strictly concave in the variables $x \in \mathbb{R}^n$ and is linear in the variables $y \in \mathbb{R}^k$. The number of linear variables k may be much larger than n, so that large-scale global minimization test problems can be constructed by the methods described.

B. Kalantari and J. B. Rosen. An algorithm for global minimization of linearly constrained concave quadratic functions. *Mathematics of Operations Research*, **12**(3), 544–561, 1987.

Abstract. We present an algorithm for the global minimization of a quadratic function $\psi(x,y) = -1/2x^T Qx + h^T x + d^T y$, over a polytope $\Omega = ((x,y) \in R^{n+k} : Ax + By \leq b, x \geq 0, y \geq 0)$, where x, $h \in \mathbb{R}^n$, $y, d \in \mathbb{R}^k$, $b \in \mathbb{R}^m$, and where A and B are $m*n$ and $m*k$ matrices, respectively, and Q is an $n*n$ symmetric positive definite matrix. The authors first consider the case where $k = 0$ and construct a 'tight' parallelepiped R, containing Omega by using an arbitrary set of Q-conjugate directions and by solving $2n$ linear programs. They then describe a branch and bound algorithm in which the linearity of the convex envelope allows efficient lower bounding for the subproblems. The results are then generalized to the case where k is nonzero and possibly much larger than n. Preliminary computational results are also presented.

A. P. Kamath and N. Karmarkar. An $O(nL)$ iteration algorithm for computing bounds in quadratic optimization problems. *In* P. M. Pardalos, ed., 'Complexity in Numerical Optimization', pp. 254–268, World Scientific, Singapore, 1993.

Abstract. We consider the problem of optimizing a quadratic function subject to integer constraints. This problem is NP-hard in the general case. We present a new polynomial time algorithm for computing bounds on the solutions to such optimization problems. We transform the problem into a problem for minimizing the trace of a matrix subject to positive definiteness condition. We then propose an interior-point method to solve this problem. We show that the algorithm takes no more than $O(nL)$ iterations (where L is the the number of bits required to represent the input). The algorithm does two matrix inversions in each iteration.

A. G. Kaplan. On one doubly-connected problem of quadratic programming. *Izvestiya Akademii Nauk Azerbaidzhanskoi SSR, Seriya Fiziko—Tekhnicheskikh i Matematicheskikh Nauk*, **2**, 158–162, 1980.

S. Kapoor and P. M. Vaidya. Fast algorithms for convex quadratic programming and multicommodity flows. *In* 'Proceedings of the Eighteenth Annual ACM Symposium on Theory of Computing', pp. 147–159, 1986.

S. Kapoor and P. M. Vaidya. An extension of Karmarkar's interior point method to convex quadratic programming. Technical report, Department of Computer Science, University of Illinois at Urbana–Champaign, Illinois, USA, 1988.

G. Karg and J. Keck. Optimal nutrition in institutional catering with the help of linear and quadratic programming. *Ernahrungs-Umschau*, **29**(8), 260–268, 1982.

T. Kashiwagi, T. Wakabayashi, Y. Hayashi, and S. Iwamoto. ELD calculation using quadratic programming based on binary search. *Stockholm Power Tech International Symposium on Electric Power Engineering IEEE, New York, NY, USA*, **5**, 55–60, 1995.

Abstract. In this paper, we propose an application of "quadratic programming based on binary search (QPBS)" to ELD (economic load dispatch) calculation for the purpose of determining the optimal generation allocation of thermal units accurately and quickly. Most conventional equal incremental methods cannot always obtain the optimal solution for ELD problems when transmission losses and/or upper and lower bounds of generator outputs are taken into account. Numerous studies have been made to solve the problems accurately with considerable complexities. The formulation of the problem considered as standard for QPBS is very similar to that of the ELD problem when transmission losses are neglected. Therefore to the ELD calculation we apply the QPBS algorithm, especially the Pardalos-Kovoor method, whose computing time order is $O(n)$ where n is the number of variables.

L. Kaufman. Solving the quadratic programming problem arising in support vector classification. *In* B. Schlkopf, C. J. C. Burges and A. J. Smola, eds, 'Advances in Kernel Methods - Support Vector Learning'. MIT Press, 1998.

R. N. Kaul, D. Bhatia, and P. Gupta. Tolerance approach to sensitivity analysis in quadratic programming problems. *Opsearch*, **36**(1), 1–9, 1999.

Abstract. Here, we apply the tolerance approach for sensitivity analysis in quadratic programming problems. We find the maximum tolerance for the simultaneous perturbation in right-hand-side terms of the problem such that the original and the perturbed problems have the same optimal basis. The maximum tolerance for the coefficients in the linear part of the objective function of the problem is also calculated.

E. L. Keller. *Quadratic optimization and linear complementarity*. PhD thesis, University of Michigan, Ann Arbor, USA, 1968.

E. L. Keller. The general quadratic programming problem. *Mathematical Programming*, **5**(3), 311–337, 1973.

A. Kennings and A. Vannelli. An efficient interior point approach for QP and LP models of the relative placement problem. *In* G. Cameron, M. Hassoun, A. Jerdee and C. Melvin, eds, 'Proceedings of the 39th Midwest Symposium on Circuits and Systems. IEEE, New York, NY, USA', Vol. 1, pp. 443–446, 1996.

Abstract. Cell placement can be performed using a combination of mathematical programming, graph partitioning and iterative improvement techniques. Mathematical programming provides the relative positions of cells throughout the placement area while ignoring several placement restrictions. We describe quadratic and linear program formulations for finding relative cell positions. Moreover, we demonstrate that both formulations can be solved using an interior point method. Numerical results are presented to demonstrate the effectiveness of the formulations and the solution methodology.

A. Kennings and A. Vannelli. VLSI placement using quadratic programming and network partitioning techniques. *International Transactions in Operational Research*, **4**(5–6), 353–364, 1997.

Abstract. VLSI cell placement involves positioning cells within a target placement geometry while minimizing the interconnecting wire length and placement area. In this paper, the placement problem is solved using a combination of quadratic programming, circuit partitioning, clustering and greedy cell interchange heuristics. The solution of a sequence of quadratic programs and circuit partitioning problems provides the general positions of cells in a high quality placement. Computational efficiency is achieved by using an interior point method for solving the sequence of quadratic programs. A very efficient clustering heuristic is used to keep important groups of cells together as the cells are spread throughout the placement area. Numerical results on a set of benchmark circuits illustrate that this new approach produces standard cell placements that are up to 17% better in wire length, 14% better in row length and up to 24 times faster than a well known simulated annealing placement heuristic.

J. L. Kennington and D. E. Fyffe. A note on the quadratic programming approach to the solution of the 0–1 integer programming problem. *Bulletin of the Operations Research Society of America*, **20**, B333–334, 1972.

Abstract. The problem $\max c^T x$ subject to $Ax \leq b$, and x integer with components either 0 or 1, can be reformulated as, max $Q(x) = a^T x - x'Cx$ subject to $Ax \leq b$, and $x \in [0,1]$, where $Q(x)$ is strictly convex. The work of Ritter has been specialized to solve this quadratic problem. Initial computational results indicate that the approach is not competitive time-wise with the implicit enumeration algorithm of Geoffrion.

G. Keri. On the minimum value of a quadratic function under linear constraints. *Studia Scientiarum Mathematicarum Hungarica*, **6**, 193–196, 1971.

G. Keri. On a class of quadratic forms. *In* A. Prekopa, ed., 'Survey of Mathematical Programming, Vol. 1', pp. 231–247, North Holland, Amsterdam, the Netherlands, 1979.

E. Kim, H. J. Kang, and M. Park. Numerical stability analysis of fuzzy control systems via quadratic programming and linear matrix inequalities. *IEEE Transactions on Systems, Man and Cybernetics, Part A (Systems and Humans)*, **29**(4), 333–346, 1999.

Abstract. This paper proposes a numerical stability analysis methodology for the singleton-type linguistic fuzzy control systems based on optimization techniques. First, it demonstrates that a singleton-type linguistic fuzzy logic controller (FLC) can be converted into a region-wise sector-bounded controller or, more generally, a polytopic system by quadratic programming (QP). Next, the convex optimization technique called linear matrix inequalities (LMI) is used to analyze the closed loop of the converted polytopic system. Finally, the applicability of the suggested methodology is highlighted via simulation results.

P. Y. Kim and C. W. Yang. Advertising decision-model—a quadratic-programming approach. *In* '1985 Proceedings of the Annual Meeting of the American Institute for Decision Sciences', Vol. 1–2, pp. 551–553, 1985a.

P. Y. Kim and C. W. Yang. A quadratic programming media choice model with repeated exposure functions. *In* R. Hanham, W. G. Vogt and M. H. Mickle, eds, 'Modeling and Simulation. Proceedings of the Sixteenth Annual Pittsburgh Conference. ISA, Research Triangle Park, NC, USA', pp. 1773–1776, 1985b.

Abstract. Highlights the fundamental limitation of the linear programming media selection model through numerous computer simulations. As an alternative, a quadratic programming model was proposed to overcome the limitation of linear programming solutions to media selection problems.

Y. H. Kim, S. Y. Kim, and J. B. Kim. Adaptive-control of a binary distillation column using quadratic programming. *Korean Journal of Chemical Engineering*, **6**(4), 306–312, 1989.

K. C. Kiwiel. A method for solving certain quadratic-programming problems arising in nonsmooth optimization. *IMA Journal of Numerical Analysis*, **6**(2), 137–152, 1986.

K. C. Kiwiel. A dual method for certain positive semidefinite quadratic programming problems. *SIAM Journal on Scientific and Statistical Computing*, **10**(1), 175–186, 1989.

Abstract. Presents a dual active set method for minimizing a sum of piecewise linear functions and a strictly convex quadratic function, subject to linear constraints. It may be used for direction finding in nondifferentiable optimization algorithms and for solving exact penalty formulations of (possibly inconsistent) strictly convex quadratic programming problems. An efficient implementation is described extending the Goldfarb and Idnani algorithm, which includes Powell's refinements. Numerical results indicate excellent accuracy of the implementation.

K. C. Kiwiel. A Cholesky dual method for proximal piecewise linear programming. *Numerische Mathematik*, **68**, 325–340, 1994.

Abstract. A quadratic programming method is given for minimizing a sum of piecewise linear functions and a proximal quadratic term, subject to simple bounds on variables. It may be used for search direction finding in nondifferentiable optimization algorithms. An efficient implementation is described that updates a Cholesky factorization of active constraints and provides good accuracy via iterative refinement. Numerical experience is reported for some large problems.

E. Klafszky and T. Terlaky. Oriented matroids, quadratic programming and the criss-cross method. *Alkalmazott Matematikai Lapok*, **14**(3–4), 365–375, 1989.

Abstract. Quadratic programming, symmetry, and positive (semi) definiteness were generalized by Morris and Todd to oriented matroids. The authors slightly modify their definitions in order to get more symmetric structures. Some generalizations of Terlaky's criss-cross method are presented for oriented matroid quadratic programming. These algorithms are based on the small subscript rule and on sign patterns, and do not preserve feasibility on any subsets. Finally two special cases (positive definite case and oriented matroid linear programming) and a modification are presented.

E. Klafszky and T. Terlaky. Some generalizations of the criss-cross method for quadratic programming. *Optimization*, **24**(1–2), 127–139, 1992.

Abstract. Three generalizations of the criss-cross method for quadratic programming are presented. Tucker's, Cottle's and Dantzig's principal pivoting methods are specialized as diagonal and exchange pivots for the linear complementarity problem obtained from a convex quadratic program. A finite criss-cross method, based on least-index resolution, is constructed for solving the LCP. In proving finiteness, orthogonality properties of pivot tableaus and positive semidefiniteness of quadratic matrices are used. In the last section some special cases and two further variants of the quadratic criss-cross method are discussed. If the matrix of the LCP has full rank, then a surprisingly simple algorithm follows, which coincides with Murty's 'Bard type schema' in the P matrix case.

D. Klatte. On the Lipschitz behaviour of optimal solutions in parametric problems of quadratic programming and linear complementarity. *Optimization*, **16**, 819–831, 1985.

J. M. Kleinhans, G. Sigl, F. M. Johannes, and K. J. Antreich. GORDIAN: VLSI placement by quadratic programming and slicing optimization. *IEEE Transactions on Computer Aided Design of Integrated Circuits and Systems*, **10**(3), 356–365, 1991.

Abstract. In this paper we present a new placement method for cell-based layout styles. It is composed of alternating and interacting global optimization and partitioning steps that are followed by an optimization of the area utilization. Methods using the divide-and-conquer paradigm usually lose the global view by generating smaller and smaller subproblems. In contrast, GORDIAN maintains the simultaneous treatment of all cells over all global optimization steps, thereby considering constraints that reflect the current dissection of the circuit. The global optimizations are performed by solving quadratic programming problems that possess unique global minima. Improved partitioning schemes for the stepwise refinement of the placement are introduced. The area utilization is optimized by an exhaustive slicing procedure. The placement method has been applied to real world problems and excellent results in terms of both placement quality and computation time have been obtained.

B. Klummer. Globale stabilität quadratischer optimierungsauprobleme. *Wissenschaftlische Zeitschrift derrHumboldt Universität zu Berlin*, **5**, 565–569, 1977.

T. Koch. Algorithm for resolving manipulator redundancy—the compact QP method in Newton-Raphson iteration scheme. *In* V. Chundy and E. Kurekova, eds, 'ISMCR '95. Proceedings of the Fourth International Symposium on Measurement and Control in Robotics. Slovak Tech. Univ, Bratislava, Slovakia', pp. 357–362, 1995.

Abstract. Kinematic redundancy occurs when a manipulator possesses more degrees of freedom than the minimum number required to execute a given task. The kinematic redundant robot offers greater flexibility, versatility and dexterity. It can simultaneously realize the main task and additional formulated tasks, like for example keeping the joints movements within a given limits, obstacle avoidance, singular configuration avoidance, etc. The principal problem of redundant robot application is to find suitable control algorithm (kinematic inverse transformation scheme). First, the paper sets up the requirements that such algorithms should fulfil for industrial applications. Next, the paper presents an algorithm that meets these requirements. It is the compact QP method in Newton-Raphson iteration scheme with some elements of the so called configuration control. Finally, simulation results from testing this algorithm in simulation system for robot workcell are given.

T. Koch and P. Kowalczewski. Control of redundant robot using the quadratic programming method. *In* 'Prace Naukowe Instytutu Cybernetyki Technicznej Politechniki Wroclawskiej, Seria: Konferencje number 43', pp. 411–418, 1996.

Abstract. The algorithm for control of redundant robots is presented. It is the compact quadratic programming method in a Newton-Raphson iteration scheme with some elements of so called configuration control. Two options of the algorithm are described. Finally the results from movements simulation of the robots with open kinematic multiple redundant chains with 11 joints are presented.

M. Kojima and L. Tunçel. Discretization and localization in successive convex relaxation methods for nonconvex quadratic optimization problems. Technical Report CORR 98-34, Department of Combinatorics and Optimization, University of Waterloo, Ontario, Canada, 1998.

H. Konno. Maximization of a convex quadratic function under linear constraints. *Mathematical Programming*, **11**(2), 117–127, 1976.

H. Konno. Maximizing a convex quadratic function over a hypercube. *Journal of the Operations Research Society of Japan*, **23**(2), 171–189, 1980.

P. Korhonen and G. Y. Yu. A reference direction approach to multiple objective quadratic-linear programming. *European Journal of Operational Research*, **102**(3), 601–610, 1997.

Abstract. In this paper, we propose an interactive procedure for solving multiple criteria problems with one quadratic objective, several linear objectives, and a set of linear constraints. The procedure is based on the use of reference directions and weighted sums. Reference directions for the linear functions, and weighted sums for combining the quadratic function with the linear ones are used as parameters to implement the free search of nondominated solutions. The idea leads to the parametric linear complementarity problem formulation. An approach to deal with this type of problems is given as well. The approach is illustrated with a numerical example.

P. Korhonen and G. Y. Yu. On computing objective function values in multiple objective quadratic-linear programming. *European Journal of Operational Research*, **106**(1), 184–190, 1998.

Abstract. We consider the computation of objective function values when a nondominated frontier is searched in multiple objective quadratic-linear programming (MOQLP). Reference directions and weighted-sums constitute a methodological basis for the search. This idea leads to a parametric linear complementarity model formulation. A critical task of making a search procedure efficient, is to compute the changes in quadratic and linear objective functions efficiently when a search direction is changed or a basis change is performed. Those changes in objective functions can be computed by a so-called direct or indirect method. The direct method is a straightforward one and based on the use of unit changes in basic decision variables. Instead, the indirect method utilizes some other basic variables of the model. We introduce the indirect method and make theoretical and empirical comparisons between the methods. Based on the comparisons, we point out that the indirect method is clearly much more efficient than the direct one.

F. Korner. Remarks on second-order conditions in connection with the algorithm of Beale for quadratic programming. *European Journal of Operational Research*, **40**(1), 85–89, 1989.

Abstract. The algorithm of Beale is considered. It is discussed for the case in which the 'solution' point of Beale's algorithm is a local minimizer. In this algorithm a general form of second-order conditions arises.

F. Korner. On the numerical realization of the exact penalty method for quadratic programming algorithms. *European Journal of Operational Research*, **46**(3), 404–408, 1990.

Abstract. Optimization problems with quadratic nonconvex objective functions and linear inequalities are considered. An active constraint strategy is used to define a sequence of equality constrained problems. Under discussion is how the subproblems can be solved efficiently.

F. Korner. A note on weakly active constraints in connection with nonconvex quadratic programming. *European Journal of Operational Research*, **57**(3), 409–411, 1992.

Abstract. In general it is very difficult to determine a true local minimizer in nonconvex quadratic programming. The main problems arise if we have so-called weakly active constraints. The author discusses an efficient method for checking the local optimality or determining a direction of descent.

F. Korner and B. Luderer. On the implementation of quadratic programming algorithms. *Systems Analysis Modelling Simulation*, **6**(9), 699–707, 1989.

Abstract. Sequences of systems of linear equations are considered which arise recursively by adding or removing one equation. It is shown how a solution can be efficiently obtained from the solution of the previous system. Systems of this kind originate for example in the solution process of general quadratic optimization problems with linear inequality constraints via reducing them to equality constrained problems. Together with the solution of the systems, the number of positive, negative and zero eigenvalues is determined. Thus, the validity of second-order Kuhn-Tucker conditions may be easily checked.

F. Korner and C. Richter. On the efficient treatment of the boolean quadratic programming problem. *Numerische Mathematik*, **40**(1), 99–109, 1982.

Abstract. The dual of the Boolean quadratic programming problem is considered. The optimal value of the objective function gives bounds for the branch and bound process which seem to be better than those known from other authors.

L. V. Korsi and V. G. Sokolov. Synthesis of a system of isotropic radiators by the method of quadratic programming. *Radio Engineering and Electronic Physics*, **17**(3), 358–364, 1972.

Abstract. An effective method of synthesizing a system of isotropic radiators is proposed. The optimizing algorithm is based on the method of projected gradient. The solutions of the problems of mixed and phase synthesis of an array of radiators are obtained by this method. Examples of numerical computation are presented illustrating different applications of the proposed method.

M. M. Kostreva. Generalization of Murty's direct algorithm to linear and convex quadratic programming. *Journal of Optimization Theory and Applications*, **62**(1), 63–76, 1989.

Abstract. Murty's algorithm for the linear complementarity problem is generalized to solve the optimality conditions for linear and convex quadratic programming problems with both equality and inequality constraints. An implementation is suggested which provides both efficiency and tight error control. Numerical experiments as well as field tests in various applications show favorable results.

O. I. Kostyukova and V. M. Raketskii. A method of reducing the suboptimality estimate in quadratic programming. *Doklady Akademii Nauk Belarusi*, **29**(12), 1072–1075, 1985.

M. Kotani and I. Nemoto. The inverse problem in magnetopneumography—use of hypothetical distributions and quadratic programming. *Nuovo Cimento D*, **2**(2), 594–607, 1983.

Abstract. The inverse problem in the magnetic measurement of the lung was studied with a method using hypothetical distributions of magnetic dust. Both unconstrained and constrained minimizations of an objective function were performed. Simulations and analysis showed the efficacy of the method.

P. F. Kough. Global solution to the indefinite quadratic programming problem. Technical report, Washington Univ., St Louis, MO, USA, 1974.

Abstract. The global solution to the indefinite quadratic problem is obtained via a generalized Bender cut procedure. The Bender cut method iteratively adds cuts to a master problem. Maximising the master problem is shown to be equivalent to maximising several convex quadratic subproblems. Each cut creates a

subproblem. This decomposition provides for the solution of the master problem by an implicit enumeration algorithm combined with Tui cuts. In order to accelerate convergence only a subset of the subproblems is solved. The formal method is then combined with the accelerated procedure to insure convergence to the global optimum.

P. F. Kough. The indefinite quadratic programming problem. *Operations Research*, **27**(3), 516–533, 1979.

Abstract. Develops several algorithms that obtain the global optimum to the indefinite quadratic programming problem. A generalized Benders cut method is employed. These algorithms all possess ε-finite convergence. To obtain finite convergence the author develops exact cuts, which are locally precise representations of a reduced objective. A finite algorithm is then constructed. Introductory computational results are presented.

I. S. Kourtev and E. G. Friedman. Clock skew scheduling for improved reliability via quadratic programming. *1999 IEEE/ACM International Conference on Computer ided Design Digest of Technical Papers. IEEE, Piscataway, NJ, USA*, pp. 239–243, 1999. Also appeared in the Proceedings of the Twelfth Annual IEEE International ASIC/SOC Conference, IEEE, Piscataway, NJ, USA, 1999, pp 210–215.

Abstract. This paper considers the problem of determining an optimal clock skew schedule for a synchronous VLSI circuit. A novel formulation of clock skew scheduling as a constrained quadratic programming (QP) problem is introduced. The concept of a permissible range, or a valid interval, for the clock skew of each local data path is key to this QP approach. From a reliability perspective, the ideal clock schedule corresponds to each clock skew within the circuit being at the center of the respective permissible range. However, this ideal clock schedule is nor practically implementable because of limitations imposed by the connectivity among the registers within the circuit. To evaluate the reliability, a quadratic cost function is introduced as the Euclidean distance between the ideal schedule and a given practically feasible clock schedule. This cost function is the minimization objective of the described algorithms for the solution of the previously mentioned quadratic program. Furthermore, the work described here substantially differs from previous research in that it permits complete control over specific clock signal delays or skews within the circuit. Specifically, the algorithms described here can be employed to obtain results with explicitly specified target values of important clock delays/skews with a circuit, such as for example, the clock delays/skews for I/O registers. An additional benefit is a potential reduction in clock period of up to 10%. An efficient mathematical algorithm is derived for the solution of the QP problem with $O(r^3)$ run time complexity and $O(r^2)$ storage complexity, where r is the number of registers in the circuit. The algorithm is implemented as a C++ program and demonstrated on the ISCAS'89 suite of benchmark circuits as well as on a number of industrial circuits. The work described here yields additional insights into the correlation between circuit structure and circuit timing by characterizing the degree to which specific signal paths limit the overall performance and reliability of a circuit. This information is directly applicable to logic and architectural synthesis.

L. B. Kovacs and M. Kotel. Indefinite quadratic programming by gradient projection method and its application to optimal control of a chemical plant. *IFAC symposium on multivariable control systems VDI/VDE Fachgruppe Reqelungstechnik, Dusseldorf, West Germany*, 1968.

Abstract. As a subproblem of the optimal control of a chemical plant an arbitrary (possibly indefinite) quadratic function is to be minimized subject to linear constraints. The problem is solved by means of two computer programs the first of which gives a local optimum starting from any feasible point. This program is based on the well-known gradient projection method utilizing the special structure of the objective function. The gradient projection method has been chosen because of its good convergence properties. The second program generates 'good' starting points for the first program and terminates the algorithm if certain criteria are satisfied making very likey the fact that the present best solution is a global maximum of the problem.

K. Koyama, K. Minato, S. Eiho, and M. Kuwahara. A method for multivariable quadratic programming possessing special structures. *Systems and Control*, **19**(7), 402–404, 1975.

M. K. Kozlov, S. P. Tarasov, and L. G. Khachiyan. Polynomial solvability of convex quadratic programming. *Doklady Akademii Nauk SSSR*, **248**(5), 1049–1051, 1979. See also, *Soviet Mathematics Doklady* volume 20, pages 1108–1111, 1979.

Abstract. An algorithm is constructed for the exact solution of a quadratic programming problem, the volume (in binary symbols) of computation being limited by the length of the input, i.e. it is shown that quadratic programming belongs to the class P of problems solvable on deterministic Turing machines, in a time expressed as a polynomial of the binary input length.

M. K. Kozlov, S. P. Tarasov, and L. G. Khachiyan. Polynomial resolution of convex quadratic programming. *Zhurnal Vychislitel'noi Matematiki i Matematicheskoi Fiziki*, **20**(5), 1319–1323, 1980.

Abstract. The input length of the problem input for a quadratic programming problem is defined as the number of binary symbols necessary to describe the input data. An exact solution algorithm for the quadratic programming problem is developed for which the number of elemental calculations is bounded by a polynomial in the input length. It is shown that quadratic programming problems belong to class P problems, which can be solved by deterministic Turing machines.

S. O. Krumke. Eine modifizierte barrieremethode fur konvexe quadratische optimierungsprobleme. Diplomathesis, University of Wurzburg, Germany, 1994.

S. O. Krumke. On the convergence of a modified barrier function method for convex quadratic and linear programming. *In* 'Proceedings of the Third International Conference on Industrial and Applied Mathematics (ICIAM'95), Hamburg, Germany', 1995.

G. Krynska. The primal algorithm using conjugate directions for quadratic programming problems with simple upper bounds. *Optimization*, **18**(4), 545–560, 1987.

Abstract. An algorithm that solves quadratic programming problems with a convex objective function and a feasible region defined by constraints $Ax = b$, $\beta \leq x \leq \alpha$ is presented. The algorithm uses the SUB method to establish a starting feasible point and to construct the starting basis matrix. A feasible direction for the k-th iteration is chosen from a set of row-vectors of the inverse of the current basis matrix. After a finite number of iterations an optimal solution is found. A conjugacy property of feasible directions, convergence of the algorithm, the question of cycling and a comparison with other algorithms from the same class of methods of feasible directions are discussed.

C. N. Kumar and K. Elango. Unit graph estimation and stabilization using quadratic programming and difference norms—comment. *Water Resources Research*, **31**(10), 2633–2634, 1995.

H. P. Künzi and W. Oettli. Integer quadratic programming. *In* R. Graves and P. Wolfe, eds, 'Recent advances in Mathematical Programming', pp. 303–308, McGraw-Hill, New York, USA, 1963.

A. Kuppurajula and K. R. Nayar. Optimal operation of distribution networks by quadratic programming methods. *Proceedings of the IEEE*, **58**(7), 1172–1174, 1970.

Abstract. The optimum tap settings for transformers at the feeder points in a distribution network which minimize (1) the total copper losses, or (2) the voltage deviations of the load points, are obtained by quadratic programming methods.

I. V. Kurdyumov, M. V. Mosolova, and V. E. Nazaikinskii. Computing algorithm for quadratic programming problems of high dimensionality. *Zhurnal Vychislitel'noi Matematiki i Matematicheskoi Fiziki*, **18**(5), 1119–1128, 1978.

Abstract. The algorithm presented is based on a modification of the coupled direction method, but requires somewhat fewer computing operations and less memory. There is also a procedure for eliminating cumulative machine error.

P. Lachout. Degeneration in Wolfe's algorithm for quadratic programming. *Ekonomicko Matematicky Obzor*, **24**(4), 463–468, 1988.

Abstract. Wolfe's Algorithm is a well-known and widely used solution technique for convex quadratic programs. This paper shows examples in which, due to degeneracy, the long form of Wolfe's algorithm fails to find the optimal solution. A new algorithm that solves every problem of convex quadratic programs is suggested.

Y. T. Lai, C. C. Kao, and W. C. Shie. A quadratic programming method for interconnection crosstalk minimization. *In* 'ISCAS'99. Proceedings of the 1999 IEEE International Symposium on Circuits and Systems VLSI . IEEE, Piscataway, NJ, USA', Vol. 6, pp. 270–273, 1999.

Abstract. Adjacent wires in a VLSI chip become closer as the fabrication technology rapidly evolves. Accordingly it becomes important to consider crosstalk caused by the coupling capacitance between adjacent wires in the layout design for the fast and safe VLSI circuits. The crosstalk is a function of coupling length and distance. The coupling length can be reduced by segment rearrangement technique. This paper presents a crosstalk minimization technique by adjusting the space between adjacent wires. An example is shown in this paper to demonstrate the effectiveness of the technique.

C. M. Lam and K. T. Fung. A quadratic programming model for optimal data distribution. *BIT*, **21**(3), 294–304, 1981.

Abstract. A quadratic programming model is developed to take into consideration a number of factors that can influence the process of optimal allocation of data among the nodes in a distributed database. The factors include communication costs, translation costs, congestion costs and storage costs. Beale's method is used to solve the resulting quadratic program. Some numerical examples are presented and the potentials of such an approach in the design and analysis of distributed databases are discussed.

A. H. Land and G. Morton. An inverse-basis method for Beale's quadratic programming algorithm. *Management Science*, **19**(5), 510–516, 1973.

Abstract. This paper presents a version of Beal's Quadratic Programming Algorithm (1959) for solving a problem of maximising a quadratic function under linear constraints. The modification discussed here makes it possible to retain the 'inverse basis' tableau which has to be augmented by additional constraints to be called 'auxiliary.' The algorithm has been successfully tested on a computer.

A. H. Land and S. Powell. *Fortran codes for mathematical programming: linear, quadratic and discrete*. J. Wiley and Sons, Chichester, England, 1973.

L. Landesa-Porras, J. O. Rubinos-Lopez, and A. Garcia-Pino. Quadratic programming and Tikhonov regularization for illumination of reflector antennas with low spillover. *In* 'Proceedings ICECOM '97. 14th International Conference on Applied Electromagetics and Communications. KoREMA, Zagreb, Croatia', pp. 63–66, 1997.

Abstract. Two new method to synthesize a feed array for the illumination of reflector antennas are presented. Spillover radiation pattern or total spillover power can be considered to obtain efficient feeds by using quadratic programming or Tikhonov regularization, respectively. Aperture or surface field optimization is considered to obtain a prescribed aperture distribution. Optimization is ensured in the global sense, and the spillover plays the role of a constraint.

M. C. Lang. Design of nonlinear phase FIR digital filters using quadratic programming. *1997 IEEE International Conference on Acoustics, Speech, and Signal Processing IEEE Comput. Soc. Press, Los Alamitos, CA, USA*, **3**, 2169–2172, 1997.

Abstract. This paper presents two methods for the design of FIR filters with arbitrary magnitude and phase responses according to a weighted mean squared error criterion with constraints on the resulting magnitude and phase errors. This constrained least square criterion allows for an arbitrary trade-off between pure L_2 filters and Chebyshev filters. The resulting nonlinear optimization problem is either converted into a standard quadratic programming problem (method 1) or exactly solved by a sequence of quadratic programs (method 2). The quadratic programming problems can be solved efficiently using standard software.

D. J. Laughhunn. Quadratic binary programming with applications to capital budgeting problems. *Operations Research*, **18**, 454–461, 1970.

R. Lazimy. Mixed-integer quadratic programming. *Mathematical Programming*, **22**(3), 332–349, 1982.

Abstract. Considers mixed-integer quadratic programs in which the objective function is quadratic in the integer and in the continuous variables, and the constraints are linear in the variables of both types. The generalized Benders' decomposition is a suitable approach for solving such programs. However, the program does not become more tractable if this method is used, since Benders' cuts are quadratic in the integer variables. A new equivalent formulation that renders the program tractable is developed, under which the dual objective function is linear in the integer variables and the dual constraint set is independent of these variables. Benders' cuts that are derived from the new formulation are linear in the integer variables, and the original problem is decomposed into a series of integer linear master problems and standard quadratic subproblems. The new formulation does not introduce new primary variables or new constraints into the computational steps of the decomposition algorithm.

R. Lazimy. Improved algorithm for mixed-integer quadratic programs and a computational study. *Mathematical Programming*, **32**(1), 100–113, 1985.

T. H. A. Le, T. Pham Dinh, and D. M. Le. A combined dc optimization-ellipsoidal branch-and-bound algorithm for solving nonconvex quadratic programming problems. *Journal of Combinatorial Optimization*, **2**(1), 9–28, 1998.

Abstract. In this paper we propose a new branch-and-bound algorithm by using an ellipsoidal partition for minimizing an indefinite quadratic function over a bounded polyhedral convex set which is not necessarily given explicitly by a system of linear inequalities and/or equalities. It is required that for this set there exists an efficient algorithm to verify whether a point is feasible, and to find a violated constraint if this point is not feasible. The algorithm is based upon the fact that the problem of minimizing an indefinite quadratic form over an ellipsoid can be efficiently solved by some available (polynomial and nonpolynomial time) algorithms. In particular, the D.C. (difference of convex functions) algorithm (DCA) with restarting procedure recently introduced by Pham Dinh Tao and L.T. Hoai An is applied to globally solving this problem. DCA is also used for locally solving the nonconvex quadratic program. It is restarted with current best feasible points in the branch-and-bound scheme, and improved them in its turn. The combined DCA-ellipsoidal branch-and-bound algorithm then enhances the convergence: it reduces considerably the upper bound and thereby a lot of ellipsoids can be eliminated from further consideration. Several numerical experiments are given.

C. Y. C. Lee, D. R. Wiff, and V. G. Rodgers. Calculating a relaxation spectrum from experimental data via quadratic programming with and without regularization. *Journal of Macromolecular Science—Physics*, **B19**(2), 211–225, 1981.

Abstract. The regularization quadratic programming approach to infer relaxation spectra from experimental mechanical data was studied. It was found that the inferior of the sum of the squared difference between the input data and the back-calculated values did not yield a satisfactory relaxation spectrum as the regularization weighting parameter α was increased. A modification was made to the function to be minimized so that all data points were equally weighted. Quadratic programming was then found to be sufficient to infer a reliable relaxation spectrum if the input experimental data had a high degree of accuracy. When the experimental data were not sufficiently accurate, regularization over-regularized the high-value region of the solution spectrum

before it could improve the low-value region. Decreasing the number of sought-after points in the solution spectrum can compensate for the noise level in the input experimental data and will allow inferring a reliable relaxation spectrum from the experimental data.

L. J. Lefkoff and S. M. Gorelick. Design and cost-analysis of rapid aquifer restoration systems using flow simulation and quadratic programming. *Ground Water*, **24**(6), 777–790, 1986.

L. L. Lei and J. E. Epperson. A regional-analysis of vegetable versus row crop production using quadratic programming. *American Journal of Agricultural Economics*, **69**(5), 1097, 1987.

C. E. Lemke. A method of solution for quadratic programs. *Management Science*, **8**, 442–453, 1962a.

C. E. Lemke. Orthogonality, duality, and quadratic type problems in mathematical programming. RPI MathRep 56, Department of Mathematics, Rensselaer Polytechnic Institute, Troy, NY, USA, 1962b.

M. L. Lenard and M. Minkoff. Randomly generated test problems for positive definite quadratic programming. *ACM Transactions on Mathematical Software*, **10**(1), 86–96, 1984.

Abstract. A procedure is described for randomly generating positive definite quadratic programming test problems. The test problems are constructed in the form of linear least-squares problems subject to linear constraints. The probability measure for the problems so generated is invariant under orthogonal transformations. The procedure allows the user to specify the size of the least-squares problem (number of unknown parameters, number of observations, and number of constraints), the relative magnitude of the residuals, the condition number of the Hessian matrix of the objective function, and the structure of the feasible region (the number of equality constraints and of inequalities which will be active at the feasible starting point and at the optimal solution). An example is given illustrating how these problems can be used to evaluate the performance of a software package.

A. Lent and Y. Censor. Extensions of Hildreth's row-action method for quadratic programming. *SIAM Journal on Control and Optimization*, **18**(4), 444–454, 1980.

Abstract. An extended version of Hildreth's iterative quadratic programming algorithm is presented, geometrically interpreted, and proved to produce a sequence of iterates that (i) converges to the solution, and (ii) has an important intermediate optimality property. This extended Hildreth algorithm is cast into a new form which more pronouncedly brings out its primal-dual nature. The application of the algorithm may be governed by an index sequence which is more general than a cyclic sequence, namely, by an almost cyclic control, and a sequence of relaxation parameters is incorporated without ruining convergence. The algorithm is a row-action method which is particularly suitable for handling large (or huge) and sparse systems.

D. J. Leo and D. J. Inman. A quadratic programming approach to the design of active-passive vibration isolation systems. *Journal of Sound and Vibration*, **220**(5), 807–825, 1999.

Abstract. A quadratic programming algorithm is presented for studying the design tradeoffs of active-passive vibration isolation systems. The novelty of the technique is that the optimal control problem is posed as a quadratic optimization with linear constraints. The quadratic cost function represents the mean square response of the payload acceleration and isolator stroke, and the linear constraints represent asymptotic tracking requirements and peak response constraints. Posing the problem as a quadratic optimization guarantees that a global optimal solution can be found if one exists, and the existence of an optimal solution guarantees that the vibration isolation system satisfies the specified design constraints. The utility of the technique is demonstrated on a comparison of passive vibration isolation and active-passive vibration isolation utilizing relative displacement feedback.

B. H. Leung. Decimation filters for oversampled analog to digital converters based on quadratic programming. *1990 IEEE International Symposium on Circuits and Systems IEEE, New York, NY, USA*, **2**, 899–901, 1990.

Abstract. A description is given of a quadratic-programming-based methodology for the design of decimation filters in CMOS technology. The methodology presented tackles the filter design problem by formulating a quadratic programming problem that minimizes the integral of the aliased noise subject to the passband and stopband constraints. This methodology offers the flexibility of incorporating nonsinusoidal quantization noise spectral density in the design process. Moreover the filter response can be optimized to meet arbitrary antialias requirements. Because the projected Hessian matrix of the objective function is positive definite, the quadratic function has a unique minimum.

B. H. Leung. Design methodology of decimation filters for oversampled adc based on quadratic programming. *IEEE Transactions on Circuits and Systems*, **38**(10), 1121–1132, 1991.

Abstract. A design methodology for oversampled analog-to-digital converter decimation filters is presented. The methodology tackles the finite-impulse-response (FIR) filter design problem by formulating a quadratic programming problem that minimizes the integral of the aliased noise subject to the passband and stopband constraints. The approach offers a design whose response is optimized to meet arbitrary quantization noise power spectral density and anti-alias requirements. Because the projected Hessian matrix of the objective function is positive definite, the quadratic function has a unique minimum. The methodology is applied to design filters for different requirements and the performance is compared to conventional approaches.

A. J. Levy. A fast quadratic programming algorithm for positive signal restoration. *IEEE Transactions on Acoustics, Speech and Signal Processing*, **31**(6), 1337–1341, 1983.

Abstract. When a signal or picture is processed by deconvolution, any additional a priori information is of prime interest since it can potentially lead to an improvement in results and to superresolution by reducing ambiguity. The author proposes a deconvolution method that assumes, first, that the unknown signal has a known finite extent, and second, that this signal is positive. The problem is stated in terms of a quadratic programming problem is stated in terms of a quadratic programming problem with positivity constraints and proposed is a new algorithm delivered from a conjugate gradient method, especially suited to this particular situation, which leads to a low cost solution. Experimental results on two-dimensional signals, emphasizing relevant superresolution are then presented.

J. T. Lewis, R. Murphy, and D. W. Tufts. Design of minimum noise digital filters subject to inequality constraints using quadratic programming. *IEEE Transactions on Acoustics, Speech and Signal Processing*, **ASSP-24**(5), 434–436, 1976.

Abstract. A numerical method is presented for designing digital filters. The method allows one to minimise the mean-square error or noise power over some intervals of frequency, while simultaneously constraining the maximum error in other intervals of frequency. Thus, for example, one can minimise noise power from a stopband of frequencies while constraining signal fidelity in a passband of frequencies by limiting the maximum passband deviation.

W. Li. Error bounds for piecewise convex quadratic programs and applications. *SIAM Journal on Control and Optimization*, **33**(5), 1510–1529, 1995.

Abstract. In this paper, we establish a local error estimate for feasible solutions of a piecewise convex quadratic program and a global error estimate for feasible solutions of a convex piecewise quadratic program. These error estimates provide a unified approach for deriving many old and new error estimates for linear programs, linear complementarity problems, convex quadratic programs, and affine variational inequality problems. The approach reveals the fact that each error estimate is a consequence of some reformulation of the original problem as a piecewise convex quadratic program or a convex piecewise quadratic program. In a sense, even Robinson's result on the upper Lipschitz continuity of a polyhedral mapping can be considered as a special case of error estimates for approximate solutions of a piecewise convex quadratic program. As

an application, we derive new (global) error estimates for iterates of the proximal point algorithm for solving a convex piecewise quadratic program.

W. Li. Differentiable piecewise quadratic exact penalty functions for quadratic programs with simple bound constraints. *SIAM Journal on Optimization*, **6**(2), 299–315, 1996.

W. Li and J. J. de Nijs. An implementation of QSPLINE method for solving convex quadratic programming problems with simple bound constraints. Technical report, Department of Mathematics and Statistics, Old Dominion University, Norfolk, Virginia, U.S.A., 2000.

Abstract. A convex quadratic programming problem with simple bound constraints can be reformulated as an unconstrained minimization problem with a convex quadratic spline (i.e., a differentiable convex piecewise quadratic function) as the objective function. This leads to a new paradigm for solving the original quadratic programming problem, in which various unconstrained minimization algorithms can be used to find a stationary point of the convex quadratic spline. In this paper, we give an implementation of a regularized Newton method (called QSPLINE Method) for finding a stationary point of the convex quadratic spline. QSPLINE Method can also be considered as an implicit active-set method with two novel features: (i) a mixed primal-dual approach for identifying active indices, and (ii) a line search strategy for a dynamic balance between the need of minimizing the original objective function and that of forcing the iterates to stay in the feasible region. The implemented version of QSPLINE Method uses a matrix up dating technique for computing Newton directions and a correction strategy for robust reduction of the convex quadratic spline when line search in a Newton direction fails. We have tested the code on a variation of the test problems used by More and Toraldo oraldo (1989). For our generated test problem, the Hessian of the objective function could be an n by n positive semidefinite matrix with rank $2n/3$, and one third of the Lagrange multipliers corresponding to active constraints at a generated optimal solution could be zero. That is, our test problems are degenerate and have highly singular Hessians. The code finds very accurate numerical solutions for all 2800 randomly generated test problems (with $n = 300$ and 500) even though all these problems are degenerate and about 60% of these problems have infinitely many solutions.

W. Li and J. Swetits. A Newton method for convex regression, data smoothing, and quadratic programming with bounded constraints. *SIAM Journal on Optimization*, **3**(3), 466–488, 1993.

W. Li and J. Swetits. A new algorithm for solving strictly convex quadratic programs. *SIAM Journal on Optimization*, **7**(3), 595–619, 1997.

Abstract. We reformulate convex quadratic programs with simple bound constraints and strictly convex quadratic programs as problems of unconstrained minimization of convex quadratic splines. Therefore, any algorithm for finding a minimizer of a convex quadratic spline can be used to solve these quadratic programming problems. In this paper, we propose a Newton method to find a minimizer of a convex quadratic spline derived from the unconstrained reformulation of a strictly convex quadratic programming problem. The Newton method is a ”natural mixture” of a descent method and an active-set method. Moreover, it is an iterative method, yet it terminates in finite operations (in exact arithmetic).

X. Li and Z. Xuan. An interior-point QP algorithm for structural optimization. *Structural Optimization*, **15**(3–4), 172–179, 1998.

Abstract. A new algorithm for convex quadratic programming (QP) is presented. Firstly, the surrogate problem for QP is developed, and the Karush-Kuhn-Tucker conditions of the surrogate problem hold if the unconstrained minimum of the objective function does not satisfy any constraints. Then, Karmarkar’s (1984) algorithm for linear programming (LP) is introduced to solve the surrogate dual problem. In addition, the case of general constraints is also discussed, and some examples of optimum truss sizing problems show that the proposed algorithm is robust and efficient.

R. H. Liang and Y. Y. Hsu. Hydroelectric generation scheduling using a quadratic programming based neural network. *In* 'IPEC '95. Proceedings of the International Power Engineering Conference. Nanyang Technol. Univ, Singapore', Vol. 2, pp. 684–689, 1995.

Abstract. A new approach based on neural networks is proposed for hydroelectric generation scheduling. The purpose of hydroelectric generation scheduling is to determine the optimal amounts of generated powers for the hydro units in the system for the next N ($N = 24$ in this paper) hours in the future. The proposed approach is basically a two-stage solution method. In the first stage, a quadratic programming based neural network is developed in order to reach a preliminary generation schedule for the hydro units. Since some practical constraints may be violated in the preliminary schedule, a heuristic rule based search algorithm is developed in the second stage to reach a feasible suboptimal schedule which satisfies all practical constraints. The proposed approach is applied to hydroelectric generation scheduling of Taiwan power system. It is concluded from the results that the proposed approach is very effective in reaching proper hydrogeneration schedules.

N. Limic and A. Mikelic. Constrained Kriging using quadratic programming. *Journal of the International Association for Mathematical Geology*, **16**(4), 423–429, 1984.

Y. Lin and J. Pang. Iterative methods for large convex quadratic programs: a survey. *SIAM Journal on Control and Optimization*, **25**, 383–411, 1987.

G. Ling. An optimal neuron evolution algorithm for constrained quadratic programming in image restoration. *IEEE Transactions on Systems, Man and Cybernetics, Part A (Systems and Humans)*, **26**(4), 513–518, 1996.

Abstract. An optimal neuron evolution algorithm for the restoration of linearly distorted images is presented in this paper. The proposed algorithm is motivated by the symmetric positive-definite quadratic programming structure inherent in restoration. Theoretical analysis and experimental results show that the algorithm not only significantly increases the convergence rate of processing, but also produces good restoration results. In addition, the algorithm provides a genuine parallel processing structure which ensures computationally feasible spatial domain image restoration.

C. T. Liu, G. C. Temes, and H. Samueli. FIR filter design for sigma-delta a/d converters using quadratic programming. *IEEE Pacific Rim Conference on Communications, Computers and Signal Processing IEEE, New York, NY, USA*, **2**, 760–763, 1991a.

Abstract. The design of overall finite impulse response (FIR) filter for sigma-delta analog-to-digital converters is formulated as a quadratic program with the total output noise as the objective function, subject to the passband and stopband constraints. The formulation is so flexible that it can suit different requirements for suppressing the quantization noise as well as rejecting the out-of-band spurious signal. Both one-stage single rate and multistage multirate filter structures are considered. For the purpose of suppressing the quantization noise, the multistage multirate FIR filter with comb filters as prefilters has much lower complexity with little performance degradation relative to the one-stage optimal filter.

C. T. Liu, G. C. Temes, and H. Samueli. FIR filter design using quadratic programming. *1991 IEEE International Symposium on Circuits and Systems IEEE, New York, NY, USA*, **1**, 148–151, 1991b.

Abstract. The optimum design of FIR (finite impulse response) filters is formulated as a quadratic program. An algorithm is described for the efficient use of computer memory and for guaranteed convergence to the solution of the original problem. Examples illustrating the design of decimation filters for sigma-delta A/D converters using the new technique are also presented. The quadratic programming approach is more flexible and often yields better results than the previously reported least-squares design approaches.

J. Q. Liu, T. T. Song, and D. Z. Du. On the necessary and sufficient condition of the local optimal solution of quadratic programming. *Chinese Annals of Mathematics Series B*, **3**(5), 625–630, 1982.

X. Liu, Y. Sun, and W. Wang. Stabilizing control of robustness for systems with maximum uncertain parameters-a quadratic programming approach. *Control Theory and Applications*, **16**(5), 729–732, 1999.

Abstract. Based on a sufficient condition for stabilization, a way to stabilize linear discrete-time systems with uncertain parameters is developed. By quadratic programming, the optimal controller of fixed order we have obtained can stabilize the system with maximum infinite norm of the uncertain system parameter vector. The conclusion is confirmed by simulation.

K. L. Lo and S. P. Zhu. A decoupled quadratic programming approach for optimal power dispatch. *Electric Power Systems Research*, **22**(1), 47–60, 1991.

Abstract. The authors present a decoupled approach for optimal active and reactive power dispatch for the economic operation of an electric utility. The proposed iterative scheme reported decomposes the optimal load flow problem into a P-subproblem and a Q-subproblem, where the control variables are the active and reactive power outputs of generators. At each iteration the subproblems are solved by quadratic programming using the flexible tolerance optimization method (FTOM). In this approach, constraints are linearized by using sensitivity analysis and the perturbation technique, in order to obtain a wider near-feasible region for this quadratic programming method. This approach has a good computation speed. The results of two test systems and three cases are presented and compared with the results of other methods.

W. Lochrie and D. Isaacs. The application of quadratic programming in adaptive filtering. *In* ‘Proceedings of the 8th annual Allerton conference on circuit and system theory. IEEE, New York, NY, USA’, pp. 101–110, 1970.

Abstract. The problem of designing an adaptive filter for a nonlinear dynamic system is formulated as a constrained estimation problem in which the measurement noise variances are estimated. An adaptive filter mechanization with an imbedded constrained estimation algorithm is then developed. A quadratic programming algorithm is incorporated in the mechanization developed for learning these variance values.

E. Loehman, D. Pingry, and A. B. Whinston. Quadratic programming and estimation problems. Technical report, Purdue Univ , Lafayette, IN, USA, 1970.

Abstract. The paper presents various problems relating to multivariate regression in a unified fashion using the quadratic programming tableau. The topics considered are multicollinearity and the generalized inverse, picking the best regression model, and constrained estimates and hypothesis testing for regression coefficients.

H. Löfgren. Liberalising Egypt’s agriculture: a quadratic programming analysis. *Journal of African Economies*, **2**(2), 238–261, 1993.

F. A. Lootsma and J. D. Pearson. An indefinite-quadratic-programming model for a continuous-production problem. *Philips Research Reports*, **25**(4), 244–254, 1970.

Abstract. A model is presented for a problem of scheduling the lengths of N production periods on one machine, which will manufacture 1 products. The problem is to choose production periods so as to minimize the sum of inventory costs for the q products in the presence of given demands. Mathematically, the problem is one of minimizing an indefinite quadratic function subject to linear constraints. A numerical example concludes the paper.

S. L. Louwes, J. C. G. Boot, and S. Wage. A quadratic programming approach to the problem of the optimal use of milk in the netherlands. *Journal of Farm Economics*, **45**, 309–317, 1963.

Z. Lu and Z. Wei. Decomposition method for quadratic programming problem with box constraints. *Mathematica Numerica Sinica*, **21**(4), 475–482, 1999.

Abstract. A decomposition method for solving quadratic programming (QP) with box constraints is presented in this paper. It is similar to the iterative method for solving linear system of equations. The main ideas of the algorithm to split the Hessian matrix Q of the QP problem into the sum of two matrices N and H such that Q=N+H and (N-H) is symmetric positive definite matrix ((N,H) is called a regular splitting of Q). A new quadratic programming problem with Hessian matrix N to replace the original Q is easier to solve than the original problem in each iteration. The convergence of the algorithm is proved under certain assumptions, and the sequence generated by the algorithm converges to optimal solution and has a linear rate of R-convergence if the matrix Q is positive definite, or a stationary point for the general indefinite matrix Q, and the numerical results are also given.

A. Lucia and J. Xu. Chemical process optimization using Newton-like methods. *Computers and Chemical Engineering*, **14**(2), 119–138, 1990.

Abstract. Various interrelated issues that effect the reliability and efficiency of Newton-like methods for chemical process optimization are studied. An algorithm for solving large, sparse quadratic programming (QP) problems that is based on an active set strategy and a symmetric, indefinite factorization is presented. The QP algorithm is fast and reliable. A simple asymmetric trust region method is proposed for improving the reliability of successive QP methods. Ill-defined QP subproblems are avoided by adjusting the size of the trust region in an automatic way. Finally, it is shown that reliable initial values of the unknown variables and multipliers can be generated automatically using generic problem information, short-cut techniques and simulation tools. Many relevant numerical results and illustrations are presented.

A. Lucia, J. Xu, and G. C. D'Couto. Sparse quadratic programming in chemical process optimization. *Annals of Operations Research*, **42**(1–4), 55–83, 1993.

Abstract. The quadratic programming aspects of a full space successive quadratic programming (SQP) method are described. In particular, fill-in, matrix factor and active set updating, numerical stability, and indefiniteness of the Hessian matrix are discussed in conjunction with a sparse modification of Bunch and Parlett factorization of symmetric indefinite (Kuhn-Tucker) matrices of the type often encountered in optimization. A new pivoting strategy, called constrained pivoting, is proposed to reduce fill-in and compared with complete, partial and threshold pivoting. It is shown that constrained pivoting often significantly reduces fill-in and thus the iterative computational burdens associated with the factorization and solution of Kuhn-Tucker conditions within the QP subproblem. Several chemical process optimization problems, with small and large degrees of freedom, are used as test problems. These include minimum work calculations for multistage isothermal compression, minimum area targeting for heat exchanger networks and distillation optimization involving some azeotropic and extractive distillations. Numerical results show uniformly that both the proposed QP and SQP algorithms, particularly the full space Newton method, are reliable and efficient. No failures were experienced at either level.

F. T. Luk and M. Pagano. Quadratic programming with M-matrices. *Linear Algebra and Its Applications*, **33**, 15–40, 1980.

Abstract. Studies the problem of quadratic programming with M-matrices. The authors describe an effective algorithm for the case where the variables are subject to a lower-bound constraint, and an analogous algorithm for the case where the variables are subject to lower-and-upper-bound constraints. The authors demonstrate the special monotone behavior of the iterate and gradient vectors. The result on the gradient vector is new. It leads us to consider a simple updating procedure which preserves the monotonicity of both vectors. The procedure uses the fact that an M-matrix has a nonnegative inverse. Two new algorithms are then constructed by incorporating this updating procedure into the two given algorithms. The authors give numerical examples which show that the new methods can be more efficient than the original ones.

L. Luksan. Dual method for solving a special problem of quadratic programming as a subproblem at linearly constrained nonlinear minimax approximation. *Kybernetika*, **20**(6), 445–457, 1984.

Abstract. Describes the dual method for solving a special problem of quadratic programming as a subproblem at linearly constrained nonlinear minimax approximation. Complete algorithm of the dual method is presented and its convergence after a finite number of steps is proved.

L. Luksan. Dual method for solving a special problem of quadratic programming as a subproblem to nonlinear minimax approximation. *Aplikace Matematiky*, **31**(5), 379–395, 1986.

Abstract. The dual method for solving a special problem of quadratic programming as a subproblem to nonlinear minimax approximation is described. Two cases are analyzed in detail; they differ in the linear dependence of the gradients of the active functions. The complete algorithm of the dual method is presented and its finite step convergence is proved.

A. D. Lutzenko and A. V. Martynov. Minimax solutions of problems in linear and quadratic programming. *Soviet Journal of Computer and Systems Sciences*, **2**, 22–27, 1968.

Abstract. A method is given for obtaining the optimum solution of monimax problems in linear and quadratic programming. It is shown that for minimax problems in linear programming the saddle-point condition is not satisfied.

C. Y. Maa and M. A. Shanblatt. Linear and quadratic programming neural network analysis. *IEEE Transactions on Neural Networks*, **3**(4), 580–594, 1992.

Abstract. Neural networks for linear and quadratic programming are analyzed. The network proposed by M. P. Kennedy and L. O. Chua (IEEE Trans. Circuits Syst., vol. 35, pp.554–562, May 1988) is justified from the viewpoint of optimization theory and the technique is extended to solve optimization problems, such as the least-squares problem. For quadratic programming, the network converges either to an equilibrium or to an exact solution, depending on whether the problem has constraints or not. The results also suggest an analytical approach to solve the linear system $Bx = b$ without calculating the matrix inverse. The results are directly applicable to optimization problems with C^2 convex objective functions and linear constraints. The dynamics and applicability of the networks are demonstrated by simulation. The distance between the equilibria of the networks and the problem solutions can be controlled by the appropriate choice of a network parameter.

J. H. Maddocks. Restricted quadratic-forms, inertia theorems, and the Schur complement. *Linear Algebra and Its Applications*, **108**, 1–36, 1988.

K. Madsen and H. Schjær-Jacobsen. Minimax optimization using quadratic programming. *In* 'Proceedings of the IEEE International Conference on Circuits and Computers ICCC 80. IEEE, New York, NY, USA', pp. 1135–1137, 1980.

Abstract. A new method for solving the non-linear minimax problem is presented. The problem is solved through a sequence of quadratic programs and no line search is used. The method may be useful in sparse problems and an algorithm for this case is briefly outlined.

K. Madsen, H. B. Nielsen, and M. C. Pinar. A finite continuation algorithm for bound constrained quadratic programming. *SIAM Journal on Optimization*, **9**(1), 62–83, 1998.

Abstract. The dual of the strictly convex quadratic programming problem with unit bounds is posed as a linear ℓ_1 minimization problem with quadratic terms. A smooth approximation to the linear ℓ_1 function is used to obtain a parametric family of piecewise-quadratic approximation problems. The unique path generated by the minimizers of these problems yields the solution to the original problem for finite values of the approximation parameter. Thus, a finite continuation algorithm is designed. The results of extensive computational experiments are reported.

K. Madsen, H. B. Nielsen, and M. C. Pinar. Bound constrained quadratic programming via piecewise quadratic functions. *Mathematical Programming*, **85**(1), 135–156, 1999.

Abstract. We consider the strictly convex quadratic programming problem with bounded variables. A dual problem is derived using Lagrange duality. The dual problem is the minimization of an unconstrained, piecewise quadratic function. It involves a lower bound of $\lambda(1)$, the smallest eigenvalue of a symmetric, positive definite matrix, and is solved by Newton iteration with line search. The paper describes the algorithm and its implementation including estimation of $\lambda(1)$, how to get a good starting point for the iteration, and up- and downdating of Cholesky factorization. Results of extensive testing and comparison with other methods for constrained QP are given.

R. A. Maggio. An exploration of the convex quadratic-programming blending model. *Modeling and Simulation: Robots and General Modeling*, **18**(5), 1617–1621, 1987.

M. Mahfouf and D. A. Linkens. Constrained multivariable generalized predictive control (GPC) for anaesthesia: the quadratic-programming approach (QP). *International Journal of Control*, **67**(4), 507–527, 1997.

Abstract. This paper considers the extension of the standard GPC algorithm to include input rate, magnitude and output constraints using the quadratic programming (QP) approach on a derived nonlinear multivariable anaesthesia model comprising simultaneous control of muscle relaxation (paralysis) and unconsciousness (in terms of blood pressure measurements). Simulation results, which are presented, analysed and discussed, demonstrate the superiority of the extended version in the deterministic and stochastic cases even when low output prediction horizons are chosen, and also the great flexibility with respect to choosing the limits on the manipulated as well as the output variables. The study also reveals that when heavy external disturbances occur, the algorithm, which combines input and output constraints, performs better than either the unconstrained one or the one that includes only input constraints. Under extreme conditions, the same algorithm reduces to an algorithm with only input constraints when the phenomenon of constraints incompatibility occurs.

G. Maier. A quadratic programming approach for certain classes of non linear structural problems. *Meccanica*, **3**, 121–130, 1968.

A. Majthay. Optimality conditions for quadratic programming. *Mathematical Programming*, **1**(3), 359–365, 1971.

A. Majthay, A. B. Whinston, and J. Coffman. Local optimisation for nonconvex quadratic programming. *Naval Research Logistics Quarterly*, **21**(3), 465–490, 1974.

Abstract. Presents an algorithm for determining a local minimum to the following general quadratic programming problem Minimize $q(x) = p^T x + 1/2x^T Qx$ subject to $Ax \geq b$ $x \geq 0$, where Q is an n by n symmetric matrix A is an m by n matrix p is an n by 1 vector b is an m by 1 vector. No additional restrictions are placed on the matrix Q. And so, the above problem is the minimization of a possibly nonconvex quadratic objective function subject to a set of linear inequality constraints.

K. Malanowski. On application of a quadratic programming procedure to optimal control problems in systems described by parabolic equations. *Control and Cybernetics*, **1**(1–2), 43–56, 1972.

Abstract. An optimal control problem of a system described by linear partial differential equation of parabolic type is considered. Both boundary and distributed controls are investigated. The performance index is a quadratic one. It is shown that this problem can be reduced to some quadratic programming problem in a Hilbert space. An iterative procedure of solving this problem is proposed. One dimensional heat conduction system is considered as an example. Some numerical results obtained using a digital computer are given.

K. Malanowski. A quadratic programming method in Hilbert space and its application to optimal control of systems described by parabolic equations. *5th IFIP Conference on Optimization Techniques (Abstracts only received). Univ Rome, Rome, Italy*, p. 37, 1973.

Abstract. Abstract only given substantially as follows. The problem of finding the minimum of a quadratic functional $J(y)$ on a closed, bounded and convex set Gamma in a Hilbert space is considered. The set Gamma may not be given in an explicit form. An iterative procedure is proposed, based on successive minimization of the functional $J(y)$ on some subsets Γ^i of the original set Γ . Three methods of constructing the subsets Γ^i are proposed. The use of the above procedure for solving optimal control problems for linear systems described by parabolic equations with quadratic performance index is proposed. Some numerical examples of optimal boundary control of one dimensional heat-transfer equation are presented. On the basis of these examples the three methods of constructing the subsets Γ^i are compared. The speed of convergence and the simplicity of the obtained control are taken into account in this comparison.

K. Man'chak and Y. A. Tomashevski. Solving quadratic programming problems on an analogue computer. *Archiwum Automatyki i Telemechanika*, **16**(2), 185–204, 1971.

Abstract. In the paper a question of solving quadratic programming problems is exemplified by a specific task of water resource allocation. Gradient methods of quadratic programming are examined. The principle of the application of an analogue computer for solving such problems is discussed. Results of simulation and experimental investigations are given. The latter are divided into two series. The first one is concerned with the statical case, the second one with the dynamical case.

D. W. Manchala, A. B. Palazzolo, A. F. Kascak, G. T. Montague, and G. V. Brown. Constrained quadratic programming, active control of rotating mass imbalance. *Journal of Sound and Vibration*, **205**(5), 561–580, 1997.

Abstract. Jet engines may experience severe vibration due to the sudden imbalance caused by blade failure. The current research investigates employment of piezoelectric actuators to suppress this using active vibration control. This requires identification of the source of the vibrations via an expert system, determination of the required phase angles and amplitudes for the correction forces, and application of the desired control signals to the piezoelectric actuators. Correction forces may exceed the physical limitations of the actuators; hence results of "constrained force" quadratic programming, least squares and multi-point correction algorithms will be compared. It is demonstrated that simply scaling down the least squares predicted correction forces to satisfy the actuator saturation constraints does not necessarily yield optimal reductions in vibration. In this paper test results are shown for sudden imbalance, and the computational time requirements and balancing effectiveness for the various approaches are compared.

O. L. Mangasarian. Duality in quadratic programming. *In* 'Nonlinear Programming', chapter 8-2, pp. 123–126. McGraw-Hill, New York, USA, 1969. Reprinted as *Classics in Applied Mathematics 10*, SIAM, Philadelphia, USA, 1994.

O. L. Mangasarian. Locally unique solutions of quadratic programs, linear and non-linear complementarity problems. *Mathematical Programming*, **19**(2), 200–212, 1980.

O. L. Mangasarian. Sparsity-preserving SOR algorithms for separable quadratic and linear programming. *Computers and Operations Research*, **11**, 105–112, 1984.

O. L. Mangasarian and H. Stone. Two-person nonzero-sum games and quadratic programming. *Journal of Mathematical Analysis and Applications*, **9**, 348–355, 1963.

H. B. Mann. Quadratic forms with linear constraints. *The American Mathematical Monthly*, **50**, 430–433, 1943.

B. D. Manos. Least square fit and quadratic programming. *Diastase*, **2**, 3–14, 1986. In Greek.

B. D. Manos and G. I. Kitsopanidis. A quadratic programming model for farm planning of a region in central macedonia. *Interfaces*, **16**(4), 2–12, 1986.

Abstract. Quadratic programming models are used in farm planning because risk and uncertainty are involved in the technical and economic coefficients used and the quantities and prices of resources. A special quadratic programming model (the E-V model) was used to plan a Greek farm region, the former Lake of Giannitsa. The resulting plan is preferred by farmers to those resulting from the linear and mixed-integer programming models and to the previously used plan because it includes crops expected to give the highest minimum total gross margin with the same total fixed costs. The farmers want plans that achieve not only the highest but also the most stable economic results.

M. A. Marino and H. A. Loaiciga. Multireservoir operation planning via quadratic programming. *In* 'Scientific Basis for Water Resources Management', Vol. 153, pp. 231–239, 1985.

H. M. Markowitz. The optimization of a quadratic function subject to constraints. *Naval Research Logistics Quarterly*, **3**, 111–133, 1956.

I. Maros and C. Meszaros. A repository of convex quadratic programming problems. *Optimization Methods and Software*, **11-12**, 671–681, 1999.

Abstract. The introduction of a standard set of linear programming problems, to be found in NETLIB/LP/DATA, had an important impact on measuring, comparing and reporting the performance of LP solvers Until recently the efficiency of new algorithmic developments has been measured using this important reference set Presently, we are witnessing an ever growing interest in the area of quadratic programming The research community is somewhat troubled by the lack of a standard format for defining a QP problem and also by the lack of a standard reference set of problems for purposes similar to that of LP In the paper we propose a standard format and announce the availability of a test set of 138 collected QP problems.

J. E. Marowitz and K. A. Fegley. System design using quadratic programming. *IEEE Transactions on Automatic Control*, **16**(3), 241–247, 1971.

Abstract. Quadratic programming is applied to the constrained design and compensation of stochastic and deterministic systems. The developed constraints control system stability, plant compensator realizability, system rise time, and bandwidth. The technique seems attractive to cases where the design information is given only in numerical form.

A. D. Martin. Mathematical programming of portfolio selections. *Management Science*, **1**(2), 152–166, 1955.

Abstract. Presents Markowitz model as a mathematical program and a solution procedure for the quadratic program

B. Martos. Quadratic programming with a quasiconvex objective function. *Operations Research*, **19**(1), 87–97, 1971.

Abstract. Gives both necessary and sufficient conditions for a quadratic function to be quasiconvex in the nonnegative orthant. Methods of pseudoconvex programming (such as those of Frank and Wolfe) can solve linearly constrained quadratic programming problems with such an objective function.

Y. K. L. Matitskas and G. S. Palubetskis. Graph cutting and 0–1 quadratic programming. *Soviet Journal of Computer and Systems Sciences*, **25**(1), 100–107, 1987.

Abstract. A problem of 0–1 programming with a quadratic objective function and two constraints in the form of inequalities, which can be interpreted as a problem of cutting a graph into two parts, is examined. An effective algorithm for cutting a tree and a branch-and-bound algorithm for an arbitrary graph are presented. The results of a computational experiment are given. The feasibility of using a reducible lower estimate of the optimum for the purpose of testing heuristic algorithms is indicated.

R. McBride and J. Yormark. Finding all solutions for a class of parametric quadratic integer programming problems. *Management Science*, **26**, 784–795, 1980.

B. A. McCarl and T. Tice. Should quadratic-programming problems be approximated. *American Journal of Agricultural Economics*, **64**(3), 585–589, 1982.

B. A. McCarl, H. Moskowitz, and H. Furtan. Quadratic programming applications. *Omega*, **5**(1), 43–55, 1977.

Abstract. This paper reviews and extends some of the methodological areas where QP is applicable, discussing and illustrating the characteristics and aspects of the accompanying solutions. Some of the methodological areas amenable include regression analysis, decision analysis, and quadratic approximations to generally complex functions. In the functional management areas, QP is applicable to problems in economics such as demand-supply response and enterprise selection. In finance, it is used in portfolio analysis; in agriculture, in crop selection.

M. P. Mckenna, J. P. Mesirov, and S. A. Zenios. Data-parallel quadratic programming on box-constrained problems. *SIAM Journal on Optimization*, **5**(3), 570–589, 1995.

Abstract. We develop designs for the data parallel solution of quadratic programming problems subject to box constraints. In particular, we consider the class of algorithms that iterate between projection steps that identify candidate active sets and conjugate gradient steps that explore the working space. Using the algorithm of More and Toraldo [Report MCS-p77–05 89, Argonne National Laboratory, Illinois, 1989] as a specific instance of this class of algorithms we show how its components can be implemented efficiently on a data-parallel SIMD computer architecture. Alternative designs are developed for both arbitrary, unstructured Hessian matrices and for structured problems. Implementations are carried out on a Connection Machine CM-2. They are shown to be very efficient, achieving a peak computing rate over 2 Chops. Problems with several hundred thousand variables are solved within one minute of solution time on the 8K CM-2. Extremely large test problems, with up to 2.89 million variables, are also solved efficiently. The data parallel implementation outperforms a benchmark implementation of interior point algorithms on an IBM 3090–6009 vector supercomputer and a successive overrelaxation algorithm on an Intel iPSC/860 hypercube.

P. Mcleanmeyinsse. Interregional flows of corn—a quadratic-programming approach. *In* 'Modeling and Simulation', Vol. 19, pp. 419–424, 1988.

W. G. Medlin and J. F. Kaiser. Bandpass digital differentiator design using quadratic programming. *ICASSP 91. 1991 International Conference on Acoustics, Speech and Signal Processing IEEE, New York, NY, USA*, **3**, 1977–1980, 1991.

Abstract. A novel technique utilizing quadratic programming for the design of bandpass FIR (finite impulse response) digital differentiators of arbitrary order is presented. The new differentiators have linear phase and are maximally accurate at the center of the differentiation band. Their design is based on a minimization procedure for the integrated square error of the frequency response over designated approximation bands. The closed-form solution for the filter coefficients is obtained by the method of Lagrange multipliers. The inclusion of stopband in the design process is also discussed. This technique has been successfully used by the authors for the design of optimal low pass differentiators. The new differentiators are important for applications where the first-, second-, or higher-order derivative of a digital signal is required to be accurate at midrange frequencies.

K. Meer. On the complexity of quadratic programming in real number models of computation. *Theoretical Computer Science*, **133**(1), 85–94, 1994.

Abstract. The complexity of linearly constrained (nonconvex) quadratic programming is analyzed within the framework of real number models, namely the one of L. Blum, M. Shub, and S. Smale (1989) and its modification recently introduced by Koiran (1993) ("weak BSS-model"). In particular we show that this problem is not NP-complete in the Koiran setting. Applications to the (full) BSS-model are discussed.

N. Megiddo and A. Tamir. Linear time algorithms for some separable quadratic programming problems. *Operations Research Letters*, **13**(4), 203–211, 1993.

Abstract. A large class of separable quadratic programming problems is presented. The problems in the class can be solved in linear time. The class includes the separable convex quadratic transportation problem with a fixed number of sources and separable convex quadratic programming with nonnegativity constraints and a fixed number of linear equality constraints.

S. Mehrotra and J. Sun. An algorithm for convex quadratic programming that requires $O(n^{3.5}L)$ arithmetic operations. *Mathematics of Operations Research*, **15**(2), 342–363, 1990.

Abstract. A new interior point method for minimizing a convex quadratic function over a polytope is developed. The authors show that the method requires $O(n^{3.5}L)$ arithmetic operations. In the algorithm they construct a sequence $P_{z_0}, P_{z_1}, \ldots .P_{z_k}, \ldots$ of nested convex sets that shrink towards the set of optimal solution(s). During iteration k they take a partial Newton step to move from an approximate analytic center of $P_{z_{k-1}}$ to an approximate analytic center of P_{z_k}. A system of linear equations is solved at each iteration to find the step direction. The solution that is available after $O(\sqrt{m}L)$ iterations can be converted to an optimal solution. The analysis indicates that inexact solutions to the linear system of equations could be used in implementing this algorithm.

A. Melman and R. Polyak. The Newton modified barrier method for QP problems. *Annals of Operations Research*, **62**, 465–519, 1996.

Abstract. The modified barrier functions (MBF) have elements of both classical Lagrangians (CL) and classical barrier functions (CBF). The MBF methods find an unconstrained minimizer of some smooth barrier function in primal space and then update the Lagrange multipliers, while the barrier parameter either remains fixed or can be updated at each step. The numerical realization of the MBF method leads to the Newton MBF method, where the primal minimizer is found by using Newton‘s method. This minimizer is then used to update the Lagrange multipliers. In this paper, the authors examine the Newton MBF method for the quadratic programming (QP) problem. It is shown that under standard second-order optimality conditions, there is a ball around the primal solution and a cut cone in the dual space such that for a set of Lagrange multipliers in this cut cone, the method converges quadratically to the primal minimizer from any point in the aforementioned ball, and continues to do so after each Lagrange multiplier update. The Lagrange multipliers remain within the cut cone and converge linearly to their optimal values. Any point in this ball will be called a ”hot start”. Starting at such ”hot start”, at most $O(\log\log\varepsilon^{-1})$ Newton steps are sufficient to perform the primal minimization which is necessary for the Lagrange multiplier update. Here, $\varepsilon > 0$ is the desired accuracy. Because of the linear convergence of the Lagrange multipliers, this means that only $O(\log\varepsilon^{-1})O(\log\log\varepsilon^{-1})$ Newton steps are required to reach an ε-approximation to the solution from any ”hot start”. In order to reach the ”hot start”, one has to perform $O(\sqrt{m}\log C)$ Newton steps, where m characterizes the size of the problem and $C > 0$ is the condition number of the QP problem. This condition number is characterized explicitly in terms of key parameters of the QP problem, which in turn depend on the input data and the size of the problem.

C. Y. Meng, S. Frimpong, and M. J. Zuo. A quadratic programming model for blast scheduling. *Journal of University of Science and Technology Beijing*, **6**(3), 165–167, 1999.

Abstract. A quadratic programming model is established to choose the blocks to be blasted in a given period. The length of this period depends on the production planning requirements. During the given period, the blocks’ parameters are available from the geological database of the mine. The objective is to minimize the deviation of the average ore grade of blasted blocks from the standard ore grade required by the mill. Transportation ability constraint, production quantity demand constraint, minimum safety bench constraint, block size constraint and block, bench precedence constraints are considered in forming the programming model. This model has more practical objective function and reasonable constraints compared with the existing model for this kind of problems.

H. M. Merrill. Failure diagnosis using quadratic programming. *IEEE Transactions on Reliability*, **R-22**(4), 207–213, 1973.

Abstract. This paper discusses the problem of determining which of a large set of possible but improbable malfunctions gave rise to a given set of measurements. The classes of systems under consideration generally lead to underdetermined sets of equations. Three methods of formulating and solving this class of problems are presented: 1) the pseudoinverse method: this leads to an easily-solved computational problem but it is not physically realistic and it tends to give poor results; 2) a pattern recognition approach based on a more realistic problem formulation: unfortunately, the computational problems associated with this formulation may be formidable; and 3) a quadratic programming approach: this is based on minimization of a physically realistic objective function.

P. Merz and B. Freisleben. Genetic algorithms for binary quadratic programming. *In* 'GECCO-99. Proceedings of the Genetic and Evolutionary Computation Conference. Joint Meeting of the Eighth International Conference on Genetic Algorithms (ICGA-99) and the Fourth Annual Genetic Programming Conference (GP-99). Morgan Kaufmann Publishers, San Francisco, CA, USA', Vol. 1, pp. 417–424, 1999.

Abstract. Genetic algorithms for the unconstrained binary quadratic programming problem (BQP) are presented. It is shown that for small problems, a simple genetic algorithm with uniform crossover is sufficient to find optimum or best-known solutions in a short time, while for problems with a high number of variables (n¿or=200), it is essential to incorporate local search to arrive at high quality solutions. A hybrid genetic algorithm incorporating local search is tested on 40 problem instances of sizes containing between n=200 and n=2500. The results of the computer experiments show that the approach is comparable to alternative heuristics such as tabu search for small instances and superior to tabu search and simulated annealing for large instances. New best solutions could be found for 14 large problem instances.

C. Meszaros. On the sparsity issues of interior point methods for quadratic programming. Technical Report WP 98-4, Laboratory of Operations Research and Decision Systems, Hungarian Academy of Sciences, 1998a.

Abstract. In this paper we will investigate how the sparsity of non-separable quadratic programming problems behaves in interior point methods. We will show that for the normal equation approach, two orderings can be performed in independent steps to reduce the fill-in during interior point iterations. One of the permutations has to be performed on the columns, whereas the other on the rows of the problem. We show that one can easily attribute the sparsity issues of non-separable quadratic programming problems to that of linear programming for which well–developed techniques are available. We describe how the fundamental structural properties of non–separable quadratic programming problems can be represented by a single matrix whose sparsity pattern can serve to determine the row permutation and to use heuristics developed for linear programming for determining which of the augmented system and normal equation approach is more advantageous. Numerical results are given on a wide variety of non–separable convex quadratic programming problems.

C. Meszaros. The separable and non-separable formulations of convex quadratic problems in interior point methods. Technical Report WP 98-3, Laboratory of Operations Research and Decision Systems, Hungarian Academy of Sciences, 1998b.

C. Meszaros. Steplengths in interior-point algorithms of quadratic programming. *Operations Research Letters*, **25**(1), 39–45, 1999.

Abstract. An approach to determine primal and dual stepsizes in the infeasible-interior-point primal-dual method for convex quadratic problems is presented. The approach reduces the primal and dual infeasibilities in each step and allows different stepsizes. The method is derived by investigating the efficient set of a multiobjective optimization problem. Computational results are also given.

M. C. Meyer. An algorithm for projections onto convex cones with applications to nonparametric regression and quadratic programming. Technical report Sta 99-12, Department of Statistics, University of Georgia, Athens, Georgia, USA, 1999.

M. Minoux. A polynomial algorithm for minimum quadratic cost flow problems. *European Journal of Operations Research*, **18**, 377–387, 1984.

B. K. Mishra and C. Das. Maximin problem and a duality theorem for mixed-integer quadratic programming. *Zeitschrift für Angewandte Mathematik und Mechanik*, **65**(7), 310–312, 1985.

G. D. Mistriotis. Quadratic programming: an efficient alternative to micro-simulation. *Bulletin of the Operations Research Society of America*, **20**, B328, 1972.

Abstract. This paper suggests an alternative to micro-simulation as a means of updating a given sample representing some background population. The problem is translated to a quadratic programming problem with linear equality constraints by the adjustment of the individual record weights of the sample. In this way the same sample with the new revised record weights would adequately represent the aged population. The philosophy of the problem formulation and its advantages are discussed. Even for large length record problems the enormously relative low cost and the low core memory requirements are illustrated. Finally, the comparative limitations are outlined.

A. A. Mitsel and A. N. Khvashchevsky. New algorithm for solving the quadratic programming problem. *Optoelectronics, Instrumentation and Data Processing Abstracts*, **3**, 1999.

Abstract. An algorithm for solving the quadratic programming problem is considered. It differs from the known algorithms by the absence of artificial variables. In some particular cases the algorithm is reduced to solving a system of linear algebraic equations.

E. Mitsopoulou. Unilateral contact, dynamic analysis of beams by a time-stepping, quadratic programming procedure. *Meccanica*, **18**(4), 254–265, 1983.

Abstract. Discusses a method for the dynamic analysis of a slender beam, undergoing 'small' deformations and contrasted without friction by a supporting profile. The relevant contact-impact (unilateral constrained) problem is studied, with reference to a discrete model in space and time, after its formulation as a quadratic programming problem (QPP) with sign constraints only. The contact-impact problem for a rigid supporting profile is first solved, and subsequently the contact problem for an elastic profile. Among available algorithms for the numerical solution, the implicit unconditionally stable algorithm of Zienkiewicz, Wood and Taylor (1980) proved to be the most efficient for the time discretization, while a modification of Hildreth-d'Esopo's algorithm with the introduction of a variable overrelaxation factor was used for the solution of the QPP.

E. Mitsopoulou and I. Doudoumis. Unilateral contact of elastic bodies by finite-element method and quadratic-programming technique. *Zeitschrift für Angewandte Mathematik und Mechanik*, **66**(5), T421–T423, 1986.

S. Mizuno and K. G. Murty. Computation of the exact solution from a near optimum solution for convex QP. Technical Report 10/92, Department of OR and IE, Cornell, New York, USA, 1992.

Abstract. Consider a convex quadratic program of size L involving m inequality constraints and possibly some equality constraints in n variables. Given a feasible solution whose objective value is within 2^{-2L} of the optimum objective value, we discuss a procedure that finds a true optimum solution in at most m iterations, where each iteration involves minimizing the quadratic objective function on an affine space (this takes at most n conjugate gradient steps).

Y. S. Mo, W. S. Lu, and A. Antoniou. An iterative quadratic programming method for multirate filter design. *In* 'ISCAS '98. Proceedings of the 1998 IEEE International Symposium on Circuits and Systems. IEEE, New York, NY, USA', Vol. 5, pp. 41–44, 1998.

Abstract. An iterative quadratic programming method for multirate filter design is described. The design problem is formulated as a 4th-order nonlinear optimization problem in which the objective function is a weighted sum of a reconstruction term and an aliasing-error term. Constraints on the filter's frequency response in the passband and stopband are imposed as a set of linear inequalities. The optimization problem is solved by iteratively minimizing a quadratic function subject to a set of linear constraints. Explicit formulas for evaluating the minimums of these quadratic functions are described, which lead to an efficient and fast algorithm. An example is included to illustrate the design method.

N. Moal and J. J. Fuchs. Sinusoids in white noise: a quadratic programming approach. *In* 'Proceedings of the 1998 IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP '98. IEEE, New York, NY, USA', Vol. 4, pp. 2221–2224, 1998.

Abstract. We address the problem of the estimation and identification of real sinusoids in white Gaussian noise using a correlation-based method. We estimate a partial covariance sequence from the data and seek a representation of these new observations as a super-position of a small number of cosines chosen from a redundant basis and the white noise contribution. We propose to minimize a quadratic program in order to choose a parsimonious decomposition among the many that allow the reconstruction. We develop optimality conditions for the criterion that can be geometrically interpreted and present a dual criterion that has an appealing physical interpretation. Some simulated examples are also presented to show the excellent performance in resolution of the approach.

I. B. Mohd and Y. Dasril. Constraint exploration method for quadratic programming problem. *Applied Mathematics and Computation*, **112**(2–3), 161–170, 2000.

Abstract. In this paper, we represent a method which is based on the violated constraints by the unconstrained minimum of the objective function of the quadratic programming problem for exploring, locating and computing the optimal solution of the problem without using additional information as have been done in most of the favourite established methods.

A. Mohri. A computational method for optimal control of a linear system by quadratic programming. *International Journal of Control*, **11**(6), 1021–1039, 1970.

Abstract. The numerical method derived is applicable to the problem, where the control u is considered to be held at a constant value during the constant time interval, and the criterion function is given by the following integral type: $J = \int(x'Qx + u'Ru)dt$. But the method introduced in this paper requires no numerical integration except to obtain the transition matrix. Therefore the computational effort is considerably saved.

J. A. Momoh, R. E. L. Adapa, and M. E. Hawary. A review of selected optimal power flow literature to 1993. I. Nonlinear and quadratic programming approaches. *IEEE Transactions on Power Systems*, **14**(1), 96–104, 1999.

Abstract. The paper presents a review of literature on optimal power flow tracing progress in this area over from 1962–93. Part 1 deals with the application of nonlinear and quadratic programming.

R. D. C. Monteiro, I. Adler, and M. G. C. Resende. A polynomial-time primal-dual affine scaling algorithm for linear and convex quadratic programming and its power series extension. *Mathematics of Operations Research*, **15**(2), 191–214, 1990.

Abstract. Describes an algorithm for linear and convex quadratic programming problems that uses power series approximation of the weighted barrier path that passes through the current iterate in order to find the next iterate. If $r \geq 1$ is the order of approximation used, the authors show that their algorithm has time complexity $O(n^{1/2^{(1+1}r)}/L^{(1+1}r))$ iterations and $O(n^3 + n^2r)$ arithmetic operations per iteration, where n is the dimension of the problem and L is the size of the input data. When $r = 1$, they show that the algorithm can be interpreted as an affine scaling algorithm in the primal-dual setup.

R. D. C. Monteiro and I. Adler. Interior path following primal-dual algorithms. 2. Convex quadratic programming. *Mathematical Programming*, **44**(1), 43–66, 1989.

R. D. C. Monteiro and T. Tsuchiya. Global convergence of the affine scaling algorithm for convex quadratic programming. *SIAM Journal on Optimization*, **8**(1), 26–58, 1998.

Summary. A global convergence proof of the second-order affine scaling algorithm for convex quadratic programming problems is given, where the new iterate is the point that minimizes the objective function over the intersection of the feasible region with the ellipsoid centered at the current point and whose radius is a fixed fraction $\beta \in (0,1)]$ of the radius of the largest "scaled" ellipsoid inscribed in the nonnegative orthant. The analysis is based on the local Karmarkar potential function introduced by Tsuchiya. For any $\beta \in (0,1)$ and without making any nondegeneracy assumption on the problem, the sequences of primal iterates and dual estimates converge to optimal solutions of the quadratic program and its dual.

J. H. Moore and A. Whinston. Experimental methods in quadratic programming. *Management Science*, **13**, 58–76, 1966.

B. Moores and A. J. Murphy. An integer quadratic-programming formulation and algorithm for the organization of nurse staffing schedules. *In* 'Third International Conference on System Science in Health Care', Vol. 34, pp. 656–659, 1984.

J. L. Morales-Pérez and R. W. H. Sargent. On the implementation and performance of an interior point method for large sparse convex quadratic programming. *Optimization Methods and Software*, **1**(2), 153–168, 1992.

J. L. Morales-Pérez and R. W. H. Sargent. Computational experience with several methods for large sparse convex quadratic programming. *Aportaciones Matematicas, Serie Comunicaciones*, **14**, 141–158, 1994.

J. J. Moré and G. Toraldo. Algorithms for bound constrained quadratic programming problems. *Numerische Mathematik*, **55**(4), 377–400, 1989.

Abstract. Presents an algorithm which combines standard active set strategies with the gradient projection method for the solution of quadratic programming problems subject to bounds. The authors show, in particular, that if the quadratic is bounded below on the feasible set then termination occurs at a stationary point in a finite number of iterations. Moreover, if all stationary points are nondegenerate, termination occurs at a local minimizer. A numerical comparison of the algorithm based on the gradient projection algorithm with a standard active set strategy shows that on mildly degenerate problems the gradient projection algorithm requires considerable less iterations and time than the active set strategy. On nondegenerate problems the number of iterations typically decreases by at least a factor of 10. For strongly degenerate problems, the performance of the gradient projection algorithm deteriorates, but it still performs better than the active set method.

J. J. Moré and G. Toraldo. On the solution of large quadratic programming problems with bound constraints. *SIAM Journal on Optimization*, **1**(1), 93–113, 1991.

J. J. Moreau. Quadratic programming in mechanics: Dynamics of one-sided constraints. *SIAM Journal on Control*, **4**, 153–158, 1996.

H. Mori and S. Tsuzuki. A robust QP-based algorithm for power system state estimation. *In* C. E. Lin, ed., 'IASTED International Conference. High Technology in the Power Industry. Power High Tech '91. ACTA Press, Calgary, Alta , Canada', pp. 130–134, 1991.

Abstract. The authors present a new robust algorithm for power system state estimation. The proposed method is based on the fact that the power flow equation can be expressed as the exact quadratic form of node voltages in rectangular coordinates. As a result, the state estimation problem is formulated as a quadratic programming problem. The advantage of the proposed method is that the obtained estimates are

very robust to gross errors of measurements or bad data due to the exact observation equation in a quadratic form. The method has been successfully applied to a sample system and its efficiency is demonstrated.

M. Morse. Subordinate quadratic forms and their complementary forms. *Revue Roumaine de Máth Pures et Appliques*, **16**, 558–569, 1971.

A. M. Morshedi, C. R. Cutler, and T. A. Skrovanek. Optimal solution of dynamic matrix control with quadratic-programming techniques (QDMC). *In* 'Advances in Instrumentation, Proceedings of the 1985 Instrument Society of America Conference and Exhibition', Vol. 20, pp. 679–689, 1985.

T. S. Motzkin. Quadratic forms positive for nonnegative variables not all zero. *Notices of the American Mathematical Society*, **12**, 1964.

F. Mraz. Local minimizer of a nonconvex quadratic programming problem. *Computing*, **45**(3), 283–289, 1990.

Abstract. A modified Beale's algorithm is described which computes the local minimizer of any quadratic objective function subject to linear constraints. Some extensions are given, first of all the possibility of movement to the neighbouring local minimizer with a reduced objective function value in some special cases.

J. A. Muckstadt. A dual decomposition algorithm for solving mixed integer-continuous quadratic programming problems. Technical report, Air Force Systems Command, Wright-Patterson AFB, OH, USA, 1969.

Abstract. The report contains a presentation of an algorithm for solving the following mixed integer-continuous mathematical programming problem: $\min c^T g + c^T x + 1/2 x^T Dx$: $(Ax+By) \geq b$, y is an element of S, $x \geq O$) where g, c, x, y, and b are column vectors of appropriate dimension; A, B, and D are matrices of appropriate dimension; and S is a closed, bounded set whose elements have integer components. The quadratic form is assumed to be positive semi-definite. The algorithm is a generalization of one developed by J. F. Benders for solving the above problem for the special case where $D = O$. The primary motivation for developing this algorithm was a desire to construct a method for solving both an aircraft maintenance scheduling problem and the unit commitment-economic dispatch scheduling problem encountered in large scale power systems.

W. Murray. An algorithm for finding a local minimum of an indefinite quadratic program. Technical Report NAC 1, National Physical Laboratory, London, England, 1971.

Abstract. An algorithm is described that will find either the global minimum of a strictly convex quadratic program or a local minimum of an indefinite quadratic program. The algorithm is a generalization of a recently published algorithm for linear programming. A feature of the new algorithm is that numerically stable factors of a matrix are recurred as opposed to its inverse or unstable factors.

K. G. Murty. On the number of solutions to the complementary quadratic programming problem. Technical report, California Univ., Operations Res Center, Berkeley, CA, USA, 1968.

Abstract. The relationship between the number of solutions to the complementary problem, the right-hand constant vector q and the matrix M is explored. Most of the proofs are based on mathematical induction. Counterexamples are given to show that the theorems fail if any of the hypotheses are not satisfied.

K. G. Murty and S. N. Kabadi. Some NP-complete problems in quadratic and nonlinear programming. *Mathematical Programming*, **39**(2), 117–129, 1987.

L. D. Muu and W. Oettli. An algorithm for indefinite quadratic programming with convex constraints. *Operations Research Letters*, **10**(6), 323–327, 1991.

Abstract. Proposes a branch-and-bound method for minimizing an indefinite quadratic function over a convex set. The bounding operation is based on a certain relaxation of the constraints. The branching is a simplex bisection.

W. C. Mylander. Nonconvex quadratic programming by a modification of Lemke's method. Technical Paper RAC-TP-414, Research Analysis Corporation, Mclean, VA, USA, 1971.

W. C. Mylander. Finite algorithms for solving quasiconvex quadratic programs. *Operations Research*, **20**, 167–173, 1972.

W. C. Mylander. *Processing nonconvex quadratic programming problems*. PhD thesis, Department of Operations Research, Stanford University, Stanford, California, USA, 1975.

N. Nabona and L. L. Freris. Optimum allocation of spinning reserve by quadratic programming. *Proceedings of the Institution of Electrical Engineers*, **122**(11), 1241–1246, 1975.

Abstract. In power system optimum dispatch problems, the spinning-reserve characteristics of the individual generators cannot be effectively linearised through the sensitivity coefficients. To solve this problem, a technique known as quadratic-approximation programming is used. Limits on the transmission-line current are also incorporated in the algorithm. It is shown that the minimum-cost dispatch problem, subject to a specified spinning reserve and the maximum-spinning-reserve problem, can be easily solved with moderate computing-time requirements.

A. Nagurney. An algorithm for the solution of a quadratic-programming problem, with application to constrained matrix and spatial price equilibrium problems. *Environment and planning A*, **21**(1), 99–114, 1989.

Y. Nakamura and S. Yamashiro. Probabilistic operation of electric power systems considering environmental constraint. X. Comparison with quadratic programming method. *Memoirs of the Kitami Institute of Technology*, **18**(2), 173–179, 1987.

Abstract. For pt.IX see ibid., volume 18, number 1 (1987). The authors describe a fast scheduling method to consider every line capacity and NO_x emission constraint when each line and thermal unit fail probabilistically. The method is an approximate one because only two thermal units control the output power to satisfy the lines capacity and only a healthy state controls the emission excepting faulty states. The usefulness of the method is confirmed by comparing it with a strict method. The strict schedule is estimated by the quadratic programming method. The report describes the outlines of both the proposed method and the quadratic programming method. Both methods are applied to a model system and the results are shown.

J. Nanda, D. P. Kothari, and S. C. Srivastava. New optimal power-dispatch algorithm using Fletcher's quadratic programming method. *IEE Proceedings C (Generation, Transmission and Distribution)*, **136**(3), 153–161, 1989.

Abstract. A fresh attempt has been made to develop a new algorithm for optimal power flow (OPF) using Fletcher's quadratic-programming method (1971). The new algorithm considers two decoupled subproblems needing minimum cost of generation and minimum system-transmission losses. These have been solved sequentially to achieve optimal allocation of real and reactive power generation and transformer tap settings with consideration of system-operating constraints on generation, busbar voltage and line-flow limits. The potential of the new algorithm for OPF has been demonstrated through system studies for two IEEE test systems and an Indian system. Results reveal that the proposed new algorithm has potential for online solving of OPF problems.

D. L. Nash and G. W. Rogers. Risk management in herd sire portfolio selection: a comparison of rounded quadratic and separable convex programming. *Journal of Dairy Science*, **79**, 301–309, 1996.

D. L. Nelson, T. O. Lewis, and T. L. Boullion. A quadratic programming technique using matrix pseudoinverses. *Industrial Mathematics*, **21**, 1–21, 1971.

Abstract. This paper is concerned with solving quadratic programming problems with various types of linear constraints. The types considered are quite general and include (1) equality constraints, (2) nonnegativity constraints, (3) inequality constraints, and (4) any combination of the first three types. In each case, it is assumed that the matrix of the quadratic objective function is at least positive semidefinite. The expressions for the solution vectors are given in terms of the matrices of the objective function and constraints, or of submatrices of these. Extensive use is made of the properties of the pseudoinverse (the generalized inverse of Penrose).

Y. Nesterov. Polynomial methods in the linear and quadratic programming. *Soviet Journal of Computer and Systems Sciences*, **26**(5), 98–101, 1988.

Abstract. New polynomial algorithms for solving problems of the linear and quadratic programming are proposed. The dependence of the estimate of the number of arithmetic operations upon the dimension of the problem has a cubic order.

Y. Nesterov. Semidefinite relaxation and nonconvex quadratic optimization. *Optimization Methods and Software*, **9**(1-3), 141–160, 1998.

Y. Nesterov. Global quadratic optimization on the sets with simplex structure. CORE Discussion Paper 9915, Center for Operations Research and Econometrics, Université catholique de Louvain, Louvain-la-Neuve, Belgium, 1999.

Y. Nesterov and A. Nemirovsky. Acceleration and parallelization of the path-following method for a linearly constrained quadratic problem. *SIAM Journal on Optimization*, **1**(4), 548–564, 1991.

B. P. Ng, M. H. Er, and C. Kot. A flexible array synthesis method using quadratic programming. *IEEE Transactions on Antennas and Propagation*, **41**(11), 1541–1550, 1993.

Abstract. A highly flexible synthesis method for an arbitrary array is proposed to best approximate a desired array pattern in a minimum-mean-square-error sense. The basic idea of the technique is to form a quadratic program with its cost function given by the mean-square error between the array response and a properly selected pattern described by a known mathematical function. This quadratic program can be a constrained or unconstrained optimization problem depending on the requirements of the desired array pattern. In formulating the quadratic program, no assumption has been made on the gain/phase response or characteristics of the individual array elements. Therefore, one can synthesize an array of arbitrary shape to any appropriate pattern with the characteristic of the array elements taken into consideration as long as one is able to model the array accurately. The proposed method is used to synthesize arrays of different shapes, linear as well as planar arrays (including rectangular and circular planar arrays), using a Chebyshev polynomial or zero function as a design template, to illustrate the feasibility of the proposed method.

H. Nicholson and M. J. H. Sterling. Optimum dispatch of active and reactive generation by quadratic programming. *IEEE Transactions on Power Apparatus and Systems*, **PAS-92**(2), 644–654, 1973.

Abstract. Optimum scheduling of generation subject to constraints on maximum and minimum levels and rates of change of generation, spare capacity and line flow limits, is studied. A linear programming formulation for system constraints is used with a quadratic cost function for generation and transmission line loss.

An optimal solution for real power dispatch is obtained by quadratic programming, and optimum allocation of reactive power is based on a gradient technique which minimises transmission loss subject to nodal voltage and reactive power limits. An ac load flow analysis is incorporated together with a load prediction program based on a spectral analysis of past load data. The feasibility of simulating the overall system problem, including load-frequency control of equivalent generation, using a combined analogue-digital process computer system linked to a large scientific digital computer by data link is investigated.

H. B. Nielsen. QPBOX user's guide, a Fortran77 package for box constrained quadratic programs. Report IMM-REP-1997-04, Department of Mathematical Modelling, Technical University of Denmark, Lyngby, Denmark, 1997.

A. Nilsberth, D. Sjelvgren, and H. Brannlund. Application of quadratic programming in probabilistic production costing. *In* 'PSCC. Proceedings of the Tenth Power Systems Computation Conference. Butterworths, London, England', pp. 678–683, 1990.

Abstract. The long term production planning of a mixed hydro-thermal power system often has to be treated as an interaction between two subproblems: Probabilistic production costing for the thermal units, and optimal scheduling of the available hydro discharges within the limits of the river system. In this interaction, marginal costs regarding the hydro energy are calculated for use in the hydro scheduling. The allocation of the hydro discharges/energies is then used for another calculation of hydro marginal costs to be used in the next iteration. In the planning system developed at the Swedish State Power Board the hydro subproblem is modeled with a time-increment of one week while the load during each week is described by three load duration curves-one for each of day-time, night-time and weekends. To calculate the correct marginal cost for the hydro energy, the available weekly energies has to be allocated optimally within each week. This optimization problem can be formulated as a quadratic programming problem with one linear equality constraint: the fixed amount of energy to be distributed. Since this method use second order derivatives the solution will converge better than for the more common methods which solve sets of linear equations.

J. Nocedal and S. J. Wright. Quadratic programming. *In* 'Numerical Optimization', Series in Operations Research, chapter 16, pp. 441–488. Springer Verlag, Heidelberg, Berlin, New York, 1999.

F. Noonan. A quadratic programming model for estimating class contribution to peak electrical demand. *Bulletin of the Operations Research Society of America*, **23**, B441, 1975.

Abstract. Within an electric utility, timely information on the contribution of peak demand from customer classes can be extremely useful in establishing both load forecasts and pricing policies. Unfortunately, a utility's normal data collection procedures does not provide this information and obtaining it, even infrequently, has entailed costly statistical sampling operations. A statistical identification model with inequality constraints on the unknown variables has been constructed as a means for providing estimates on the desired quantities. Model inputs include monthly customer billing data and aggregate peak demands. Test case results demonstrating the quality of the estimates are presented.

S. Nordebo and I. Claesson. A well-conditioned quadratic program for unique design of two-dimensional weighted Chebyshev FIR filters. *In* '1996 IEEE International Conference on Acoustics, Speech, and Signal Processing Conference Proceedings. IEEE, New York, NY, USA', Vol. 3, pp. 1355–1358, 1996.

Abstract. The weighted Chebyshev design of two-dimensional FIR filters is in general not unique since the Haar condition is not generally satisfied. However, for a design based on the discrete frequency domain, the Haar condition might be fulfilled. The question of uniqueness is, however, rather extensive to investigate. It is therefore desirable to define some simple additional constraints to the Chebyshev design in order to obtain a unique solution. The weighted Chebyshev solution of minimum Euclidean filter weight norm is always unique, and represents a sensible additional constraint since it implies minimum white noise amplification.

It is shown that this unique Chebyshev solution can always be obtained by using an efficient quadratic programming formulation with a strictly convex objective function and linear constraints. An example where a conventional Chebyshev solution is non-unique is discussed.

S. Nordebo, I. Claesson, and S. Nordholm. Weighted Chebyshev approximation for the design of broadband beamformers using quadratic programming. *IEEE Signal Processing Letters*, **1**(7), 103–105, 1994.

Abstract. A method to solve a general broadband beamformer design problem is formulated as a quadratic program. As a special case, the minimax near-field design problem of a broadband beamformer is solved as a quadratic programming formulation of the weighted Chebyshev approximation problem. The method can also be applied to the design of multidimensional digital FIR filters with an arbitrarily specified amplitude and phase. For linear phase multidimensional digital FIR filters, the quadratic program becomes a linear program. Examples are given that demonstrate the minimax near-field behavior of the beamformers designed.

S. Nordebo, I. Claesson, and S. Nordholm. Quadratic programming for the design of two-dimensional weighted Chebyshev FIR filters with incomplete specifications. *Nonlinear Analysis-Theory Methods and Applications*, **30**(7), 4345–4355, 1997.

S. Nordebo, I. Claesson, and Z. Zang. Optimum window design by semi-infinite quadratic programming. *IEEE Signal Processing Letters*, **6**(10), 262–265, 1999.

Abstract. This letter presents a new extended active set strategy for optimum finite impulse response (FIR) window design by semi-infinite quadratic programming. The windows may be asymmetric corresponding to frequency responses with general nonlinear phase. The optimality criterion is to minimize the sidelobe energy (L_2-norm) subject to a peak sidelobe magnitude-constraint (L_∞-constraint). Additional linear constraints are used to form the mainlobe (unity DC gain). Numerical examples involving group delay specifications are used to illustrate the usefulness of the algorithm.

A. Nou. An algorithm for a singly constrained quadratic program subject to lower bounds. Technical Report TRITA/MAT-97-OS2, Department of Mathematics, KTH, Stockholm, Sweden, 1997.

B. Novak and I. Novak. Quadratic programming applied to short term hydrothermal scheduling. *In* H. B. Siguerdidjane and P. Bernhard, eds, 'Control Applications of Nonlinear Programming and Optimization 1989. Proceedings of the 8th IFAC Workshop. Pergamon, Oxford, England', pp. 57–62, 1990.

Abstract. The purpose of optimal short term scheduling is to maximize electrical energy production depending on natural water inflows of a river, according to daily load duration curve and operation of thermal power plants with minimal oscillating load. Thermal power plants have a long start-up time and cannot follow rapid changes in load demand, so they are used to cover the constant part of the load, while peaks in load demand are covered by hydro units. A mathematical model of a hydro plant chain is developed which uses only two instead of three variables. The optimization algorithm uses Newton's method, which is applied on the exact ℓ_1 penalty function, allowing a nonfeasible starting point so that no phase I simplex or projection method is needed at the beginning of optimization. The program was applied to the hydro-chain on the river Drava.

I. Nowak. Some heuristics and test problems for nonconvex quadratic programming over a simplex. Preprint series, Institut für Mathematik, Humboldt-Universität zu Berlin, Germany, 1998.

Abstract. In this paper we compare two methods for estimating a global minimizer of an indefinite quadratic form over a simplex. The first method is based on the enumeration of local minimizers of a so-called control polytope. The second method is based on an approximation of the convex envelope using semidefinite programming. In order to test the algorithms a method for generating random test problems is presented where the optimal solution is known and the number of binding constraints is prescribed. Moreover, it is investigated if some modifications of the objective function influence the performance of the algorithms. Numerical experiments are reported.

I. Nowak. A global optimality criterion for nonconvex quadratic programming over a simplex. Preprint series, Institut für Mathematik, Humboldt-Universität zu Berlin, Germany, 1998b.

Abstract. In this paper we propose a global optimality criterion for globally minimizing a quadratic form over the standard simplex, which in addition provides a sharp lower bound for the optimal value. The approach is based on the solution of a semidefinite program (SDP) and a convex quadratic program (QP). Since there exist fast (polynomial time) algorithms for solving SDP's and QP's the computational time for checking the global optimality criterion and for computing the lower bound is reasonable. Numerical experiments on random test examples up to 30 variables indicate that the optimality criterion verifies a global solution in almost all instances.

D. Oelschlagel and H. Susse. Error estimations for Wolfe methods of quadratic programming problem solutions with the help of interval arithmetic. *Mathematische Operationsforschung und Statistik, Serie Statistics*, **9**(3), 389–396, 1978.

Abstract. The estimations are given by the solution of a linear system of equation by means of the methods of interval arithmetic. Therefore, it is quite certain that the round off errors are also included in these estimations.

J. H. Oh and E. Feron. Primal-dual quadratic programming approach to multiple conflict resolution. *In* 'Proceedings of the 1998 American Control Conference. American Autom. Control Council, Evanston, IL, USA', Vol. 5, pp. 2802–2806, 1998.

Abstract. This paper considers a multiple conflict resolution problem for air traffic control systems. The time required to optimally solve aircraft conflicts is known to grow exponentially with the number of aircraft involved and may become prohibitive when large numbers of aircraft are involved. As an attempt to circumvent this issue, a heuristic polynomial-time conflict resolution algorithm is proposed on the basis of analysis results on the relationship between primal and dual quadratic programs.

Y. Ohashi, H. Kando, H. Ukai, and T. Iwazumi. Optimal design of FIR linear phase digital filters via convex quadratic programming method. *International Journal of Systems Science*, **19**(8), 1469–1482, 1988.

Abstract. Many methods exist for designing FIR linear phase digital filters by using various optimization techniques. An optimization design method via the convex quadratic programming method is investigated in the paper. The aim of this method is to design digital filters with certain desired frequency responses that keep the passband ripple to a minimum while maintaining a balance between opposite characteristics of frequency responses, i.e. the narrow transition band and the great stopband attenuation. In order to verify the effectiveness of this method, low-pass filters are designed and comparison studies with other typical optimization methods are given.

S. Ohtaki and K. I. Hata. Plane elastic-plastic stress analysis using quadratic programming. adoption of the yield function containing the third invariant of the deviatoric stress tensor. *Memoirs of the Hokkaido Institute of Technology*, **15**, 35–41, 1987.

Abstract. The finite element elastic-plastic analysis based on the theory of plasticity taking into account the second order effect using the quadratic programming method is applied to the plane stress problem. This second order effect is caused from the yield function derived from Prager's theory, which is assumed to be expressed in terms of not only the second invariant of the deviatoric stress tensor J_2' but also of the third one J_3'. Matrices concerning with this method are formulated under the plane stress state. A numerical calculation is presented for the stress concentration problem of a thin rectangular plate with semi-circular side notches subjected to a uni-axial tension. Results obtained by using the second order plastic theory are compared with those of the ordinary J_2 theory.

S. Ohtaki and A. Kurimura. An analysis of two-dimensional elastic-plastic stress problem using the quadratic programming. (The case of adopting Prandtl-Reuss equation and von Mises yield function). *Bulletin of the Japan Society of Mechanical Engineers*, **28**(243), 1864–1867, 1985.

Abstract. G. Maier (1971) investigated problems in which normality conditions with regard to plastic strain increments pertaining to the yield surface in stress space are not satisfied and work-softening effects are considered with the aid of quadratic programming concepts. The authors applied this method for solving the elastic-plastic plane stress problem as a quadratic optimization in terms of displacement rates and plastic multiplier rates. The matrices are formulated in explicit forms for the problem of a perforated strip under tension. A comparison is made between the results obtained by the present method and by the ordinary matrix method. The elastic-plastic problem is fairly tractable to the present method.

A. Ohuchi and I. Kaji. Algorithms for optimal allocation problems having quadratic objective function. *Journal of the Operations Research Society of Japan*, **23**(1), 64–79, 1980.

Abstract. An optimal allocation problem (APQ) with a quadratic objective function, a total resource constraint and an upper and lower bound constraint is considered. The APQ is a very basic and simple model but it can serve as a subproblem in the solution of the generalized allocation problem. Applying the Lagrange relaxation method an explicit expression of the dual function associated with the APQ and an equation which the optimal dual variable must satisfy and derived first. Then, some properties of the equation are discussed. Finally, three algorithms for solving the equation are proposed, and some computational results for the APQ are given. These results reveal the effectiveness of the algorithm.

A. Ohuchi and I. Kaji. An algorithm for the Hitchcock transportation problems with quadratic cost functions. *Journal of the Operations Research Society of Japan*, **24**(2), 170–181, 1981.

Abstract. In this paper an algorithm is given for the Hitchcock transportation problems (HTPQ). A procedure has been proposed which involves successively minimising the Lagrangian with respect to each of its dual coordinates. The fact that the algorithms for optimal allocation problems having quadratic objective functions (APQ) can be used for the maximising procedure in network transportation problems with quadratic costs functions was investigated, and APQ studied in detail. The application of algorithms proposed by the authors to solving HTPQ is considered. In particular several examples are given of relatively large-scale problems, with computational times.

M. E. O'Kelly. A quadratic integer-program for the location of interacting hub facilities. *European Journal of Operational Research*, **32**(3), 393–404, 1987.

D. P. O'Leary. A generalized conjugate gradient algorithm for solving a class of quadratic programming problems. *Linear Algebra and Its Applications*, **34**, 371–399, 1980.

Abstract. Applies matrix splitting techniques and a conjugate gradient algorithm to the problem of minimizing a convex quadratic form subject to upper and lower bounds on the variables. This method exploits sparsity structure in the matrix of the quadratic form. Choices of the splitting operator are discussed, and convergence results are established.

D. P. O'Leary. Sparse quadratic programming without matrix updating. Technical Report TR-1200, Computer Science Department, University of Maryland, USA, 1982.

D. P. O'Leary and W. H. Yang. Elasto-plastic torsion by quadratic programming. *Computer Methods in Applied Mechanics and Engineering*, **16**(3), 361–368, 1978.

Abstract. A finite element scheme (together with a conjugate gradient algorithm) is demonstrated to be a very effective method for analyzing elastoplastic torsion of prismatic bars posed as quadratic programming problems. Solutions for bars with elliptical and Sokolovsky's oval cross-sections are presented. The solutions for the elliptical bars agree with the existing elastic and limit plastic solutions at the two extremes of the elastic-plastic range. The algorithm also reproduces accurately the Sokovsky solution and extends it beyond its limitations.

T. Onoda and H. Sekimoto. A neutron spectrometry unfolding code based on quadratic programming. *Nuclear Instruments and Methods in Physics Research, Section A (Accelerators, Spectrometers, Detectors and Associated Equipment)*, **272**(3), 844–846, 1988.

Abstract. A new unfolding code based on quadratic programming has been developed for precise treatment of the nonnegativity constraint of the neutron spectrum. This code does not require any initial guess and enables a global optimum solution to be derived.

A. Orden. Stationary points of quadratic functions under linear constraints. *Computer Journal*, **7**, 238–242, 1974.

S. Ou and M. Pedram. Timing-driven bipartitioning with replication using iterative quadratic programming. *In* 'Proceedings of the ASP-DAC '99 Asia and South Pacific Design Automation Conference 1999. IEEE, Piscatatway, NJ, USA', Vol. 1, pp. 105–108, 1999.

Abstract. We present an algorithm for solving a general min-cut, two-way partitioning problem subject to timing constraints. The problem is formulated as a constrained programming problem and solved in two phases: cut-set minimization and timing satisfaction. A mathematical programming technique based on iterative quadratic programming (TPIQ) is used to find an approximate solution to the constrained problem. When the timing constraints are too strict to have a feasible solution, node replication is used to satisfy the constraints. Experimental results on the ISCAS89 benchmark suite show that TPIQ can solve the timing-driven bipartitioning problem with little impact on the chip size.

K. A. Palaniswamy, J. K. Sharma, and K. B. Misra. Minimization of load curtailment in power system using quadratic programming. *Journal of the Institution of Engineers (India) Electrical Engineering Division*, **65**, 213–218, 1985.

Abstract. The authors describe the problems of steady state optimal load shedding in power systems and present a method for minimizing load curtailment under a given set of emergency operating conditions. Electric power consumers are inconvenienced if there is an inadequate level of supply voltage and load curtailment. The inconvenience is to be minimized subject to the system power-flow equations and the limits on real and reactive power generation along with the limits on the line flows. This optimization problem is solved using quadratic programming which has been found effective in the assignment of distribution of load curtailment over the entire system. The method is illustrated on a sample system for generation-outage and line-outage conditions.

V. N. Palyaeva. A linearization method with quadratic programming for linearly constrained approximation-problems. *Vestnik Leningradskogo Universiteta Seriya Matematika Mekhanika Astronomiya*, pp. 30–36, 1983.

J. S. Pang. An equivalence between two algorithms for quadratic programming. *Mathematical Programming*, **20**(2), 152–165, 1981.

Abstract. This paper demonstrates that the Van de Panne-Whinston symmetric simplex method when applied to a certain implicit formulation of a quadratic program generates the same sequence of primal feasible vectors as does the Von Hohenbalken simplicial decomposition algorithm specialized to the same program. Such an equivalence of the two algorithms extends earlier results for a least-distance program due to Cottle-Djang.

J. S. Pang. Methods for quadratic programming: a survey. *Computers and Chemical Engineering*, **7**(5), 583–594, 1983.

Abstract. This paper presents a survey on methods for solving the general quadratic programming problem. The discussion is centered on (i) the unification of several classes of finite methods, (ii) recent developments of iterative methods and (iii) the numerical implementation of several finite and iterative methods for largescale applications.

P. M. Pardalos. Generation of large-scale quadratic programs for use as global optimization test problems. *ACM Transactions on Mathematical Software*, **13**(2), 133–137, 1987.

P. M. Pardalos. Quadratic problems defined on a convex hull of points. *BIT*, **28**, 323–328, 1988.

P. M. Pardalos. Polynomial time algorithms for some classes of constrained nonconvex quadratic problems. *Optimization*, **21**(6), 843–853, 1990.

P. M. Pardalos. Construction of test problems in quadratic bivalent programming. *ACM Transactions on Mathematical Software*, **17**(1), 74–87, 1991a.

P. M. Pardalos. Global optimization algorithms for linearly constrained indefinite quadratic problems. *Computational Mathematics and Applications*, **21**(6/7), 87–97, 1991b.

P. M. Pardalos and N. Kovoor. An algorithm for a singly constrained class of quadratic programs subject to upper and lower bounds. *Mathematical Programming*, **46**(3), 321–328, 1990.

P. M. Pardalos and G. P. Rodgers. Parallel branch and bound algorithms for unconstrained quadratic 0–1 programming. *In* R. Sharda, ed., 'Impact of Recent Computer Advances on Operations Research', pp. 131–143, North Holland, Amsterdam, the Netherlands, 1989.

P. M. Pardalos and G. P. Rodgers. Computational aspects of a branch and bound algorithm for quadratic 0–1 programming. *Computing*, **45**, 131–144, 1990a.

P. M. Pardalos and G. P. Rodgers. Parallel branch and bound algorithms for quadratic 0–1 programs on the hypercube architecture. *Annals of Operations Research*, **22**, 271–292, 1990b.

P. M. Pardalos and G. Schnitger. Checking local optimality in constrained quadratic programming is NP-hard. *Operations Research Letters*, **7**(1), 33–35, 1988.

Abstract. Proves that the problem of checking local optimality for a feasible point and the problem of checking if a local minimum is strict, are NP-hard problems. As a consequence the problem of checking whether a function is locally strictly convex is also NP-hard.

P. M. Pardalos and S. A. Vavasis. Quadratic programming with one negative eigenvalue is NP-hard. *Journal of Global Optimization*, **1**, 15–23, 1991.

P. M. Pardalos, M. V. Ramana, and Y. Yajima. Cuts and semidefinite relaxations for nonconvex quadratic problems. *SIAM Journal on Optimization*, **(submitted)**, 1998.

P. M. Pardalos, Y. Ye, and C. G. Han. An interior point algorithm for large-scale quadratic problems with box constraints. *In* A. Bensoussan and J. L. Lions, eds, 'Analysis and Optimization of Systems : Proceedings of the 9th International Conference, Antibes, France, 1990', Vol. 144 of *Lecture Notes in Control and Information*, pp. 413–422, Springer Verlag, Heidelberg, Berlin, New York, 1990.

K. C. Park, P. H. Chang, and S. H. Kim. The enhanced compact QP method for redundant manipulators using practical inequality constraints. *In* ‘Proceedings. 1998 IEEE International Conference on Robotics and Automation. IEEE, New York, NY, USA’, Vol. 1, pp. 107–114, 1998.

Abstract. For resolving the manipulator redundancy under inequality constraints, the compact QP method (including the improved Compact QP method) is very effective and efficient. From a viewpoint of practical applications, however, it turns out, we found, to have some performance limitations such as unrealistically high torque due to joint angle limit constraints and tracking errors due to joint torque limit constraints under parameter variation and disturbance. Remedying the limitations, the enhanced compact QP method is developed by using the practical inequality constraints with p-step-ahead predictor and time delay estimation. Through dynamic simulation results, it is verified that the enhanced compact QP method significantly improves the compact QP method in terms of efficiency and effectiveness for the real-time control of redundant manipulators under physical limits.

R. T. Parry. The application of quadratic programming to the portfolio selection problem: a review. Special Studies Paper 5, Board of Governors of the Federal Reserve System, USA, 1970.

T. D. Parsons. *A combinatorial approach to convex quadratic programming*. PhD thesis, Princeton University, 1966.

T. D. Parsons. A combinatorial approach to convex quadratic programming. *Linear Algebra and Its Applications*, **3**(3), 359–378, 1970.

Abstract. The problem of minimizing a convex quadratic function of many variables constrained by linear inequalities has been studied extensively, and a number of algorithms have been developed. Most of them are similar and are closely related to the simplex method for linear programming. This paper is essentially an extension of A. W. Tucker’s Combinatorial Theory underlying linear programs to convex quadratic programs. As such, it shares the purpose of that paper ’to discover the underlying theoretical structure’ of the problem, and to unite this with the actual steps of computation to make clear the limits of generality.

N. Pelillo and A. Jagota. Feasible and infeasible maxima in a quadratic program for maximum clique. *Journal of Artificial Neural Networks*, **2**(4), 411–420, 1995.

Abstract. T. S. Motzkin and E. G. Straus (1965) related global maxima of a certain quadratic program to the maximum clique size in a certain graph. We extend this result to relate strict local maxima of this program to certain maximal cliques, and certain maxima to non cliques. Our results are useful to a companion paper which employs this QP in a neural net model to find large cliques in graphs.

A. F. Perold. An alternative method for a global analysis of quadratic programs in a finite number of steps. *Mathematical Programming*, **15**(1), 105–109, 1978.

P. Pfaff. Portfolio selection using quadratic programming on a microcomputer. *In* ‘Modeling and Simulation: Proceedings of the Twelfth Annual Pittsburgh Conference’, Vol. 1–4, pp. 993–997, 1981.

T. Pham Dinh, T. Q. Phong, R. Horaud, and L. Quan. Stability of Lagrangian duality for nonconvex quadratic programming. solution methods and applications in computer vision. *RAIRO-Mathematical Modelling and Numerical Analysis—Modélisation Mathématique et Analyse Numérique*, **31**(1), 57–90, 1997a.

Abstract. The problem of minimizing a quadratic form over a ball centered at the origin is considered. The stability of Lagrangian duality is established and complete characterizations of a global optimal solution are

given. On the basis of this theoretical study, two principal solution methods are presented. An important application of nonconvex quadratic programming is the computation of rite step to a new iterate in the Trust Region (TR) approach methods which are known to be efficient for nonlinear optimization problems. Also, we discuss the mathematical models of some important problems encountered in Computer Vision. Most of them can be formulated as a minimization of a sum of squares of nonlinear functions. A practical TR-based algorithm is proposed for nonlinear least squares problem which seems to be well suited for our applications.

T. Pham Dinh, P. Thai Quynh, R. Horaud, and L. Quan. Stability of Lagrangian duality for nonconvex quadratic programming. Solution methods and applications in computer vision. *Mathematical Modelling and Numerical Analysis*, **31**(1), 57–90, 1997b.

Abstract. The problem of minimizing a quadratic form over a ball centered at the origin is considered. The stability of Lagrangian duality is established and complete characterizations of a global optimal solution are given. On the basis of this theoretical study, two principal solution methods are presented An important application of nonconvex quadratic programming is the computation of the step to a new iterate in the Trust Region (TR) approach methods which are known to be efficient for nonlinear optimization problems. Also, we discuss the mathematical models of some important problems encountered in Computer Vision. Most of them can be formulated as a minimization of a sum of squares of nonlinear functions. A practical TR-based algorithm is proposed for nonlinear least squares problem which seems to be well suited for our applications.

J. Philip. Restoration of pictures by quadratic programming and by Fourier transformation in the complex domain. *Proceedings of the IEEE*, **61**(4), 468–469, 1973.

Abstract. Two methods for digital restoration of pictures blurred by imperfections of the optical system and by random noise are suggested. Method one is based on the a priori information that a picture has nonnegative intensity and leads to quadratic programming. In method two, the finite extent of the picture is shown to make Fourier transformation in the complex domain useful.

J. Philip. Digital image and spectrum restoration by quadratic programming and by modified Fourier transformation. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **PAMI-1**(4), 385–399, 1979.

Abstract. Considers the convolution equation $f*h+e=d$, where f is sought, h is a known 'point spread function', e represents random errors and d is the measured data. All these functions are defined on the integers mod(N). A mathematical-statistical formulation of the problem leads to $min_f \|f*h-d\|_A$, where the A-norm is derived from the statistical distribution of e. If f is known to be nonnegative, this is a quadratic programming problem. Using the discrete Fourier transforms (DFTs) F,H, and D of f,h, and d, the author arrives at a minimization in another norm: $\min_F \|F.H-D\|_\alpha$. A solution would be $F=D/H$, but H has zeros. He considers the theoretical and practical difficulties that arise from these zeros and describes two methods for calculating F numerically also when H has zeros. Numerical tests of methods are presented, in particular tests with one of the methods, called 'the derivative method', where d is a blurred image.

A. T. Phillips and J. B. Rosen. A parallel algorithm for constrained concave quadratic global minimization. *Mathematical Programming, Series B*, **42**(2), 421–448, 1988.

A. T. Phillips and J. B. Rosen. Guaranteed ffl-approximate solution for indefinite quadratic global minimization. *Naval Research Logistics Quarterly*, **37**, 499–514, 1990.

H. X. Phu and N. D. Yen. On the stability of solutions to quadratic programming problems. *Mathematical Programming*, **89**(3), 385–394, 2001.

Abstract. We consider the parametric programming problem (Q_p) of minimizing the quadratic function $f(x,p) := x^T Ax + b^T x$ subject to the constraint $Cx \le d$, where $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, $C \in \mathbb{R}^{m\times n}$, $d \in \mathbb{R}^m$, and $p := (A,b,C,d)$ is the parameter. Here, the matrix A is not assumed to be positive semidefinite. The set of the global minimizers and the set of the local minimizers to (Q_p) are denoted by $M(p)$ and $M^{loc}(p)$, respectively. It is proved that if the point-to-set mapping $M^{loc}(\cdot)$ is lower semicontinuous at p then $M^{loc}(p)$

is a nonempty set which consists of at most $\mathcal{A}_{m,n}$ points, where $\mathcal{A}_{m,n} = \binom{m}{\min\{[m/2], n\}}$ is the maximal cardinality of the antichains of distinct subsets of $\{1, 2, \dots, m\}$ which have at most n elements. It is proved also that the lower semicontinuity of $M(\cdot)$ at p implies that $M(p)$ is a singleton. Under some regularity assumption, these necessary conditions become the sufficient ones.

J. W. Pitton. Positive time-frequency distributions via quadratic programming. *Multidimensional Systems and Signal Processing*, **9**(4), 439–445, 1998.

Abstract. A new method for computing positive time-frequency distributions (TFD) for nonstationary signals is presented. This work extends the earlier work of the author and his colleagues in computing positive TFD (1994). This paper describes a general quadratic programming approach to the problem of computing these signal-dependent distributions. The method is based on an evolutionary spectrum formulation of positive TFD. The minimization problem reduces to a linearly-constrained quadratic programming problem, for which standard solutions are widely available.

S. Poljak and H. Wolkowicz. Convex relaxations of (0,1)-quadratic programming. *Mathematics of Operations Research*, **20**(3), 550–561, 1995.

Abstract. We consider three parametric relaxations of the (0,1)-quadratic programming problem. These relaxations are to: quadratic maximization over simple box constraints, quadratic maximization over the sphere, and the maximum eigenvalue of a bordered matrix. When minimized over the parameter, each of the relaxations provides an upper bound on the original discrete problem. Moreover, these bounds are efficiently computable. Our main result is that, surprisingly, all three bounds are equal.

S. Poljak, F. Rendl, and H. Wolkowicz. A recipe for semidefinite relaxation for (0,1)-quadratic programming. *Journal of Global Optimization*, **7**(1), 51–73, 1995.

Abstract. We review various relaxations of (0,1)-quadratic programming problems. These include semidefinite programs, parametric trust region problems and concave quadratic maximization. All relaxations that we consider lead to efficiently solvable problems. The main contributions of the paper are the following. Using Lagrangian duality, we prove equivalence of the relaxations in a unified and simple way. Some of these equivalences have been known previously, but our approach leads to short and transparent proofs. Moreover we extend the approach to the case of equality constrained problems by taking the squared linear constraints into the objective function. We show how this technique can be applied to the Quadratic Assignment Problem, the Graph Partition Problem and the Max- Clique Problem. Finally we show our relaxation to be best possible among all quadratic majorants with zero trace.

D. B. Ponceleón. *Barrier methods for large-scale quadratic programming*. PhD thesis, Department of Computer Science, Stanford University, Stanford, California, USA, 1990.

M. Pontil, S. Rogai, and A. Verri. Support vector machines: a very large QP. *In* R. D. Leone, A. Murli, P. M. Pardalos and G. Toraldo, eds, 'High Performance Algorithms and Software in Nonlinear Optimization', pp. 315–336, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1998.

D. N. Pope, G. L. Curry, and D. T. Phillips. Closed form solutions to non-serial, nonconvex quadratic-programming problems using dynamic-programming. *Journal of Mathematical Analysis and Applications*, **86**(2), 628–647, 1982.

M. J. D. Powell. An upper triangular matrix method for quadratic programming. *In* O. L. Mangasarian, R. R. Meyer and S. M. Robinson, eds, 'Nonlinear Programming, 2', Academic Press, London, 1981.

M. J. D. Powell. ZQPCVX, a Fortran subroutine for convex quadratic programming. Technical Report DAMTP/NA17, Department of Applied Mathematics and Theoretical Physics, Cambridge University, Cambridge, England, 1983.

M. J. D. Powell. On the quadratic programming algorithm of Goldfarb and Idnani. *Mathematical Programming Studies*, **25**, 46–61, 1985.

Abstract. Two implementations of the algorithm of Goldfarb and Idnani (1983) for convex quadratic programming are considered. A pathological example shows that the faster one can be unstable, but numerical testing on some difficult problems indicates that both implementations give excellent accuracy. Therefore the author (1983) has provided for general use a FORTRAN subroutine that applies the faster implementation. This subroutine is compared with two widely available quadratic programming subroutines that employ feasible point methods, namely QPSOL (Gill, Murray, Saunders, and Wright, 1983) and VEO2A (Fletcher, 1970). The author concludes that the algorithm of Goldfarb and Idnani is very suitable in practice for convex quadratic programming calculations.

J. C. Preisig. Copositivity and the minimization of quadratic functions with nonnegativity and quadratic equality constraints. *SIAM Journal on Control and Optimization*, **34**(4), 1135–1150, 1996.

L. G. Proll. A computer routine for quadratic and linear programming problems (algorithm r431). *Collected Algorithms from ACM*, **2589**, 1974.

M. L. Psiaki and K. Park. Parallel orthogonal factorization null-space method for dynamic quadratic programming. *Journal of Optimization Theory and Applications*, **85**(2), 409–434, 1995.

Abstract. An algorithm has been developed to solve quadratic programs that have a dynamic programming structure. It has been developed for use as part of a parallel trajectory optimization algorithm and aims to achieve significant speed without sacrificing numerical stability. The algorithm makes use of the dynamic programming problem structure and the domain decomposition approach. It parallelizes the orthogonal factorization null-space method of quadratic programming by developing a parallel orthogonal factorization and a parallel Cholesky factorization. Tests of the algorithm on a 32-node INTEL iPSC/2 hypercube demonstrate speedup factors as large as 10 in comparison to the fastest known equivalent serial algorithm.

R. Pytlak. A range-space method for piecewise-linear quadratic programming: an application to optimal control algorithms. *In* ‘Proceedings of the 33rd IEEE Conference on Decision and Control. IEEE, New York, NY, USA’, Vol. 2, pp. 1462–1463, 1994.

Abstract. A new method for solving a convex optimization problem with box constraints is presented. The objective function has a positive-definite quadratic term and a piecewise-linear term. The method is derived from a range space method for QP problems. The approach is particularly efficient if the piecewise-linear term has few breakpoints. Numerical comparisons with an efficient implementation of a null-space active-set algorithm (LSSOL) are also presented.

A. J. Quist, E. Deklerk, C. Roos, and T. Terlaky. Copositive relaxation for general quadratic programming. *Optimization Methods and Software*, **9**(1–3), 185–208, 1998.

Abstract. We consider general, typically nonconvex, Quadratic Programming Problems. The Semi-definite relaxation proposed by Shor provides bounds on the optical solution. but it does not always provide sufficiently strong bounds if linear constraints are also involved. To get rid of the linear side-constraints. another, stronger convex relaxation is derived. This relaxation uses copositive matrices. Special cases are discussed for which both relaxations are equal. At the end of the paper, the complexity and solvability of the relaxations are discussed.

S. R. Radhakrishnan. Capital budgeting and mixed 0–1 integer quadratic programming. *Bulletin of the Operations Research Society of America*, **20**, B331, 1972.

Abstract. The capital budgeting problem is treated as a part of the general theory of choice, where utility is to be maximized subject to the opportunities and constraints. The utility function is assumed to be quadratic to take care of any risk-averse behaviour of the shareholders and a quadratic mixed 0–1 integer programming model is developed. The duality concepts in discrete programming are used to derive properties of optimal solutions to the model. Special solution techniques for the mixed 0–1 integer quadratic programming model are discussed.

M. R. Rakhimov. On some methods of solving a problem of linearly-quadratic programming for systems with distributed parameters. *Zhurnal Vychislitel'noi Matematiki i Matematicheskoi Fiziki*, **26**(12), 1797–1812, 1986.

Abstract. The author formulates a theory of the sufficient conditions of optimality of a certain class of nonlinear equations of the second order. By the use of spectral expansion of a non-self-conjugate operator, a solution of the problem of linearly-quadratic programming is constructed in closed form. A proof is obtained of a series of theorems of the properties of the solution of a Rikkati equation by substantiation of the method of dynamic programming. The results obtained are based on the properties of a Riss basis formed by the eigen and adjoint elements of a stationary operator forming the principal part of the dynamic equation examined. It is shown that the approximate solution of the problem converges to an exact one.

M. R. Rakhimov. The use of methods of spectrum series and dynamic-programming for studying the problems of linear-quadratic programming. *Doklady Akademii nauk SSSR*, **292**(4), 818–821, 1987.

R. Ramabhadran and J. K. Antonio. Fast solution techniques for a class of optimal trajectory planning problems with applications to automated spray coating. Technical Report TR-EE 95–9, Perdue Department of ECE, Perdue University, ???, USA, 1995.

Abstract. Optimal trajectory planning problems are often formulated as constrained variational problems. In general, solutions to variational problems are determined by appropriately discretizing the underlying objective functional and solving the resulting nonlinear differential equation(s) and/or nonlinear programming problem(s) numerically. These general solution techniques often require a significant amount of time to be computed, and therefore are of limited value when optimal trajectories need to be frequently computed and/or re-computed. In this paper, a realistic class of optimal trajectory planning problems is defined for which the existence of fast numerical solution techniques are demonstrated. To illustrate the practicality of this class of trajectory planning problems and the proposed solution techniques, three optimal trajectory planning problems for spray coating applications are formulated and solved. Based on the proposed discretization technique, it is shown that these problems can be reduced to either a linear program or a quadratic program, which are readily solved. In contrast, using the standard discretization of these problems generally leads to nonconvex nonlinear programming problems that require a significant amount of computation to arrive at a (possibly) locally optimal solution.

H. V. Ramakrishnan. Large scale energy system planning using quadratic programming. *Journal of the Institution of Engineers (India) Electrical Engineering Division*, **68**, 170–176, 1988.

Abstract. A quadratic programming approach to the problem of planning a large scale energy system is presented. The approach determines optimal plant location, type and capacity of generating units to be added to any existing system by minimizing the objective function consisting of annual amortized cost of generation and transmission of the system, simultaneously satisfying the constraints of peak and average demands. An illustrative example from published literature is worked out. The effect of additional constraints for minimum generation is also studied. Highlights of the results, a critical comparison with previous studies, and important conclusions are given.

J. R. J. Rao and R. Bradlaw. A unified characterization of nonunique response in elastic/perfectly-plastic and elastic/locking structures using constraint qualification. Technical Report 9, Department of Mechanical Engineering, University of Houston, Texas, USA, 1995.

Abstract. Mathematical programming methods have traditionally been used extensively in the analysis and design of elastic/plastic structures. However, some of the recent results from parametric nonlinear programming have not been fully exploited in mechanical applications, particularly those relating to the causes of singularities due to parametric variations. In this paper, we reexamine the phenomenon of nonunique displacements in elastic/perfectly-plastic structures under proportional loading. Once the analysis model as derived from a minimum energy principle is formulated as a quadratic program, it turns out that the response can be characterized as being unique or nonunique depending on the satisfaction of the linear independence or the Mangasarian-Fromovitz constraint qualification. A completely analogous result is then also derived for elastic/locking structures, thus indicating the usefulness of constraint qualifications in analyzing the nonunique behavior in other applications as well.

A. Ravindran. A computer routine for quadratic and linear programming problems (algorithm a431). *Collected Algorithms from ACM*, **2285**, 1972.

A. Ravindran and H. K. Lee. Computer experiments on quadratic programming algorithms. *European Journal of Operational Research*, **8**(2), 166–174, 1981.

Abstract. Compares the computational performance of five quadratic programming algorithms. These include Wolfe's simplex method, Lemke's complementary pivot method, convex simplex method and quadratic differential algorithm. Execution time and iteration count are used as the major criteria for comparison. Since Lemke's algorithm out-performed all other methods in the study, a detailed statistical analysis was performed to determine the relative importance of problem parameters on the efficiency of Lemke's algorithm. An analysis of variance showed that the number of variables, the percent of positive linear terms in the objective, the number of constraints, and their interactions were the significant factors for both iteration count and execution time. Finally, regression equations for iteration count and execution time are derived as a function of fifteen problem parameters.

B. S. Razumikhin. Method of physical modelling in mathematical programming and economics. III. Solution algorithms for problems of linear and quadratic programming and for equilibrium problems of exchange models. *Automation and Remote Control*, **33**(6), 992–1001, 1972.

Abstract. For pt. II see Autom. and Remote Control (USA), 33(4) pp. 628–39, (1972). Recurrence algorithms for numerical solution of linear and quadratic programming problems and equilibrium problems of linear exchange models are given. The algorithms follow from the physical properties of the corresponding problems and the method of redundant relations.

B. D. Reddy and G. P. Mitchell. The analysis of elastic-plastic plates: a quadratic programming problem and its solution by finite elements. *Computer Methods in Applied Mechanics and Engineering*, **41**(2), 237–248, 1983.

Abstract. An extended kinematic minimum principle in classical plasticity is used as the basis for the finite element formulation of the rate problem for elastic-plastic plates. A simple algorithm is used to solve the resulting quadratic programming problem. The numerical solution of the problem is carried out in two ways: one method involves load step sizes which are scaled so that one or more gauss points just become plastic, and the other method involves load step sizes which are fixed once and for all at the outset. Examples are given and discussed.

K. P. Reddy, A. K. Mittal, and S. K. Gupta. Bivalent quadratic programming problem-a computational study. *Opsearch*, **21**(3), 153–166, 1984.

Abstract. The problem considered is the minimisation of a quadratic function of n 0–1 variables. Effective bounding strategies are developed and incorporated in a branch-and-bound algorithm. The algorithm is used to identify the relatively difficult problems. A heuristic algorithm is also proposed to solve difficult and/or large size problems. Computational results pertaining to the effectiveness of the proposed algorithms are reported.

A. T. Redpath, B. L. Vickery, and D. H. Wright. A new technique for radiotherapy planning using quadratic programming. *Physics in Medicine and Biology*, **N1**(5), 781–791, 1976.

Abstract. Previous attempts at optimisation of radiotherapy planning are described and criticised, and consideration of these attempts has resulted in the development of a new technique using quadratic programming. Uniformity of tumour dose is selected as the most important feature of any plan, and this is achieved by minimising the variance of the dose to preselected points within the tumour. The dose to vulnerable regions can be constrained not to exceed a given percentage of the mean tumour dose. Optimisation of field weight and field type is possible. The operation of the system is described and some typical results are given.

G. F. Reid and L. Hasdorff. Economic dispatch using quadratic programming. *IEEE Transactions on Power Apparatus and Systems*, **PAS-92**(6), 2015–2023, 1973.

Abstract. The economic dispatch problem is formulated as a quadratic programming problem and solved using Wolfe's algorithm. The method is capable of handling both equality and inequality constraints on p, q, and v and can solve the load flow as well as the economic dispatch problem. The quadratic programming algorithm does not require the use of penalty factors or the determination of gradient step size which can cause convergence difficulties. Convergence was obtained in three iterations for all test systems considered and solution time is small enough to allow the method to be used for on-line dispatching at practical time intervals. Results are presented for 5, 14, 30, 57, and 118 bus test systems.

R. Rhode and R. Weber. Multiple objective quadratic-linear programming. *In* J. P. Brans, ed., 'Operational Research '81. Proceedings of the Ninth IFORS International Conference. North-Holland, Amsterdam, Netherlands', pp. 405–420, 1981.

Abstract. A vector optimization problem is formulated and discussed which contains one quadratic and several linear objective functions. It is shown that such problems may be solved easily in different ways. Algorithms for the strictly concave case are presented.

F. Riciniello. A survey on the methods of linear and quadratic programming. *Note Recensioni e Notizie*, **22**(2–3), 270–315, 1973.

Abstract. Some methods of linear and quadratic programming are reviewed. Attention is focused on the algorithms which have been recognized as the most suitable for computer implementation because of their limited memory occupancy and low sensitivity to roundoff errors.

N. L. Ricker. Use of quadratic programming for constrained internal model control. *Industrial and Engineering Chemistry, Process Design and Development*, **24**(4), 925–936, 1985.

Abstract. Absolute bounds on control action and other control system constraints can be handled conveniently through a combination of quadratic-programming and multivariable Internal Model Control (IMC). Three alternative simplex-type quadratic programming (QP) algorithms are considered, and the resulting IMC algorithms are applied to the control of a simulated multiple-effect evaporation process. Overall control quality is excellent; it degrades very little when the process is operated near a constraint on the control action. Calculations required for an adjustment of the control action are essentially instantaneous on a PDP 11/60 minicomputer. The results of the QP comparison suggest that certain QP algorithms can take advantage of the special structure of the IMC QP problem.

K. Rim, R. Chand, and E. J. Haug. Analysis of unbonded contact problems by means of quadratic programming. *Journal of Optimization Theory and Applications*, **20**(2), 171–189, 1976.

Abstract. Two-body, elastic, unbonded contact problems are formulated as quadratic programming problems. Uniqueness theorems of quadratic programming theory are applied to show that the solution of a contact problem, if one exists, is unique and can be readily found by the modified simplex method of quadratic programming. A solution technique that is compatible with finite-element methods is developed, so that contact problems with complex boundary configurations can be routinely solved. A number of classical and nonclassical problems are solved. Good agreement is found for problems with previously known solutions.

K. Ritter. *Über das maximum-problem für nichtkonkave quadratische funcktionen*. Doctoral dissertation, Albert-Ludwigs University, Frieberg, Germany, 1964.

K. Ritter. Stationary points of quadratic maximum problems. *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, **4**, 149–158, 1965.

K. Ritter. A method for solving maximum problems with a nonconcave quadratic objective function. *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, **4**, 340–351, 1966.

K. Ritter. A decomposition method for structured quadratic programming problems. *Journal of Computer and System Sciences*, **1**(3), 241–260, 1967.

K. Ritter. On parametric linear and quadratic programming problems. *In* R. W. Cottle, M. L. Kelmanson and B. Korte, eds, 'Mathematical Programming. Proceedings of the International Congress on Mathematical Programming. North-Holland, Amsterdam, Netherlands', pp. 307–335, 1984.

Abstract. An algorithm is described for determining the optimal solution of parametric linear and quadratic programming problems as an explicit piecewise linear function of the parameter. Each linear function is uniquely determined by an appropriate subset of active constraints. For every critical value of the parameter a new subset has to be determined. A simple rule is given for adding and deleting constraints from this subset.

A. G. Robinson, N. Jiang, and C. S. Lerme. On the continuous quadratic knapsack-problem. *Mathematical Programming*, **55**(1), 99–108, 1992.

R. T. Rockafellar. Linear-quadratic programming and optimal control. *SIAM Journal on Control and Optimization*, **25**(3), 781–814, 1987.

Abstract. A generalized approach is taken to linear and quadratic programming in which dual as well as primal variables may be subjected to bounds, and constraints may be represented through penalties. Corresponding problem models in optimal control related to continuous-time programming are then set up and theorems on duality and the existence of solutions are derived. Optimality conditions are obtained in the form of a global saddle point property which decomposes into an instantaneous saddle point condition on the primal and dual control vectors at each time, along with an endpoint condition.

R. T. Rockafellar. Computational schemes for large-scale problems in extended linear-quadratic programming. *Mathematical Programming, Series B*, **48**(3), 447–474, 1990.

Abstract. Numerical approaches are developed for solving large-scale problems of extended linear-quadratic programming that exhibit Lagrangian separability in both primal and dual variables simultaneously. Such problems are kin to large-scale linear complementarity models as derived from applications of variational inequalities, and they arise from general models in multistage stochastic programming and discrete-time optimal control. Because their objective functions are merely piecewise linear-quadratic, due to the presence of penalty terms, they do not fit a conventional quadratic programming framework. They have potentially advantageous features, however, which so far have not been exploited in solution procedures. These features are laid out and analyzed for their computational potential. In particular, a new class of algorithms, called

finite-envelope methods, is described that does take advantage of the structure. Such methods reduce the solution of a high-dimensional extended linear-quadratic program to that of a sequence of low-dimensional ordinary quadratic programs.

R. T. Rockafellar. Large-scale extended linear-quadratic programming and multistage optimization. *In* 'Advances in Numerical Partial Differential Equations and Optimization: Proceedings of the Fifth Mexico-United States Workshop', Vol. 2, pp. 247–261, 1991.

R. T. Rockafellar. A simplex-active-set algorithm for piecewise quadratic programming. *In* D.-Z. Du and J. Sun, eds, 'Advances in Optimization and Approximation', pp. 275–292, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1994.

R. T. Rockafellar and R. J.-B. Wets. A dual solution procedure for quadratic stochastic programs with simple recourse. Technical Report CP-83-017, International Institute for Applied Systems Analysis, Vienna, Austria, 1983.

R. T. Rockafellar and R. J. B. Wets. Linear-quadratic programming-problems with stochastic penalties—the finite generation algorithm. *Lecture Notes in Control and Information Sciences*, **81**, 545–560, 1986.

J. E. Rogers, P. T. Boggs, and P. D. Domich. A predictor-corrector-o3d formulation for quadratic programming. Technical Report NISTIR 5020, National Institute of Standards and Technology, USA, 1994.

J. S. Rogers and M. Rolko. A quadratic programming model for planning generation and inter-utility transmission. *International Journal of Electrical Power and Energy Systems*, **14**(1), 18–22, 1992.

Abstract. A new quadratic programming approach to planning the coordinated expansion of generation and transmission between two electric utilities is described. Earlier methods use a step function to approximate the load duration curve. The resulting linear programming models work well if the shape of the load curve is not changed by the installation of equipment. However, expansion of inter-utility transmission changes the shape of the load curve which must be supplied by the generation of each utility, and so a more flexible type of approximation is needed. The authors use a piecewise linear approximation to the load curve to construct a quadratic programming model. Real system data are used to compare the results from the new model with those from the earlier type and with results from an analytical model that gives an exact solution.

J. B. Rosen and P. M. Pardalos. Global minimization of large-scale constrained concave quadratic problems by separable programming. *Mathematical Programming*, **34**(2), 163–174, 1986.

V. Ruggiero and L. Zanni. A modified projection algorithm for large strictly-convex quadratic programs. *Journal of Optimization Theory and Applications*, **104**(2), 281–299, 2000.

Abstract. In this paper, we propose a modified projection-type method for solving strictly-convex quadratic programs. This iterative scheme requires essentially the solution of an easy quadratic programming subproblem and a matrix-vector multiplication at each iteration. The main feature of the method consists in updating the Hessian matrix of the subproblems by a convenient scaling parameter. The convergence of the scheme is obtained by introducing a correction formula for the solution of the subproblems and very weak conditions on the scaling parameter. A practical nonexpensive updating rule for the scaling parameter is suggested. The results of numerical experimentation enable this approach to be compared with some classical projection-type methods and its effectiveness as a solver of large and very sparse quadratic programs to be evaluated.

M. H. Rusin. A revised Simplex method for quadratic programming. *SIAM Journal on Applied Mathematics*, **20**(2), 143–160, 1971. See also, Bulletin of the Operations Research Society of America, volume 18, page B17, 1970.

Abstract. A computational method for solving quadratic programming problems is presented which reduces to the revised simplex method for linear programming when the objective function is linear. The basis matrix which is maintained is symmetric and may vary in size from iteration to iteration. Comparisons on test problems indicate that the method is more efficient than other available methods.

R. S. Sacher. A decomposition algorithm for quadratic programming. *Mathematical Programming*, **18**(1), 16–30, 1980.

Abstract. A decomposition algorithm using, Lemke's method is proposed for the solution of quadratic programming problems having possibly unbounded feasible regions. The feasible region for each master program is a generalized simplex of minimal size. TThis property is maintained by a dropping procedure which does not affect the finiteness of the convergence. The details of the matrix transformations associated with an efficient implementation of the algorithm are given. Encouraging preliminary computational experience is presented.

H. D. Sahinoglou, P. M. Pardalos, and I. M. Roussos. Characterizations of global minima in nonconvex quadratic programming. *In* 'Proceedings of the 19th Annual Pittsburh Conference on Modelling and Simulation Modeling and Simulation', pp. 1823–1828, 1988.

T. Satoh, T. Okita, and M. Itoh. Fast capacity method for solving convex quadratic programming problem. *Transactions of the Society of Instrument and Control Engineers*, **32**(4), 587–595, 1996.

Abstract. The quadratic programming (QP) problem is a natural generalization of well-known linear programming (LP) problem. The QP problem is formulated as a problem to minimize a linear objective function subject to a system of linear equalities and inequalities. The continuation method is a powerful tool for solving a system of nonlinear equations. In the continuation method, the continuation parameter beta is introduced into the original problem P and a parameterized problem P(beta) is constructed. Assuming that the optimal solution to P(P), for beta =0, can be trivially or easily found, and that the optimal solution to $P(\beta)$, for $\beta = 1$, will be the desired solution to the original problem P, a trajectory of the solution to $P(\beta)$ is followed by increasing the value of beta from 0 to 1. In this paper, based on the continuation method, a path following method called the capacity method is constructed for solving the QP problem. The basic idea of the capacity method is to follow a trajectory of the Kuhn-Tucker point when the value of beta is increased from 0 to 1. It is shown that the trajectory is expressed as a piecewise-linear function of beta . Therefore the optimal solution can be found very efficiently by following the piecewise-linear trajectory. In this paper, firstly, the assumption that the right-hand-side vector of the constraint equations must be positive, which restricts the applicability of the capacity method, is dropped and a method for transforming any QP problem to the standard form of the capacity method is presented. Secondly, the basis matrix of QP is factorized by using an LDL^T factorization, and the factorization is efficiently updated by using matrix modification techniques. Then the proposed method is applied to some test problems, and numerical results indicate the effectiveness of the method.

M. A. Saunders. Stable reduction to KKT systems in barrier methods for linear and quadratic programming. Technical Report SOL 96-3, Department of Operations Research, Stanford University, California, USA, 1996.

C. Schmid and L. T. Biegler. Quadratic programming methods for reduced Hessian SQP. *Computers and Chemical Engineering*, **18**(9), 817–832, 1994.

Abstract. Reduced Hessian successive quadratic programming (SQP) is well suited for the solution of large-scale process optimization problems with many variables and constraints but only few degrees of freedom.

The preprocessing phase determines an initial consistent point, to select a nonsingular set of basis variables and to identify linear dependency among the equality constraints. Fourer's (1985, 1989) piecewise-linear simplex techniques allow us to solve a smaller initialization problem more efficiently than is possible with standard simplex techniques. We present a new QP solver, QPKWIK, based on a dual algorithm which only requires the inverse Cholesky factor of the Hessian matrix to be supplied. The resulting solution technique for the QP subproblem is $O(n^2)$ with respect to the degrees of freedom of the problem. Further, the unconstrained optimum is dual feasible, which precludes the need for phase I calculations, and makes this method superior even for problems with few degrees of freedom. QPKWIK has been implemented so as to enhance the efficiency of the active set identification and is also able to determine a search direction when infeasible QP subproblems are encountered by relaxing the equality constraints without violating the simple bounds on the variables. Finally, numerical results are included to illustrate the advantages of the proposed techniques and to assess the overall performance of the reduced Hessian method. This approach is especially well-suited for process and real-time optimization problems. We demonstrate this on several distillation and fractionation problems.

U. Schmitz, A. Donati, T. L. James, N. B. Ulyanov, and L. J. Yao. Small structural ensembles for a 17-nucleotide mimic of the tRNA TYC-loop via fitting of dipolar relaxation rates with the quadratic programming algorithm. *Biopolymers*, **46**, 329–342, 1998.

I. E. Schochetman and R. L. Smith. Solution existence in infinite quadratic programming. *In* A. V. Fiacco, ed., 'Lecture Notes in Pure and Applied Mathematics', Vol. 195. Marcel Dekker, 1998.

Abstract. We consider an infinite quadratic programming problem with positive semi-definite quadratic costs, equality constraints and unbounded variables. Sufficient conditions are given for there to exist an optimal solution. Specifically, we require that (1) the cost operator be strictly positive definite when restricted to the orthogonal complement of its kernel, and (2) the constraint operator have closed range when restricted to the kernel of the cost operator. Condition (1) is shown to be equivalent to the spectrum of the restricted cost operator being bounded away from zero. Similarly, condition (2) is equivalent to the minimum modulus of the restricted constraint operator being positive. In the presence of separability, we give a sufficient condition for (2) to hold in terms of finite dimensional truncations of the restricted constraint operator. We apply our results to a broad class of infinite horizon optimization problems. In this setting, the finite dimensional truncations can be considered to be finite dimensional approximations to our problem whose limit, in a somewhat formal sense, is our infinite dimensional problem. Each of these approximations has properties (1) and (2) by virtue of their finite-dimensionality, i.e., each admits an optimal solution. However, our infinite dimensional problem may not. Thus, we give sufficient conditions for our problem to also admit an optimal solution. Finally, we illustrate this application in the case of an infinite horizon LQ regulator problem (a production planning problem).

I. E. Schochetman, R. L. Smith, and S. K. Tsui. Solution existence for time-varying infinite-horizon quadratic programming. *Journal of Mathematical Analysis and Applications*, **195**(1), 135–147, 1995.

Abstract. We consider a general, time-varying, infinite horizon, pure quadratic programming problem with positive-definite cost matrices and unbounded decision variables. Sufficient conditions are provided for there to exist an optimal solution. Specifically, we show that if the eigenvalues of the cost matrices are bounded away from zero, then a (unique) optimal solution exists. We apply our results to the infinite horizon LQ tracker problem in optimal control theory.

M. Schocm. Über die äquivalenz der allgeminen quadratischer optimierungsaufgabe zu einer parametrischen komplimentaritären optimierungsaufgabe. *Mathematische Operationsforschung und Statistik, Serie Optimization*, **15**, 211–216, 1984.

L. Schrage. *Integer, and Quadratic Programming with LINDO*. The Scientific Press, third edn, 1986.

P. Scobey and D. G. Kabe. Direct solutions to some linear and quadratic programming problems. *Industrial Mathematics*, **29**(2), 59–75, 1979.

Abstract. Some results of univariate normal linear regression theory are used to present direct solutions to some linear and quadratic programming problems. The linear programming problems include the knapsack problem, the extreme point linear programming problem, and the interval linear programming problem. Some inaccuracies in interval linear programming problems and quadratic programming problems are pointed out.

P. D. Scott. A quadratic programming dual algorithm for minimax control. *IEEE Transactions on Automatic Control*, **AC-20**(3), 434–435, 1975.

Abstract. The optimal control problem with peak weighting on trajectory error is related to quadratic programming through a duality transformation. A series of finite dimensional quadratic programs yields finitely convergent solutions from which the optimal control may be recovered. Each program yields upper and lower bounds on the optimal cost.

J. Semple. Infinite positive-definite quadratic programming in a Hilbert space. *Journal of Optimization Theory and Applications*, **88**(3), 743–749, 1996.

Abstract. This note generalizes the results of Benson, Smith, Schochetman, and Bean (1995) regarding the minimization of a positive-definite functional over the countable intersection of closed convex sets in a Hilbert space. A finite approximating subproblem for the general case is shown to have the same strong convergence properties of the earlier work without any of the specialized structures imposed therein. In particular, the current development does not rely on any properties of L^2 and does not require the Hilbert space to be separable.

M. Serna and F. Xhafa. On the parallel approximability of some classes of quadratic programming. Technical Report R95-58, Departament de Llenguatges i Sistemes Informátics, Universitat Politècnica de Catalunya, Spain, 1995.

Abstract. In this paper we analyze the parallel approximability of two special classes of Quadratic Programming. First, we consider Convex Quadratic Programming . We show that the problem of Approximating Convex Quadratic Programming is P-complete. We also consider two approximation problems related to it, Solution Approximation and Value Approximation and show both of these cannot be solved in NC, unless P=NC. Secondly, on the positive side, we show that we have an NC Approximation Scheme for those instances of Quadratic Programming that are ”smooth” and ”positive.” Then we show how to extend the result for positive instances of bounded degree Smooth Integer Programming problems. Finally, we formulate several combinatorial problems as positive QP (or positive Integer Programs) in packing/covering form and show that the presented techniques can be used to obtain NC Approximation Schemes for ”dense” instances of such problems.

M. Serna and F. Xhafa. The parallel approximability of a subclass of quadratic programming. *In* ‘Proceedings. 1997 International Conference on Parallel and Distributed Systems. IEEE Comput. Soc, Los Alamitos, CA, USA’, pp. 474–481, 1997.

Abstract. In this paper we deal with the parallel approximability of a special class of Quadratic Programming (QP), called Smooth Positive Quadratic Programming. This subclass of QP is obtained by imposing restrictions on the coefficients of the QP instance. The Smoothness condition restricts the magnitudes of the coefficients while the positiveness requires that all the coefficients be non-negative. Interestingly, even with these restrictions several combinatorial problems can be modeled by Smooth QP. We show NC Approximation Schemes for the instances of Smooth Positive QP. This is done by reducing the instance of QP to an instance of Positive Linear Programming, finding in NC an approximate fractional solution to the obtained program, and then rounding the fractional solution to an integer approximate solution for the original

problem. Then we show how to extend the result for positive instances of bounded degree to Smooth Integer Programming problems. Finally, we formulate several important combinatorial problems as Positive Quadratic Programs (or Positive Integer Programs) in packing/covering form and show that the techniques presented can be used to obtain NC Approximation Schemes for ”dense” instances of such problems.

D. G. Shankland. Quadratic programming using generalized inverses. Technical report, Air Force Inst. Tech , Wright Patterson AFB, OH, USA, 1975.

Abstract. A method for computing the optimum value of a quadratic functional subject to linear inequalities rapidly ascertains which, if any, of the inequalities are binding at the optimum point. The method resembles that of H. Theil and C. Van de Panne, but no combinatorial analysis need be performed to isolate the binding constraints. All violated constraints are imposed as equalities, and those with positive Lagrangian multipliers are retained. Contradictory equalities are automatically resolved by the use of the generalized inverse. The method appears most useful in systems with large numbers of variables and constraints.

D. G. Shankland. A numerically efficient procedure for the Theil-Van de Panne quadratic programming method. *Journal of Optimization Theory and Applications*, **31**(1), 117–123, 1980.

Abstract. A procedure is given for implementing the Theil-Van de Panne algorithm, which utilizes the Cholesky decomposition to reduce the computations involved in matrix inversions.

J. K. Sharma and S. Kanti. Indefinite quadratic programming and transportation technique. *Indian Journal of Pure and Applied Mathematics*, **8**(9), 1029–1031, 1977.

Abstract. A technique, similar to the transportation technique in linear programming, is described to minimize a locally indefinite quadratic function. The problem is attacked directly starting from a basic feasible solution and the conditions under which the solution can be improved have been indicated. Conditions for local optimality have been obtained.

J. F. Sharp and K. M. Suk. A quadratic programming planning model. *Engineering Economist*, **20**(1), 71–77, 1974.

Abstract. Various authors have suggested using a quadratic programming model of the firm. A quadratic programming approach has been used to study the optimal use of milk in the Netherlands. There seems to be a lack of other actual applications reported in the literature. The authors have successfully applied a quadratic programming marketing model to an industrial firm. It has several features not included in the milk application. These include: more than one market for the same product, limits on the range of applicability, and legal restrictions. A simplified version of this model is given below.

R. Shen. Reactive power optimization in power system quadratic programming method. *Proceedings of the Chinese Society of Electrical Engineering.*, **6**(5), 40–48, 1986.

Abstract. A quadratic programming method of comprehensive reactive power optimization in power systems is proposed. With real power loss as the objective function and operating variables as constraints, a mathematical model for comprehensive reactive power optimization is set up by sensitivity relations between control and state variables in the power system. The optimal locations and capabilities of compensated reactive power, optimal terminal voltages of generators and optimal ratios of on-load transformer tap-changers are determined by optimization. Based on economic analysis, a mathematical model of multiobjective programming for comprehensive reactive power optimization is set up with real power loss and total compensation capability as objective functions. Three systems are optimized by these methods and results are given and compared with those of linear programming methods.

H. D. Sherali and A. Alameddine. Reformulation-linearization technique for bilinear programming problems. *Journal of Global Optimization*, **2**(4), 379–410, 1992.

H. D. Sherali and C. H. Tuncbilek. A reformulation-convexification approach for solving nonconvex quadratic programming problems. *Journal of Global Optimization*, **7**(1), 1–31, 1995.

Abstract. In this paper, we consider the class of linearly constrained nonconvex quadratic programming problems, and present a new approach based on a novel Reformulation- Linearization/Convexification Technique. In this approach, a tight linear (or convex) programming relaxation, or outer- approximation to the convex envelope of the objective function over the constrained region, is constructed for the problem by generating new constraints through the process of employing suitable products of constraints and using variable redefinitions. Various such relaxations are considered and analyzed, including ones that retain some useful nonlinear relationships. Efficient solution techniques are then explored for solving these relaxations in order to derive lower and upper bounds on the problem, and appropriate branching/partitioning strategies are used in concert with these bounding techniques to derive a convergent algorithm. Computational results are presented on a set of test problems from the literature to demonstrate the efficiency of the approach. (One of these test problems had not previously been solved to optimality.) It is shown that for many problems, the initial relaxation itself produces an optimal solution.

W. M. Shi. Comparison of optimization with linear and quadratic programming in 2nd-order design. *In* 'Collection of 8th International Symposium on Geodetic Computation', Vol. 9, pp. 157–163, 1991.

J. K. Shim. A survey of quadratic programming applications to business and economics. *International Journal of Systems Science*, **14**(1), 105–115, 1983.

Abstract. One of the shortcomings of linear programming lies in its linearity assumption, primarily in the objective function. This compels one to work with a constant marginal rate of substitution and constant return to scale. However, this assumption is at variance with the economists' preference postulates. The paper presents a survey of quadratic programming practices in corporate and economic planning formulations. It examines a variety of applications of quadratic programming-portfolio selection, monopolists' profit maximization, inequality constrained least-squares estimation, spatial equilibrium analysis, goal programming with quadratic preferences, and optimal decision rules.

Y. Shimazu, M. Fukushima, and T. Ibaraki. A successive over-relaxation method for quadratic programming problems with interval constraints. *Journal of the Operations Research Society of Japan*, **36**(2), 73–89, 1993.

Abstract. Hildreth's algorithm (1957) is a classical iterative method for solving strictly convex quadratic programming problems, which uses the rows of constraint matrix just one at a time. This algorithm is particularly suited to large and sparse problems, because it acts upon the given problem data directly and the coefficient matrix is never modified in the course of the iterations. The original Hildreth's algorithm is mathematically equivalent to Gauss-Seidel method applied to the dual of the given quadratic programming problem. In this paper, we propose a successive overrelaxation modification of Hildreth's algorithm for solving interval constrained quadratic programming problems. We prove global convergence of the algorithm and show that the rate of convergence is linear. Computational results are also presented to demonstrate the effectiveness of the algorithm.

R. I. Shrager. Quadratic programming for nonlinear regression. *Communications of the ACM*, **15**(1), 41–45, 1972. See also, *Collected Algorithms from ACM*, 2395.

Abstract. A quadratic programming algorithm is described for use with the magnified diagonal method of nonlinear regression with linear constraints. The regression method is published in JACM, July 1970.

V. Sima. Algorithm podqp—Solving positive definite quadratic programming problems. *Studies in Informatics and Control*, **1**(1), 1992.

Abstract. An efficient and numerically stable dual algorithm- PODQP- for solving positive definite quadratic programming problems is briefly described. Both inequality and constraints are dealt wiyh. The algorithm employs an active set strategy and can exploit a priori information about an initial active set. In particular, the algorithm is strongly recommended for solving nonlinearly constrained optimization problems using projected Lagrangian techniques. Orthogonal transformations are used for updating the matrices and matrix factorizations after each change performed in the active set.

S. Simunovic and S. Saigal. Frictionless contact with BEM using quadratic programming. *Journal of Engineering Mechanics-ASCE*, **118**(9), 1876–1891, 1992.

Abstract. The contact surface, with its accompanying load transfer, may well constitute the critical factor in a structural member. Thus it is essential to be able to perform contact stress analysis of a component accurately and efficiently. Considerable research effort is represented in the literature for contact analysis using finite elements. To obtain reliable results in the contact zone, it is necessary to provide a very fine discretization in that zone. Many distinct contact zones may exist. which may force the entire domain, and not just the contact zones, to be discretized finely. This generally leads to an excessive number of degrees of freedom (dof), resulting in an uneconomical, and sometimes intractable, analysis. The boundary element method (BEM), which deals with the discretization of only the boundary of the structure being analyzed, may be used to circumvent these difficulties and provide accurate, economical results. While a fine discretization of the contact zone is still unavoidable, the BEM leads to a smaller number of dof's because the rest of the boundary need not have a fine mesh. The problem of frictionless contact between an elastic body and a rigid surface is formulated as an optimization problem. Three distinct functions are defined in terms of the unknown variables (displacements and tractions) corresponding to the contact surface, and expressed as quadratic objective functions that are to be minimized. The solution is obtained using the standard quadratic programming techniques of optimization. A number of example problems with straight and curved contact boundaries were solved. The present formulations were validated through comparison of the test problems with existing alternative solutions.

S. Simunovic and S. Saigal. Frictionless contact with BEM using quadratic programming—closure. *Journal of Engineering Mechanics-ASCE*, **119**(12), 2540, 1993.

S. Simunovic and S. Saigal. Frictional contact formulation using quadratic programming. *Computational Mechanics*, **15**(2), 173–187, 1994.

Abstract. A new solution procedure for contact problems in elasticity with prescribed normal tractions on contact surface has been proposed in this paper. The procedure is based on the boundary element method and quadratic programming. It is next used in a two step solution algorithm for the analysis of contact problems with friction. Several numerical examples are presented and compared with results obtained using alternative solution methods.

S. Simunovic and S. Saigal. Quadratic-programming contact formulation for elastic bodies using boundary-element method. *AIAA Journal*, **33**(2), 325–331, 1995.

Abstract. A method for the analysis of contact of deformable bodies based on the boundary element method (BEM) has been presented in this paper. The contact problem is stated in the form of a convex quadratic programming (QP) problem written in terms of the contact tractions on the contact surface. A strategy for the incorporation of the BEM contact analysis into models whose domain may be discretized using the finite element method (FEM) has been investigated. A discussion concerning the merits of the proposed approach is provided and several examples are presented to illustrate the validity of the method.

F. W. Sinden. A geometric representation for pairs of dual quadratic or linear programs. *Journal of Mathematical Analysis and Applications*, **5**, 378–402, 1963.

J. Skorin-Kapov. On strongly polynomial algorithms for some classes of quadratic programming problems. *Mathematical Communications*, **2**(2), 95–105, 1997.

Abstract. In this paper we survey some results concerning polynomial and/or strongly polynomial solvability of some classes of quadratic programming problems. The discussion on polynomial solvability of continuous convex quadratic programming is followed by a couple of models for quadratic integer programming which, due to their special structure, allow polynomial (or even strongly polynomial) solvability. The theoretical merit of those results stems from the fact that a running time (i.e. the number of elementary arithmetic operations) of a strongly polynomial algorithm is independent of the input size of the problem.

M. Skutella. Convex quadratic programming relaxations for network scheduling problems. *In* J. Nesetril, ed., 'Algorithms—ESA'99', Vol. 1643 of *Lecture Notes in Computer Science*, pp. 127–138, Springer Verlag, Heidelberg, Berlin, New York, 1999.

L. H. Smith. A note on quadratic programming in activation analysis. *Operations Research*, **18**(2), 290–299, 1970. See also, *Bulletin de la Societe Royale Belge des Electriciens*, Volume 16, page 62, 1968.

Abstract. Various statistical techniques have been employed in 'activation analysis' in order to provide a better means of estimating the amounts of various pure chemical elements contained in an unknown mixture. In particular, the method of least squares has been employed extensively. However, for the most part, the usual least squares applications in activation analysis have utilized the ordinary matrix model $Y = Xbeta + e$, under the 'error' assumptions (a) zero means, (b) variances proportional to Y, and (c) zero covariances. In addition to the fact that assumptions (b) and (c) may lead to erroneous results, the usual applications allow only point estimation, with no provision for confidence intervals and tests for model goodness of fit. Further, the usual applications fail to eliminate the drawback that negative coefficients are sometimes obtained. The present paper sets forth an iterative quadratic programming estimation procedure that not only eliminates the necessity for assumptions (b) and (c), but also alleviates the other above-mentioned difficulties.

J. E. Sohl. An application of quadratic programming to the deregulation of natural gas. *In* 'Proceedings of the 16th Annual Meeting of the American Institute for Decision Sciences. American Inst. Decision Sci, Atlanta, GA, USA', pp. 686–689, 1984.

Abstract. Examines the issue of natural gas deregulation within the context of the linear complementarity programming (LCP) format. The LCP model forecasts United States oil, natural gas, and coal prices and quantities for both the demand and the supply side. Forecasts are presented on a regionalized level. Validation of the LCP model is achieved by forecasting historical prices and quantities. In addition, the model examines several scenarios concerning the various policy options for the decontrol of natural gas.

V. N. Solov'ev. Algorithms of solution of problems of quadratic programming and of optimal guaranteed estimation. *Automation and Remote Control*, **51**(9), 1213–1218, 1990.

Abstract. An algorithm of solution is proposed for a quadratic programming problem with a Stieltjes matrix that is often encountered after inverting a matrix with nonnegative elements. This algorithm is applied to optimal guaranteed estimation.

J. Sommerschuh. Properties of the general quadratic optimization problem and the corresponding linear complementarity problem. *Optimization*, **18**(1), 31–39, 1987.

Abstract. The quadratic nonconvex function $\psi(x) = 1/2x^T Cx + r^T x$ (C symmetric, arbitrary) is considered on the convex polyhedral set $G = (x \in \mathbb{R}^n | Gx \leq g)$, and neighbourhoods of local minima are described. Using the corresponding Kuhn-Tucker conditions the author found a cut excluding a neighbourhood of a previously determined local minimum.

C. B. Somuah and F. C. Schweppe. Minimum frequency constrained generation margin allocation using quadratic programming. *International Journal of Electrical Power and Energy Systems*, **9**(2), 105–112, 1987.

Abstract. The problem of allocating the total system margin among all the generators in a power systems, taking into consideration the maximum post-disturbance frequency deviation, is formulated as a quadratic programming problem. The optimization problem uses the sensitivity of the frequency deviation to changes in each generator's margin in the allocation of total system margin. The quadratic programming problem is solved via the Dantzig-Wolfe decomposition technique using a sequence of linear programming sub-problems. A significant improvement is found in the post-disturbance performance of the power system in comparison with conventional margin allocation. Numerical examples on two test systems are included. Results of sensitivity studies relating changes in power plant margin and size of postulated disturbance to changes in system frequency deviation and generation cost are also discussed.

P. Spellucci. Numerical experiments with modern methods for large scale QP problems. *In* 'Recent Advances in Optimization', Vol. 452 of *Lecture Notes in Economics and Mathematical Systems*, 1997.

F. R. Spena. A quadratic programming approach to the collapse for pure torsion of beams with thin-walled multiply-connected section. *Journal of Information and Optimization Sciences*, **12**(3), 419–429, 1991.

Abstract. Referring to a thin-walled, homogeneous, isotropic, perfectly, plastic prismatical beam with a multiply-connected cross-section, the collapse value for the torque for an equilibrium stress state of pure torsion is determined. It is shown that the mathematical formulation leads to a quadratic programming problem for which the objective function is represented by the collapse torque. It is also proved that the objective function to be maximized is strictly concave, such that the optimal solution of the problem exists and it is unique.

P. S. J. Spencer, L. Kersley, and S. E. Pryse. A new solution to the problem of ionospheric tomography using quadratic programming. *Radio Science*, **33**, 607–616, 1998.

G. E. Staats and J. P. Dittman. Surrogate quadratic programming. *Bulletin of the Operations Research Society of America*, **20**, B–140, 1972.

Abstract. In this paper, a highly efficient algorithm for solving quadratic programming problems is presented. The algorithm is based on the results obtained when the constraint set is replaced by a surrogate constraint. Kuhn-Tucker Conditions are then applied to obtain either an unconstrained optimization problem or an equality constrained problem. This results in the need to solve only a system of linear equations in order to obtain the solution to the surrogated problem. An iterative procedure is given for obtaining a surrogated problem which has the same solution as the original quadratic programming problem.

A. Stachurski. Monotone sequences of feasible solutions for quadratic-programming problems with *M*-matrices and box constraints. *Lecture Notes in Control and Information Sciences*, **84**, 896–902, 1986.

A. Stachurski. Analysis of the Newton projection method in application to quadratic programming problems with *M*-matrices and box constraints. *Archiwum Automatyki i Telemechanika*, **34**(1–2), 155–164, 1989.

Abstract. This paper is devoted to the analysis of the Newton projection method, when this is applied to solve a special QP problem with an *M*-matrix as the Hessian and simple box constraints. The problem arises as the result of an approximation of a partial Dirichlet problem with obstacle by means of the finite element method. It is proved that the properties of the Hessian (being an *M*-matrix) imply that if the upper bounds upon variables does not appear in the problem and the starting point is equal to the lower bound upon variables then the number of iterations of the Newton projection method does not exceed n (where n denotes the size of the problem); that if the upper bounds do not exist, and the starting point is feasible, and ε^0 (parameter used in the definition of the working set of active constraints) is forced to be equal to 0 in step

1, then the number of iterations is not greater than $n+1$; and that if there exist lower and upper bounds on the problem variables and if some additional requirements are introduced into the algorithm then the Newton projection method generates the same sequence of points as that of J. S. Pang (1976). The analysis carried out indicates some corollaries allowing a simplification of the formulation of the Newton projection method for the considered class of problems.

A. Stachurski. An equivalence between two algorithms for a class of quadratic programming problems with *M*-matrices. *Optimization*, **21**(6), 871–878, 1990.

Abstract. The behaviour of the gradient projection method for the quadratic programming with M-matrices and lower bounds on variables is analysed. It is shown that the Chandrasekaran method for solving such problems is simply a realization of the Newton projection method if for the latter one the starting point has components equal to lower bounds on variables.

V. I. Starostenko. Constant algorithms of quadratic programming and solution of the inverse problems of gravimetry relative to densities. *Geofizicheskii-Sbornik.*, **64**, 52–57, 1975.

Abstract. The problems of quadratic and linear programming are formulated and their solutions by means of quadratic programming algorithms regulating according to A. N. Tikhonov are presented.

F. Steiner and L. Zilahysebess. Gravity interpretation with the aid of quadratic programming—discussion. *Geophysics*, **48**(10), 1413–1414, 1983.

T. Sugimoto, M. Fukushima, and T. Ibaraki. A parallel relaxation method for quadratic programming problems with interval constraints. *Journal of Computational and Applied Mathematics*, **60**(1–2), 219–236, 1995.

Abstract. Optimization problems with interval constraints are encountered in various fields such as network flows and computer tomography. As these problems are usually very large, they are not easy to solve without taking their sparsity into account. Recently ”row-action methods”, which originate from the classical Hildreth's method for quadratic programming problems, have drawn much attention, since they are particularly useful for large and sparse problems. Various row-action methods have already been proposed for optimization problems with interval constraints, but they mostly belong to the class of sequential methods based on the Gauss-Seidel and SOR methods. In this paper, we propose a highly parallelizable method for solving those problems, which may be regarded as an application of the Jacobi method to the dual of the original problems. We prove convergence of the algorithm and report some computational results to demonstrate its effectiveness.

M. F. Sukhinin. Step-by-step quadratic penalty solution of a quadratic-programming problem. *Computational Mathematics and Mathematical Physics*, **34**(8–9), 1125–1131, 1994.

Abstract. An numerical algorithm for a quadratic programming problem is considered. Results of experiments on the transportation problem are given.

J. L. Sullivan, J. W. Adams, and R. Roosta. The design of constrained minimax FIR digital filters using quadratic programming. *Conference Record of The Twenty Ninth Asilomar Conference on Signals, Systems and Computers. IEEE Comput. Soc Press, Los Alamitos, CA, USA*, **2**, 826–830, 1996.

Abstract. We (see Adams et al., ibid., pp. 314, 1994) presented in an earlier article a new quadratic programming algorithm that can design constrained least-squares FIR digital filters. In this article we present a new quadratic programming algorithm for designing constrained minimax FIR filters. The new algorithm is better than the Parks-McClellan (1973) algorithm because it can design a much wider variety of filters. In particular, it can do minimax optimization subject to arbitrary equality and inequality constraints.

J. Sun. A convergence proof for an affine-scaling algorithm for convex quadratic programming without nondegeneracy assumptions. *Mathematical Programming*, **60**(1), 69–79, 1993.

Abstract. This paper presents a theoretical result on convergence of a primal affine-scaling method for convex quadratic programs. It is shown that, as long as the stepsize is less than a threshold value which depends on the input data only, Ye and Tse's interior ellipsoid algorithm for convex quadratic programming is globally convergent without nondegeneracy assumptions. In addition, its local convergence rate is at least linear and the dual iterates have an ergodically convergent property.

J. Sun. On piecewise quadratic Newton and trust-region problems. *Mathematical Programming*, **76**(3), 451–468, 1997.

Abstract. Some recent algorithms for nonsmooth optimization require solutions to certain piecewise quadratic programming subproblems. Two types of subproblems are considered in this paper. The first type seeks the minimization of a continuously differentiable and strictly convex piecewise quadratic function subject to linear equality constraints. We prove that a nonsmooth version of Newton's method is globally and finitely convergent in this case. The second type involves the minimization of a possibly nonconvex and non-differentiable piecewise quadratic function over a Euclidean ball. Characterizations of the global minimizer are studied under various conditions. The results extend a classical result on the trust region problem.

J. Sun and H. Kuo. Applying a Newton method to strictly convex separable network quadratic programs. *SIAM Journal on Optimization*, **8**(3), 728–745, 1998.

Abstract. By introducing quadratic penalty terms, a strictly convex separable network quadratic program can be reduced to an unconstrained optimization problem whose objective is a continuously differentiable piecewise quadratic function. A recently developed nonsmooth version of Newton's method is applied to the reduced problem. The generalized Newton direction is computed by an iterative procedure which exploits the special network data structures that originated from the network simplex method. New features of the algorithm include the use of min-max bases and a dynamic strategy in computation of the Newton directions. Some preliminary computational results are presented. The results suggest the use of ”warm start” instead of ”cold start”.

J. Sun and J. Zhu. A predictor-corrector method for extended linear-quadratic programming. *Computers and Operations Research*, **23**(8), 755–767, 1996.

Abstract. The saddle point form of extended linear-quadratic programs can be solved by an interior point path-following method in polynomial time. The algorithm may take advantage of the block structures of certain problems arising from optimal control and stochastic programming. In addition, it needs no line searches and treats fully or not fully quadratic problems equally. Preliminary computational results apparently show that the algorithm is effective in solving a class of two-stage stochastic programming problems.

S. M. Sun, H. S. Tzou, and M. C. Natori. Parameteric quadratic programming method for dynamic contact problems with friction. *AIAA Journal*, **32**(2), 371–378, 1994.

Abstract. Based on the parametric variational principle, a general but effective parameteric quadratic programming technique satisfying various contact conditions if established for dynamic analysis of contact problems with friction and damping. The discretization with respect to time and space leads to a static linear complementary problem (LCP) for each time step which is solved by a quadratic optimization algorithm such as the Lemke algorithm, etc. Thus, the convergence and numerical stability of the solution can be guaranteed. The substructure condensation technique is implemented to handle the unknown contact boundary condition so that the computation effect is considerably reduced. An an application of the method presented, three dynamic contact examples and numerical results are given.

S. Sunder and V. Ramachandran. Design of recursive differentiators using quadratic programming. *In* ‘Proceedings of the 36th Midwest Symposium on Circuits and Systems. IEEE, New York, NY, USA’, Vol. 1, pp. 774–777, 1993.

Abstract. A method for the design of first and higher-degree recursive differentiators with constant group-delay characteristics using a least-squares approach is presented. In this method, a mean-square error based on the difference between the desired and actual frequency response is formulated in a quadratic form.

Quadratic programming is employed wherein the constraint on stability is accommodated to design stable differentiators. Our method is compared with the linear programming (LP) approach in terms of the computational complexity and the variation of magnitude and group-delay errors with frequency. It is shown that the differentiators designed using our method have a much lower computational complexity and smaller variation of the magnitude and group-delay error with frequency than those designed using the LP approach.

W. R. S. Sutherland, H. Wolkowicz, and V. Zeidan. An explicit linear solution for the quadratic dynamic- programming problem. *Journal of Optimization Theory and Applications*, **58**(2), 319–330, 1988.

K. Swarup. Indefinite quadratic programming. *Cahiers du Centre d'Etudes de Recherche Opérationalle*, **8**, 217–222, 1966a.

K. Swarup. Programming with indefinite quadratic function with linear constraints. *Cahiers du Centre d'Etudes de Recherche Opérationalle*, **8**, 132–136, 1966b.

K. Swarup. Quadratic programming. *Cahiers du Centre d'Etudes de Recherche Opérationalle*, **8**, 223–234, 1966c.

B. A. Szabo and T. T. Chung. The quadratic programming approach to the finite element method. *International Journal for Numerical Methods in Engineering*, **5**(3), 375–381, 1973.

Abstract. A finite element approximation technique is described in which the potential energy function is minimized subject to linear constraints. The constraints require satisfaction of all kinematic boundary conditions and interelement continuity conditions. An example, indicating the efficiency of the proposed method, is presented.

K. Tabushi, S. Itoh, M. Sakura, Y. Kutsutaninakamura, T. A. Iinuma, and T. Arai. A method for calculating the optimum irradiation condition for intracavitary radiotherapy using quadratic programming. *Physics in Medicine and Biology*, **33**(5), 515–527, 1988.

Abstract. A method of calculating optimum irradiation conditions for intracavitary radiotherapy using quadratic programming has been formulated and then modified for practical application. The allowable range of obtained dose, which is usually fixed in advance, is automatically computed to be as small as possible. The variance of the product of the activity and the irradiation time of the tandem source is also minimised to avoid the occurrence of cold and/or hot spots. Optimum irradiation conditions for conventional intracavitary radiotherapy of carcinoma of the uterine cervix were obtained on the basis of isodose curves passed through the points A of the Manchester system. Those for carcinoma of the other organs and special cases of carcinoma of the uterine cervix can be determined after consideration of the tumour state.

M. Takamoto, N. Yamada, Y. Kobayashi, H. Nonaka, and S. Okoshi. 0–1 quadratic programming algorithm for resource leveling of manufacturing process schedules. *Systems and Computers in Japan*, **26**(10), 68–76, 1995. See also, Transactions of the Institute of Electronics, Information and Communication Engineers, volume J77D-II(10) pages 2075–2082, 1994.

Abstract. In industrial plant construction scheduling, it is necessary to minimize the fluctuation, the maximum peak of the fluctuation and the maximum peak value of the amount of daily resources, which is calculated as the sum of daily resources for each process. Minimization of the fluctuation or the maximum peak value corresponds to leveling the pile of resources. To perform this resource leveling, we need to decide on an objective function which is a monotone function that simply increases with the degree of resource leveling and then solve the optimization problem by fixing the process start dates while minimizing the objective function. Industrial plant construction is, however, in many cases a large-scale scheduling with an entire period of more than 1000 days and more than 100 processes, so it is very difficult to obtain a global optimization solution. We have developed an algorithm which solves a large-scale optimization problem to level necessary resources. This algorithm can quickly search for a good suboptimal solution close to the global optimal solution of a 0–1 quadratic programming problem. The algorithm searches by repeating a pivot operation using variable selection rules for resource leveling. We applied this algorithm to large-scale scheduling for an actual plant construction schedule, and successfully obtained a practical suboptimal solution within a few minutes (CPU power: 28MIPS). The results suggest that the algorithm is practical for resource leveling of large-scale construction scheduling.

T. Takayama and N. Uri. A note on spatial and temporal price and allocation modeling—quadratic programming or linear complementarity programming. *Regional Science and Urban Economics*, **13**(4), 455–470, 1983.

A. Takeda, Y. Dai, M. Fukuda, and M. Kojima. Towards the implementation of successive convex relaxation method for nonconvex quadratic optimization problems. Research Report on Information Sciences B-347, Department of Mathematical and Computing Sciences, Tokyo Institute of Technology, Japan, 1999.

Abstract. Recently Kojima and Tunçel proposed new successive convex relaxation methods and their localized-discretized variants for general nonconvex quadratic programs. Although an upper bound of the objective function value within *a prior* precision can be found theoretically by solving a finite number of linear programs, several important implementation problems remain unsolved. In this paper we discuss these issues, present practically implementable algorithms and report numerical results.

N. N. Tam and N. D. Yen. Continuity properties of the Karush-Kuhn-Tucker point set in quadratic programming problems. *Mathematical Programming*, **85**(1), 193–206, 1999.

Abstract. We obtain necessary and sufficient conditions for the set of the Karush-Kuhn-Tucker points in a canonical quadratic programming problem to be upper semicontinuous or lower semicontinuous with respect to the problem parameters.

K. Tammer. Possibilities for the application of the results of parametric programming to solving indefinite quadratic programming problems. *Mathematische Operationsforschung und Statistik*, **7**(2), 209–222, 1976.

Abstract. Some possibilities for the application of the results of the convex and nonconvex parametric quadratic programming to the computation of nonconvex quadratic programming problems are given. In the first case the problem can be solved by solving a strongly convex parametric quadratic problem with in general more than one parameter and some smaller quadratic problems, in the second case by solving a one parametric nonconvex quadratic problem.

H. Tamura and Y. Kihara. Multistage quadratic programming for discrete-time optimal control with state and control constraints. *Systems and Control*, **21**(12), 702–709, 1977.

Abstract. A multistage quadratic programming technique is developed for discrete-time linear-quadratic (L-Q) optimal control problem with state and control constraints. The problem is converted to a linear program with bilinear constraints. Then, taking advantage of the staircase-structure of equality constraints (system equation), Dantzig-Wolfe decomposition principle is applied repeatedly in each stage, where the decomposition technique for the bilinear constraints is newly developed in this paper. The significant advantage of the multistage quadratic programming technique in this paper is that it can handle a large number of stages without increasing the computation time and storage requirements enormously, and that it can handle state and control constraints without difficulty. Therefore, a substantial reduction of computational burden is obtained from the existing discrete-time optimal control algorithms. Numerical examples for comparison with Wolfe's quadratic programming method are included.

Y. Tan and C. Deng. Solving for a quadratic programming with a quadratic constraint based on a neural network frame. *Neurocomputing*, **30**(1–4), 117–128, 2000.

Abstract. In many applications, a class of optimization problems called quadratic programming with a special quadratic constraint (QPQC) often occurs, such as in the fields of maximum-entropy spectral estimation, FIR filter design with time-frequency constraints, and the design of an FIR filter bank with a perfect reconstruction property. In order to deal with this kind of optimization problem and to be inspired by the computational virtue of analog or dynamic neural networks, a feedback neural network is proposed for solving for this class of QPQC computation problems in real time. The stability, convergence and computational performance of the proposed neural network have also been analyzed and proved in detail, so as to theoretically guarantee the computational effectiveness and capability of the network. From the theoretical analyses, it turns out that the solution of a QPQC problem is just the generalized minimum eigenvector of the objective matrix with respect to the constrained matrix. A number of simulation experiments have been given to further support our theoretical analysis and to illustrate the computational performance of the proposed network.

H. Tanaka and H. Lee. Interval regression analysis by quadratic programming approach. *IEEE Transactions on Fuzzy Systems*, **6**(4), 473–481, 1998.

Abstract. When we use linear programming in possibilistic regression analysis, some coefficients tend to become crisp because of the characteristic of linear programming. On the other hand, a quadratic programming approach gives more diverse spread coefficients than a linear programming one. Therefore, to overcome the crisp characteristic of linear programming, we propose interval regression analysis based on a quadratic programming approach. Another advantage of adopting a quadratic programming approach is to be able to integrate both the property of central tendency in least squares and the possibilistic property in fuzzy regression. By changing the weights of the quadratic function, we can analyze the given data from different viewpoints. For data with crisp inputs and interval outputs, the possibility and necessity models can be considered. Therefore, the unified quadratic programming approach obtaining the possibility and necessity regression models simultaneously is proposed. Even though there always exist possibility estimation models, the existence of necessity estimation models is not guaranteed if we fail to assume a proper function fitting to the given data as a regression model. Thus, we consider polynomials as regression models since any curve can be represented by the polynomial approximation. Using polynomials, we discuss how to obtain approximation models which fit well to the given data where the measure of fitness is newly defined to gauge the similarity between the possibility and the necessity models. Furthermore, from the obtained possibility and necessity regression models, a trapezoidal fuzzy output can be constructed.

H. Tanaka, K. Koyama, and H. Lee. Interval regression analysis based on quadratic programming. *In* ‘Proceedings of the Fifth IEEE International Conference on Fuzzy Systems. FUZZ-IEEE ’96. IEEE, New York, NY, USA’, Vol. 1, pp. 325–329, 1996.

Abstract. We propose an approach to interval regression analysis based on quadratic programming. Secondly we propose a unified approach for the possibility and the necessity regression models in interval regression. From the obtained two models, fuzzy output can be defined. Moreover, a performance index for the unified approach to measure the fitness of the given data to the obtained models is defined.

J. Tang and D. Wang. An interactive approach based on a genetic algorithm for a type of quadratic programming problems with fuzzy objective and resources. *Computers and Operations Research*, **24**(5), 413–422, 1997.

Abstract. A type of model of fuzzy quadratic programming problems (FQP) is proposed. The model describes the fuzzy objective and resource constraints with different types of membership functions according to different types of fuzzy objective and fuzzy resource constraints in actual production problems. This article develops an inexact approach to solve this type of model of quadratic programming problems with fuzzy objective and resource constraints. Instead of finding an exact optimal solution, we use a genetic algorithm (GA) with mutation along the weighted gradient direction to find a family of solutions with acceptable membership degrees. Then by means of the human-computer interaction, the solutions preferred by the DM under different criteria can be achieved.

W. S. Tang. A discrete-time recurrent neural network for solving quadratic programs with application to fir filter synthesis. *In* 'SMC 2000 Conference Proceedings. 2000 IEEE International Conference on Systems, Man and Cybernetics. 'Cybernetics Evolving to Systems, Humans, Organizations, and their Complex Interactions, IEEE, Piscataway, NJ, USA', Vol. 4, pp. 2491–2496, 2000.

Abstract. A discrete-time recurrent neural network is presented for solving convex quadratic programs. It is the discrete-time version of its continuous-time counterpart which was developed by J. Wang and H. Li (1994). Sharing the same characteristic with its continuous-time counterpart, the proposed discrete-time neural network could compute the exact optimal solution to a quadratic program without using any penalty parameter. However, the discrete-time version is more desirable in practical realization in view of the availability of digital hardware and the good compatibility to computer. The condition for the neural network globally converging to the optimal solution of a quadratic program is given. The neural network is applied to FIR filter synthesis for illustrating its effectiveness.

W. S. Tang and J. Wang. A discrete-time lagrangian network for solving constrained quadratic programs. *International Journal of Neural Systems*, **10**(4), 261–265, 2000.

Abstract. A discrete-time recurrent neural network which is called the discrete-time Lagrangian network is proposed for solving convex quadratic programs. It is developed based on the classical Lagrange optimization method and solves quadratic programs without using any penalty parameter. The condition for the neural network to globally converge to the optimal solution of the quadratic program is given. Simulation results are presented to illustrate its performance.

Q. Tao and D. Sun. The simplification of neural network for quadratic programming problems and its applications in optimal control. *In* 'Proceedings of the 3rd World Congress on Intelligent Control and Automation, IEEE, Piscataway, NJ, USA', Vol. 5, pp. 3504–3508, 2000.

Abstract. In this paper, a kind of neural network for quadratic programming problems is first simplified. The simplification is necessary for high accuracy solutions and low cost implementation; the simplified model has high performance. The proposed neural network is used to solve quadratic control problems for discrete linear systems with constraints. The simulation proved the rationality of the results obtained.

R. M. Teny and A. K. Kochhar. Solution of the aggregate production planning problem in a multi-stage-multi-product manufacturing system using functional space analysis and quadratic programming approaches. *International Journal of Systems Science*, **14**(3), 325–342, 1983.

Abstract. It is shown that both of these approaches can be used to determine the production planning strategies for the number of periods under consideration. Statistical inference techniques are used to determine whether the functional space analysis technique results in a global minimum. Tests carried out on two different sets of data show that the quadratic programming approach always results in a minimum cost solution, although it requires a large amount of computing power. Although complicated mathematics is required to formulate the problem and solve it, the resulting solution can be easily understood and applied to a practical manufacturing system.

T. Terlaky. A new algorithm for quadratic programming. *European Journal of Operational Research*, **32**(2), 294–301, 1987. See also, *Alkalmazott Matematikai Lapok.* 12(3–4):283–293, 1986.

Abstract. Presents a new finite algorithm for quadratic programming. The algorithm is based on the solution procedures of linear programming (pivoting, Bland's rule, Hungarian Methods, criss-cross method), however this method does not require the enlargement of the basic tableau as Frank-Wolfe method does. It can be considered as a feasible point active-set method. This algorithm is a straightforward generalization of Klafszky's and Terlaky's Hungarian method. It has nearly the same structure as Ritter's algorithm (which is based on conjugate directions), but it does not use conjugate directions.

H. Theil and C. van De Panne. Quadratic programming as an extension of conventional quadratic maximization. *Management Science*, **7**, 1–20, 1960.

A. L. Tits and J. L. Zhou. A simple, quadratically convergent interior-point algorithm for linear-programming and convex quadratic programming. *In* 'Large Scale Optimization: State of the Art', Vol. 1, pp. 411–427, 1994.

M. J. Todd. Linear and quadratic programming in oriented matroids. *Journal of Combinatorial Theory, Series B*, **39**(2), 105–133, 1985.

Abstract. Duality theorems of linear and quadratic programming are proven constructively in the combinatorial setting of oriented matroids. One version of the algorithm for linear programming has the interesting feature of maintaining feasibility. The development of the quadratic programming duality result suggests the study of properties of square matrices such as symmetry and positive semidefiniteness in the context of oriented matroids.

M. J. Todd and Y. Wang. A projective algorithm for convex quadratic programming. Technical report, School of Operations Research and Industrial Engineering, Cornell University, Ithaca, New York, USA, 1991.

O. N. Tokareva. The estimates of the rate of convergence of algorithms based on barrier functions with the use of quadratic programming problems. *Cybernetics*, **17**(5), 688–695, 1981.

J. A. Tomlin and M. A. Saunders. Stable reduction to KKT systems in barrier methods for linear and quadratic programming. Research Report RJ 10039, IBM Almaden Research Center, San Jose, California, USA, 1996.

J. J. Torsti. Inversion of heat capacity by quadratic programming in the cases of KCl and Cu. *Annales Academiae Scientiarum Fennicae, Series AVI (Physica)*, **372**, 1–20, 1971.

Abstract. By using experimental specific heat data and by separating the specific heat into two parts, lattice specific heat and anharmonic contributions, a method has been presented for the calculation of the lattice vibration spectrum and of the anharmonic contributions to the specific heat with their confidence limits. The calculated frequency spectrum of KCl was in qualitative agreement with the frequency spectrum by Copley et al. which corresponded to the inelastic neutron scattering measurements. The highest peak in the spectra was at $4.6 * 10^{12}\ s^{-1}$. Likewise, the calculated frequency spectrum of Cu was found to be in agreement with Varshni's and Shukla's theoretical spectrum (abstr. A8094 of 1966) which is based on the axial symmetric model for lattice dynamics. The highest peak was at $6.3 * 10^{12}\ s^{-1}$. If the volume dependence of the frequencies is taken into account, the results for both KCl and Cu are in better agreement with the experimental data than when using the harmonic approximation. The confidence limits of the frequency spectra were comparatively large at higher frequencies. By contrast, the limits of coefficients of anharmonic contributions to the specific heat were narrow.

J. J. Torsti and A. M. Aurela. A fast quadratic programming method for solving ill-conditioned systems of equations. *Journal of Mathematical Analysis and Applications*, **38**(1), 193–204, 1972.

T. B. Trafalis and N. P. Couellan. Neural-network training via quadratic programming. *In* 'Operations Research/Computer Science Interfaces Series, Advances in Metaheuristics, Optimization, and Stochastic Modeling Technologies', pp. 123–139, 1997.

E. Triantaphyllou. A quadratic programming approach in estimating similarity relations. *IEEE Transactions on Fuzzy Systems*, **1**(2), 138–145, 1993.

Abstract. The problem of estimating how similar N objects are when they are compared with each other is investigated, using comparative judgments of all possible pairs of the N objects as data. The pairwise comparisons focus on the similarity relations instead of the relative importance of each object. A quadratic programming model is also proposed. It processes the similarity-based pairwise comparisons and determines the similarity relations among the N objects. The model has linear constraints; therefore it can be solved easily by transferring it into a system of linear equations.

T. Tsuchiya. Global convergence of the affine scaling algorithm for primal degenerate strictly convex quadratic programming problems. *Annals of Operations Research*, **46–47**(1–4), 509–539, 1993.

Abstract. Deals with global convergence of the affine scaling algorithm for strictly convex QP problems satisfying a dual nondegeneracy condition. By means of the local Karmarkar potential function which was successfully applied to demonstrate global convergence of the affine scaling algorithm for LP, the author shows global convergence of the algorithm when the step-size 1/8 is adopted without requiring any primal nondegeneracy condition.

A. W. Tucker. A least-distance approach to quadratic programming. *In* G. B. Dantzig and A. F. Veinott, eds, 'Lectutes in Applied Mathematics II', number 1 *in* 'Mathematics of the Decision Sciences', pp. 163–176, American Mathematical Society, Providence, Rhode Island, USA, 1968.

M. Tuma. *Large and Sparse Quadratic Programming*. PhD thesis, UIVT CSAV, Prague, Czeck Republic, 1989.

M. Tuma. A quadratic programming algorithm for large and sparse problems. *Kybernetika*, **27**(2), 155–167, 1991.

Abstract. Describes a primal feasible convergent algorithm for the quadratic programming problem. It is especially designed to cope with the large and sparse instances of this type with their structural and numerical algorithmic specialties. It makes use of improvements that have not been used for the general quadratic programming or that have not been used so far.

G. Uebe. Note on the paper of H. Amato and G. Mensch: rank restriction on the quadratic form in indefinite quadratic programming. *Zeitschrift für Operations Research, Serie A (Theorie)*, **16**(3), 113–114, 1972.

H. Väliaho. New algorithms for quadratic programming. *Commentationes Physico Mathematicae*, **36**(9), 111–136, 1970.

Abstract. A new class of algorithms for quadratic programming is constructed, applying the theory of stepwise regression analysis introduced in Väliaho (1969), A Synthetic Approach to Stepwise Regression Analysis. The basis of the algorithms is the fact that a feasible vector is the solution if and only if the Lagrangians attached to the binding constraints are negative. The basic algorithm starts from the free minimum. At any stage, a violated constraint is turned into a desideratum and the weight of this desideratum is increased gradually from 0 to infinity , meanwhile removing those binding constraints, the Lagrangians attached to which become zero. A modified algorithm is also proposed leading to the solution more rapidly than the basic algorithm. As further developments, interval and simple constraints are considered. A procedure for changing the quadratic programming problem is introduced, making it possible to cope with semidefinite cases. Finally a new algorithm for linear programming is constructed based on the fact that linear programming is an elementary particular case of quadratic programming.

H. Väliaho. The method of approximate constraints for quadratic programming. *Commentationes Physico Mathematicae*, **41**(3), 257–280, 1971a.

Abstract. The fundamentals of the method of approximate constraints are derived without appealing to the terminology of regression analysis. Some previous errors are corrected, and some contributions made: The connection between pre- and post-loops is revealed; algorithms are derived without using desiderata; the effect of changing a constraint on the solution is examined; Houthakker's capacity method is carried through making use of the scheme of the Method of Approximate Constraints: a special case of indefinite quadratic programming is solved.

H. Väliaho. Notes on some algorithms for quadratic programming. *Commentationes Physico Mathematicae*, **41**(3), 247–255, 1971b.

Abstract. A monotonic version of the basic algorithm is introduced, using non-diagonal pivotal operations. This approach makes it possible to derive the basic algorithm without using the concepts 'modified constraint' or 'desideratum'. The efficiencies of some algorithms for quadratic programming are compared in the light of two numerical examples.

H. Väliaho. On the definity of quadratic forms subject to linear constraints. *Journal of Optimization Theory and Applications*, **38**(1), 143–145, 1982.

H. Väliaho. A unified approach to one-parametric general quadratic programming. *Mathematical Programming*, **33**(3), 318–338, 1985.

Abstract. A method is proposed for finding local minima to the parametric general quadratic programming problem where all the coefficients are linear or polynomial functions of a scalar parameter. The local minimum vector and the local minimum value are determined explicitly as rational functions of the parameter. A numerical example is given.

H. Väliaho. Quadratic-programming criteria for copositive matrices. *Linear Algebra and its Applications*, **119**, 163–182, 1989.

H. Väliaho. A new proof for the criss-cross method for quadratic programming. *Optimization*, **25**(4), 391–400, 1992.

Abstract. In a working paper Chang (1979) has introduced a simple method for the linear complementarity problem involving a nonnegative definite matrix, based on principal pivoting and the least-index rule of Bland (1977). This method has two important special cases, namely the criss-cross methods for linear programming and for convex quadratic programming, invented later independently by Terlaky and Klafszky. The proof of Chang is very complicated. Terlaky and Klafszky provide much simpler proofs for the criss-cross method. We propose another simple proof for the criss-cross method for quadratic programming. It appears from an example by Roos (1990) that the number of pivots required by the criss-cross method may grow exponentially with the number of the constraints of the problem. We use two additional examples, due essentially to Fathi (1979) and Murty (1988), to analyse the worst-case behaviour of the method.

H. Väliaho. Cycling can occur in Mraz's algorithm for nonconvex quadratic programming. *Computing*, **51**(2), 183–184, 1993.

Abstract. F. Mraz (ibid., vol, 45, pp, 283–9, 1990) proposed a modified version of Beale's algorithm for nonconvex quadratic programming. He states that after a finite number of steps the algorithm detects that the problem is infeasible or has an unbounded solution, or finds a true local minimum. We give a counterexample which shows that cycling can occur in Mraz's algorithm.

C. van De Panne. A non-artificial Simplex method for quadratic programming. Report 6229, Economic Institute of the Netherlands School of Economics, Rotterdam, Netherlands, 1962.

C. van De Panne. *Methods for Linear and Quadratic Programming*. North Holland, Amsterdam, the Netherlands, 1975.

C. van De Panne and A. B. Whinston. The Simplex and the dual method for quadratic programming. *Operational Research Quarterly Journal*, **15**, 355–388, 1964a.

C. van De Panne and A. B. Whinston. Simplicial methods for quadratic programming. *Naval Research Logistics Quarterly*, **11**(4), 273–302, 1964b.

C. van De Panne and A. B. Whinston. A comparison of two methods for quadratic programming. *Operations Research*, **14**, 422–441, 1966.

C. Van de Panne and A. B. Whinston. The symmetric formulation of the Simplex method for quadratic programming. *Econometrica*, **37**, 507–527, 1969.

R. J. Vanderbei. LOQO: an interior point code for quadratic programming. Technical Report SOR 94–15, Program in Statistics and Operations,Research, Princeton University, New Jersey, USA, 1994.

V. S. Vassiliadis. Application of the modified barrier method in large-scale quadratic programming problems. *Computers and Chemical Engineering*, **20**, S243–248, 1996.

Abstract. Presents the application of the penalty/modified barrier function (PE/MBF) method in the solution of large-scale quadratic programming (QP) problems. A review of the recent literature on PE/MBF methods is presented and the choice of this method for QP problems is justified by previous experience in very large-scale bound-constrained problems. The proposed algorithm performs two types of iterations: an outer iteration in which the Lagrange multipliers of the bounds are adjusted, and an inner iteration for the solution of an equality constrained subproblem. The inner iteration solves a modified problem, containing PE/MBF terms for the bounds in the objective and is subject to equality constraints only. The equality constraints are handled directly via the use of additional Lagrange multipliers during the inner iteration and thus, instead of an unconstrained problem, the inner iteration solves a modified equality constrained problem. Any inequality constraints, other than bounds, are formulated as equalities via the use of slack variables. Computational results show this method to be promising, and motivate further investigation for the general case of nonlinear programming problems.

V. S. Vassiliadis and S. A. Brooks. Application of the modified barrier method in large-scale quadratic programming problems. *Computers and Chemical Engineering*, **22**(9), 1197–1205, 1998.

Abstract. The application of the penalty/modified barrier function method (PE/MBF) is presented for the solution of large-scale positive-semidefinite quadratic programming problems (QP). A review of the recent literature on QP methods is presented and the choice of the PE/MBF method for QP problems is justified by previous experience in very large-scale bound-constrained problems. The proposed algorithm performs two types of iterations: an outer iteration in which the Lagrange multipliers of the bounds are adjusted, and an inner iteration for the solution of an equality constrained subproblem. The inner iteration solves a modified problem, containing penalty/modified barrier terms for the bounds in the objective, and is subject to equality constraints only. The equality constraints are handled directly via the use of additional Lagrange multipliers during the inner iteration and thus, instead of an unconstrained problem, the inner iteration solves a modified equality constrained problem. Any inequality constraints, other than bounds, are formulated as equalities via the use of slack variables. Computational results show this method to be promising, and motivate further investigation for the general case of nonlinear programming problems.

S. A. Vavasis. Quadratic programming is in NP. *Information Processing Letters*, **36**(2), 73–77, 1990.

Abstract. Quadratic programming is an important example of optimization with applications to engineering design, combinatorial optimization, game theory, and economics. Garey and Johnson (1979) state that quadratic programming is NP-hard. The author shows that it lies in NP, thereby proving that it is NP-complete.

S. A. Vavasis. Convex quadratic programming. *In* 'Nonlinear Optimization: Complexity Issues', pp. 36–75, Oxford University Press, Oxford, England, 1991a.

S. A. Vavasis. Nonconvex quadratic programming. *In* 'Nonlinear Optimization: Complexity Issues', pp. 76–102, Oxford University Press, Oxford, England, 1991b.

S. A. Vavasis. Approximation algorithms for indefinite quadratic programming. *Mathematical Programming*, **57**(2), 279–311, 1992a.

Abstract. We consider ε-approximation schemes for indefinite quadratic programming. We argue that such an approximation can be found in polynomial time for fixed ε and t, where t denotes the number of negative eigenvalues of the quadratic term. Our algorithm is polynomial in $1/\varepsilon$ for fixed t, and exponential in t for fixed ε. We next look at the special case of knapsack problems, showing that a more efficient (polynomial in t) approximation algorithm exists.

S. A. Vavasis. Local minima for indefinite quadratic knapsack-problems. *Mathematical Programming*, **54**(2), 127–153, 1992b.

J. R. Vera. Ill-posedness and quadratic programming: stability and complexity issues. Technical Report 95-47, Center for Operations Research and Economics, University Catholique de Louvain, Belgium, 1995.

F. H. Verhoff. A quadratic programming technique with application to kinetic rate constant determination. *Chemical Engineering Science*, **33**(3), 263–269, 1978.

Abstract. A technique for the solution of the quadratic programming problem by a series of unconstrained optimizations in subspaces of the original space is developed. This technique uses a previously known transformation to convert the problem into an optimization in the non-negative orthant. The optimization can then be solved by optimizations in subspaces. The algorithm is shown to always decrease the objective function and to converge in a finite number of iterations. The algorithm is easily programmed on a computer. An example, previously used by others, is solved with this technique in a fewer number of iterations. An illustration of this technique for kinetic rate constant determination is included. Other applications for this simple quadratic programming scheme are discussed.

S. A. Vinogradov and D. F. Wilson. Phosphorescence lifetime analysis with a quadratic programming algorithm for determining quencher distributions in heterogeneous systems. *Biophysical Journal*, **67**(5), 2048–2059, 1994.

Abstract. A new method for analysis of phosphorescence lifetime distributions in heterogeneous systems has been developed. This method is based on decomposition of the data vector to a linearly independent set of exponentials and uses quadratic programming principles for χ^2 minimization. Solution of the resulting algorithm requires a finite number of calculations (it is not iterative) and is computationally fast and robust. The algorithm has been tested on various simulated decays and for analysis of phosphorescence measurements of experimental systems with discrete distributions of lifetimes. Critical analysis of the effect of signal-to-noise on the resolving capability of the algorithm is presented. This technique is recommended for resolution of the distributions of quencher concentration in heterogeneous samples, of which oxygen distributions in tissue is an important example. Phosphors of practical importance for biological oxygen measurements: Pd-meso-tetra (4-oarboxyphenyl) porphyrin (PdTCPP) and Pd-meso-porphyrin (PdMP) have been used to provide experimental test of the algorithm.

V. Visweswaran and C. A. Floudas. New properties and computational improvement of the GOP algorithm for problems with quadratic objective function and constraints. *Journal of Global Optimization*, **3**(3), 439–462, 1993.

O. N. Vylegzhanin. Comparative analysis of the possibility of linear and quadratic programming in the quantitative spectrophotometry of mixtures of incompletely known composition. *Journal of Applied Spectroscopy*, **52**(6), 659–663, 1990.

Abstract. The author previously formulated (1989) a statement of the problem of quantitative multicomponent spectrophotometry for the case when an unknown additive is present in the investigated object along with components having known spectra. The methods of linear (LP) or quadratic (QP) programming are usually used for the determination of the content in such mixtures of components with known spectra. In the present work a comparative numerical investigation of the possibilities of the methods of LP and QP for the solution of the indicated problem is carried out.

Y. H. Wan. On the average speed of Lemke's algorithm for quadratic programming. *Mathematical Programming*, **35**(2), 236–246, 1986.

Abstract. The author shows that the average number of steps of Lemke's algorithm for quadratic programming problems grows at most linearly in the number of variables while fixing the number of constraints. The result and method were motivated by Smale's result on linear programming problems (1983). The author gives the probability that a quadratic programming problem indeed possesses a finite optimal solution.

J. Wang. Recurrent neural network for solving quadratic programming problems with equality constraints. *Electronics Letters*, **28**(14), 1345–1347, 1992.

Abstract. A recurrent neural network for solving quadratic programming problems with equality constraints is presented. The proposed recurrent neural network is asymptotically stable and able to generate optimal solutions to quadratic programs with equality constraints. An opamp based analogue circuit realisation of the recurrent neural network is described. An illustrative example is also discussed to demonstrate the performance and characteristics of the analogue neural network.

J. Wang and Y. Xia. A dual neural network solving quadratic programming problems. *In* 'IJCNN'99. International Joint Conference on Neural Networks. Proceedings, IEEE, Piscataway, NJ, USA', Vol. 1, pp. 588–593, 1999.

Abstract. We propose a dual neural network with globally exponential stability for solving quadratic programming problems with unique solutions. Compared with the Bouzerdoum-Pattison network (1993), there is no need for choosing the self-feedback or lateral connection matrices in the present network. Moreover, the size of the dual network is less than that of the original problem.

H. H. Weber. Some thoughts on vertical price determination and price connection, using the theory of quadratic programming. *Operations Research Verfahren*, **26**, 780–782, 1977.

Abstract. Considers the method of fixing wholesale and retail prices. It is shown that price-fixing benefits all concerned.

H. Wei and H. Sasaki. Large-scale optimal power-flow based on interior-point quadratic programming of solving symmetrical indefinite system. *In* 'Large Scale Systems: Theory and Applications 1995', pp. 377–382, 1995.

H. Wei, H. Sasaki, T. Nagata, and R. Yokoyama. An application of interior point quadratic programming algorithm to power system optimization problems. *Transactions of the Institute of Electrical Engineers of Japan, Part B*, **116-B**(2), 174–180, 1996. See also, Conference Proceedings. 1995 IEEE Power Industry Computer Application Conference, IEEE, New York, NY, USA, pages 98–104, 1995, and IEEE Transactions on Power Systems, volume 11, number 1, pages 260–266, 1996.

Abstract. This paper presents a new algorithm of the interior point quadratic programming which can solve power system optimization problems with profoundly less computational efforts. The proposed algorithm has the following two special features. First, it is based on the path-following interior point algorithm, the search direction of which is taken as Newton direction, thus having quadratic convergence. In the second place, it solves directly a symmetric indefinite system instead of reducing it to the usual system with a positive-definite matrix. Under this artifice, the algorithm avoids the formation of ADA^T and generates fewer fill-ins than the case of factorizing the positive definite system matrix for large scale systems, realizing its impressive speed-up. Since the formula of the interior point method have been deduced more generally, the proposed algorithm can start from either a feasible (interior point) or an infeasible point (noninterior point). Numerical results on the IEEE test systems and a Japanese 344 bus system (2100 variables, 2088 equality constraints and 1400 inequality constraints) have verified that the proposed algorithm possesses enough robustness and needs significantly less solution time compared with already reported applications of the interior point method.

S. H. Wei. A method for strictly convex quadratic programming problems. *Opsearch*, **14**(2), 118–124, 1977.

Abstract. There are many existing excellent methods for the quadratic programs. Each one has its own feature. It is the purpose of this paper to develop an algorithm for the quadratic programs in which the features of those existing methods are utilized to solve the subprograms involved.

Z. L. Wei. Subspace search method for quadratic programming with box constraints. *Journal of Computational Mathematics*, **17**(3), 307–314, 1999.

Abstract. A subspace search method for solving quadratic programming with box constraints is presented in this paper. The original problem is divided into many independent subproblem at an initial point, and a search direction is obtained by solving each of the subproblem, as well as a new iterative point is determined such that the value of objective function is decreasing. The convergence of the algorithm is proved under certain assumptions, and the numerical results are also given.

V. Weispfenning. Parametric linear and quadratic optimization by elimination. Technical Report MIP-9404, Passau University, Germany, 1994.

U. Werner. Multiple branching in parametric quadratic programming. *Optimization Methods and Software*, **4**(4), 307–316, 1995.

J. Weston and W. Barenek. Programming investment portfolio construction. *Analysts Journal*, **11**, 51–55, 1955.

Abstract. Explains Markowitz model for securities analysts. Mentions the importance of operations research for solving the quadratic programming problem

R. Wetzel. *Untersuchungen zu einem algorithmus für quadratischer optimierungsaufgaben unter verwendung linearer komplimentaritätsprobleme*. Diploma thesis, Karl-Marx-Universät, Leipzig, Germany, 1981.

T. Whalen and G. Wang. Optimizing ordinal utility using quadratic programming and psychophysical theory. *In* ‘PeachFuzz 2000. 19th International Conference of the North American Fuzzy Information Processing Society, IEEE, Piscataway, NJ, USA’, pp. 307–310, 2000.

Abstract. This paper presents a method for supporting the decision making process for multiattribute decisions by applying quadratic programming to screen alternatives when the degree to which various alternatives satisfy, each attribute and/or the relative importance of the attributes are only roughly specified. In particular, we will work with verbal specifications of these quantities, like low, medium, and high. These specifications will be interpreted as extended ordinal constraints using concepts of ”just noticeable difference” derived from psychophysics.

A. B. Whinston. A dual decomposition algorithm for quadratic programming. *Cahiers du Centre d'Etudes de Recherche Opérationalle*, **6**(4), 188–201, 1964.

A. B. Whinston. The bounded variable problem—an application of the dual method for quadratic programming. *Naval Research Logistics Quarterly*, **12**(2), 173–179, 1965.

A. B. Whinston. A decomposition algorithm for quadratic programming. *Cahiers du Centre d'Etudes de Recherche Opérationalle*, **8**(2), 112–131, 1966.

G. Wilson. Quadratic programming analogs. *IEEE Transactions on Circuits and Systems*, **CAS-33**(9), 907–911, 1986.

Abstract. It is shown that a recently proposed analog of the quadratic programming problem is (L. O. Chua and G. N. Lin, ibid., vol. CAS-31, pp. 182–8, Feb. 1984) is only applicable to a subset of such problems. The characteristics required of a generalized analog are detailed and a suitable form of implementation is described. Circuits implementing the general linearly constrained quadratic program are described and it is found that experimental simulations can match computer predictions with accuracies on the order of 0.1%. The presence of quenchable large-scale oscillations is also noted and their origin is traced to diode nonidealities.

P. Wolfe. The Simplex method for quadratic programming. *Econometrica*, **27**, 382–398, 1959.

Y. K. Wong and S. P. Chan. Quadratic programming approach to the solution of economic dispatch problem. *Technical Papers from the Seventh Conference on Electric Power Supply Industry. SW Queensland Electricity Board, Brisbane, Queensland, Australia*, **4**, 4.19/1–14, 1988.

Abstract. The economic dispatch problem can be solved by an easy variation of linear programming (i.e. quadratic programming). The equations set up are suitable for computer-based solution. The LP/PROTRAN code is especially suitable to solve such unit commitment problems since it can handle (mixed) integer programming, and the ordinary LP/PROTRAN and FORTRAN codes can also be intermixed.

E. Woomer and W. Pilkey. The balancing of rotating shafts by quadratic programming. *Journal of Mechanical Design-Transactions of the ASME*, **103**(4), 831–834, 1981.

S. J. Wright. A fast algorithm for equality-constrained quadratic programming on the Alliant FX/8. *Annals of Operations Research*, **14**(1–4), 225–243, 1988.

Abstract. An efficient implementation of the null-space method for quadratic programming on the Alliant FX/8 computer is described. The most computationally significant operations in this method are the orthogonal factorization of the constraint matrix and corresponding similarity transformation of the Hessian, and the Cholesky factorization of the reduced Hessian matrix. It is shown how these can be implemented in such a way as to take full advantage of the Alliant's parallel/vector capabilities and memory hierarchy. Timing results are given on a set of test problems for which the data can be easily accommodated in core memory.

S. J. Wright. On reduced convex qp formulations of monotone lcp problems. Technical Report MCS-P808-0400, Argonne National Laboratory, Illinois, USA, 2000.

L. Wu. A conjugate gradient method for the unconstrained minimization of strictly convex quadratic splines. *Mathematical Programming, Series A*, **72**(1), 17–32, 1996.

X. Y. Wu, Y. S. Xia, J. Li, and W. K. Chen. A high-performance neural network for solving linear and quadratic programming problems. *IEEE Transactions on Neural Networks*, **7**(3), 643–651, 1996.

Abstract. Two classes of high-performance neural networks for solving linear and quadratic programming problems are given. We prove that the new system converges globally to the solutions of the linear and quadratic programming problems. In a neural network, network parameters are usually not specified. The proposed models can overcome numerical difficulty caused by neural networks with network parameters and obtain desired approximate solutions of the linear and quadratic programming problems.

Y. Xia. A new neural network for solving linear and quadratic programming problems. *IEEE Transactions on Neural Networks*, **7**(6), 1544–1548, 1996.

Abstract. A new neural network for solving linear and quadratic programming problems is presented and is shown to be globally convergent. The new neural network improves existing neural networks for solving these problems: it avoids the parameter turning problem, it is capable of achieving the exact solutions, and it uses only simple hardware in which no analog multipliers for variables are required. Furthermore, the network solves both the primal problems and their dual problems simultaneously.

Y. Xia and J. Wang. Primal neural networks for solving convex quadratic programs. *In* 'IJCNN'99. International Joint Conference on Neural Networks. Proceedings, IEEE, Piscataway, NJ, USA', Vol. 1, pp. 582–587, 1999.

Abstract. We propose two primal neural networks with globally exponential stability for solving quadratic programming problems. Both the self-feedback and lateral connection matrices in the present network are compared with the Bonzerdoum-Pattison network (1993). Moreover, the size of the proposed networks is same as that of the original problem, smaller than that of primal-dual networks.

Y. S. Xia and X. Y. Wu. An augmented neural network for solving linear and quadratic programming problems. *Acta Electronica Sinica*, **23**(1), 67–72, 1995.

Abstract. In this paper, an augmented neural network for solving linear and quadratic programming problems is given. It is shown that the new network system is globally convergent to a solution to the problem. The network is capable of achieving the exact solution and solving both the primal problem and dual problem simultaneously.

Z. Xia. The ABS class and quadratic programming II: General contraints. Technical Report 91-5, Department of Mathematics, Statistics, Computer Science and Applications, University of Bergamo, Italy, 1991a.

Abstract. This paper, following part I, considers the use of ABS methods in quadratic programming problems with equality and inequality constraints.

Z. Xia. Quadratic programming via the ABS algorithm I: Equality contraints. Technical Report 91-1, Department of Mathematics, Statistics, Computer Science and Applications, University of Bergamo, Italy, 1991b.

Abstract. Some methods for solving quadratic programming problems with equality constraints are reformulated in terms of the ABS class of algorithms. This paper provides the background for further study in applying the ABS algorithms to solve quadratic programming and nonlinearly constrained programming.

N. H. Xiu. A class of improved active set methods for nonconvex quadratic programming problem. *Mathematica Numerica Sinica*, **16**(4), 406–417, 1994.

Abstract. In this paper, a new algorithm for the solution of nonconvex quadratic programming problems is proposed. Compared with the method of Best (1984), the main improvement of this method is the introduction of ”projected correct direction” in search direction, which may drop or add many active constraints at each iteration. The convergence of the method is proven under the nonconvexity condition. Its special cases are also developed, and their theoretical analysis is given. The numerical examples show its effectiveness.

W. L. Xu and A. G. Journel. Histogram and scattergram smoothing using convex quadratic programming. *Mathematical Geology*, **27**(1), 83–103, 1995.

Abstract. An algorithm, based on convex quadratic programming, is proposed to smooth sample histograms. The resulting smoothed histogram remains close to the original histogram and honors target mean and variance. The algorithm is extended to smooth sample scattergrams. The resulting smoothed scattergram remains close to the original scattergram shape and the two previously smoothed marginal distributions and honors the target correlation coefficient.

B. L. Xue and D. W. Li. Comments on “new conditions for global stability of neural networks with application to linear and quadratic programming problems”. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, **44**(11), 1099–1101, 1997.

Abstract. For original paper, see M. Forti and A. Tesi, ibid., 42, pp. 354–66 (1995). This letter makes the following comments: 1) the assumption of all neuron activation functions to vanish at the origin, which is utilized in the proof of the result implying the existence and uniqueness of the network equilibrium point, can be actually omitted; 2) in the infinite sector case, the result of global asymptotic stability (GAS) remains true with respect to the class of increasing (not necessarily strictly) activations, as in the finite sector case. Consequently, a result about absolute stability (ABST) of neural networks, which can represent a generalization of the existing related ones, is also obtained.

H. Yabe, S. Takahashi, and N. Yamaki. Application of resolution-type quasi-Newton update formula for serial quadratic programming. *15th Numerical Analysis Symposium Nihon Univ, Tokyo, Japan*, pp. 60–63, 1986.

Abstract. Among various numerical solutions for constrained minimization problems, serial quadratic programming has been recently attracting interest. Though this is an effective method, it is necessary to maintain the partial quadratic programming problem as a convex quadratic programming problem in a narrow sense, in order to secure global convergence. The authors propose an algorithm such that the partial problem always becomes a convex quadratic programming problem in a narrow sense by a sort of relaxation of secant conditions. In the algorithm forming process, the resolution-type quasi-Newton update formula is discussed. This resolution-type formula can be adapted well to the Goldfarb-Idnani method to solve the partial quadratic programming problem.

A. B. Yadykin. A parametric method for quadratic programming problems with semi-definite quadratic forms. *Zhurnal Vychislitel’noi Matematiki i Matematicheskoi Fiziki*, **15**(6), 1436–1446, 1975.

Abstract. A finite method of solution of such problems is proposed which is based on parametrisation of original problems. The existence of a continuous segmentally-linear trajectory towards the minimum is established. The construction of this trajectory is given.

A. B. Yadykin. Method of factoring in solution of problems of parametric quadratic programming. *Automation and Remote Control*, **38**(11), 1687–1695, 1977.

Abstract. A technique for transforming the Cholesky factors (LDL^T expansion) of a positive semi-definite matrix is considered. The technique is used for solving a class of parametric quadratic programming problems of high dimensionality.

Y. Yajima and T. Fujie. A polyhedral approach for nonconvex quadratic programming problems with box constraints. *Journal of Global Optimization*, **13**(2), 151–170, 1998. See also, Research Reports on Mathematical and Computing Sciences, Series B (Operations Research), number 323, pages 1–23, 1996.

Abstract. We apply a linearization technique for nonconvex quadratic problems with box constraints. We show that cutting plane algorithms can be designed to solve the equivalent problems which minimize a linear function over a convex region. We propose several classes of valid inequalities of the convex region which are closely related to the Boolean quadric polytope. We also describe heuristic procedures for generating cutting planes. Results of preliminary computational experiments show that our inequalities generate a polytope which is a fairly tight approximation of the convex region.

E. Yamakawa and M. Fukushima. A block-parallel conjugate gradient method for separable quadratic programming problems. *Journal of the Operations Research Society of Japan*, **39**(3), 407–427, 1996.

Abstract. For a large-scale quadratic programming problem with separable objective function, a variant of the conjugate gradient method can effectively be applied to the dual problem. In this paper, we consider a block-parallel modification of the conjugate gradient method, which is suitable for implementation on a parallel computer. More precisely, the method proceeds in a block Jacobi manner and executes the conjugate gradient iteration to solve quadratic programming subproblems associated with respective blocks. We implement the method on a connection machine model CM-5 in the single-program multiple-data model of computation. We report some numerical results, which show that the proposed method is effective particularly for problems with some block structure.

E. Yamakawa, E. Maki, and M. Fukushima. An asynchronous block-parallel algorithm for separable quadratic programming problems. *Transactions of the Institute of Systems, Control and Information Engineers*, **10**(5), 248–258, 1997.

Abstract. An asynchronous modification of the block-parallel algorithm is presented for quadratic programming problems with separable objective functions. In particular, an implementation strategy is proposed to effectively decentralize the communication overhead associated with a specific processor of a parallel computer. Through extensive numerical experiments on the Connection Machine CM-5, the efficiency of the proposed method is examined for large scale problems with some block structure.

C. W. Yang. An impact analysis of a quadratic-programming model—a case of the Appalachian coal market with the pollution tax. *In* 'Use of Computers in the Coal Industry', Vol. 1983, pp. 665–669, 1983.

E. K. Yang and J. W. Tolle. A class of methods for solving large, convex quadratic programs subject to box constraints. *Mathematical Programming*, **51**(2), 223–228, 1991.

Y. Ye. Further development on the interior algorithm for convex quadratic programming. Working paper, Deparment of Engineering Economic Systems, Stanford University, California, USA, 1987a.

Y. Ye. *Interior point algorithms for linear, quadratic and linearly constrained convex programming*. PhD thesis, Engineering and Economic Systems Department, Stanford University, California 94305, USA, 1987b.

Y. Ye. An extension of Kamarkar's algorithm and the trust region method for quadratic-programming. *In* N. Megiddo, ed., 'Progress in Mathematical Programming', pp. 49–63, Springer Verlag, Heidelberg, Berlin, New York, 1989.

Abstract. An extension of Karmakar (1984)'s algorithm and the trust region method is developed for solving quadratic programming problems. This extension is based on the affine scaling technique, followed by optimization over a trust ellipsoidal region. It creates a sequence of interior feasible points that converge to the optimal feasible solution. The initial computational results reported here suggest the potential usefulness of this algorithm in practice.

Y. Ye. Interior-point algorithms for quadratic programming. *In* S. Kumar, ed., 'Recent Developments in Mathematical Programming', Gordon and Breach Scientific Publishers, Philadelphia, 1991.

Y. Ye. On affine scaling algorithms for nonconvex quadratic programming. *Mathematical Programming*, **56**(3), 285–300, 1992.

Abstract. We investigate the use of interior algorithms, especially the affine-scaling algorithm, to solve nonconvex—indefinite or negative definite—quadratic programming (QP) problems. Although the nonconvex QP with a polytope constraint is a "hard" problem, we show that the problem with an ellipsoidal constraint is "easy". When the "hard" QP is solved by successively solving the "easy" QP, the sequence of points monotonically converge to a feasible point satisfying both the first and the second order optimality conditions.

Y. Ye. Approximating quadratic programming with bound constraints. Working paper, Department of Management Sciences, University of Iowa, Iowa City, USA, 1997a.

Abstract. We consider the problem of approximating the global maximum of a quadratic program with n variables subject to bound constraints. Based on the results of Goemans and Williamson (1996) and Nesterov (1997), we show that a $4/7$ approximate solution can be obtained in polynomial time

Y. Ye. Indefinite quadratic programming. *In* 'Interior-Point Algorithm: Theory and Analysis', chapter 9.4–9.5, pp. 310–331. J. Wiley and Sons, New York, USA, 1997b.

Y. Ye. On the complexity of approximating a kkt point of quadratic programming. Working paper, Department of Management Science, University of Iowa, Iowa City, USA, 1997c.

Abstract. We present a potential reduction algorithm to approximate a Karush-Kuhn-Tucker (KKT) point of general quadratic programming. We show that the algorithm is a fully polynomial-time approximation scheme, and its running-time dependency on accuracy $\varepsilon \in (0,1)$ is $O((1/\varepsilon)\log(1/\varepsilon)\log(\log(1/\varepsilon)))$, compared to the previously best-known result $O((1/\varepsilon)^2)$. Furthermore, the limit of the KKT point satisfies the second-order necessary optimality condition of being a local minimizer.

Y. Ye and E. Tse. An extension of Karmarkar's projective algorithm for convex quadratic programming. *Mathematical Programming*, **44**(2), 157–179, 1989.

T. L. Yen, C. K. Chi, and C. S. Wu. A quadratic programming method for interconnection crosstalk minimization. *In* 'ISCAS'99. Proceedings of the 1999 IEEE International Symposium on Circuits and Systems VLSI, IEEE, Piscataway', Vol. 6, pp. 270–273, 1999.

Abstract. Adjacent wires in a VLSI chip become closer as the fabrication technology rapidly evolves. Accordingly it becomes important to consider crosstalk caused by the coupling capacitance between adjacent wires in the layout design for the fast and safe VLSI circuits. The crosstalk is a function of coupling length and distance. The coupling length can be reduced by segment rearrangement technique. This paper presents a crosstalk minimization technique by adjusting the space between adjacent wires. An example is shown in this paper to demonstrate the effectiveness of the technique.

C. M. Ying and B. Joseph. Performance and stability analysis of LP-MPC and QP-MPC cascade control systems. *AICHE Journal*, **45**(7), 1521–1534, 1999.

Abstract. Model predictive control (MPC) is used extensively in industry to optimally control constrained, multivariable processes. For nonsquare systems (with more inputs than outputs), extra degrees of freedom can be used to dynamically drive the process to its economic optimum operating conditions. This is accomplished by cascading a local linear programming (LP) or quadratic programming (QP) controller using steady-state models. Such a cascade control scheme (LP-MPC or QP-MPC) continuously computes and updates the set points used by the lower-level MPC algorithm. While this methodology has been in use by industry for many years, its properties have not been addressed in the literature. The properties of such cascaded MPC systems are analyzed from the point of view of implementation strategies, stability properties, and economic and dynamic performance. Some theoretical results on stability are derived along with a case study involving the Shell control problem.

C. M. Ying, S. Voorakaranam, and B. Joseph. Analysis and performance of the LP-MPC and QP-MPC cascade control system. *In* 'Proceedings of the 1998 American Control Conference. American Autom. Control Council, Evanston, IL, USA', Vol. 2, pp. 806–810, 1998.

Abstract. The objective of this paper is to address some of the theoretical and practical issues related to the implementation of two stage model predictive control (MPC) schemes such as linear program-MPC and quadratic program-MPC. Single stage QDMC has the drawbacks of steady state offsets, inconsistent set-points and does not take into account economic objectives. The addition of another stage on the top of QDMC is used to address these problems. Results of the application of two stage MPC to the Shell Challenge problem are presented. A comparison is made with the single stage conventional QDMC and some interesting observations are reported. Stability and dynamic performance issues are also considered in this paper.

D. B. Yudin. Stochastic quadratic programming. *Engineering-Cybernetics*, **3**, 1–7, 1969.

Abstract. Two models of control under conditions of incomplete information are presented. Economic methods for the solution of the corresponding stochastic quadratic programming problems are given.

S. Zahl. A deformation method for quadratic programming. *Journal of the Royal Statistical Society*, **26B**, 141–160, 1964.

S. Zahl. Supplement to 'a deformation method for quadratic programming'. *Journal of the Royal Satistical Society*, **26B**, 166–168, 1965.

A. Zelner. Linear regression with inequality constraints on the coefficients: an application of quadratic programming and linear decision rules. Report 6109, Economic Institute of the Netherlands School of Economics, Rotterdam, Netherlands, 1961.

H. Zhang, W. X. Zhong, and Y. Gu. A combined parametric quadratic programming and iteration method for 3-D elastic-plastic frictional contact problem analysis. *Computer Methods in Applied Mechanics and Engineering*, **155**(3–4), 307–324, 1998.

Abstract. The paper is concerned with the formulation and numerical realization of elastic-plastic frictional contact problems. Compared with two-dimensional elastic-plastic contact problems, three-dimensional elastic-plastic contact problems with friction are more difficult to deal with, because of the unknown slip direction of the tangential force and exhausting computing time when the double nonlinear problems are considered. In order to avoid these difficulties, a combined PQP (parametric quadratic programming) and iteration method is constructed. The stiffness matrix of a 3D contact element is introduced by means of a penalty function expression of the contact pressure and frictional force. The contact condition and the flow rule are expressed by the same form as in a non-associated plastic flow problem, and the penalty factors can be eliminated by using a definite numerical technique. The iteration algorithm, whose functions are to give a precise discretization of space Coulomb's friction law, is used along with the PQP method and alleviates the difficulty of unknown slip direction and cuts down computing costs. Also, the additional complementary conditions are discussed, so that the non-physical ray solutions caused by the original mathematical model can be eliminated. Numerical examples are given to demonstrate the validity of the present method.

X. S. Zhang and H. C. Zhu. A neural network model for quadratic programming with simple upper and lower bounds and its application to linear programming. *In* D. Z. Du and X. S. Zhang, eds, 'Algorithms and Computation. 5th International Symposium, ISAAC '94 Proceedings. Springer-Verlag, Berlin, Germany', pp. 119–127, 1994.

Abstract. We put forward a neural network model for quadratic programming problems with simple upper and lower bounds and analyze the properties of solutions obtained by the model. It is shown that linear programming problems can be transferred into such quadratic programming problems and can be solved by the model.

W. X. Zhong and S. M. Sun. A finite element method for elasto-plastic structures and contact problems by parametric quadratic programming. *International Journal for Numerical Methods in Engineering*, **26**(12), 2723–2738, 1988.

Abstract. The stiffness matrix of a contact element is introduced by means of a penalty function expression of the contact pressure and frictional force. The contact condition and the flow rule are expressed by the same form as in a non-associated plastic flow problem. A unified PQP (parametric quadratic programming) model related to contact problems as well as to elasto-plastic structures is constructed. A series of PQP formulae for contact problems and elastic-plastic structures is derived in the text, and some numerical examples are illustrated as well.

W. X. Zhong and S. M. Sun. A parametric quadratic programming approach to elastic contact problems with friction. *Computers and Structures*, **32**(1), 37–43, 1989. See also, Computer Modelling in Ocean Engineering. 1:651-658, 1988.

Abstract. A numerical procedure is developed for the solution of contact problems with friction. The stiffness matrix of the contact element is introduced by means of the penalty function expressions of the contact pressure and frictional force. Furthermore, the parametric quadratic programming (PQP) method based on the finite element method for frictional contact problems is obtained, in analogy with that for elasto-plastic problems. Applying some special techniques, the penalty parameters can be eliminated. The accuracy and efficiency of the method has been verified in several applications.

W. X. Zhong and R. L. Zhang. Quadratic programming with parametric vector in plasticity and geomechanics. *In* 'NUMETA 87: Transient/Dynamic Analysis and Constitutive Laws for Engineering Materials', Vol. 2, pp. 595–602, 1987.

W. X. Zhong and R. L. Zhang. Parametric variational principles and their quadratic programming solutions in plasticity. *Computers and Structures*, **30**(4), 887–896, 1988.

Abstract. Two parametric variational principles, the parametric minimum potential energy principle and the parametric minimum complementary energy principle, are presented. These principles can be used to solve incremental problems in plasticity and geomechanics in a direct way and problems where the materials are inconsistent with the Drucker postulate of stability, such as in nonassociated flow or softening problems. Examples of applications include problems solved by both analytical and parametric quadratic programming approaches.

C. Y. Zhu. On the primal-dual steepest descent algorithm for extended linear-quadratic programming. *SIAM Journal on Optimization*, **5**(1), 114–128, 1995.

Abstract. The aim of this paper is two-fold. First, new variants are proposed for the primal-dual steepest descent algorithm as due in the family of primal-dual projected gradient algorithms developed by Zhu and Rockafellar [SIAM J. Optim., 3 (1993), pp. 751–783] for large-scale extended linear-quadratic programming. The variants include a second update scheme for the iterates, where the primal-dual feedback is arranged in a new pattern, as well as alternatives for the "perfect line search" in the original version of the reference. Second, new linear convergence results are proved for all these variants of the algorithm, including the original version as a special case, without the additional assumptions used by Zhu and Rockafellar. For the variants with the second update scheme, a much sharper estimation for the rate of convergence is obtained due to the new primal-dual feedback pattern.

K. Zimmermann and W. Achtziger. On time optimal piecewise linear schedules for LSGP- and LGPS-partitionings of array processors via quadratic programming. Technical report 200, Institute of Applied Mathematics, Univeristy of Erlangen-Nuremberg, Germany, 1996a.

K. Zimmermann and W. Achtziger. Synthesizing regular arrays from single affine recurrences via quadratic and branching parametric linear programming. *In* 'Parcella '96, Akademie Verlag, Leipzig', pp. 224–231, 1996b.

K. Zimmermann and W. Achtziger. On time optimal implementation of uniform recurrences onto array processors via quadratic programming. *Journal of VLSI Signal Processing Systems for Signal, Image, and Video Technology*, **19**(1), 19–38, 1998.

Abstract. Many important algorithms can be described by n-dimensional uniform recurrences. The computations are then indexed by integral vectors of length n and the data dependencies between computations can be described by the difference vector of the corresponding indexes which are independent of the indexes. This paper addresses the following optimization problem. Given an n-dimensional uniform recurrence whose computation indexes are mapped by a linear function onto the processors of an array processor embedded in k-space ($1 \leq k \leq n$). Find an optimal linear function for the computation indexes. We study a continuous approximation of this problem by passing from linear to quasi-linear timing functions. The resultant problem formulation is then a quadratic programming problem which can be solved by standard algorithms for quadratic or general nonlinear optimization problems. We demonstrate the effectiveness of our approach by several nontrivial test examples.

G. Zioutas, L. Camarinopoulos, and E. Bora Senta. Robust autoregressive estimates using quadratic programming. *European Journal of Operational Research*, **101**(3), 486–498, 1997.

Abstract. The robust estimation of autoregressive parameters is formulated in terms of a quadratic programming problem. This article's main contribution is to present an estimator that down weights both types of outliers in time series and improves the forecasting results. New robust estimates are yielded by combining optimally two weight functions suitable for innovation and additive outliers in time series. The technique which is developed here is based on an approach of mathematical programming application to ℓ_p-approximation. The behavior of the estimators are illustrated numerically, under the additive outlier generating model. Monte Carlo results show that the proposed estimators compared favorably with respect to M-estimators and bounded influence estimators. Based on these results we conclude that one can improve the robust properties of AR(p) estimators using quadratic programming.