Asymptotic approximation method and its convergence on semi-infinite programming*

Zhong-Ping Wan†
School of Mathematics and Statistics,
Wuhan Univ., Wuhan 430072 China

Xian-Jia Wang
Institute of Systems Engineering
Wuhan Univ., Wuhan 430072 China

Guang-Min Wang

School of Mathematics and Statistics,
Wuhan Univ., Wuhan 430072 China

Abstract

The aim of this paper is to discuss an asymptotic approximation model and its convergence for the minimax semi-infinite programming problem. An asymptotic surrogate constraints method for the minimax semi-infinite programming problem is presented making use of two general discrete approximation methods. Simultaneously, we discuss the consistence and the epi-convergence of the asymptotic approximation problem.
Key words: semi-infinite programming; asymptotic approximation; convergence.
AMS Subject Classification 90C34

1 Introduction

This paper mainly consider the following semi-infinite programming problem[7]:

$$\begin{aligned} SIMP: \qquad & \text{Min} \quad \Psi^0(x) \\ & \text{s.t.} \quad x \in X \overset{\triangle}{=} \{x \in \mathrm{D} \subset R^n \mid \Psi^j(x) \leq 0, j \in Q\}. \end{aligned} \tag{1}$$

For any $j \in \bar{q} \overset{\triangle}{=} \{0\} \bigcup Q = \{0, 1, \cdots, q\}$, the function $\Psi^j(x)$ is defined by

$$\Psi^j(x) = \max_{y_j \in Y_j} \phi^j(x, y_j).$$

If let

$$\Psi(x) = \max_{j \in Q} \Psi^j(x). \tag{2}$$

*Supported by the National Natural Science Foundation(60274048) and the Doctoral Foundation in Ministry of Education of China(20020486035).
†Corresponding author. E-mail:zpwan@public.wh.hb.cn

Then the problem*SIMP*is equivalent to

$$\begin{aligned} \textit{SIMP}: \qquad & \text{Min} \quad \Psi^0(x) \\ & \text{s.t.} \quad x \in X = \{x \in \mathrm{D} \subset R^n \mid \Psi(x) \le 0\}. \end{aligned} \tag{3}$$

First ,we make the following assumption

Assumption 1 *1. For any $j \in \bar{q}$,the function $\phi^j(\cdot,\cdot)$ is continuous. and its gradient $\nabla_x \phi^j(\cdot,\cdot)$ exists and is continuous;*

2. The subset D is a bounded close set, and Y_j is a compact set;

3. The solution to the problem (SIMP) *exists.*

The approximate consistence of the problem (1) and the algorithm based on the optimal function were discussed in [7]. This optimal function is non-differentiable, a global convergence algorithm to some semi-infinite programming problem was presented in [10]. The maximum-entropy asymptotic approximation method to SIMP was gived in [1, 4, 11], In [2], authors summarized theories and computing methods about the semi-infinite programming problem, and so on. Researches about the optimal theory for the SIMP refer to[2, 7].

Here, an asymptotic surrogate constraints method for the semi-infinite programming problem is presented making use of two general discrete approximation methods. Simultaneously, we discuss the consistence and the convergence of the problem.

2 Asymptotic approximation and its convergence

We mainly discuss the convergence of asymptotic approximation of the parametric set Y_j $(j \in \bar{q})$ in the sense of *Kuratowski* This approximate model is different from those of [2, 7, 8, 9], and is their extension to some extent.

Approximation process: Discrete the compact set Y_j by some approximation model,to make its approximate sequence $\{Y_j^k\}$ convergence to Y_j in the sense of *Kuratowski*.

Definition 1 *The set sequence $\{Y_j^k\}$ converges to Y_j in the sense of* Kuratowski*, if for any $j \in \bar{q}$, we can construct the set sequence $\{Y_j^k\}$,such that*

$$\liminf Y_j^k \supset Y_j \supset \limsup Y_j^k$$

denoted by $Y_j^k \xrightarrow{\mathrm{K}} Y_j$. Where

$$\liminf Y_j^k = \{y_j \mid \textit{there is } y_j^k \textit{ such that } y_j^k \to y_j \textit{ except for finite number of } k, y_j^k \in Y_j^k\},$$

$$\limsup Y_j^k = \{y_j \mid \textit{there is the subsequence } \{k_i\} \textit{ such that } y_j^{k_i} \to y_j \textit{ and } y_j^{k_i} \in Y_j^{k_i}\}.$$

For example, for all $j \in \bar{q}$, Y_j is an unit cube in R^n, i.e.$Y_j = \mathcal{I}^{m_j}$,(m_j is an arbitrarily positive integer) where $\mathcal{I} \triangleq [0,\ 1]$. We define $Y_j^k = \mathcal{I}_k^{m_j}$, where

$$\mathcal{I}_k = \left\{0,\ \frac{1}{a(k)},\ \frac{2}{a(k)},\ \cdots,\ \frac{a(k)-1}{a(k)}\right\},$$

and $a(k) = k_0 2^{k-k_0}$ (for some fixed positive integer k_0). It's not difficult to verify $\mathcal{I}_k \xrightarrow{K} \mathcal{I}$. When the compact set Y_j is irregular,by the approximation model,we can adopt the networks of lines paralleling to the coordinate axis to give a partition of Y_j,which brings great convenience to approximation.

Thus,we can get the approximation problem of *SIMP*:

$$\begin{aligned} SIMP_k: \qquad & \text{Min} \quad \Psi_k^0(x) \\ & \text{s.t.} \quad x \in X_k = \{x \in \mathrm{D} \subset R^n \mid \Psi_k^j(x) \le 0, j \in Q\}, \end{aligned} \tag{4}$$

where

$$\Psi_k^j = \max\nolimits_{j \in Y_j^k} \phi^j(x, y_j).$$

If let

$$\Psi_k(x) = \max\nolimits_{j \in Q} \Psi_k^j(x). \tag{5}$$

The problem $SIMP_k$ is equivalent to

$$\begin{aligned} SIMP_k: \qquad & \text{Min} \quad \Psi_k^0(x) \\ & \text{s.t.} \quad x \in X_k \triangleq \{x \in \mathrm{D} \subset R^n \mid \Psi_k(x) \le 0\}. \end{aligned} \tag{6}$$

To obtain the iterated point x^{k+1},the general algorithm resolving the problem (3) is described as follows:

The general algorithm model:

Step 0. $k \leftarrow 0$, and fix $x^0 \in D$.

Step 1. $k \leftarrow k+1$, compute Ψ_k^0 and Ψ_k.

Step 2. If the programming $SIMP_k$ is not feasible, then $x^{k+1} \leftarrow x^k$. Otherwise,compute

$$x^{k+1} \in \text{argmin}\{\Psi_k^0(x) | \Psi_k(x) \le 0,\ x \in D\}.$$

repeat Step 1.

By the approximation process, $Y_j^k \xrightarrow{K} Y_j$, thus for any $y \in Y_j$, there is always a sequence $\{y^k\}$ such that $\{y^k\} \to y$ and for the large enough k, $\{y^k\} \subset Y_j^k$ always holds, then for any large enough k there is always the subsequence Y_j^k(we can put it as itself), such that as $y^{'} \in Y_j^k$

$$\|y - y^{'}\| \leq K \bigtriangleup (k)$$

holds, where the monotone decreasing function $\bigtriangleup : k \to R$ and $\bigtriangleup(k) \searrow 0\,(k \to +\infty)$. For convenience,we make the further assumption.

Assumption 2 *:*

1. *For each k, the problem SIMP_k has solutions;*
2. *There is a monotone decreasing function $\bigtriangleup : k \to R$ and $\bigtriangleup(k) \searrow 0\,(k \to +\infty)$, Simultaneously,there exists k_0,for each $y \in Y_j$;there is $y^{'} \in Y_j^k\ (k > k_0)$ such that*

$$\|y - y^{'}\| \leq K \bigtriangleup (k).$$

Definition 2 *The sequence $\{f_k\}$ is said to epiconverge to f ([5, 6]etc. if the following conditions are satisfied, denoted by $f_k \overset{\text{epi}}{\to} f$,*

$$\textit{for any} x, \textit{there is always a sequence}\{x_k\}\textit{converging to } x,$$

$$\textit{such that} f(x) \geq \limsup_{k\to\infty} f_k(x_k); \tag{7}$$

$$\textit{for any} x \textit{ and any sequence } \{x_k\} \textit{ converging to } x, f(x) \leq \liminf_{k\to\infty} f_k(x_k). \tag{8}$$

Definition 3 *The approximation problem sequence $\{\mathrm{SIMP}_k\}$ epiconverges to the problem* SIMP, *If Ψ_k^0 epiconverges to Ψ^0 and the set sequence $\{X_k\}$ converges to X in the sense of* Kuratowski.

Definition 4: The problems $\{\mathrm{SIMP}_k\}$ is consistent approximation, if the problem $\{\mathrm{SIMP}_k\}$ epiconverges to the problem SIMP. *Denoted by* $\mathrm{SIMP}_k \overset{\text{epi}}{\to} \mathrm{SIMP}$.

According to the above approximation process, we have

Lemma 1 *Assume there is a Lipschtize constant L,for any x, $j \in \bar{q}$*

$$\left|\phi^j(x, y_j) - \phi^j(x, y_j^{'})\right| \leq L \bigtriangleup (k),$$

$$\left\|\nabla\phi^j(x, y_j) - \nabla\phi^j(x, y_j^{'})\right\| \leq L \bigtriangleup (k).$$

Then, under the conditions 1 and 2, there is a constant C, such that, for all $x \in D$,

$$|\Psi_k(x) - \Psi(x)| \leq C \bigtriangleup (k) \tag{9}$$

$$\left|\Psi_k^0(x) - \Psi^0(x)\right| \leq C \bigtriangleup (k). \tag{10}$$

Proof: Since $Y_j^k \subseteq Y_j$, then $\Psi_k(x) \leq \Psi(x)$.For any $x \in D$ there is j_x and $y_x \in Y_{j_x}$, such that

$$\Psi(x) = \phi^{j_x}(x, y_x).$$

By assumption 2, there is a $y_x^{'} \in Y_{j_x}^k$, such that $\| y_x - y_x^{'} \| \leq K \triangle (k)$. Hence

$$\Psi_k(x) \geq \phi^{j_x}(x, y_{j_x}^{'}) \geq \phi^{j_x}(x, y_x) - LK\triangle(k) = \Psi(x) - LK\triangle(k).$$

Take $C = LK$,then (9) holds. In the similar way, we can prove (10)holds. $\sharp$

Lemma 2 *Under the assumption conditions in Lemma 1, assume $\bar{x}$ is the accumulation point of the sequence $\{x^k\}$, then*

$$\lim_{k \in \mathcal{N}} \Psi_k^0(x^k) = \Psi^0(\bar{x}) \tag{11}$$

$$\lim_{k \in \mathcal{N}} \Psi_k(x^k) = \Psi(\bar{x}) \tag{12}$$

If the sequence $\{x^k\}$ has the unique accumulation point $\bar{x}$,then

$$\lim_{k \in \mathcal{N}} [\Psi_k(x^k) - \Psi_{k-1}(x^k)] = 0. \tag{13}$$

Where $\mathcal{N}$ is an index set satisfying that the sequence $\{x^k\}$ converges to $\bar{x}$, i.e. $\mathcal{N} : x^k \to \bar{x}$.

Proof: Obviously, we have

$$|\Psi_k^0(x^k) - \Psi^0(\bar{x})| \leq |\Psi_k^0(x^k) - \Psi^0(x^k)| + |\Psi^0(x^k) - \Psi^0(\bar{x})|.$$

Then by the assumption condition ,for any $\varepsilon > 0$,there is an integer K_1, as $k > K_1$

$$|\Psi^0(x^k) - \Psi^0(\bar{x})| < \frac{\varepsilon}{2}.$$

By Lemma 1,there is an integer $K_2 > 0$, as $k > K_2$,$\Delta(k) < \frac{\varepsilon}{2C}$. Hence, as $k \geq \max\{K_1, K_2\}$

$$\left|\Psi_k^0(x^k) - \Psi^0(\bar{x})\right| < \varepsilon.$$

In the similar way,we can prove (12) holds.
By (12), we have

$$\Psi_k(x) - C\Delta(k) \leq \Psi(x) \leq \Psi_k(x) + C\Delta(k).$$

Hence,

$$\Psi_{k-1}(x) - C\Delta(k-1) \leq \Psi)x) \leq \Psi_{k-1}(x) + C\Delta(k-1).$$

Since $\Delta(k-1) \geq \Delta(k)$, so that we have

$$\begin{aligned} \Psi_k(x^k) - \Psi_{k-1}(x^k) &\leq \Psi_k(x^k) - \Psi(x^k) + C\Delta(k-1) \\ &= \Psi_k(x^k) - \Psi(\bar{x}) + \Psi(\bar{x}) - \Psi(x^k) + C\Delta(k-1). \end{aligned}$$

By (12) and the continuity of $\Psi(x)$,and in terms of as $k \to \infty$, $\Delta(k-1) \searrow 0$, we have

$$\lim_{k \in \mathcal{N}} [\Psi_k(x^k) - \Psi_{k-1}(x^k)] \leq 0.$$

Thus

$$\overline{\lim_{k \in \mathcal{N}}} [\Psi_k(x^k) - \Psi_{k-1}(x^k)] \leq 0.$$

Further,by Lemma 1,we have $\Psi_{k-1}(x) \leq \Psi(x) + C\Delta(k-1)$. So that

$$\begin{aligned} \Psi_k(x^k) - \Psi_{k-1}(x^k) &\geq \Psi_k(x^k) - \Psi(x^k) - C\Delta(k-1) \\ &= \Psi_k(x^k) - \Psi(\bar{x}) + \Psi(\bar{x}) - \Psi(x^k) - C\Delta(k-1). \end{aligned}$$

Then

$$\lim_{k \in \mathcal{N}} [\Psi_k(x^k) - \Psi_{k-1}(x^k)] \geq 0.$$

Therefore

$$\underline{\lim}_{k \in \mathcal{N}} [\Psi_k(x^k) - \Psi_{k-1}(x^k)] \geq 0.$$

It follows that

$$\lim_{k \in \mathcal{N}} [\Psi_k(x^k) - \Psi_{k-1}(x^k)] = 0. \qquad \sharp$$

Clearly,the sequences $\{\Psi_k^0\}$ and $\{\Psi_k\}$ continuously converges to Ψ^0 and Ψ, respectively. So by (11) and (12) in lemma 2,and according to the theorem 5 of [5] and [6],we can easily conclude as follows:

Proposition 1 *Under the assumption conditions in Lemma 1,we have*

$$\lim_{k \to \infty} \Psi_k^0 \overset{\text{epi}}{=} \Psi^0 \ \ \textit{and} \ \ \lim_{k \to \infty} \Psi_k \overset{\text{epi}}{=} \Psi. \qquad \sharp$$

Lemma 3 *Assume there is $\tilde{x} \in D$,such that $\Psi(\tilde{x}) < 0$ holds. and for the sequence $\{x^k\}$ such that (12) and (13) hold. In addition, as $\triangle(k-1) \to 0$,($k \to \infty$) then the accumulation point $\bar{x}$ of $\{x^k\}$ must satisfy $\Psi(\bar{x}) \leq 0$ and $x \in D$.*

Proof: Since $\Psi(\tilde{x}) < 0$, then by lemma 2, we have $\lim_{k \to \infty} \Psi_k(\tilde{x}) < 0$. Hence the problem $SIMP_k$ is not feasible at most finite number of points. Thus in terms of (13), for large enough k, we have

$$\Psi_{k-1}(x^k) = [\Psi_{k-1}(x^k) - \Psi_k(x^k)] + \Psi_k(x^k) \leq 0,$$

then we have

$$\Psi(\bar{x}) = \lim_{k \in \mathcal{N}} \Psi_k(x^k) = \lim_{k \in \mathcal{N}} \{[\Psi_k(x^k) - \Psi_{k-1}(x^k)] + \Psi_{k-1}(x^k)\} \leq 0,$$

where $\mathcal{N}$ is an index set such that the sequence $\{x^k\}$ converges to $\bar{x}$, i.e. $\mathcal{N} : x^k \to \bar{x}$. $\sharp$

If denote the index function of X by δ_X, we can easily draw a conclusion as follows[6]:

$$X \subseteq \liminf_k X_k$$

if and only if δ_{X_k} and δ_X satisfy (7); and

$$\limsup_k X_k \subseteq X$$

if and only if δ_{X_k} and δ_X satisfy (8).

Thus, according to the results above,we have

Proposition 2 *The set sequence $\{X_k\}$ converges to X in sense of* Kuratowski. *Therefore,the problem $\{\mathrm{SIMP}_k\}$ is consistent approximation.*

Lemma 3 guarantees the asymptotic feasibility of the iterated sequence $\{x^k\}$. The following lemma 4 guarantees the asymptotic optimality of the problem.

Lemma 4 *Assume $\{x^k\}$ is the solution sequence of the problem* SIMP_k*, $\Psi(x)$ is a convex function, and there is $\tilde{x} \in D$ such that $\Psi(\tilde{x}) < 0$. In addition, as $k \to \infty$, $\triangle(k-1) \to 0$, then for each $x \in \Omega \triangleq \{x \in D \mid \Psi(x) \leq 0\}$,*

$$\overline{\lim_{k\to\infty}} \Psi_k^0(x^k) \leq \Psi^0(x).$$

Proof: For any $x \in \Omega$, let $z^s = \frac{1}{2^s}\tilde{x} + (1 - \frac{1}{2^s})x$, as $s \to \infty$, then $z^s \to x$. Since $\Psi(x)$ is convex, then for all positive integers s, we have $\Psi(z^s) < 0$. Let $\mathcal{N}$ be the index set satisfying the following condition:

$$\lim_{k\in\mathcal{K}} \Psi_k^0(x^k) = \overline{\lim_{k\to\infty}} \Psi_k^0(x^k).$$

Since $\Psi(z^s) < 0$, then there is an index set sequence $\{k_s\}$, such that for all $k \geq k_s$,$\Psi_k(z^s) < 0$. And since

$$\Psi_{k-1}(x) = [\Psi_{k-1}(x) - \Psi_k(x)] + [\Psi_k(x) - \Psi_k(z^s)] + \Psi_k(z^s),$$

making use of lemma 2 and the results above, we have $\Psi_{k-1}(x) \leq 0$. Thus we have $\Psi_k^0(x^k) \leq \Psi_k^0(x)$. Since for all $k \geq k_s$, $\Psi_{k-1}(z^s) = [\Psi_{k-1}(z^s) - \Psi_k(z^s)] + \Psi_k(z^s) \leq 0$, It follows that for all $k \geq k_s$ and any s we have $\Psi_k^0(x^k) \leq \Psi_k^0(z^s)$. Therefore, for each s, there is a $k \in \mathcal{N}$ such that $k \geq k_s$, there is an index set $\mathcal{K} \subseteq \mathcal{N}$ and a sequence $z^k \to x$, such that for any $k \in \mathcal{K} \subseteq \mathcal{N}$,$\Psi_k^0(x^k) \leq \Psi_k^0(z^k)$. Thus we have

$$\overline{\lim_{k\to\infty}} \Psi_k^0(x^k) = \lim_{k\in\mathcal{K}} \Psi_k^0(x^k) \leq \lim_{k\in\mathcal{K}} \Psi_k^0(z^k) = \Psi^0(x).$$

Since x is arbitrary, the result of the lemma is correct. $\sharp$

It follows that we have the following convergence theorem.

Theorem 1 *Assume that*

1) $\{x^k\}$ *is the problem sequence of the problem* SIMP_k*;*

2) The assumption conditions in lemma 2 are satisfied;

3) For any $j \in Q,\ y_j \in Y_j,\ \phi^j(x, y_j)$ *is a convex function;*

4) There is $\tilde{x} \in D$ *such that* $\Psi(\tilde{x}) < 0$ *holds;*

5) As $k \to \infty,\ \triangle(k-1) \to 0,$

Then,every accumulation point $\bar{x}$ *of the sequence* $\{x^k\}$ *generated by the algorithm must be the optimal solution of the problem* SIMP*.*

Proof: By lemma 4, for every accumulation point $\bar{x}$ of the sequence $\{x^k\}$, we have $\bar{x} \in D$ and satisfies $\Psi(\bar{x}) \le 0$. By lemma 6every accumulation point of the sequence $\{x_k\}$ belongs to the solution set $\mathrm{argmin}\{\Psi^0(x)|x \in X\}$, where $x_k \in \mathrm{argmin}\{\Psi^0_k(x)|x \in X_k\}$(for all k). By [12], We need only prove $F_k \overset{epi}{\to} F$, $F_k(x) = \Psi^0_k(x) + \delta_{X_k}$ and $F(x) = \Psi^0(x) + \delta_X$.

If $x \in X$, and the sequence $\{x_k\}$ satisfies $x_k \to x$. Then by $\Psi^0_k \overset{epi}{\to} \Psi^0$. We have

$$\liminf_{k\to\infty} F_k(x_k) \ge \liminf_{k\to\infty} \Psi^0_k(x_k) \ge \Psi^0(x) = F(x).$$

If $x \notin X$, then $F(x) = \infty$. For any sequence $\{x_k\}$ converging to x. By Lemma 1 in [3], we know, there is at most finite number of $x_k \in X_k,$ in the sequence $\{x_k\}$. Thus, for large enough k, we have

$$\liminf_{k\to\infty} F_k(x_k) = \infty = F(x).$$

Therefore we have

$$\liminf_{k\to\infty} F_k(x_k) \ge F(x).$$

On the other hand, assume $x \in intX$(denote the inner points of X). Since$\Psi^0_k \overset{epi}{\to} \Psi^0$, then, there is a sequence $\{x_k\}$ converging to x. Hence by assumption condition 4) there is a subsequence $\{x_{k_i}\} \subseteq X_{k_i}$ such that $x_{k_i} \to x$. Thus, we have

$$\limsup_{i\to\infty} F_{k_i}(x_{k_i}) = \limsup_{i\to\infty} \Psi^0_{k_i}(x_{k_i}) \le \Psi^0(x) = F(x).$$

If $x \notin S$, then for any sequence $\{x_k\}$ converging to x, Clearly, for large enough k, we have

$$\limsup_{k\to\infty} F_k(x_k) = \infty = F(x).$$

If $x \in \partial X$ (denote the edge set of X), let $z^t = \frac{1}{2^t}\bar{x} + (1 - \frac{1}{2^t})x$. By convexity of $\Psi(x)$, then

$$\Psi(z^t) < 0 \text{ for all positive integers } t.$$

Assume $\mathcal{N}$ denotes the index set satisfying the following:

$$\lim_{k \in \mathcal{N}} \Psi_k^0(x_k) = \limsup_{k \in \mathcal{N}} \Psi_k^0(x_k).$$

then there is a subsequence $\{k_t\}$ such that for all $k > k_t$, $\Psi_k(z^t) < 0$. By 4) again, we have $\Psi_{k-1}(z^t) \leq 0$. For each t, there is k such that $k \geq k_t$, Thus there exists an index set $\tilde{N} \subseteq N$ and a sequence $\{z_k\}_{k \in \tilde{N}} \to x$, such that for any $k \in \tilde{N}$, $\Psi_k^0(x_k) \leq \Psi_k^0(y_k)$. Thus by $\Psi_k^0 \xrightarrow{epi} \Psi^0$, we have

$$\limsup_{k \in \tilde{N}} \Psi_k^0(x_k) \leq \Psi^0(z_k) \leq \liminf_{k \in \tilde{N}} \Psi_k^0(x_k).$$

By continuity, we have $\lim_{k \in \tilde{N}} \Psi_k^0(x_k) = \Psi^0(y_k)$.

For simplicity,let z_k^i denote$z_k (k \in \tilde{N})$. Now we construct the following dual sequences:

$$\begin{array}{ccccc}
(z_1^{(1)}, \Psi_1^0(z_1^{(1)})) & (z_1^{(2)}, \Psi_1^0(z_1^{(2)})) & \cdots & \to & (z_1, \Psi_1^0(z_1)) \\
(z_2^{(1)}, \Psi_2^0(z_2^{(1)})) & (z_2^{(2)}, \Psi_2^0(z_2^{(2)})) & \cdots & \to & (z_2, \Psi_2^0(z_2)) \\
\vdots & \vdots & & & \vdots
\end{array}$$

According to diagonalization process theorem, there is a mapping $i \to k(i)$ such that

$$(z_i^{(k(i))}, \Psi_i^0(z_i^{(k(i))})) \to (x, \Psi^0(x)).$$

Therefore, for the sequence $\{z_i^{(k(i))}\}$, $z_i^{(k(i))} \to x$, and

$$\limsup_{i \to \infty} \Psi_i^0(z_i^{(k(i))}) = \Psi^0(x), \text{ for all } z_i^{(k(i))} \in X_{k(i)}.$$

Then

$$F_i(z_i^{(k(i))}) = \Psi_i^0(z_i^{(k(i))}).$$

Thus we have

$$\limsup_{i \to \infty} F_i(z_i^{(k(i))}) = F(x). \quad \sharp$$

3 Conclusion

Here,we primarily discuss the asymptotic feasibility and the asymptotic optimality of general asymptotic discrete model of the minimax semi-infinite programming problem, and prove that any accumulation point of the solution sequence to the discrete programming problem must be the optimal solution to the original programming problem. Further discussion and research are still needed about the question. For instance, detailed design discrete method and multitude of numerical experimental question; Convergence speed rate of the algorithm etc.

Acknowlegments: The author wish to acknowledge the continuous encouragement and the guidance of Professor Jinde Wang and Professor Pusheng Fei over many years.

References

[1] S.-C. Fang, J.R. Rajasekera and H.-S. J. Tsao (1997): *Entropy Optimization and Mathematical Programming*, Kluwer Academic Publisheres, Boston

[2] R. Hettich and K. Kortanek (1993): *Semi-infinite programming:Theory, methods and applications*, SIAM Rev. 35, 3, 380-429

[3] J.L.Higle and S.Sen(1995): *Statistical approximations for recourse constrained stochastic programs*,Annals of Operations Research 50, 157-175.

[4] C. Ji, Z. Wan and M.A. Marino (1999): *Maximum entropy approach method for semi-infinite minimax problem*, Advances in Modelling and Analysis A 35, 1, 35-42

[5] P. Kall (1986): *Approximation to Optimization Problems: An Elementary Review*, Math. of Operations Research, 11, 1, 9-17

[6] M.B. Lignola and J. Morgan (1994): *Epiconvergence and Continuous Convergence*, Atti. Sem. Mat. Fis. Univ. Modesa,XLII, 467-475

[7] E. Ploak (1997): *Optimization, Applications and consistent Approximations*, Springer, Berlin

[8] R. Reemtsen (1991): *Discretization Methods for the Solution of Semi-infinite Programming Problems*, JOTA,Vol. 71 85-103

[9] G. Still (2001): *Discretization in semi-infinite programming the rate of convergence*, Math. Program. 91, 53-69

[10] G. Zhou, C. Wang, Q. Sun(1998): *A Globally Convergent Method for the Semi-Infinite Minimax Problems*(in Chinese)OR Transactions, 2, 2, 42-51

[11] Z. Wan and G. Wu (1999): *Asymptotic surrogate constraint method and its convergence for a class of semi-infinite programming*, Appl. Math. J. Chinese Univ., Ser.B 14, 4, 485-491

[12] J. Wang1988): *Some results on epigraph convergence,* Appl. Math. J. Chinese Univ., Ser.A(in Chinese) 3, 4, 520-527.