On a new collection of stochastic linear programming test problems

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Abstract

The purpose of this paper is to introduce a new test problem collection for stochastic linear programming that the authors have recently begun to assemble. While there are existing stochastic programming test problem collections, our new collection has three features that distinguish it from existing collections. First, our collection is web-based with free public access, and we intend to enrich it as new test problems become available. Indeed, we encourage submissions of new test problems. Second, for each test problem class we provide a short description, a mathematical problem statement and a notational reconciliation to a standard format. Third, for each test problem instance in our collection we provide numerical data in the SMPS [4] format. In a companion effort we have developed a data structure for implementing algorithms for stochastic linear programs, and C-routines that convert data in SMPS format to that data structure. These routines will also be freely available from the authors soon.

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1 Introduction

Stochastic programming has grown in importance in recent decades, because it allows the modeler to accurately represent planning under uncertainty. With strong interest in solving such problems and in finding more efficient solution techniques, there has arisen a need for a test set of stochastic programming problems.

One of the most popular forms of stochastic programming problems is the multistage stochastic linear program with recourse (MSSLP). See Section 2, for a precise statement of the MSSLP problem. While MSSLPs are growing in popularity, many of the applications are proprietary, and therefore the models are not publicly available. Test set collections of MSSLPs exist [6, 7]. However, they need to be enriched with newer applications. In fact, in many cases, the application associated with the existing test case is not known. Also, it would be helpful if the original applications are described in the notation of the original model, and related to a unified notation such as in Section 2. In addition, it would be desirable if the data for the test problems is available in SMPS [4], the (emerging) standard for specifying input to software for MSSLPs.

To address the above needs, we have collected a group of eleven problem classes from a variety of settings. They are all MSSLPs, but of various structures and sizes, with randomness occuring in different parts in different problems. In some cases, problem instances were explicitly stated in the literature. In other cases, we created the problems based solely on the problem description in the literature, and in some cases, there is not yet any sample problem.

For each model application, we present a problem description, a concise problem statement, and, if available, a numerical example given by the model authors. We have attempted to stay as close to the authors' notation as possible in these subsections. Additionally, where feasible, we present a notational reconciliation, which shows how to transform the notation of the problem into that in Section 2.

Each problem class may be used to generate one or more instances of MSSLPs. We have created 15 such instances. The data for these 15 test problems, as well as six other test problems that we did not create, are available in SMPS format [4] from the authors at `http://www.uwsp.edu/math/afelt/slptestset.html`.

Also available at the web site is a chapter of problem descriptions. Each section covers a single problem class. At the beginning of each section, we give a citation to the original application, a brief description of the problem structure, and if applicable, the names of the SMPS files for the associated problem instances. We provide one such section as an example in Section 3.

It is the intention of the authors to update the classes of applications and the test problem instances as new application areas become prominent, and to make the information that we present for each application area, as well as SMPS inputs for each test problem instance freely available to the stochastic programming research community.

In that spirit, we encourage colleagues to submit new problem data with an accompanying description. Such submissions should include the following:

1. description of the application and problem notation,
2. problem statement, in the same notation,
3. numerical example, if practical,
4. reconciliation to the notation of Section 2,
5. data files in SMPS format for each instance, and
6. optimal solutions for each instance and example.

We end this section by mentioning, that the initial motivation for the present work came from the need to carefully test the new polynomial interior cutting plane algorithms for stochastic programs developed in [1]. The design of our test problem collection was motivated and influenced by the elegant work of Moré et al. [9, 2] on test problem collections for several classes of (deterministic) nonlinear optimization problems, and the impact such collections have had on the development of software for such problem classes [10, 8, 3]. As part of testing the algorithms developed in [1], we have also developed C-routines that convert SMPS data into a data structure suitable for implementing algorithms for stochastic programs. These routines will soon be made available as open source from the authors.

2 Notation for multistage stochastic linear programs

In this section we state a generic form of the multistage stochastic linear program (MSSLP). Our notation here is motivated by the implementation of algorithms for the MSSLP, especially those based on cutting plane notions.

We begin by describing the underlying probability structure. We have N sequential discrete time stages with stage 1 representing the present. Time stages $2, 3, \ldots, N$ occur in the future sequentially in that order, at which realizations of random variables[1] $\boldsymbol{\xi_2}, \boldsymbol{\xi_3} \ldots, \boldsymbol{\xi_N}$ become available respectively.

The random variable $\boldsymbol{\xi_2}$ has a known discrete distribution with a finite number of realizations. At stage 2, a realization of $\boldsymbol{\xi_2}$ becomes available, and the system moves forward to stage 3 at which a realization of $\boldsymbol{\xi_3}$ becomes available. The conditional distribution of $\boldsymbol{\xi_3}$ given that ξ_2 has been observed is discrete with a finite number of realizations, and is known. Note that the pair ξ_2, ξ_3 may be termed a *partial scenario* since they represent only the realizations from the stages 2 and 3 of the N-stage process. We shall write $\sigma_3 := (\xi_2, \xi_3)$, to indicate a partial scenario with realizations up to stage 3 consisting of realizations ξ_2 and ξ_3 at stages 2 and 3 respectively. Note that we may write $\sigma_2 := \xi_2$.

Now suppose that we are at stage $t-1$ $(3 \leq t \leq N)$ and that realizations $\xi_2, \xi_3, \ldots, \xi_{t-1}$ have become available in stages 2 through $t-1$ respectively. Let $\sigma_{t-1} := (\xi_2, \xi_3, \ldots, \xi_{t-1})$. The system now moves forward to stage t at which a realization of $\boldsymbol{\xi_t}$ becomes available. The conditional distribution of $\boldsymbol{\xi_t}$ given that the partial scenario σ_{t-1} has been observed is discrete with a finite number of realizations, and is known.

Before proceeding further we pause for some comments. Let ξ_N be the realization of $\boldsymbol{\xi_N}$ observed in stage N and let $\sigma_N := (\xi_2, \xi_3, \ldots, \xi_N)$. We call σ_N a *scenario*. We let $\mathcal{S}_t$ be the set of partial scenarios with realizations up to stage t for $t = 2, 3, \ldots, N$.

An MSSLP is a mathematical formulation of the decision process we now describe in association with the above probability structure. In the linear case that leads to MSSLPs, x_1 is a decision that has to be chosen in stage 1 from the set $\{x_1 \in \mathbb{R}^{n_1} : A_1 x_1 = b_1, x_1 \geq 0\}$ at a direct cost $c_1^\top x_1$, where $c_1 \in \mathbb{R}^{n_1}$, $b_1 \in \mathbb{R}^{m_1}$, and $A_1 \in \mathbb{R}^{m_1 \times n_1}$ constitute the first-

[1] In this paper, random variables will be represented in boldface. The expected value with respect to $\boldsymbol{x}$ will be written $\underset{\{\mathbf{x}\}}{E}[\cdot]$, and the conditional expected value (conditioned on y) will be written $\underset{\{\mathbf{x}|y\}}{E}[\cdot]$.

stage deterministic data. In addition, depending on the decision x_1 taken at present and the realizations of $\boldsymbol{\xi_2}, \boldsymbol{\xi_3}, \ldots, \boldsymbol{\xi_N}$ that would become available in the future, there would be an indirect cost due to *recourse* actions that may become necessary. In an MSSLP the objective function at stage 1 is to minimize the sum of the direct cost $c_1^\mathsf{T} x_1$ and the expectation $\mathcal{Q}_2(x_1)$ of this indirect cost. The computation of $\mathcal{Q}_2(x_1)$ requires recursion and is as specified below.

In an MSSLP, $\boldsymbol{\xi}_2 := (\boldsymbol{c_2}, \boldsymbol{b_2}, \boldsymbol{A_2}, \boldsymbol{T_2})$ and the distribution of $\boldsymbol{\xi_2}$ is such that $\boldsymbol{c_2}$, $\boldsymbol{b_2}$, $\boldsymbol{A_2}$ and $\boldsymbol{T_2}$ have realizations in $\mathbb{R}^{n_2}$, $\mathbb{R}^{m_2}$, $\mathbb{R}^{m_2 \times n_2}$ and $\mathbb{R}^{m_2 \times n_1}$ respectively. If the realizatation observed is $\xi_2 := (c_2, b_2, A_2, T_2)$ then the recourse decision x_2 is chosen from the set $\{x_2 \in \mathbb{R}^{n_2} : A_2 x_2 = b_2 - T_2 x_1, x_2 \geq 0\}$. The direct cost of this recourse action is $c_2^\mathsf{T} x_2$. The conditional expected cost (conditioned on ξ_2) of the recourse action is the sum of $c_2^\mathsf{T} x_2$ and the expectation $\mathcal{Q}_{3,\xi_2}(x_2)$ of the indirect cost of future recourse actions, and $\mathcal{Q}_2(x_1)$ is the expectation of this sum over $\boldsymbol{\xi_2}$.

Now suppose that we are at stage $t-1$ $(3 \leq t \leq N-1)$ and that the partial scenario that has been observed is $\sigma_{t-1} := (\xi_2, \xi_3, \ldots, \xi_{t-1})$. In an MSSLP, $\boldsymbol{\xi}_t := (\boldsymbol{c_t}, \boldsymbol{b_t}, \boldsymbol{A_t}, \boldsymbol{T_t})$ and the distribution of $\boldsymbol{\xi_t}$ given σ_{t-1} is such that $\boldsymbol{c_t}$, $\boldsymbol{b_t}$, $\boldsymbol{A_t}$ and $\boldsymbol{T_t}$ have realizations in $\mathbb{R}^{n_t}$, $\mathbb{R}^{m_t}$, $\mathbb{R}^{m_t \times n_t}$ and $\mathbb{R}^{m_t \times n_{t-1}}$ respectively. If the realization to be observed at stage t is $\xi_t = (c_t, b_t, A_t, T_t)$ then the recourse decision x_t is chosen from the set $\{x_t \in \mathbb{R}^{n_t} : A_t x_t = b_t - T_t x_{t-1}, x_t \geq 0\}$. The direct cost of this recourse action is $c_t^\mathsf{T} x_t$. The conditional expected cost (conditioned on σ_{t-1}) of the recourse action is the sum of $c_t^\mathsf{T} x_t$ and the expectation $\mathcal{Q}_{t+1,\sigma_t}(x_t)$ of the indirect cost of future recourse actions, where $\sigma_t := (\sigma_{t-1}, \xi_t)$. The value $\mathcal{Q}_{t,\sigma_{t-1}}(x_{t-1})$ is the expectation of this sum over partial scenarios in $\mathcal{S}_{t-1}$.

Now suppose that we are at stage $N-1$, and that we have observed the partial scenario $\sigma_{N-1} := (\xi_2, \xi_3, \ldots, \xi_{N-1})$. In an MSSLP, $\boldsymbol{\xi_N} := (\boldsymbol{c_N}, \boldsymbol{b_N}, \boldsymbol{A_N}, \boldsymbol{T_N})$ and the distribution of $\boldsymbol{\xi_N}$ given σ_{N-1} is sucth that $\boldsymbol{c_N}, \boldsymbol{b_N}, \boldsymbol{A_N}$, and $\boldsymbol{T_N}$ have realizations in $\mathbb{R}^{n_N}$, $\mathbb{R}^{m_N}$, $\mathbb{R}^{m_N \times n_N}$ and $\mathbb{R}^{m_N \times n_{N-1}}$, respectively. The function $\mathcal{Q}_{N,\sigma_{N-1}}$ is specified by the statements in previous paragraphs with t replaced by N, and by setting $\mathcal{Q}_{N+1,\sigma_N} \equiv 0$, since the process has only N stages. This recursion can be used to specify functions $\mathcal{Q}_{t,\sigma_{t-1}}$ for $t = 2, 3, \ldots, N$.

The preceding description leads to the following statement of the multistage stochastic linear program with recourse.

$$\begin{array}{llll} \text{Minimize} & Z(x_1) := & c_1^\top x_1 + \mathcal{Q}_2(x_1) & \\ \text{subject to} & & A_1 x_1 & = b_1 \\ & & x_1 & \geq 0, \end{array}$$

where

$$\begin{aligned}
\mathcal{Q}_2(x_1) &:= \underset{\{\boldsymbol{\xi_2}\}}{E} \left[Q_2(x_2, \boldsymbol{\xi_2}) \right], \\
Q_2(x_1, \xi_2) &:= \inf_{x_2 \in \mathbb{R}^{n_2}} \left\{ c_2^\top x_2 + \mathcal{Q}_{3,\sigma_2}(x_2) : A_2 x_2 = b_2 - T_2 x_1, x_2 \geq 0 \right\} \\
&\quad \text{(with } \sigma_2 := \xi_2\text{)}, \\
\mathcal{Q}_{t,\sigma_{t-1}}(x_{t-1}) &:= \underset{\{\boldsymbol{\xi_t} | \sigma_{t-1}\}}{E} \left[Q_t(x_{t-1}, \boldsymbol{\xi_t}) \right] \text{ for } t = 3, 4, \ldots, N, \\
Q_t(x_{t-1}, \xi_t) &:= \inf_{x_t \in \mathbb{R}^{n_t}} \left\{ c_t^\top x_t + \mathcal{Q}_{t+1,\sigma_t}(x_t) : A_t x_t = b_t - T_t x_{t-1}, x_t \geq 0 \right\} \\
&\quad \text{(with } \sigma_t := (\sigma_{t-1}, \xi_t)\text{) for } t = 3, 4, \ldots, N-1,
\end{aligned}$$

and

$$Q_N(x_{n-1}, \xi_N) := \inf_{x_N \in \mathbb{R}^{n_N}} \left\{ c_N^\top x_N : A_N x_N = b_N - T_N x_{N-1}, x_N \geq 0 \right\}. \tag{1}$$

Note that the data for the MSSLP above consist of:

- first stage deterministic data c_1, b_1, A_1,
- the distribution of $(\boldsymbol{c_2}, \boldsymbol{b_2}, \boldsymbol{A_2}, \boldsymbol{T_2})$, and
- for all $\sigma_{t-1} \in \mathcal{S}_{t-1}$ the conditional distribution of $(\boldsymbol{c_t}, \boldsymbol{b_t}, \boldsymbol{A_t}, \boldsymbol{T_t})$, conditioned on σ_{t-1}, for $t = 3, 4, \ldots, N$.

We refer to the case where the distribution of $\boldsymbol{\xi_t}$ is independent of σ_{t-1} for $t = 3, 4, \ldots, N$ as the *independent* case. In the independent case (1) simplifies to the following form, which we state for convenient reference.

$$\begin{array}{lllll} \text{Minimize} & Z(x_1) := & c_1^\mathsf{T} x_1 & + \mathcal{Q}_2(x_1) & \\ \text{subject to} & & A_1 x_1 & & = b_1 \\ & & x_1 & & \geq 0, \end{array}$$

where

$$\mathcal{Q}_t(x_{t-1}) := \underset{\{\boldsymbol{c_t}, \boldsymbol{b_t}, \boldsymbol{A_t}, \boldsymbol{T_t}\}}{E} [Q_t(x_{t-1}, \boldsymbol{c_t}, \boldsymbol{b_t}, \boldsymbol{A_t}, \boldsymbol{T_t})] \text{ for } t = 2, 3, \ldots, N,$$

$$Q_t(x_{t-1}, c_t, b_t, A_t, T_t) := \inf_{x_t \in \mathbb{R}^{n_t}} \left\{ c_t^\mathsf{T} x_t + \mathcal{Q}_{t+1}(x_t) : A_t x_t = b_t - T_t x_{t-1}, x_t \geq 0 \right\} \quad t = 2, 3, \ldots, N-1,$$

and

$$Q_N(x_{N-1}, c_N, b_N, A_N, T_N) := \inf_{x_N \in \mathbb{R}^{n_N}} \left\{ c_N^\mathsf{T} x_N : A_N x_N = b_N - T_N x_{N-1}, x_N \geq 0 \right\}. \tag{2}$$

Note that in this independent case the data for the MSSLP consist of:

- first stage deterministic data c_1, b_1, A_1, and
- the distribution of $(\boldsymbol{c_t}, \boldsymbol{b_t}, \boldsymbol{A_t}, \boldsymbol{T_t})$ for $t = 2, 3, \ldots, N$.

The most common data input standard for problems of type (1) and (2) is the SMPS [4] standard. This standard requires that one realization or scenario of the *entire* problem (i.e. all stages) be specified first. The other realizations in the scenario tree may then be described in several formats, including description of independent realizations for scalars or vectors, and description of branching scenarios.

Such flexibility of input format has advantages and disadvantages. While problems of type (1) or (2) may be easily described, it is dificult to write computer routines for reading such flexible input. In a companion effort, the authors will realease open source routines for reading SMPS data and placing the data into appropriate internal computer data structures.

3 Energy and environmental planning

Due to Fragnière [5]
(Multistage, linear stochastic problem)

$$\texttt{/environ/env.cor},\ \texttt{/env.tim},\ \begin{cases} \texttt{/env\_det.aggr} \\ \texttt{/env.sto.imp} \\ \texttt{/env.sto.loose} \\ \texttt{/env.sto.lrge} \\ \texttt{/env.sto.xlrge} \end{cases}$$

3.1 Description

The model by Fragnière [5] assists the Canton of Geneva in planning its energy supply infrastructure and policies. The model is based on the MARKAL (market allocation) model. This is quite an extensive model, containing a great degree of realism. Included is the possibility that emissions of greenhouse gases will be required to decrease. This possibility is expressed in a discrete random distribution.

The model includes equilibrium constraints, capacity expansion constraints, demand constraints, production constraints, and environmental constraints. Energy is supplied by many different technologies, including hydro power, cogeneration, fossil fuels, urban waste incineration, and imported electricity. Demands are also classified by technology. Examples are electricity for industrial use, gas furnaces in existing houses, and wood stoves in new houses. Variables expressed in upper case letters are decision variables.

An energy balance may be performed on the supply grid, for each energy type. For types k which are neither electricity nor low temperature heat, the balance yields

$$\begin{aligned} &\sum_{\substack{i\in TCH\\ i\notin DMD}} out_{ki}(t)P_i(t) + \sum_{i\in DMD} out_{ki}(t)cf_i(t)C_i(t) + \sum_s IMP_{ks}(t) \\ &\geq \sum_{\substack{i\in TCH\\ i\notin DMD}} inp_{ki}(t)P_i(t) + \sum_{i\in DMD} inp_{ki}(t)cf_i(t)C_i(t) + \sum_s EXP_{ks}(t), \\ &\qquad\qquad\qquad\qquad\qquad\qquad \forall k\in ENC, \forall t\in T, \quad (3) \end{aligned}$$

where the variables and sets are defined in Table 1 and Table 2, respectively. Note that for $i \in DMD$, the term $C_i(t)$ refers to the installed *delivery* capacity, whereas for production type technologies, it refers to the installed *production* capacity.

For electricity and low temperature (district) heat, the energy balances are

$$\eta\left[\sum_{i\in ELA} P_{izy}(t)+\sum_{s} IMPELC_{szy}(t)\right]\geq \sum_{i\in PRC} inp_{ELC,i}(t)q_{zy}P_i(t)$$
$$+\sum_{i\in DMD} inp_{ELC,i}(t)cf_i(t)fr_{j(i)zy}C_i(t)+\sum_{k} EXPELC_{kzy}(t)$$
$$+\eta\sum_{\substack{i\in STG\\ \ni y=n}} e_iP_{izd}(t),\quad \forall z\in Z,\forall y\in Y,\forall t\in T,$$

and

$$\gamma\sum_{i\in HPL} P_{iz}(t)\geq \sum_{i\in DMD} inp_{LTH,i}(t)cf_i(t)C_i(t)\sum_{y\in Y} fr_{j(i)zy},$$
$$\forall z\in Z,\forall t\in T,$$

respectively.

Table 1: Variable definitions

$P_i(t)$	the activity, or utilization, of technology i, in period t
$P_{izy}(t)$	the production of electricity from technology i, in period t, season z, and part of the day y
$P_{iz}(t)$	the production of low temperature heat from technology i, in period t, season z
$C_i(t)$	the total installed capacity of technology i in period t
$M_{iz}(t)$	production lost due to regular maintenance of technology i in season z, period t
$IMP_{ks}(t)$	imported energy of type k in period t, from source s
$IMPELC_{szy}(t)$	imported electricity, from source s, in period t, season z, and part of the day y
$EXP_{ks}(t)$	exported energy of type k in period t, to destination s
$EXPELC_{szy}(t)$	exported electricity, to destination s, in period t, season z, and part of the day y
$out_{ki}(t)$	output of energy type k in period t, per unit activity from technology $i\notin DMD$, or per unit capacity from $i\in DMD$
	(continued on the next page)

Variable definitions (continued)	
$out_{ik}(t)$	fraction of demand technology i which supplies utility demand $k \in DM$ in period t
$inp_{ki}(t)$	input of energy type k in period t, per unit activity from technology $i \notin DMD$, or per unit capacity from $i \in DMD$
$cf_i(t)$	mean utilization factor of the total installed capacity for technology $i \in DMD$ in period t
q_{zy}	the fraction of a year covered by season z, part of the day y
$j(i)$	utility demand category $j(i) \in DM$, for $i \in DMD$
$fr_{j(i)zy}$	fraction of the utility demand from category $j(i)$ which comes in season z, time of day y
e_i	the electricity input required at night to produce one unit of electricity in the daytime from technology $i \in STG$
η	efficiency coefficient for electrical distribution
ρ	efficiency coefficient for low temperature heat distribution
γ	efficiency coefficient for district (low temperature) heat distribution
l_i	duration of equipment i, in time stages
$I_i(t)$	new capacity purchased for technology i, starting in period t
$resid_i(t)$	capacity which existed at the beginning of the optimization problem
$demand_k(t)$	demand for utility $k \in DM$ in period t
$af_i(t)$	availability factor of technology i in period t
fo_i	the fraction of a year that technology i is lost for production, due to one unit of unavailability
u_i	conversion factor from units of capacity to units of production
er	reserve capacity necessary to cover daily peak demand for electricity
hr	reserve capacity necessary to cover daily peak demand for low temperature heat
$pk_i(t)$	fraction of installed capacity for production technology i, available to satisfy peak demand in period t
$epk_i(t)$	fraction of electrical consumption for production technology i, which corresponds to peak consumption in period t
	(continued on the next page)

Variable definitions (continued)	
$elf_{j(i)}(t)$	fraction of capacity for demand technology i, which corresponds to the peak consumption in period t
bl	maximum fraction of nighttime electrical production from technologies $i \in BAS$
α	annual discount rate
n	number of years per period
$invcost_i(t)$	cost per unit investment in technology i, period t
$fixom_i(t)$	fixed annual operation and maintenance costs for technology i, period t, per unit capacity
$varom_i(t)$	variable annual operation and maintenance costs, per unit production, for non-demand technology i, period t
$cost_{ks}(t)$	unit cost of energy type k, purchased from source s in period t
$cost_{ELC,s}(t)$	unit cost of electricity, purchased from source s in period t
$price_{ks}(t)$	unit price of energy type k, sold to source s in period t
$price_{ks}(t)$	unit price of electricity, sold to source s in period t
$co2_i(t)$	carbon dioxide emissions per unit capacity, from technology i, period t
$limit_{CO2}(t)$	limit imposed on carbon dioxide emissions in period t

The capacity of each technology was either installed after the beginning of the optimization problem, or it was there from the beginning. From this, we get the constraint

$$C_i(t) = \sum_{m=\mathrm{Max}\{1,t-l_i+1\}}^{t} I_i(m) + resid_i(t), \quad \forall t \in T, \forall i.$$

We must meet the demand for each utility in each round. Thus,

$$\sum_{i \in DMD(k)} C_i(t) + \sum_{\substack{i \in DMD \\ i \notin DMD(k)}} out_{ik}(t) C_i(t) \geq demand_k(t),$$

$$\forall k \in DM, \forall t \in T.$$

Of course, we cannot produce more than the capacity. For general production technologies, this constraint is

$$P_i(t) \leq af_i(t) C_i(t), \quad \forall i \in PRC, \forall t \in T.$$

Table 2: Set definitions

ENC	the energy types, except electricity (ELC) and low temperature heat (LTH)
T	time periods
TCH	supply and demand technologies
DMD	demand technologies
$DMD(k)$	demand technologies which can only supply utility demand $k \in DM$
DM	utility demands
Y	parts of the day (d for daytime, n for nighttime)
Z	seasons of the year (w for winter, s for summer, i for intermediate)
ELA	technologies that produce electricity
PRC	energy production technologies
STG	technologies that effectively allow the storage of electricity
HPL	technologies which produce low temperature heat (LTH)
CON	technologies which produce electricity and/or low temperature heat
BAS	electrical production technologies which produce only at a steady rate, day and night
$CO2$	technologies which emit carbon dioxide

For technologies that produce electricity, the production constraint is

$$P_{izy}(t) + \left(\frac{q_{zy}}{q_{zd} + q_{zn}}\right) \leq u_i q_{zy} \left(1 - [1 - af_i(t)]\, fo_i\right) C_i(t),$$
$$\forall i \in ELA, \forall z \in Z, \forall y \in Y, \forall t \in T.$$

The second term is the production lost due to maintenance.

Similarly for technologies that produce low temperature heat,

$$P_{iz}(t) + M_{iz}(t) \leq u_i (q_{zd} + q_{zn}) \left(1 - [1 - af_i(t)]\, fo_i\right) C_i(t),$$
$$\forall i \in HPL, \forall z \in Z, \forall t \in T.$$

The following constraint pertains to maintenance.

$$\sum_{z \in Z} M_{ix}(t) \geq [1 - af_i(t)][1 - fo_i] u_i C_i(t), \quad \forall i \in CON, \forall t \in T.$$

On any given day, the peak demand level is, of course, higher than the daily average demand. The capacity for production of electricity must be sufficient to cover peak demands, which occur during the day in both winter and summer. The constant er sets how much higher than daily average demand levels the peak can be. The peak constraint for electricity is

$$\frac{\eta}{1 + er} \left[\sum_{i \in ELA} u_i pk_i(t) C_i(t) + \frac{1}{q_{zd}} \sum_{s} IMPELC_{szd}(t) \right] \geq$$
$$\sum_{i \in PRC} inp_{ELC,i}(t) epk_i(t) P_i(t) + \frac{1}{q_{zd}} \sum_{s} EXPELC_{szd}(t)$$
$$+ \sum_{i \in DMD} inp_{ELC,i}(t) elf_{j(i)}(t) cf_i(t) \left(\frac{fr_{j(i)zd}}{q_{zd}} \right) C_i(t),$$
$$\forall z \in \{w, s\}, \forall t \in T.$$

The peak demand constraint for district heat is

$$\frac{\rho}{1 + hr} \sum_{i \in HPL} u_i pk_i(t) C_i(t) \geq$$
$$\sum_{i \in DMD} inp_{LTH,i}(t) cf_i(t) \left(\frac{fr_{j(i)wd} + fr_{j(i)wn}}{q_{wd} + q_{wn}} \right) C_i(t), \quad \forall t \in T,$$

where hr is the analog to er for electricity.

Some types of electrical production technologies, here called BAS, can only operate at a constant production level, day and night. We may desire to limit the percentage of production from such technologies, since they do not give hour to hour operation flexibility. The upper bound, bl is used in the following constraint:

$$\sum_{i \in BAS} P_{izn}(t) + \sum_{s} \eta IMPELC_{szn}(t) - EXPELC_{szn}(t)$$
$$\leq bl \left[\sum_{i \in ELA} P_{izn}(t) + \sum_{s} \eta IMPELC_{szn}(t) - EXPELC_{szn}(t) \right],$$
$$\forall z \in Z, \forall t \in T.$$

Fragnière [5] states that the production of greenhouse gases is limited, but we were unable to find an explicitly stated constraint. Therefore, we propose our own of the form

$$\sum_{i \in CO2} co2_i(t) C_i(t) + \boldsymbol{\delta} \sum_{s} \sum_{z \in Z} \sum_{y \in Y} IMPELC_{szy}(t) \leq \boldsymbol{limit_{CO2}}(t), \quad \forall t \in T. \tag{4}$$

The second term on the left hand side represents the possibility of imported electricity counting toward the CO_2 limit. Random $\boldsymbol{\delta}(t) \in (0,1)$ represents the probability of such a rule. Of course, $\delta(1) = 0$ with probability one.

The objective is to minimize capital and operating costs, which can be expressed as

$$\sum_{t \in T} \frac{1}{(1+\alpha)^{n(t-1)}} \sum_{i \in TCH} invcost_i(t) I_i(t) + \left(\sum_{m=1}^{n} (1+\alpha)^{1-m} \right)$$
$$\sum_{t \in T} \frac{1}{(1+\alpha)^{n(t-1)}} \left[\sum_{i \in TCH} fixom_i(t) C_i(t) + \sum_{i \in PRC} varom_i(t) P_i(t) + \right.$$
$$\sum_{i \in HPL} \sum_{z \in Z} varom_i(t) P_{iz}(t) + \sum_{i \in ELA} \sum_{z \in Z} \sum_{y \in Y} varom_i(t) P_{izy}(t) +$$
$$\sum_{k \in ENC} \sum_{s} cost_{ks}(t) IMP_{ks}(t) + \sum_{s} \sum_{z \in Z} \sum_{y \in Y} cost_{ELC,s}(t) IMPELC_{szy}(t) -$$
$$\sum_{k \in ENC} \sum_{s} price_{ks}(t) EXP_{ks}(t) -$$
$$\left. \sum_{s} \sum_{z \in Z} \sum_{y \in Y} price_{ELC,s}(t) EXPELC_{szy}(t) \right].$$

3.2 Problem statement

We present a problem that is not as elaborate as that created by Fragnière [5]. It corresponds to the numerical examples given in the "Numerical examples" section.

Minimize

$$\sum_{t\in T}\frac{1}{(1+\alpha)^{n(t-1)}}\sum_{i\in TCH} invcost_i(t)I_i(t) + \left(\sum_{m=1}^{n}(1+\alpha)^{1-m}\right)$$

$$\sum_{t\in T}\frac{1}{(1+\alpha)^{n(t-1)}}\left[\sum_{i\in TCH} fixom_i(t)C_i(t) + \sum_{i\in PRC} varom_i(t)P_i(t) + \right.$$

$$\sum_{i\in HPL}\sum_{z\in Z} varom_i(t)P_{iz}(t) + \sum_{i\in ELA}\sum_{z\in Z}\sum_{y\in Y} varom_i(t)P_{izy}(t) +$$

$$\sum_{k\in ENC}\sum_{s} cost_{ks}(t)IMP_{ks}(t) + \sum_{s}\sum_{z\in Z}\sum_{y\in Y} cost_{ELC,s}(t)IMPELC_{szy}(t) -$$

$$\sum_{k\in ENC}\sum_{s} price_{ks}(t)EXP_{ks}(t) -$$

$$\left.\sum_{s}\sum_{z\in Z}\sum_{y\in Y} price_{ELC,s}(t)EXPELC_{szy}(t)\right].$$

subject to

$$\sum_{\substack{i\in TCH\\ i\notin DMD}} out_{ki}(t)P_i(t) + \sum_{i\in DMD} out_{ki}(t)cf_i(t)C_i(t) + \sum_{s} IMP_{ks}(t)$$

$$\geq \sum_{\substack{i\in TCH\\ i\notin DMD}} inp_{ki}(t)P_i(t) + \sum_{i\in DMD} inp_{ki}(t)cf_i(t)C_i(t) + \sum_{s} EXP_{ks}(t),$$

$$\forall k\in ENC, \forall t\in T,$$

$$\eta\left[\sum_{i\in ELA} P_{izy}(t) + \sum_{s} IMPELC_{szy}(t)\right] \geq \sum_{i\in PRC} inp_{ELC,i}(t)q_{zy}P_i(t)$$

$$+ \sum_{i\in DMD} inp_{ELC,i}(t)cf_i(t)fr_{j(i)zy}C_i(t) + \sum_{k} EXPELC_{kzy}(t)$$

$$+\,\eta \sum_{\substack{i\in STG\\ \ni y=n}} e_iP_{izd}(t), \quad \forall z\in Z, \forall y\in Y, \forall t\in T,$$

$$\gamma \sum_{i \in HPL} P_{iz}(t) \geq \sum_{i \in DMD} inp_{LTH,i}(t) cf_i(t) C_i(t) \sum_{y \in Y} fr_{j(i)zy},$$
$$\forall z \in Z, \forall t \in T,$$

$$C_i(t) = \sum_{m=\text{Max}\{1, t-l_i+1\}}^{t} I_i(m) + resid_i(t), \quad \forall t \in T, \forall i,$$

$$\sum_{i \in DMD(k)} C_i(t) + \sum_{\substack{i \in DMD \\ i \notin DMD(k)}} out_{ik}(t) C_i(t) \geq demand_k(t),$$
$$\forall k \in DM, \forall t \in T,$$

$$P_i(t) \leq af_i(t) C_i(t), \quad \forall i \in PRC, \forall t \in T,$$

$$P_{izy}(t) + \left(\frac{q_{zy}}{q_{zd} + q_{zn}}\right) \leq u_i q_{zy} \left(1 - [1 - af_i(t)]\, fo_i\right) C_i(t),$$
$$\forall i \in ELA, \forall z \in Z, \forall y \in Y, \forall t \in T,$$

$$\sum_{z \in Z} M_{ix}(t) \geq [1 - af_i(t)][1 - fo_i] u_i C_i(t), \quad \forall i \in CON, \forall t \in T,$$

$$\frac{\eta}{1 + er} \left[\sum_{i \in ELA} u_i pk_i(t) C_i(t) + \frac{1}{q_{zd}} \sum_s IMPELC_{szd}(t)\right] \geq$$
$$\sum_{i \in PRC} inp_{ELC,i}(t) epk_i(t) P_i(t) + \frac{1}{q_{zd}} \sum_s EXPELC_{szd}(t)$$
$$+ \sum_{i \in DMD} inp_{ELC,i}(t) elf_{j(i)}(t) cf_i(t) \left(\frac{fr_{j(i)zd}}{q_{zd}}\right) C_i(t),$$
$$\forall z \in \{w, s\}, \forall t \in T,$$

$$\frac{\rho}{1 + hr} \sum_{i \in HPL} u_i pk_i(t) C_i(t) \geq$$
$$\sum_{i \in DMD} inp_{LTH,i}(t) cf_i(t) \left(\frac{fr_{j(i)wd} + fr_{j(i)wn}}{q_{wd} + q_{wn}}\right) C_i(t), \quad \forall t \in T,$$

$$\sum_{i \in BAS} P_{izn}(t) + \sum_{s} \eta IMPELC_{szn}(t) - EXPELC_{szn}(t)$$
$$\leq bl \left[\sum_{i \in ELA} P_{izn}(t) + \sum_{s} \eta IMPELC_{szn}(t) - EXPELC_{szn}(t) \right],$$
$$\forall z \in Z, \forall t \in T,$$

$$\sum_{i \in CO2} co2_i(t) C_i(t) + \boldsymbol{\delta}(t) \sum_{s} \sum_{z \in Z} \sum_{y \in Y} IMPELC_{szy}(t) \leq \boldsymbol{limit_{CO2}}(t), \quad \forall t \in T$$

and for all $t \in T$,

$$P_i(t) \geq 0,\ \forall i \in PRC, \quad I_i(t) \geq 0,\ \forall i \in TCH,$$
$$C_i(t) \geq 0,\ \forall i \in TCH, \quad P_{iz}(t) \geq 0,\ \forall i \in HPL, \forall z \in Z,$$
$$P_{izy}(t) \geq 0,\ \forall i \in ELA, \forall z \in Z, \forall y \in Y, \quad IMP_{ks}(t) \geq 0, \forall k \in ENC \forall s,$$
$$IMPELC_{szy}(t) \geq 0, \forall s, \forall z \in Z, \forall y \in Y, \quad EXP_{ks}(t) \geq 0, \forall k \in ENC \forall s,$$
$$EXPELC_{szy}(t) \geq 0, \forall s, \forall z \in Z, \forall y \in Y, \quad \delta(1) = 0.$$

3.3 Numerical examples

The problem created by Fragnière [5] for the Canton of Geneva is extremely large and complex, and the input data format is not SMPS. Therefore, we have created our own sample problems of this kind. The numbers in this example are based on the authors' judgment, not actual economic data.

The example creates a situation similar to that experienced in the United States, where oil imports (OIL) are the largest source of energy. Other imports are coal (COL), natural gas (NGS), propane (PRO), nuclear fuel (NUF), and electricity (ELC). There are no exports in this example.

The energy types allowed are electricity (ELC), gasoline (GAS), coal (COL), heating oil and diesel (HOL), natural gas (NGS), propane/LPG (LPG), jet fuel (JET), and nuclear fuel (NUC). When one unit of oil is imported, the following portions of hydrocarbon based energy types are assumed to be gained: 0.45 gasoline, 0.25 heating oil/diesel, 0.10 natural gas, 0.10 jet fuel, and 0.10 propane/LPG. The inequalities of type (3) must take this into account. For example, the inequality balancing natural gas is

$$0.10 IMP_{\mathrm{OIL}}(t) + IMP_{\mathrm{NG}} \geq inp_{\mathrm{NGS,HNG}}(t) cf_{\mathrm{HNG}}(t) C_{\mathrm{HNG}}(t)$$
$$+ inp_{\mathrm{NGS,NEL}}(t) cf_{\mathrm{NEL}}(t) C_{\mathrm{NEL}}(t).$$

The available technologies are listed in Table 6, along with their associated coefficients for the example problem. Other coefficients are listed in Table 3, Table 4 and Table 5.

There are several two stage versions of this problem in the test set. They differ in how stochasticity is introduced. The problem *env:loose*, using the stochastic file `env.sto.loose`, simply assumes very non-challenging (i.e. loose) CO_2 limits. The problem *env:aggressive* (`env.sto.aggr`) sets aggressive CO_2 limits. Each of these has five random realizations, and the parameter $\boldsymbol{\delta}(t)$ takes a value 0 with probability one.

The problem *env:import* (`env.sto.imp`) uses the aggressive CO_2 limits, and, in addition, considers the possibility that imported electricity (IMPELC) will be counted toward such limits in period two. That is, $\boldsymbol{\delta}(2)$ takes a nonzero value with nonzero probability. This problem has fifteen random realizations.

The problem *env:large* (`env.sto.lrge`) builds on *env:import* by making random the costs of various energy sources. The number of realizations is $8,232$. The problem *env:xlarge* (`env.sto.xlrge`) is a larger version still, mostly to test distributed memory capabilities of the solver.

Table 3: Example problem seasonal coefficients

	summer		winter	
	day	night	day	night
q_{zy}	0.60	0.40	0.40	0.60
$cost_{\mathrm{ELC}}$	5.2	5.0	4.8	4.6
$fr_{\mathrm{ELC},zy}$	0.35	0.25	0.10	0.30

Table 4: Example problem demands

k	$demand_k(1)$	$demand_k(2)$
ELC	170	230
HHO	30	30
NG	15	25
GAS	60	80
LPG	3	3
JET	10	20

Table 5: Example problem coefficients

α	0.05
n	5
η	0.80
e_{HYD}	0.10
er	0.20
$cost_{\mathrm{OIL}}$	0.8
$cost_{\mathrm{COAL}}$	0.7
$cost_{\mathrm{NG}}$	0.6
$cost_{\mathrm{PRO}}$	0.7
$cost_{\mathrm{NUF}}$	0.9

Table 6: Example problem technologies and associated coefficients

description	i	$cf_i(t)$	$inp_{ki}(t)^\dagger$	$\left(\begin{array}{c} resid_i(1): \\ resid_i(2) \end{array}\right)$	$u_i(t)$	$pk_i(t)$	$invcost_i(1)^\ddagger$	$fixom_i(t)$	$varom_i(t)$	$co2_i(t)$
industrial electricity	ELI	0.8	3.0	(100 : 100)		0.1	30	3.0	0.1	0.0
domestic electricity	ELD	0.8	2.8	(110 : 110)		0.2	50	4.0	0.1	0.0
heating oil/diesel	HHO	0.4	1.0	(15 : 15)			20	2.0	0.5	1.4
household nat. gas	HNG	0.4	0.5	(25 : 25)			25	2.0	0.5	1.0
automobiles	CAR	0.5	0.7	(80 : 80)			40	2.0	0.5	1.5
household propane	HLP	0.4	0.6	(5 : 5)			30	2.5	0.8	1.1
elec. from coal	CEL	0.75	0.7	(110 : 100)	$\frac{1}{0,7}$	0.1	30	3.0	0.4	1.2
elec. from NG	NEL	0.7	0.4	(40 : 40)	$\frac{1}{0.4}$	0.3	35	2.0	0.5	0.9
jet fuel prod.	AIR	0.75	0.8	(15 : 15)			40	2.0	0.5	1.5
diesel trucks	TRK	0.75	1.0	(30 : 30)			40	2.0	05	1.7
elec. from nucl.	NUL	0.8	0.1	(40 : 20)	$\frac{1}{0,7}$	0.10	80	4	0.7	0.0
elec. from hydr.	HYD	0.8	0.8	(10 : 8)	$\frac{1}{0.8}$	0.2	80	4	0.1	0.0

†For the obvious k.

‡For $t = 2$, multiply value by 1.5.

3.4 Notational reconciliation

Because of the size of the problem, we reconcile only the problem from the Numerical examples section to the format of (2). In this section, we will use "ELI ... HYD" to denote the set of technologies listed in Table 6, in the order presented. We will also use "OIL ... ELC" to denote the imports, and "ELC ... JET" for the demands in Table 4. Additionally, "WD ... SN" will mean the sequence "WD, WN, SD, SN," and "CEL ... HYD" will stand for the sequence of electricity producers "CEL, NEL, NUL, HYD." These abbreviations will make our arrays smaller to print.

We will also use the notation e_i to mean the unit vector in the ith direction from the space $\mathbb{R}^{12}$.

For $t = 1, 2$, make the following definitions:

$$
x_t := \left[\begin{array}{c}
I_{\mathrm{ELI}}(t) \\ \vdots \\ I_{\mathrm{HYD}}(t) \\ \hline
C_{\mathrm{ELI}}(t) \\ \vdots \\ C_{\mathrm{HYD}}(t) \\ \hline
\left[\begin{array}{c} P_{\mathrm{CEL,WD}}(t) \\ \vdots \\ P_{\mathrm{CEL,SN}}(t) \end{array}\right] \\ \vdots \\
\left[\begin{array}{c} P_{\mathrm{HYD,WD}}(t) \\ \vdots \\ P_{\mathrm{HYD,SN}}(t) \end{array}\right] \\ \hline
IMP_{\mathrm{OIL}}(t) \\ \vdots \\ IMP_{\mathrm{NUF}}(t) \\ \hline
IMPELC_{\mathrm{WD}}(t) \\ \vdots \\ IMPELC_{\mathrm{SN}}(t)
\end{array}\right], \qquad
c_t := \left[\begin{array}{c}
r(t)invcost_{\mathrm{ELI}}(t) \\ \vdots \\ r(t)invcost_{\mathrm{HYD}}(t) \\ \hline
s(t)fixom_{\mathrm{ELI}}(t) \\ \vdots \\ s(t)fixom_{\mathrm{HYD}}(t) \\ \hline
\left[\begin{array}{c} s(t)varom_{\mathrm{CEL}}(t) \\ \vdots \\ s(t)varom_{\mathrm{CEL}}(t) \end{array}\right] \\ \vdots \\
\left[\begin{array}{c} s(t)varom_{\mathrm{HYD}}(t) \\ \vdots \\ s(t)varom_{\mathrm{HYD}}(t) \end{array}\right] \\ \hline
s(t)cost_{\mathrm{OIL}} \\ \vdots \\ s(t)cost_{\mathrm{NUF}} \\ \hline
s(t)cost_{\mathrm{ELC,WD}} \\ \vdots \\ s(t)cost_{\mathrm{ELC,SN}}
\end{array}\right],
$$

where $r(t) := (1+\alpha)^{-n(t-1)}$ and $s(t) := \left(\sum_{m=1}^{n} (1+\alpha)^{1-m} \right) r(t)$.

Define the following matrices:

$$BA(t) := \begin{bmatrix} inp_{\text{GAS,CAR}}(t)cf_{\text{CAR}}(t)e_5^\mathsf{T} \\ inp_{\text{COL,CEL}}(t)cf_{\text{CEL}}(t)e_7^\mathsf{T} + inp_{\text{HOL,TRK}}(t)cf_{\text{TRK}}(t)e_{10}^\mathsf{T} \\ inp_{\text{HOL,HHO}}(t)cf_{\text{HHO}}(t)e_3^\mathsf{T} + inp_{\text{NGS,NEL}}(t)cf_{\text{NEL}}(t)e_8^\mathsf{T} \\ inp_{\text{NGS,HNG}}(t)cf_{\text{HNG}}(t)e_4^\mathsf{T} \\ inp_{\text{LPG,HLP}}(t)cf_{\text{HLP}}(t)e_6^\mathsf{T} \\ inp_{\text{JET,AIR}}(t)cf_{\text{AIR}}(t)e_9^\mathsf{T} \\ inp_{\text{NUC,NUL}}(t)cf_{\text{NUL}}(t)e_{11}^\mathsf{T} \end{bmatrix},$$

$$BB(t) := \begin{bmatrix} -0.45 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ -0.25 & 0 & 0 & 0 & 0 & 0 \\ -0.10 & 0 & 0 & 0 & 0 & 0 \\ -0.10 & 0 & 0 & 0 & 0 & 0 \\ -0.10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix},$$

$$BC(t) := -\eta \begin{bmatrix} I^{4\times 4} & I^{4\times 4} & I^{4\times 4} & I^{4\times 4} \end{bmatrix}, \qquad BD(t) := -\eta \left[I^{4\times 4} \right]$$

$$BE(t) :=$$

$$\left[\begin{array}{cc|c} inp_{\text{ELC,ELI}}(t)cf_{\text{ELI}}(t)fr_{\text{ELC,WD}} & inp_{\text{ELC,ELD}}(t)cf_{\text{ELD}}(t)fr_{\text{ELC,WD}} & \\ \vdots & \vdots & 0^{4\times 10} \\ inp_{\text{ELC,ELI}}(t)cf_{\text{ELI}}(t)fr_{\text{ELC,SN}} & inp_{\text{ELC,ELD}}(t)cf_{\text{ELD}}(t)fr_{\text{ELC,SN}} & \end{array} \right],$$

$$BF(t) := \eta e_{\text{HYD}} \begin{bmatrix} 0^{4\times 12} & I^{4\times 4} \end{bmatrix}, \qquad BH(t) := -\begin{bmatrix} (e_1+e_2)^\mathsf{T} \\ (e_3+e_{10})^\mathsf{T} \\ e_4^\mathsf{T} \\ e_5^\mathsf{T} \\ e_6^\mathsf{T} \\ e_9^\mathsf{T} \end{bmatrix},$$

$$BM := \begin{bmatrix} \left(\frac{1}{q_{\text{WD}}}\right) e_1^\mathsf{T} \\ \left(\frac{1}{q_{\text{SD}}}\right) e_2^\mathsf{T} \end{bmatrix}, \qquad BN := [co2_{\text{ELI}} \cdots co2_{\text{HYD}}],$$

$$
BK := -\begin{bmatrix} \begin{bmatrix} u_{\mathrm{CEL}} q_{\mathrm{WD}} e_7^{\mathsf{T}} \\ \vdots \\ u_{\mathrm{CEL}} q_{\mathrm{SN}} e_7^{\mathsf{T}} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} u_{\mathrm{HYD}} q_{\mathrm{WD}} e_{12}^{\mathsf{T}} \\ \vdots \\ u_{\mathrm{HYD}} q_{\mathrm{SN}} e_{12}^{\mathsf{T}} \end{bmatrix} \end{bmatrix}, \qquad bk := -\begin{bmatrix} q_{\mathrm{WD}}/(q_{\mathrm{WD}} + q_{\mathrm{WN}}) \\ q_{\mathrm{WN}}/(q_{\mathrm{WD}} + q_{\mathrm{WN}}) \\ q_{\mathrm{SD}}/(q_{\mathrm{SD}} + q_{\mathrm{SN}}) \\ q_{\mathrm{SN}}/(q_{\mathrm{SD}} + q_{\mathrm{SN}}) \end{bmatrix},
$$

$$
\begin{aligned}
BL(t) := \begin{bmatrix} 1 \\ 1 \end{bmatrix} \left(\frac{n}{1+er} \right) [&-u_{\mathrm{CEL}} pk_{\mathrm{CEL}}(t) e_7 - \\
&u_{\mathrm{NEL}} pk_{\mathrm{NEL}}(t) e_8 - u_{\mathrm{NUL}} pk_{\mathrm{NUL}}(t) e_{11} - u_{\mathrm{HYD}} pk_{\mathrm{HYD}}(t) e_{12}]^{\mathsf{T}} + \\
&cf_{\mathrm{ELI}}(t) \begin{bmatrix} elf_{\mathrm{ELC,WD}}(t) fr_{\mathrm{ELC,WD}}/q_{\mathrm{WD}} \\ elf_{\mathrm{ELC,SD}}(t) fr_{\mathrm{ELC,SD}}/q_{\mathrm{SD}} \end{bmatrix} (inp_{\mathrm{ELC,ELI}}(t) e_1^{\mathsf{T}} + \\
&\qquad\qquad inp_{\mathrm{ELC,ELD}}(t) e_2^{\mathsf{T}}),
\end{aligned}
$$

and the random

$$
\boldsymbol{BP}(t) := \begin{bmatrix} \boldsymbol{\delta}(t) & \boldsymbol{\delta}(t) & \boldsymbol{\delta}(t) & \boldsymbol{\delta}(t) \end{bmatrix}.
$$

Then, finally, we can assign $\boldsymbol{A_t}$, for $t = 1, 2$, and T_2 in blocks. Let

$$
\boldsymbol{A_t} := \left[\begin{array}{c|c|c|c|c}
 & BA(t) & & BB(t) & \\ \hline
 & BE(t) & (BC(t) + BF(t)) & & BD(t) \\ \hline
-I^{12\times 12} & I^{12\times 12} & & & \\ \hline
 & BH & & & \\ \hline
 & BK & I^{16\times 16} & & \\ \hline
 & BL(t) & & & BM \\ \hline
 & BN & & & \boldsymbol{BP}
\end{array} \right],
$$

and

$$
T_2 := \left[\begin{array}{c}
0^{7\times 50} \\ \hline
0^{4\times 50} \\ \hline
\begin{array}{c|c|c|c|c} & -I^{12\times 12} & & & \end{array} \\ \hline
0^{6\times 50} \\ \hline
0^{16\times 50} \\ \hline
0^{2\times 50} \\ \hline
0^{1\times 50}
\end{array} \right].
$$

We define the random right hand side as

$$
\boldsymbol{b}_t := \begin{bmatrix} 0^7 \\ 0^4 \\ resid_{\mathrm{ELI}}(t) \\ \vdots \\ resid_{\mathrm{HYD}}(t) \\ -demand_{\mathrm{ELC}}(t) \\ -demand_{\mathrm{HHO}}(t) \\ -demand_{\mathrm{NGS}}(t) \\ -demand_{\mathrm{GAS}}(t) \\ -demand_{\mathrm{LPG}}(t) \\ -demand_{\mathrm{JET}}(t) \\ b_k \\ b_k \\ b_k \\ b_k \\ 0^2 \\ \boldsymbol{limit_{CO2}}(t) \end{bmatrix}.
$$

If the user then appends slack variables in the blocks corresponding to $BA(t)$, $BE(t)$, $BH(t)$, $BK(t)$, $BL(t)$ and $BN(t)$, we will have the problem in the form of (2).

References

[1] K. A. Ariyawansa and P. L. Jiang. Polynomial cutting plane algorithms for two-stage stochastic linear programs based on ellipsoids, volumetric centers and analytic centers. Technical report, Department of Pure and Applied Mathematics, Washington State University, 1996.

[2] B. M. Averick, R. G. Carter, and J. J. Moré. The Minpack-2 test problem collection (preliminary version). Technical Report Technical Memorandum ANL/MCS-TM-150, Mathematics and Computer Science Division, Argonne National Laboratory, Argonne, IL, 1991.

[3] Steven J. Benson, Lois Curfman McInnes, and J. J. Moré. GPCG: a case study in the performance and scalability of optimization algorithms. Technical Report ANL/MCS-P768-0799, Mathematics and Computer Science Division Argonne National Laboratory, Sep 2000.

[4] J.R. Birge, M.A.H. Dempster, H.I. Gassmann, E.A. Gunn, A.J. King, and S.W. Wallace. A standard input format for multiperiod stochastic linear programs. *COAL Newsletter*, (17):1–19, 1987. URL: `http://ttg.sba.dal.ca/sba/profs/hgassmann/smps.html`.

[5] E. Fragnière. *Choix énergétiques et environnementaux pour le Canton de Genève*. PhD thesis, Université de Genève, April 1995. Thèse no 412.

[6] H. I. Gassmann. Optimal harvest of a forest in the presence of uncertainty. *Canadian Journal of Forest Research*, 19:1267–1274, 1989.

[7] D. Holmes. A (PO)rtable (S)tochastic programming (T)est (S)et (POSTS). URL: `http://www-personal.umich.edu/~jrbirge/dholmes/post.html`, Jan 1997.

[8] C. Lin and J. J. Moré. Newton's method for large bound-constrained optimization method. *SIAM J. Optim.*, (9):1100–1127, 1999.

[9] J. J. Moré. A collection of nonlinear model problems. In *Computational solution of nonlinear systems of equations*, volume 26 of *Lecture notes in applied mathematics*, pages 723–762. American Mathematical Society, Providence, RI, 1990.

[10] J. J. Moré and G. Toraldo. On the solution of large quadratic programming problems with bound constraints. *SIAM J. Optim.*, (1):93–113, 1991.