# Integrating design and production planning considerations in multi-bay manufacturing facility layout

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**Abstract**

This paper develops a new mathematical model that integrates layout design and production planning to prescribe efficient multi-bay manufacturing facilities. The model addresses the need to distribute department replicas throughout the facility and extends the use of product and process requirements as problem parameters in order to increase process routing flexibility. In addition, the model allows for the consideration of practical material handling and production features such as alternate process routings, product flow production patterns, and department replica capacities. Computational results demonstrate that the run time required to solve our test problems is quite acceptable given the long-term nature of facility layout decisions. Moreover, comparative results indicate that department replication can reduce material movement significantly while maintaining the existing production capacity.

**Keywords:** facilities planning and design, production, multi-bay manufacturing.

## 1 Introduction

The multi-bay manufacturing facility layout problem is concerned with determining the most efficient assignment of processing departments to bays in a facility that is defined by parallel bays arranged along a spine and served by an automated material handling system (see Figure 1). This problem arises in a wide variety of manufacturing contexts; e.g., heavy manufacturing and semiconductor fabrication (Meller 1997, Yang and Peters 1997). A common characteristic of this facility layout problem is that inter-bay material handling costs dominate the material handling costs within the bay. Thus, at the facility layout design stage, efficiency is measured in terms of the inter-bay material movement.

Traditionally, in the multi-bay manufacturing facility layout problem, it is assumed that all machines of the same type share the same department. That is, machines are grouped together in departments according to the process or function that they perform as in a functional layout. In general, functional layouts can be efficient provided high product variety and/or low product volumes. Functional layouts, however, force high inter-bay material movement, often resulting in long lead times, poor resource utilization, and limited throughput rates.

In this paper, we consider a distributed multi-bay manufacturing facility layout problem in which replicas of a given department type may exist in the facility. We allow these replicas to be assigned to different bays and for product flow allocation between pairs of individual department replicas to be made as function of the facility layout. Thus, design and production planning considerations are integrated at the facility layout design stage. The motivation is that the distribution of department replicas throughout the facility increases the accessibility of these department types from different bays. This, in turn, decreases inter-bay material movement. As a result, more efficient flows can be found for a given set of products and process routings. In addition, each bay can be

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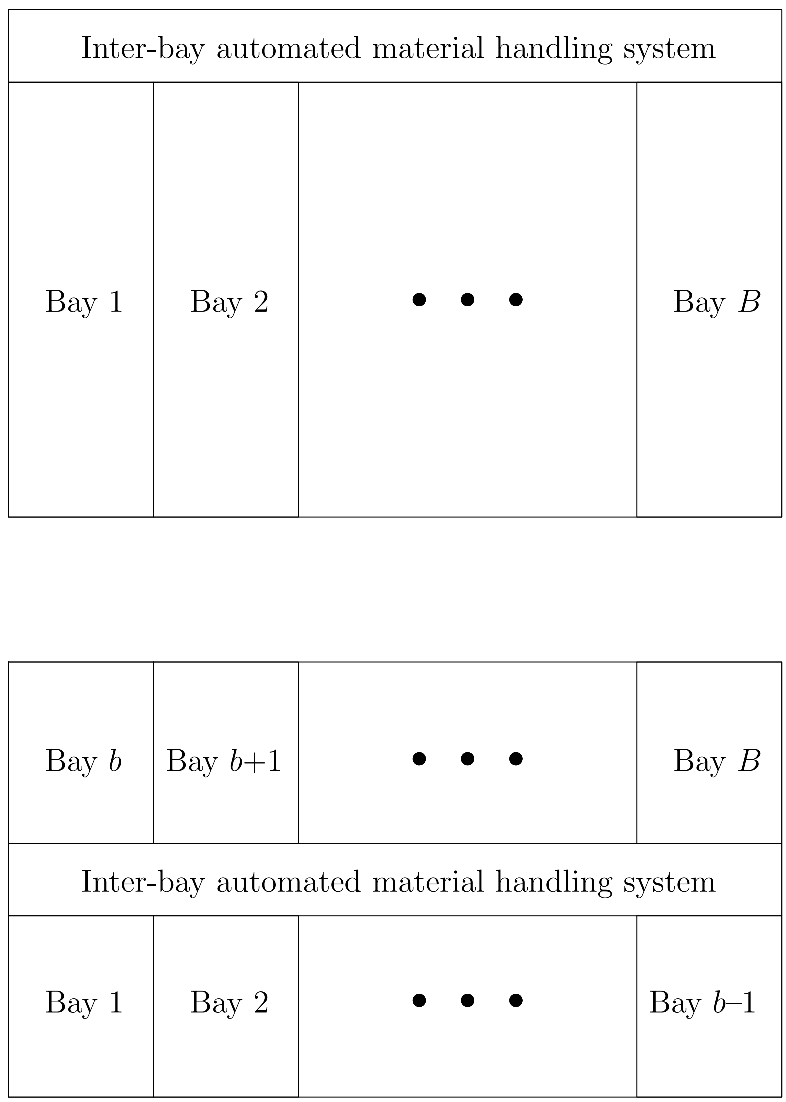


Figure 1: Two possible multi-bay manufacturing facility configurations.

organized much more efficiently; thus, allowing each bay to take advantage of their specific product and process requirements.

In the context of the multi-bay manufacturing facility layout problem, the contributions of this paper are twofold:

(1) It extends the design of bays to be composed of processing units beyond those that have been traditionally studied and implemented (i.e., we encourage not to group processing units as in a functional layout) in order to increase the efficiency of the allocation of resources throughout the facility.

(2) It extends the use of product and process requirements as problem parameters (i.e., we encourage not to aggregate product and process requirements into an undirected flow matrix) in order to increase process routing flexibility.

Notice that the use of specific process requirements and department disaggregation allows for the consideration of alternate process routings, department replica capacities, and product flow production patterns: production planning issues that, traditionally, have not been considered at the facility layout design stage.

## 2 Background

Research on the multi-bay manufacturing facility layout problem has been limited to Meller (1997). There is no mention of this problem in the most recent literature surveys (Kusiak and Heragu 1987, Meller and Gau 1996). There are, however, some similarities between the multi-bay manufacturing facility layout problem, the spine layout problem (Langevin et al. 1994), and the multi-floor facility layout problem. The reader is referred to Meller (1997) for such a comparison.

The multi-bay manufacturing facility layout problem exhibits the following characteristics:

- The number of bays in the facility and the bay areas are known.
- Inter-bay material movement is limited to the end of the bays.
- Inter-bay material handling costs dominate the material handling costs within the bay.
- The bay structure is typically designed to accommodate a linear or U-shaped product flow production pattern within each bay.

Clearly, these characteristics impose certain restrictions on the layout solution. However, while there may be other, perhaps more efficient layout solutions, many companies are reluctant to implement these alternative solutions since the multi-bay configuration offers many advantages (Peters 1994). These characteristics led Meller (1997) to a two-stage solution methodology, where in Stage 1 departments are assigned to bays using a mixed-integer program and in Stage 2 the layout within each bay is determined by solving independent linear ordering problems.

In this paper, we consider a distributed multi-bay manufacturing facility layout problem in which replicas of a given department type may exist in the facility. Note that department replication does not necessarily require the acquisition of additional resources. It can simply be achieved by disaggregating existing departments into smaller processing units that need not be placed together. In this paper, each processing unit is referred to as a department replica. Thus, a department replica is a collection of one or more machines of the same type. As a result, department replicas of the same type may have different capacities, which allows a more efficient allocation of resources throughout the facility.

The idea of breaking up departments into smaller processing units is not a new concept. Benjaafar and Sheikhzadeh (2000), Montreuil et al. (1999), Urban et al. (2000), and Venkatadri et al. (1997) have shown that department disaggregation can be a useful strategy for a wide variety of manufacturing settings. In the context of the multi-bay manufacturing facility layout problem, department disaggregation allows each bay to be organized much more efficiently. The proposed approach, in contrast to product family (or cellular) layouts, does not require the identification of product families *a priori*. Thus avoiding the subjective use of somewhat arbitrary measures of similarity or *ad hoc* methods to identify bottleneck resources and exceptional elements. Moreover, alternative within-bay layout configurations can be designed to take advantage of the specific product and process requirements for each bay (Tompkins et al. 1996).

The solution methodology for the distributed multi-bay manufacturing facility layout problem consists of two stages. In Stage 1, we solve for the assignment of department replicas to bays and for the product flow allocation among department replicas. In Stage 2, the layout within each bay is determined.

# 3 Stage 1: Assigning department replicas to bays and allocating product flow between department replicas

Since inter-bay material handling costs dominate, we define the distributed bay assignment problem to be solved in Stage 1 as the assignment of department replicas to bays and the allocation of the product flow between department replicas so that inter-bay material movement is minimized. This problem integrates design and production planning considerations and differs from traditional layout problems in four aspects:

(1) The number of bays in the facility and the bay areas are known.

(2) Each department type may have several replicas.

(3) Department replicas of the same type may have different capacities.

(4) The product flow between departments is not fixed *a priori*.

The specification of the number of bays and their areas, in some facility layout problems, is often dictated by floor plan dimensions and product and process requirements. For example, a robot may be used to handle products and tend departments in a bay. Since a robot, whether fixed position, track mounted, or gantry, has a work envelope of finite size, it can only tend a limited number of departments. Thus, in some facility layout problems, the size of the work envelope may dictate the number of bays and their areas.

In other facility layout problems, the number of bays and their areas are considered to be additional decision variables. If one assumes that bays are to have equal areas (i.e., the number of bays is the single additional decision variable), one could do an exhaustive search solving distributed multi-bay manufacturing facility layout problems with the number of bays varying from two to the total number of department replicas. The optimal or near-optimal number of bays will be determined using the objective function values of Stage 1 and Stage 2. If bays are not required to have rectangular shapes, the mathematical model and solution methodology proposed in Castillo and Peters (2003) could be used to solve the problem when the number of bays and their areas are also decision variables.

Consider the following parameters:

$P$: number of product types.

$R_p$: demand for product type $p$.

$B$: number of bays.

$A_b$: maximum area available in bay $b$.

$N$: number of department types.

$N_i$: number of replicas of department type $i$.

$M = \sum_{i=1}^{N} N_i$: total number of department replicas.

$a_{n_i}$: minimum required area for the $n$th replica of department type $i$.

$C_{n_i}$: capacity of the $n$th replica of department type $i$.

$c_{n_ip}$: the time required to process product $p$ on the $n$th replica of department type $i$.

$\Lambda_{ijp}$: total product flow from departments of type $i$ to departments of type $j$ for product type $p$.

Note that the total product flow between each pair of department types $i$ and $j$ for product type $p$, $\Lambda_{ijp}$, is determined from the product demand and process routing. This results in an asymmetric multi-product directed flow matrix, which is used as problem parameter. Also, note that if setups are significant, department replica capacities must be reduced and the number of replicas must be increased to ensure sufficient production capacity. This may not be trivial since the number of different product types that a department replica will process is not determined *a priori*. As we will see in Section 5, however, the optimal solution tends to favor product flow allocations that consolidate the production of each product type in as few department replicas as possible. Hence, if setups were to occur, they would be reduced. Moreover, the effect of department disaggregation on the optimal solution is of the diminishing kind, with most of the benefits realized with modest disaggregation. Thus, material movement can significantly be reduced without a major loss of pooling synergy.

The decision variables are the variables denoting the assignment of each department replica to a bay and the variables denoting the product flow allocation from the $n$th replica of department type $i$ to the $m$th replica of department type $j$ for product type $p$; that is,

$$x_{n_i b} = \begin{cases} 1 & \text{if the } n\text{th replica of department type } i \text{ is assigned to bay } b, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

and

$$\lambda_{n_i m_j p}. \tag{2}$$

The following constraints must be used for the assignment of department replicas to bays:

$$\sum_{b=1}^{B} x_{n_i b} = 1 \qquad \forall n_i, \forall i, \tag{3}$$

$$\sum_{i=1}^{N} \sum_{n_i=1}^{N_i} a_{n_i} x_{n_i b} \leq A_b \qquad \forall b. \tag{4}$$

Constraint (3) restricts a department replica to only one bay; while constraint (4) represents the bay area limit.

The following constraints must be used for the allocation of product flow between department replicas:

$$\sum_{n_i=1}^{N_i} \sum_{m_j=1}^{N_j} \lambda_{n_i m_j p} = \Lambda_{ijp} \qquad \forall i, j, \forall p, \tag{5}$$

$$\sum_{i=0}^{N} \sum_{n_i=1}^{N_i} \lambda_{n_i m_j p} = \sum_{q=1}^{N+1} \sum_{r_q=1}^{N_q} \lambda_{m_j r_q p} \qquad \forall m_j, \forall j, \forall p, \tag{6}$$

$$\sum_{p=1}^{P} \sum_{i=0}^{N} \sum_{n_i=1}^{N_i} \lambda_{n_i m_j p} c_{m_j p} \leq C_{m_j} \qquad \forall m_j, \forall j. \tag{7}$$

Constraint (5) equates the product flow between replicas of departments $i$ and $j$ to the total product flow between department types $i$ and $j$ as dictated by the multi-product directed flow matrix. Constraint (6) assures product flow conservation. Note that, for modeling convenience, departments $i = 0$ and $q = N + 1$ represent the input and output relationships with the outside, respectively. Constraint (7) assures that the product flow allocated to a department replica does not exceed its capacity.

Recall that we allow the number of bays and their areas to be problem parameters. Thus, the distance between bays is known in advance. Let $D_{ab} \geq 0$ represent the distance between bay $a$ and bay $b$. We define the formulation of the distributed bay assignment problem with the following cubic mixed-integer program:

**Problem DBAP1**

$$\min \quad \sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{n_i=1}^{N_i}\sum_{m_j=1}^{N_j}\sum_{a=1}^{B}\sum_{b=1}^{B} D_{ab}\lambda_{n_i m_j p} x_{n_i b} x_{m_j a} \tag{8}$$

$$\text{s.t.} \quad (3)-(7)$$

$$x_{n_i b} \in \{0,1\} \qquad \forall n_i, \forall i, \forall b, \tag{9}$$

$$\lambda_{n_i m_j p} \geq 0 \qquad \forall n_i, m_j, \forall i, j, \forall p. \tag{10}$$

The formulation presented above is a generalization of the bay assignment formulation presented by Meller (1997). In the bay assignment formulation, product flow between departments is fixed *a priori* since there is only one replica of each department type. In the distributed bay assignment formulation, both department replica assignment and product flow allocation are decision variables. Our formulation reduces to the bay assignment formulation if there is only one replica of each department type. Thus, the manufacturing facility layout obtained from optimally solving our distributed formulation will be at least as good as the one obtained from optimally solving the bay assignment formulation.

The pertinent literature indicates that basically two approaches are employed in solving nonlinear mixed-integer programs; namely, heuristic procedures and exact solution methods. Exact solution methods usually rely on either an implicit enumeration algorithm or on a linearization method followed by the solution of a mixed-integer program (Bazaraa et al. 1993). The present paper falls in both groups because it uses a method of linearization and also proposes a heuristic solution procedure.

In order to linearize the cubic objective function (8), let $w_{n_i b m_j a p} = \lambda_{n_i m_j p} x_{n_i b} x_{m_j a}$ and define the parameter $\omega = \max_p\{R_p\}$, such that $\omega$ is at least as large as any value of $\lambda_{n_i m_j p}$ and, in turn, any value of $w_{n_i b m_j a p}$. The linear mixed-integer program is:

**Problem DBAP2**

$$\min \quad \sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{n_i=1}^{N_i}\sum_{m_j=1}^{N_j}\sum_{a=1}^{B}\sum_{b=1}^{B} D_{ab} w_{n_i b m_j a p} \tag{11}$$

$$\text{s.t.} \quad (3)-(7),\ (9),\ (10)$$

$$w_{n_i b m_j a p} \geq \lambda_{n_i m_j p} + \omega x_{n_i b} + \omega x_{m_j a} - 2\omega \qquad \forall a, b, \forall n_i, m_j, \forall i, j, \forall p, \tag{12}$$

$$w_{n_i b m_j a p} \geq 0 \qquad \forall a, b, \forall n_i, m_j, \forall i, j, \forall p. \tag{13}$$

**Proposition 1** *DBAP1 and DBAP2 are equivalent in the sense that they have the same optimal solutions.*

**Proof:** First, consider an optimal solution to problem DBAP1: an optimal solution to DBAP1 satisfies all of the constraints of DBAP2 and the two problems have the same solution since $w_{n_i b m_j a p} = \lambda_{n_i m_j p} x_{n_i b} x_{m_j a}$. Second, consider an optimal solution to problem DBAP2: $w_{n_i b m_j a p} \geq \lambda_{n_i m_j p}$ only if $x_{n_i b} = x_{m_j a} = 1$ and constraint (12) is satisfied since, by definition, $\omega$ is at least as large as all $\lambda_{n_i m_j p}$. Moreover, we are assured that $w_{n_i b m_j a p}$ will not exceed $\lambda_{n_i m_j p}$ due to the minimization objective. On the other hand, if $x_{n_i b} = x_{m_j a} = 1$ and $w_{n_i b m_j a p} < \lambda_{n_i m_j p}$ constraint (12) is violated. Thus, $w_{n_i b m_j a p} = \lambda_{n_i m_j p}$ if and only if $x_{n_i b} = x_{m_j a} = 1$, which is expressed as $w_{n_i b m_j a p} = \lambda_{n_i m_j p} x_{n_i b} x_{m_j a}$, and the two problems have the same solution. □

## 3.1 Heuristic solution procedure

As evident by the constraint set of DBAP2, the distributed multi-bay manufacturing facility layout problem is an aggregation of a design problem (constraints (3) and (4)) and a production planning problem (constraints (5) - (7)). Therefore, in order to obtain a solution, it is possible to exploit the special structure of the formulation using a heuristic procedure that solves a bay assignment problem with fixed product flows (Step 1: design problem) followed by a product flow allocation problem with fixed facility layout (Step 2: production planning problem). Figure 2 outlines our overall heuristic solution procedure.

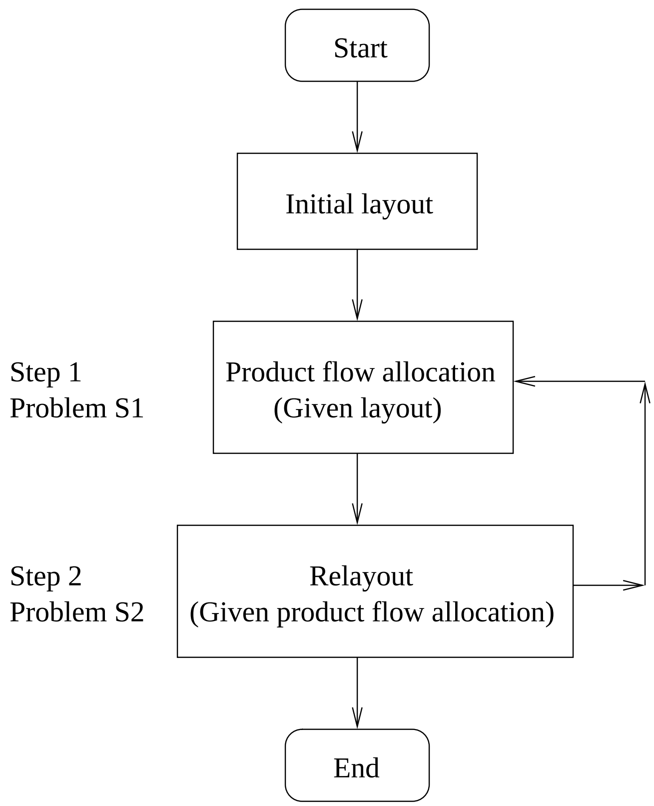


Figure 2: Flowchart of heuristic solution procedure.

The heuristic procedure iterates between Step 1 and Step 2 until convergence is achieved. Notice that the solution obtained using the iterative heuristic procedure is not guaranteed to be the optimal solution for problem DBAP2. However, if both S1 and S2 are solved optimally, the product flow allocation solution of Step 1 cannot be improved by changing the assignment of department replicas to bays and the assignment solution of Step 2 cannot be improved by changing the product flow allocation (Benjaafar and Sheikhzadeh 2000, Cooper 1963, Venkatadri et al. 1997).

The basic steps of the heuristic procedure are:

**Step 1.** Given a fixed assignment of department replicas to bays, find a product flow allocation between department replicas that minimizes inter-bay material movement.

**Problem S1**

$$\begin{aligned} \min \quad & \sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{n_i=1}^{N_i}\sum_{m_j=1}^{N_j}\sum_{a=1}^{B}\sum_{b=1}^{B} D_{ab}\bar{x}_{n_i b}\bar{x}_{m_j a}\lambda_{n_i m_j p} \qquad (14)\\ \text{s.t.} \quad & (5)-(7),\ (10). \end{aligned}$$

where the assignments of each department replica to a bay $\bar{x}_{n_i b}, \forall b, \forall n_i, \forall i$, are fixed.

**Step 2.** Given a fixed product flow allocation, find an assignment of department replicas to bays that minimizes inter-bay material movement.

**Problem S2**

$$\begin{aligned} \min \quad & \sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{n_i=1}^{N_i}\sum_{m_j=1}^{N_j}\sum_{a=1}^{B}\sum_{b=1}^{B} D_{ab}\bar{\lambda}_{n_i m_j p} x_{n_i b} x_{m_j a} \qquad (15)\\ \text{s.t.} \quad & (3),\ (4),\ (9). \end{aligned}$$

where the product flows $\bar{\lambda}_{n_i m_j p}, \forall n_i, m_j, \forall i, j, \forall p$, are fixed.

Furthermore, if we denote the feasible domain of S2 by $\Delta$, then the problem can be written as:

$$\begin{aligned} \min \quad & \mathbf{x}^T\mathbf{Z}\mathbf{x} \\ \text{s.t.} \quad & \mathbf{x} \in \Delta, \end{aligned}$$

where the entries of the $(M \times M)$-matrix $\mathbf{Z}$ are the products $D_{ab}\sum_p \bar{\lambda}_{n_i m_j p}$.

**Proposition 2** *S2 is equivalent to the following 0-1 concave programming problem:*

$$\begin{aligned} \min \quad & \mathbf{x}^T\mathbf{Q}\mathbf{x} \\ \text{s.t.} \quad & \mathbf{x} \in \Omega, \end{aligned}$$

*where* $\mathbf{Q} = \mathbf{Z} - \alpha\mathbf{I}$ *is negative definite,* $\alpha > ||\mathbf{Z}||_\infty$ *(where* $||\mathbf{Z}||_\infty$ *is the row norm of the* $(M \times M)$*-matrix* $\mathbf{Z}$*),* $\mathbf{I}$ *is the* $M \times M$ *unit matrix, and* $\Omega$ *is the set of all* $x_{n_i b}$ *satisfying constraints (3), (4), and (9).*

**Proof:** (Horst et al. 2000) Let $\mathbf{x} = (x_{1_1 1}, x_{2_1 1}, \ldots, x_{N_N B})^T = (x_1, x_2, \ldots, x_M)^T$ and consider the quadratic form $\mathbf{x}^T\mathbf{Q}\mathbf{x}$. Assume, without loss of generality, that $\mathbf{Q}$ is symmetric. Clearly, $\mathbf{Q}$ is symmetric when $\mathbf{Z}$ is symmetric. However, we obtained $\mathbf{Z}$ using the distances between bays and the asymmetric multi-product directed flow matrix; thus, we need to replace the asymmetric matrix $\mathbf{Q}$ by $\mathbf{Q}' = \frac{1}{2}(\mathbf{Q} + \mathbf{Q}^T)$, which is symmetric and satisfies $\mathbf{x}^T\mathbf{Q}'\mathbf{x} = \mathbf{x}^T\mathbf{Q}\mathbf{x}$ since $\mathbf{x}^T\mathbf{Q}^T\mathbf{x} = (\mathbf{x}^T\mathbf{Q}^T\mathbf{x})^T = \mathbf{x}^T\mathbf{Q}\mathbf{x}$. We obtain:

$$\begin{aligned}
\mathbf{x}^T\mathbf{Q}\mathbf{x} &= \sum_{r=1}^{M} q_{rr}x_r^2 + 2\sum_{r=1}^{M-1}\sum_{s=r+1}^{M} q_{rs}x_rx_s \\
&= \sum_{r=1}^{M}(q_{rr} + \sum_{s=1(s\neq r)}^{M} q_{rs})x_r^2 - \sum_{r=1}^{M-1}\sum_{s=r+1}^{M} q_{rs}(x_r - x_s)^2 \\
&= \sum_{r=1}^{M}(-\alpha + \sum_{s=1}^{M} z_{rs})x_r^2 - \sum_{r=1}^{M-1}\sum_{s=r+1}^{M} z_{rs}(x_r - x_s)^2 \\
&\leq \sum_{r=1}^{M}(-\alpha + \sum_{s=1}^{M} z_{rs})x_r^2.
\end{aligned}$$

Therefore, $\mathbf{x}^T\mathbf{Q}\mathbf{x} < 0$ for any $x_r \neq 0$ and the matrix $\mathbf{Q}$ is negative definite. □

The use of the equivalent 0-1 concave programming problem allows us to take advantage of specialized, state-of-the-art algorithms in order to solve Step 2 of the heuristic solution procedure efficiently. The reader is referred to Andersen and Ye (1998, 1999) for the details of such algorithms. The heuristic solution procedure described above was implemented in MATLAB and interfaced with TOMLAB 3.0 (Holmström 2001). Computational results are presented in the sequel after we discuss the solution methodology for Stage 2, that is, determining the layout for each bay.

In addition, notice that Step 1 of the iterative heuristic solution procedure (the production planning problem) is a linear program that can be solved efficiently regardless of the size of the problem instance. With respect to Step 2 of the procedure (the design problem), if the total number of department replicas and the number of products are very large then the run time required to solve the distributed bay assignment problem may be quite long. In such instances a meta-heuristic such as simulated annealing or tabu search could be used to solve the design problem.

We conclude this section by noting that the use of a single optimization program that integrates design and production planning considerations is quite suitable for a green-field type of design, for which no current design exists. If a current design exists, however, the use of a single optimization program allows assessing a situation from a 'best-case' perspective. Furthermore, notice that we can fix the design problem (Step 2) if necessary and solve the remaining production planning problem (Step 1) using our own approach. This issue is quite important since the production planning problem is likely to be solved more frequently that the design problem.

# 4 Stage 2: Determining within-bay layouts

In Stage 2, the layout for each bay is determined so as to minimize the material handling costs within the bays, which includes product flows to and from the automated material handling system. We assume that there is one material handling system and that the within-bay costs are dependent on the within-bay distances between department replicas $n_i$ and $m_j$, $d_{n_im_j}$, which are determined by the solution to Stage 1 and Stage 2. If department replicas are assigned to the same bay ($x_{n_ib} = x_{m_jb}$), the within-bay distance is merely the vertical distance traveled between department replicas. If department replicas, however, are not assigned to the same bay ($x_{n_ib} \neq x_{m_jb}$), then we can define the distance as $d_{n_im_j} = d^V_{n_i\text{MHS}} + d^V_{m_j\text{MHS}}$, where $d^V_{n_i\text{MHS}}$ corresponds to the vertical distance between department replica $n_i$ and the material handling system.

The objective function for the layout problem in Stage 2 is:

$$\min \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{n_i=1}^{N_i}\sum_{m_j=1}^{N_j} f_{n_im_j}d_{n_im_j}, \tag{16}$$

where $f_{n_im_j} = \sum_{p=1}^{P}\bar{\lambda}_{n_im_jp}$. It must be noted that, in general, even single-bay layout problems are difficult to solve. As in Meller (1997), we are considering the special case where department replicas are arranged in a linear ordering. Such problems can be solved efficiently for the size of single-bay problems that we are likely to encounter.

The problem in Stage 2 is to determine $B$ within-bay layouts by solving $B$ independent linear ordering problems. In order to do so, we introduce $f_{n_i\mathrm{MHS}}$ as the product flow from department replica $n_i$ to the automated material handling system and $f_{\mathrm{MHS}n_i}$ as the product flow from the automated material handling system to department replica $n_i$. That is,

$$f_{n_i\mathrm{MHS}} = \sum_{j=1}^{N}\sum_{m_j=1}^{N_j} f_{n_i m_j}\Big(1-\sum_{b=1}^{B} x_{n_i b}x_{m_j b}\Big) \tag{17}$$

$$f_{\mathrm{MHS}n_i} = \sum_{j=1}^{N}\sum_{m_j=1}^{N_j} f_{m_j n_i}\Big(1-\sum_{b=1}^{B} x_{n_i b}x_{m_j b}\Big) \tag{18}$$

The length of department replica $n_i$ is equal to the area of the department replica, $a_{n_i}$, divided by the width of the bay the department replica is assigned to. We assume all distances are measured rectilinearly between department replica centroids. The solution of Stage 2 was also implemented in MATLAB and interfaced with TOMLAB 3.0. The implementation platform is a Windows-based PC with a Pentium III 933MHz processor and 512MB of RAM.

# 5 Computational results

The two-stage solution methodology proceeds as follows: in Stage 1, we use the heuristic solution procedure developed in Section 3.1 to solve for the assignment of department replicas to bays and for the product flow allocation among department replicas, and in Stage 2 we determine the layout within each bay by solving independent linear ordering problems. Since the distributed multi-bay manufacturing facility layout problem has not been previously presented, we have no examples from the literature to use. Thus, to evaluate the performance of our approach, we modified 18 problem data sets from Urban et al. (2000) to include bay area, department area, and product process routing information. (The modified data sets are available upon request from the corresponding author.) The problems range in size from two to four bays, four to ten department types, six to thirty total department replicas, and from three to nine product types.

By way of a more detailed example, we apply our approach to a problem data set with two bays, six department types, twelve total department replicas, and nine product types (problem 9-12). We assume that each bay has an area of 24 ($4 \times 6$) square distance units. The data set for this problem is presented in Table 1. Note that the capacities and areas of the replicas are the same for each department type. The solution is presented in tables 2 and 3. In the tables, the notation $n_i$ refers to the assignment of the $n$th replica of department type $i$ to a particular bay and the notation $n_i \rightarrow m_j = \lambda$ refers to the allocation of $\lambda$ units of product flow from the $n$th replica of department type $i$ to the $m$th replica of department type $j$ for a particular product type. In Table 3, the linear ordering of department replicas within each bay is such that the material handling system is located at the 'left' of the list. For Stage 1, the problem was solved with a solution of 846.78 inter-bay distance units (assuming 1 inter-bay distance unit between each bay) in 0.4 seconds, and, for Stage 2, the problem was solved with a solution of 4176.09 distance units in 1.4 seconds and 8557.02 distance units in 1.2 seconds for bays 1 and 2, respectively.

Figure 3 shows the layout solution for the detailed example (problem 9-12). Note that the dashed area in the figure is unoccupied. The product flow allocation solution for product 9 is depicted. Such product flow allocation solution is interpreted as follows: 88.3 units of the product demand are processed in department replica $1_2$ and then are moved to department replica $1_6$. The remaining 1.7 units of the product demand are processed in department replica $2_2$ and then are also moved to department replica $1_6$. At department replica $1_6$, 90 units of the product demand are processed and then moved to department replica $2_4$. Such 90 units are processed in department replica $2_4$ and then are moved to department replica $1_1$ for final processing.

For comparison purposes, the problem was also solved assuming that all machines of the same type share the same department and are located adjacent to each other; that is, assuming a functional layout. In this case, departments 3, 4, and 5 were assigned to bay 1 and departments 1, 2, and 6 were assigned to bay 2. The functional

Table 1: Data set for problem 9-12.

| | Department type | | | | | | |
|---|---|---|---|---|---|---|---|
| Product | 1 | 2 | 3 | 4 | 5 | 6 | Demand |
| 1 | 1 (5) | 20 (4) | – | 10 (1) | 25 (2) | 3 (3) | 200 |
| 2 | 2 (1) | 24 (5) | 24 (4) | 5 (3) | – | 3 (2) | 120 |
| 3 | 4 (2) | – | 32 (1) | – | 25 (3) | 3 (4) | 60 |
| 4 | 1 (4) | 16 (2) | 36 (3) | 10 (6) | 25 (5) | 3 (1) | 150 |
| 5 | 2 (5) | – | 30 (1) | 15 (2) | 25 (4) | 3 (3) | 100 |
| 6 | 3 (4) | 20 (2) | – | 15 (1) | – | 3 (3) | 30 |
| 7 | 4 (4) | 20 (1) | 26 (3) | – | 25 (2) | 3 (5) | 300 |
| 8 | 3 (2) | – | 22 (4) | 5 (5) | 25 (3) | 3 (1) | 240 |
| 9 | 5 (4) | 24 (1) | – | 10 (3) | – | 3 (2) | 90 |
| | Processing time (Routing order) | | | | | | |
| Replicas | 1 | 2 | 3 | 2 | 3 | 1 | |
| Capacity | 4320 | 9120 | 9120 | 4320 | 9120 | 4320 | |
| Area | 5 | 5 | 2 | 5 | 2 | 5 | |

layout problem was solved with a solution of 2420 inter-bay distance units (assuming 1 inter-bay distance unit between each bay) in 0.2 seconds for Stage 1 and with a solution of 7749.20 distance units in 0.3 seconds and 6896.49 distance units in 0.3 seconds for bays 1 and 2, respectively, for Stage 2. Recall that if a functional layout is assumed, there is no decision regarding the allocation of product flow between departments.

Also, for comparison purposes, in Table 4, we present Stage 1 solutions to the 18 problem data sets considering the optimal solution to the mixed-integer program DBAP2 and the solution found by the heuristic procedure outlined above. Since the heuristic solution procedure requires an initial layout, our computational strategy was to solve a problem data set many times, using a different starting layout each time. For every data set, we solved the problem 50 times with different random starting layouts. We sorted the objective function values resulting from the 50 runs and counted how many times the best objective function was found. Although we solved each problem 50 times, the best solution was always found within the first 5 attempts. In Table 4, we report the best objective function value, the average solution time (in seconds) for the 50 runs, and the percentage deviation from the optimal solution. Notice that for seven problem data sets we were able to verify that the heuristic solution procedure found the optimal solution. The average percentage deviation from the optimal solution is less than 1%.

In tables 5, 6, and 7, we extend our comparative results regarding distributed and functional layouts. Stage 1 results are presented in Table 5, while Stage 2 results are presented in tables 6 and 7. In the tables, we also present run times (in seconds) on our workstation. Even for thirty total department replicas and nine product types the run time is quite acceptable given facility layout decisions are planned on a long-term horizon of 5 to 10 years. As expected, the solutions to the distributed layouts are always better than the solutions to the functional layouts. Considering both Stage 1 and Stage 2, we achieved an average percentage improvement of over 38% relative to traditional functional facility layout approaches. As expected, it is clear from Table 5 that the larger number of total department replicas (i.e., the larger the layout size), the larger the potential percentage improvement. We note, however, that the percentage improvement depends on the amount of department disaggregation and on the product and process characteristics. Our problem data sets and comparative results are based on the maximum amount of department disaggregation possible between functional and distributed layouts. In practice, the amount of department disaggregation must be carefully planned in order to cope with the loss of pooling synergy.

In order to examine the effect of department disaggregation on the comparative results between functional and distributed layouts, we conducted a series of experiments with 10 problem data sets under different levels of disaggregation. We examined four levels of department disaggregation. In the first level, we assumed that all machines of the same type share the same department and are located adjacent to each other; that is, assuming a functional layout. In the second level, the department type with the highest production capacity (the largest total

Table 2: Product flow allocation solution for problem 9-12.

| Product | Product flow | | | | |
|---|---|---|---|---|---|
| 1 | $2_4 \rightarrow 1_5 = 200.0$ | $1_5 \rightarrow 1_6 = 200.0$ | $1_6 \rightarrow 1_2 = 200.0$ | $1_2 \rightarrow 1_1 = 200.0$ | |
| 2 | $1_1 \rightarrow 1_6 = 120.0$ | $1_6 \rightarrow 1_4 = 120.0$ | $1_4 \rightarrow 3_3 = 120.0$ | $3_3 \rightarrow 2_2 = 120.0$ | |
| 3 | $2_3 \rightarrow 1_1 = 58.6$ | $1_1 \rightarrow 1_5 = 60.0$ | $1_5 \rightarrow 1_6 = 60.0$ | | |
| | $3_3 \rightarrow 1_1 = 1.4$ | | | | |
| 4 | $1_6 \rightarrow 1_2 = 150.0$ | $1_2 \rightarrow 2_3 = 150.0$ | $2_3 \rightarrow 1_1 = 150.0$ | $1_1 \rightarrow 1_5 = 4.8$ | $1_5 \rightarrow 2_4 = 4.8$ |
| | | | | $1_1 \rightarrow 2_5 = 145.2$ | $2_5 \rightarrow 1_4 = 145.2$ |
| 5 | $1_3 \rightarrow 1_4 = 38.5$ | $1_4 \rightarrow 1_6 = 38.5$ | $1_6 \rightarrow 1_5 = 100.0$ | $1_5 \rightarrow 1_1 = 100.0$ | |
| | $2_3 \rightarrow 2_4 = 61.5$ | $2_4 \rightarrow 1_6 = 61.5$ | | | |
| 6 | $2_4 \rightarrow 1_2 = 30.0$ | $1_2 \rightarrow 1_6 = 30.0$ | $1_6 \rightarrow 1_1 = 30.0$ | | |
| 7 | $2_2 \rightarrow 2_5 = 196.8$ | $2_5 \rightarrow 3_3 = 196.8$ | $1_3 \rightarrow 1_1 = 103.2$ | $1_1 \rightarrow 1_6 = 300.0$ | |
| | $2_2 \rightarrow 3_5 = 103.2$ | $3_5 \rightarrow 1_3 = 103.2$ | $3_3 \rightarrow 1_1 = 196.8$ | | |
| 8 | $1_6 \rightarrow 1_1 = 240.0$ | $1_1 \rightarrow 2_5 = 22.8$ | $2_5 \rightarrow 1_3 = 22.8$ | $1_3 \rightarrow 1_4 = 240.0$ | |
| | | $1_1 \rightarrow 3_5 = 217.2$ | $3_5 \rightarrow 1_3 = 217.2$ | | |
| 9 | $1_2 \rightarrow 1_6 = 88.3$ | $1_6 \rightarrow 2_4 = 90.0$ | $2_4 \rightarrow 1_1 = 90.0$ | | |
| | $2_2 \rightarrow 1_6 = 1.7$ | | | | |

Table 3: Layout solution for problem 9-12.

| | | Stage 1: Heuristic | | Stage 2 | |
|---|---|---|---|---|---|
| Bay # | Department replica assignment | Distance | Time | Distance | Time |
| 1 | $2_5, 3_5, 3_3, 1_4, 1_3, 2_2$ | 846.78 | 0.4 | 4176.09 | 1.4 |
| 2 | $2_4, 1_5, 1_6, 2_3, 1_1, 1_2$ | | | 8557.02 | 1.2 |

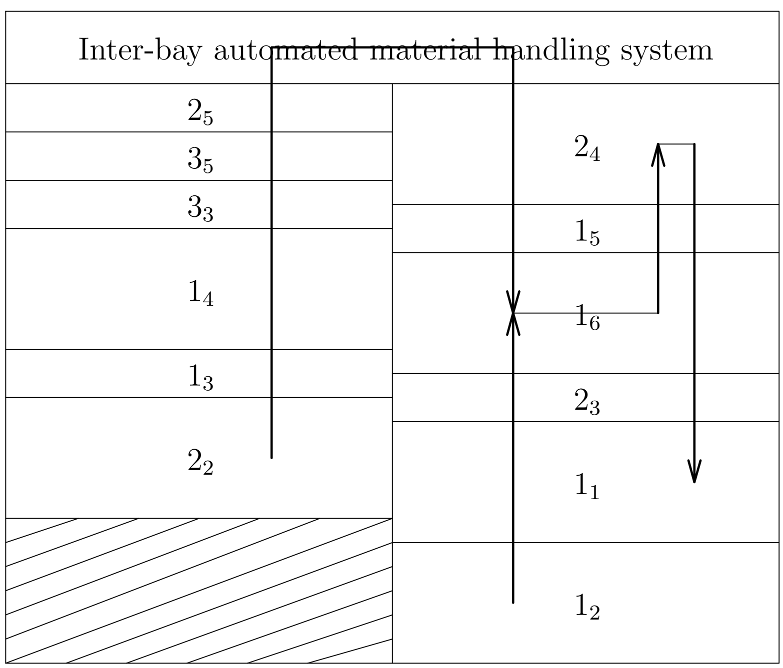


Figure 3: Layout solution for problem 9-12. The dashed area is unoccupied. The product flow allocation solution for product 9 is depicted.

area) is duplicated (i.e., the department type is disaggregated into two department replicas) having half the total production capacity and occupying half the total area. In the third level, the department type with the highest production capacity (the largest total area) is disaggregated into three department replicas. Finally, in the fourth level, the department type with the highest production capacity (the largest total area) is disaggregated into six department replicas. Results based on disaggregating the department type with the highest production capacity are presented in Table 8, while results based on disaggregation the department type with the largest total area are presented in Table 9.

As we can see, the manufacturing facility layouts obtained from optimally solving a distributed formulation are least as good as the ones obtained from optimally solving the bay assignment formulation. That is, disaggregation is never detrimental in terms of the inter-bay material movement. More importantly, we can see that the effect of department disaggregation on the optimal solution is of the diminishing kind, with most of the benefits realized with modest disaggregation (a single department type disaggregated into two department replicas). In fact, even with such a modest disaggregation, inter-bay material movement can be reduced by 52% relative to traditional functional facility layout approaches.

# 6 Conclusions

In this paper, we consider a distributed multi-bay manufacturing facility layout problem in which replicas of a given department type may exist in the facility. We accepted the premise that the distribution of department replicas throughout the facility can be a useful strategy that can be exploited to find more efficient layout solutions. Thus, we allowed these replicas, in contrast to traditional practice, to be assigned to different bays and product flow allocation between pairs of individual department replicas to be made as function of the facility layout. Thus, integrating design and production planning considerations at the facility layout design stage while considering practical material handling and production features such as alternate process routings, product flow production patterns, and department replica capacities.

The special structure of the distributed multi-bay manufacturing facility layout problem led to a two-stage solution methodology, where in Stage 1 we solve for the assignment of department replicas to bays and for the product flow allocation among department replicas, and in Stage 2 we determine the layout within each bay. Our comparative results indicate that there is a compelling argument for the need to consider department replication in multi-bay manufacturing facility layout. Indeed, computational results on our problem data sets indicate that department replication can reduce material movement significantly while maintaining the existing production capacity.

The major contributions of this paper lie in the following aspects:

(1) We formulated a mixed-integer program that integrates design and production planning considerations. The design problem represents an attempt to achieve distribution of individual department replicas throughout the facility in a meaningful manner. In addition, the production planning problem allocates product flow (and consequently improves layout solutions) as a function of the product demand realization to the best available routes.

(2) We extended the design of bays to be composed of processing units beyond those that have been traditionally studied and implemented. In addition, we extended the use of product and process requirements as problem parameters in order to increase process routing flexibility.

(3) As with any combinatorial optimization problem that is not amenable to optimal solution procedures, a heuristic is necessary for the solution of Stage 1. We developed a decomposition approach that exploits the structure of the formulation using a heuristic procedure that solves a distributed bay assignment problem with fixed product flows followed by a product flow allocation problem with fixed facility layout. In our suite of test problems, the average percentage deviation from the optimal solution is less than 1%.

(4) We achieved an average percentage improvement of over 38% relative to traditional functional facility layout approaches. That is, we can substantiate the premise that the distribution of department replicas throughout the facility increases the accessibility of these replica types from different bays and, in turn, decreases material movement.

Table 4: Stage 1: Computational results for test problems.

| Problem | Products | Total department replicas | Department types | Bays | DBAP2 Distance | DBAP2 Time | Heuristic Best distance | Heuristic Avg. Time | Heuristic No. of times best solution is obtained | Percentage deviation from optimal |
|---|---|---|---|---|---|---|---|---|---|---|
| 3-6 | 3 | 6 | 4 | 2 | 68.33 | 0.0 | 68.33 | 0.2 | 11 | 0.00% |
| 6-6 | 6 | 6 | 5 | 2 | 430.00 | 0.1 | 430.00 | 0.2 | 13 | 0.00% |
| 9-6 | 9 | 6 | 4 | 2 | 80.00 | 0.1 | 80.00 | 0.2 | 25 | 0.00% |
| 3-9 | 3 | 9 | 4 | 2 | 40.50 | 0.2 | 40.50 | 0.2 | 11 | 0.00% |
| 6-9 | 6 | 9 | 4 | 2 | 24.40 | 1.0 | 24.89 | 0.2 | 14 | 2.01% |
| 9-9 | 9 | 9 | 6 | 2 | 806.67 | 0.6 | 806.67 | 0.3 | 13 | 0.00% |
| 3-12 | 3 | 12 | 5 | 2 | 14.00 | 0.4 | 14.45 | 0.3 | 21 | 3.21% |
| 6-12 | 6 | 12 | 7 | 2 | 156.17 | 4.8 | 162.00 | 0.4 | 9 | 3.73% |
| 9-12 | 9 | 12 | 6 | 2 | 846.78 | 6.8 | 846.78 | 0.4 | 11 | 0.00% |
| 3-15 | 3 | 15 | 8 | 3 | 1583.91 | 966.7 | 1583.91 | 1.0 | 14 | 0.00% |
| 6-15 | 6 | 15 | 6 | 3 | 603.00* | – | 644.05 | 2.6 | 21 | – |
| 9-15 | 9 | 15 | 5 | 3 | 184.60* | – | 184.60 | 3.1 | 10 | – |
| 3-20 | 3 | 20 | 7 | 3 | 178.41* | – | 207.00 | 3.3 | 12 | – |
| 6-20 | 6 | 20 | 6 | 3 | 83.98* | – | 86.98 | 5.3 | 9 | – |
| 9-20 | 9 | 20 | 8 | 3 | 3568.53* | – | 3498.77 | 6.4 | 9 | – |
| 3-30 | 3 | 30 | 6 | 4 | 269.03* | – | 241.04 | 466.4 | 10 | – |
| 6-30 | 6 | 30 | 10 | 4 | 632.13* | – | 497.92 | 965.3 | 12 | – |
| 9-30 | 9 | 30 | 8 | 4 | 302.83* | – | 275.10 | 1051.7 | 9 | – |

* Best solution found within 1 hour.

Table 5: Stage 1: Comparative results for test problems.

| | Functional layout | | Distributed layout | | Percentage |
|---|---|---|---|---|---|
| Problem | Distance | Time | Distance | Time | improvement |
| 3-6 | 115.00 | 0.0 | 68.33 | 0.1 | 40.58% |
| 6-6 | 430.00 | 0.1 | 430.00 | 0.1 | 0.00% |
| 9-6 | 170.00 | 0.1 | 80.00 | 0.1 | 52.94% |
| 3-9 | 90.00 | 0.1 | 40.50 | 0.1 | 55.00% |
| 6-9 | 30.00 | 0.1 | 29.79 | 0.1 | 0.70% |
| 9-9 | 1105.00 | 0.1 | 806.67 | 0.2 | 27.00% |
| 3-12 | 18.00 | 0.2 | 14.45 | 0.3 | 19.72% |
| 6-12 | 405.00 | 0.3 | 162.00 | 0.4 | 60.00% |
| 9-12 | 2420.00 | 0.2 | 846.78 | 0.4 | 65.01% |
| 3-15 | 4280.00 | 0.5 | 1583.91 | 1.0 | 62.99% |
| 6-15 | 1590.00 | 0.3 | 644.05 | 2.6 | 59.49% |
| 9-15 | 408.00 | 0.2 | 184.60 | 3.1 | 54.75% |
| 3-20 | 649.00 | 0.3 | 207.00 | 3.3 | 68.10% |
| 6-20 | 307.00 | 0.3 | 86.98 | 5.3 | 71.67% |
| 9-20 | 10275.00 | 0.5 | 3498.77 | 6.4 | 65.95% |
| 3-30 | 545.00 | 0.2 | 241.04 | 466.4 | 55.77% |
| 6-30 | 1688.00 | 2.1 | 497.92 | 965.3 | 70.50% |
| 9-30 | 1028.00 | 0.9 | 275.10 | 1051.7 | 73.24% |

Table 6: Stage 2: Comparative results for test problems.

| | | Functional layout | | | Distributed layout | | | Percentage |
|---|---|---|---|---|---|---|---|---|
| Problem | Bay # | Layout solution | Distance | Time | Layout solution | Distance | Time | improvement |
| 3-6 | 1 | $1_4, 1_3$ | 224.38 | 0.2 | $1_4, 1_1, 2_2$ | 332.32 | 0.2 | 16.44% |
| | 2 | $1_1, 1_2$ | 370.68 | 0.1 | $1_2, 1_3, 2_3$ | 164.92 | 0.2 | |
| 6-6 | 1 | $1_1, 1_4, 1_3$ | 1802.62 | 0.3 | $1_1, 1_4, 1_3$ | 1802.62 | 0.2 | 3.95% |
| | 2 | $1_2$ | 1021.41 | 0.1 | $1_2, 3_2, 2_2$ | 910.00 | 0.2 | |
| 9-6 | 1 | $1_1, 1_2$ | 385.01 | 0.2 | $1_1, 1_4, 2_3$ | 415.04 | 0.2 | 25.05% |
| | 2 | $1_4, 1_3$ | 396.26 | 0.2 | $1_2, 1_3, 2_2$ | 170.53 | 0.3 | |
| 3-9 | 1 | $1_5, 1_1, 1_3$ | 231.25 | 0.2 | $1_2, 2_3, 3_2, 1_4$ | 71.92 | 0.3 | 14.89% |
| | 2 | $1_4, 1_2$ | 276.26 | 0.2 | $2_4, 1_3, 1_5, 1_1, 2_2$ | 360.04 | 0.2 | |
| 6-9 | 1 | $1_2, 1_1$ | 108.75 | 0.1 | $1_2, 2_3, 3_2, 1_4$ | 75.59 | 0.3 | 20.80% |
| | 2 | $1_3, 1_4$ | 137.78 | 0.2 | $1_5, 1_1, 2_2, 2_4, 1_3$ | 119.65 | 0.4 | |
| 9-9 | 1 | $1_1, 1_6, 1_5, 1_4$ | 4730.62 | 0.2 | $3_2, 1_1, 1_6, 1_5, 1_4$ | 4414.43 | 0.3 | 15.58% |
| | 2 | $1_3, 1_2$ | 2850.67 | 0.2 | $1_3, 1_2, 2_4, 2_2$ | 1985.94 | 0.3 | |
| 3-12 | 1 | $1_5, 1_4, 1_3$ | 64.75 | 0.3 | $2_3, 2_2, 5_4, 1_4, 3_2$ | 34.45 | 0.3 | 10.23% |
| | 2 | $1_1, 1_2$ | 78.50 | 0.2 | $3_4, 1_5, 1_3, 2_4, 4_4, 1_1, 1_2$ | 94.15 | 0.3 | |
| 6-12 | 1 | $1_7, 1_1, 1_3, 1_2$ | 1495.34 | 0.2 | $2_3, 1_7, 1_3, 1_2, 1_1, 2_4, 2_5$ | 1294.27 | 0.3 | 30.98% |
| | 2 | $1_5, 1_4, 1_6$ | 1299.75 | 0.3 | $1_6, 1_5, 3_4, 2_2, 1_4$ | 634.82 | 0.3 | |
| 9-12 | 1 | $1_5, 1_3, 1_4$ | 7749.20 | 0.3 | $2_5, 3_5, 3_3, 1_4, 1_3, 2_2$ | 4176.09 | 1.4 | 13.06% |
| | 2 | $1_1, 1_6, 1_2$ | 6896.49 | 0.3 | $2_4, 1_5, 1_6, 2_3, 1_1, 1_2$ | 8557.02 | 1.2 | |
| 3-15 | 1 | $1_1, 1_8, 1_7$ | 5882.39 | 0.3 | $2_3, 2_2, 2_6, 1_7, 1_4$ | 2497.39 | 0.6 | 49.87% |
| | 2 | $1_4, 1_6, 1_5$ | 9080.54 | 0.3 | $1_8, 1_1, 1_5, 1_2$ | 4840.08 | 0.7 | |
| | 3 | $1_2, 1_3$ | 4945.78 | 0.2 | $3_4, 3_2, 1_6, 3_6, 2_4, 1_3$ | 2643.27 | 0.6 | |
| 6-15 | 1 | $1_2, 1_3$ | 3108.31 | 0.2 | $2_2, 4_4, 4_2, 1_4, 3_4, 3_5$ | 1210.68 | 0.7 | 42.30% |
| | 2 | $1_6, 1_1, 1_4$ | 2744.93 | 0.4 | $1_1, 1_6, 3_2, 1_3, 2_5$ | 2030.96 | 0.8 | |
| | 3 | $1_5$ | 892.14 | 0.1 | $2_3, 1_2, 2_4, 1_5$ | 650.13 | 0.7 | |
| 9-15 | 1 | $1_1, 1_3$ | 692.00 | 0.2 | $3_2, 2_2, 2_3, 4_2$ | 200.33 | 1.1 | 28.92% |
| | 2 | $1_5, 1_4$ | 486.10 | 0.2 | $1_1, 1_5, 2_4, 1_3, 1_2$ | 580.63 | 1.4 | |
| | 3 | $1_2$ | 346.55 | 0.1 | $6_4, 4_4, 3_3, 3_4, 5_4, 1_4$ | 302.77 | 1.3 | |

Table 7: Stage 2: Comparative results for test problems (cont.).

| | | Functional layout | | | Distributed layout | | | Percentage |
|---|---|---|---|---|---|---|---|---|
| Problem | Bay # | Layout solution | Distance | Time | Layout solution | Distance | Time | improvement |
| 3-20 | 1 | $1_3, 1_6$ | 423.85 | 0.2 | $1_2, 2_4, 2_5, 3_4, 1_3, 3_3, 1_6$ | 453.17 | 0.7 | 38.23% |
| | 2 | $1_1, 1_5, 1_2$ | 1470.33 | 0.3 | $3_2, 6_3, 1_5, 1_7, 1_1, 4_3, 5_3$ | 947.26 | 0.6 | |
| | 3 | $1_7, 1_4$ | 720.10 | 0.2 | $2_3, 3_6, 4_4, 2_2, 1_4, 2_6$ | 214.30 | 0.6 | |
| 6-20 | 1 | $1_1, 1_5$ | 343.30 | 0.2 | $3_4, 2_5, 3_3, 2_4, 2_2, 1_5, 3_2$ | 127.49 | 0.7 | 16.22% |
| | 2 | $1_4, 1_3$ | 343.19 | 0.2 | $3_5, 4_4, 1_6, 1_3, 1_1, 5_2$ | 487.35 | 0.7 | |
| | 3 | $1_6, 1_2$ | 237.64 | 0.2 | $4_5, 2_3, 6_5, 5_5, 4_2, 1_4, 1_2$ | 159.36 | 0.6 | |
| 9-20 | 1 | $1_1, 1_6$ | 4110.69 | 0.2 | $1_4, 3_3, 3_6, 4_6, 1_5, 3_4, 2_3$ | 7840.80 | 1.4 | 34.85% |
| | 2 | $1_8, 1_7, 1_5$ | 15840.31 | 0.3 | $1_8, 1_1, 1_2, 2_4, 1_6, 2_7, 1_7$ | 14621.00 | 1.7 | |
| | 3 | $1_2, 1_3, 1_4$ | 14778.18 | 0.3 | $2_6, 1_3, 2_5, 2_2, 4_3, 3_5$ | 6677.98 | 1.3 | |
| 3-30 | 1 | $1_5$ | 131.31 | 0.1 | $4_4, 3_3, 1_5, 3_2, 1_3, 7_5, 4_3, 3_4$ | 348.97 | 0.5 | 43.50% |
| | 2 | $1_6, 1_3$ | 1360.23 | 0.2 | $1_1, 1_6, 2_3, 5_3, 2_4, 6_4, 5_4$ | 892.44 | 0.6 | |
| | 3 | $1_1, 1_4$ | 859.92 | 0.2 | $4_2, 1_2, 7_3, 6_3, 8_3, 1_4, 5_5$ | 320.51 | 0.6 | |
| | 4 | $1_2$ | 433.19 | 0.1 | $8_5, 2_2, 6_5, 10_5, 2_5, 3_5, 4_5, 9_5$ | 11.54 | 0.6 | |
| 6-30 | 1 | $1_2, 1_5, 1_7$ | 1180.20 | 0.3 | $1_8, 1_9, 2_4, 4_3, 2_8, 2_5, 1_6, 3_6, 1_4$ | 1243.85 | 0.6 | 40.45% |
| | 2 | $1_1, 1_9, 1_10$ | 2251.24 | 0.3 | $1_5, 1_1, 1_10, 3_3, 2_9, 1_2$ | 1805.70 | 0.8 | |
| | 3 | $1_8, 1_3, 1_4$ | 2286.40 | 0.3 | $3_5, 3_9, 3_7, 3_8, 1_3, 4_4, 2_6, 2_2, 1_7$ | 888.43 | 0.7 | |
| | 4 | $1_6$ | 984.53 | 0.1 | $3_4, 2_3, 4_8, 4_6, 2_7, 5_6$ | 53.59 | 0.6 | |
| 9-30 | 1 | $1_1, 1_2$ | 565.02 | 0.2 | $5_6, 1_7, 2_3, 6_6, 3_4, 4_4, 3_3, 1_4$ | 496.98 | 1.4 | 38.36% |
| | 2 | $1_8, 1_5, 1_3$ | 1159.54 | 0.3 | $2_7, 4_6, 1_8, 1_5, 8_6, 1_1, 4_3, 1_2$ | 1086.19 | 1.3 | |
| | 3 | $1_7, 1_6$ | 859.25 | 0.2 | $1_3, 2_2, 2_4, 2_5, 2_6, 3_2$ | 324.95 | 1.7 | |
| | 4 | $1_4$ | 593.34 | 0.1 | $5_4, 3_6, 3_7, 4_2, 7_6, 5_3, 1_6, 6_3$ | 50.18 | 1.1 | |

Table 8: Effect of highest production capacity department disaggregation on inter-bay material movement.

| | Disaggregation level | | | |
|---|---|---|---|---|
| Problem | 1 | 2 | 3 | 4 |
| 3-6 | 115.00 | 68.33 | 55.00 | 55.00 |
| 6-6 | 430.00 | 430.00 | 430.00 | 430.00 |
| 9-6 | 170.00 | 80.00 | 80.00 | 80.00 |
| 3-9 | 90.00 | 90.00 | 90.00 | 90.00 |
| 6-9 | 30.00 | 30.00 | 30.00 | 30.00 |
| 9-9 | 1105.00 | 1105.00 | 1105.00 | 1105.00 |
| 3-12 | 18.00 | 16.55 | 14.73 | 14.73 |
| 6-12 | 405.00 | 354.00 | 354.00 | 342.00 |
| 9-12 | 2420.00 | 2060.00 | 2060.00 | 2060.00 |
| 3-15 | 4280.00 | 3370.00 | 2980.00 | 2980.00 |

Furthermore, the overall modeling approach and solution procedure presented in this paper is a prototype that could be embellished to incorporate even broader issues such as prescribing the number of bays and their areas, and the types of material handling devices. In addition, other constraints could be incorporated, for instance, to limit inter-bay material movement, the fraction of a product type that flows through more than one bay, or the utilization of department replicas.

We conclude by stating that a considerable amount of research has focused on developing facility layout approaches that follow the product family (or cellular) layout paradigm or the functional layout paradigm. Recent studies have been proposed to capture the benefits of product family (or cellular) layouts while attempting to overcome the associated shortcomings. See, for example, Askin et al. (1999), Montreuil and Venkatardi (1991), and Montreuil et al. (1993). Note that the purpose of the previous work was different, namely, the goal was the even distribution of department replicas in order to provide flexibility; whereas our purpose is to allow some disaggregation as a means of reducing material handling costs.

Our results establish an important trade-off between investing in additional production capacity or disaggregating existing departments into smaller processing units, perhaps located in different bays, at the cost of additional material movement. In the proposed approach, it is not necessary to identify product families *a priori* as in earlier approaches that require either the subjective use of somewhat arbitrary measures of similarity or *ad hoc* methods to identify bottleneck resources and exceptional elements. Nonetheless, department disaggregation should not be pursued blindly, especially if setups are significant. We found, however, that the optimal solution tends to favor product flow allocations that consolidate the production of each product type in as few department replicas as possible, and that the effect of department disaggregation on the optimal solution is of the diminishing kind, with most of the benefits realized with modest disaggregation. Thus, even in the presence of setups, material movement can significantly be reduced without a major loss of pooling synergy.

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Table 9: Effect of largest total area department disaggregation on inter-bay material movement.

| | Disaggregation level | | | |
|---|---|---|---|---|
| Problem | 1 | 2 | 3 | 4 |
| 3-6 | 115.00 | 68.33 | 55.00 | 55.00 |
| 6-6 | 430.00 | 430.00 | 430.00 | 430.00 |
| 9-6 | 170.00 | 105.00 | 105.00 | 105.00 |
| 3-9 | 90.00 | 65.00 | 65.00 | 65.00 |
| 6-9 | 30.00 | 30.00 | 30.00 | 30.00 |
| 9-9 | 1105.00 | 1105.00 | 1105.00 | 1105.00 |
| 3-12 | 18.00 | 16.55 | 14.73 | 14.73 |
| 6-12 | 405.00 | 354.00 | 354.00 | 342.00 |
| 9-12 | 2420.00 | 2114.00 | 1980.00 | 1980.00 |
| 3-15 | 4280.00 | 3110.00 | 3110.00 | 3110.00 |


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