

# Conditional Risk Mappings

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We introduce an axiomatic definition of a conditional convex risk mapping and we derive its properties. In particular, we prove a representation theorem for conditional risk mappings in terms of conditional expectations. We also develop dynamic programming relations for multistage optimization problems involving conditional risk mappings.

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