ON THE COMPLEXITY OF SCHEDULING WITH ELASTIC TIMES

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ABSTRACT. We consider the problem of scheduling hypermedia documents with elastic times. The objects have variable durations characterized by ideal, minimum, and maximum values. A schedule is a set of tensions on the arcs of the precedence graph which also represents synchronization constraints. We consider the problem of deciding if there exists a schedule in which the durations of at most a given number of objects is not set at their ideal values. We prove the NP-completeness of this problem, which is also NP-complete for series-parallel graphs.

1. Problem Formulation

We recall in this section the problem of scheduling hypermedia objects with elastic times previously formulated in [4]. Hypermedia documents are composed of a set of media segment presentations, called presentation events. The author of the document specifies for each presentation event a minimum, an ideal, and a maximum duration. He may also specify a cost function associated with each event duration, and a set of relationships (precedences and synchronizations) between these events. A hypermedia system (or a *formatter*) attempts to schedule these events, respecting the precedence and synchronization constraints. It provides at compile time a first solution. This schedule eventually is modified at run time, due to the occurence of unexpected events.

A hypermedia document is represented by a directed acyclic graph $G = [N, A]$, which is called a temporal graph. The set of nodes N represents synchronization points, where presentation events either start or stop. Two particular nodes represent the start and the end of the document. The arcs between the synchronization points represent presentation events (dummy events may be added to account for the inclusion of possible delays). Minimum, ideal, and maximum durations d_{ij}^{min}, d_{ij}^{ideal}, and d_{ij}^{max}, respectively, are associated with each presentation event corresponding to an arc $(i,j) \in A$. Accordingly, $d_{ij}^{min} = 0$, $d_{ij}^{ideal} = 0$ (or any non-negative value), and $d_{ij}^{max} = \infty$ for all dummy arcs.

A continuous variable $t_i \geq 0$ is associated with each node $i \in N$. This variable represents the time at which synchronization takes place at this node. The duration of each arc $(i,j) \in A$ is represented by a continuous variable x_{ij}. Variables t_i define a potential for the set of nodes, while vector x defines a *tension* on the graph. The duration of an arc can be seen as a difference of potentials $x_{ij} = t_j - t_i$. The tension model is introduced in [3]. The generic *Minimum Cost Tension Problem* is formulated as:

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$$\text{MCTP:}\begin{cases} z^* = & \min \sum_{(i,j)\in A} f_{ij}(x_{ij}) \\ & \text{subject to:} \\ & t_j - t_i = x_{ij} \qquad\qquad \forall (i,j) \in A \\ & d_{ij}^{min} \le x_{ij} \le d_{ij}^{max} \quad \forall (i,j) \in A, \end{cases}$$

where each $f_{ij}(.)$ is a cost function associated with arc $(i,j) \in A$.

There are two main optimization criteria usually associated with this scheduling problem [4]:

(1) The minimization of the total cost of shrinking or stretching objects.
(2) The minimization of the number of presentation events with a different duration from the ideal one.

The first criterion aims at maximizing the overall presentation quality (author oriented criterion). In this case, the associated cost functions $f_{ij}(.)$ are usually assumed to be continuous and convex for all arcs. Thus, MCTP may be solved efficiently by its transformation to a minimum cost flow problem; see [1]. The second criterion has the goal of facilitating the work for the formatter (formatter oriented criterion). The cost functions $f_{ij}(.)$ are discrete functions of the event durations and the associate problem is NP-hard, as shown in the next section:

$$f_{ij}(x_{ij}) = \begin{cases} 1, \text{ if } x_{ij} \neq d_{ij}^{ideal} \\ 0, \text{ if } x_{ij} = d_{ij}^{ideal}. \end{cases}$$

The two above criteria are often conflicting. Ideally, the best solutions in practice take both of them into account. A suitable strategy to do so would consist in selecting the best solution with respect to the second objective, among all those which optimize the first. Methods considering trade-offs between the two criteria would be the most attractive to be implemented in a formatter. The combinatorial nature of the minimization of the number of objects whose duration is modified makes the second objective the most difficult to be tackled by optimization algorithms.

2. Proof of NP-Completeness

We consider the following variant of the partition decision problem, which is NP-Complete according to [2]:

Partition2:

- Input: A finite set $S = \{a_1, a_2, \ldots, a_{2i-1}, a_{2i}, \ldots, a_{2n-1}, a_{2n}\}$ of $2n$ non negative integer numbers.
- Question: Does it exist a subset S' of S containing exactly one integer from each pair (a_{2i-1}, a_{2i}), for $i = 1, \ldots, n$, and such that $\sum_{a\in S'} a = \sum_{b \in S\setminus S'} b$?

We notice that if two consecutive integers a_{2i-1} and a_{2i} are equal, then any of them may be indifferently chosen. Thus, we can suppose without loss of generality that the integer numbers in set S are pairwise distinct.

We now consider the following decision problem, associated with that of scheduling the presentation events so as to minimize the number of events not at their ideal durations:

SchedulingWithElasticTimes:

- Input: A temporal graph $G = [N, A]$ with minimum and maximal durations d_{ij}^{min} and d_{ij}^{max} associated with each arc $(i,j) \in A$, a subset $A' \subseteq A$ of arcs with known ideal durations d_{ij}^{ideal} for every $(i,j) \in A'$, and some integer K.

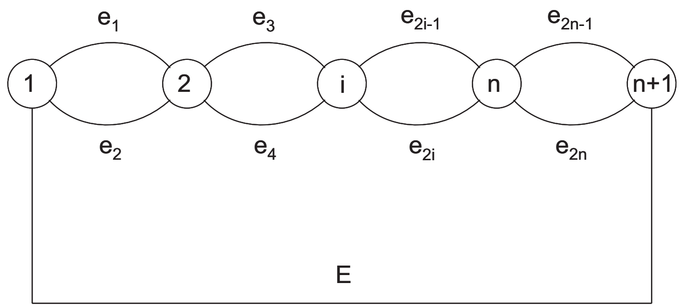

FIGURE 1. Graph associated with an instance of **Partition2** (all arcs are oriented from left to right).

- Question: Does it exist a tension on G such that the number of arcs of A' not at their ideal durations is less than or equal to K?

We first show that **Partition2** polynomially reduces to **SchedulingWithElasticTimes**:

Theorem 1. **Partition2** *polynomially reduces to* **SchedulingWithElasticTimes**.

Proof. Let us consider an instance of **Partition2**. We build a series-parallel temporal graph $G = [N, A]$ containing exactly $n+1$ vertices labelled $1, \ldots, n+1$ and $2n+1$ arcs denoted $e_1, e_2, \ldots, e_{2n-1}, e_{2n}$, and E, as depicted in Figure 2. Every arc has an ideal duration associated with it, i.e. $A' = A$. Arcs e_{2i-1} and e_{2i} have the same extremities i and $i+1$, and ideal durations equal to a_{2i-1} and a_{2i}, respectively, for $i = 1, \ldots, n$. The last arc $E = (1, n+1)$ has an ideal duration $D = (1/2)\sum_{i=1}^{2n} a_i$. The minimum and maximum durations are set to 0 and $+\infty$, for all arcs. Let $K = n+1$.

Two parallel arcs e_{2i-1} and e_{2i} can not both be at their ideal duration, as by hypothesis $a_{2i-1} \neq a_{2i}$. A tension with $K = n+1$ arcs at their ideal duration must necessarily have the duration of arc E set at its ideal value D, and exactly one arc for each pair e_{2i-1}, e_{2i} also at its ideal duration, for $i = 1, \ldots, n$. Let N' be the set formed by the indices of the n parallel arcs at their ideal values (then, $E \notin N'$). Thus, the sum of their durations must be exactly equal to D. Hence, if we can build a solution for the tension problem **SchedulingWithElasticTimes**, we have also a solution for the partition problem: $S' = \{a_i \in S : \ e_i \in N', i = 1, \ldots, n\}$. The reduction is polynomial, since G has $n+1$ nodes and $2n+1$ arcs. ∎

The NP-completeness of **SchedulingWithElasticTimes** follows immediately from the above result.

Corollary 1. **SchedulingWithElasticTimes** *is NP-Complete.*

Proof. **SchedulingWithElasticTimes** belongs to NP, since the tensions x_{ij} can be computed in time $O(|A|)$ and the lower and upper bounds can also be checked in time $O(|A|)$. **Partition** and its variant **Partition2** are NP-complete. Since **SchedulingWithElasticTimes** reduces to **Partition2** in polynomial time, then the former is NP-complete. ∎

The proof of the theorem reduces **SchedulingWithElasticTimes** to **Partition2** in a series-parallel graph. Hence, finding an optimal tension for MCTP with the second optimization criterion in a series-parallel temporal graph is already NP-hard. This result is somewhat surprising, since many difficult scheduling problems are easy for series-parallel graphs.

References

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