Construction project scheduling problem with uncertain resource constraints[1]

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Abstract: This paper discusses that major problem is the construction project scheduling mathematical model and a simple algorithm in the uncertain resource environments. The project scheduling problem with uncertain resource constraints comprised mainly three parties: one of which its maximal limited capacity is fixed throughout the project duration; second maximal limited resource capacity is random variable; thirdly resource is fuzzy. The objective function is taken as the total completion duration of project to be minimized. A proper uncertain mathematical model is firstly constructed for pre-given confidence level and the important degree of the activity by using of chance-constrained programming method. And then the original uncertain problem is converted into an equivalent deterministic nonlinear programming problem by means of some efficient measures. Finally a simple two-stage solving method is proposed, the evaluation and adjustment strategy may be applied to obtain a modifying solution according as solution to deterministic programming problem. A numerical example is presented.
Key words: project scheduling; resource constraints under uncertain environment; chance-constrained programming.

1 Introduction

The problem of project scheduling activities under resource and precedence(or generalized precedence) restrictions with the objective of minimizing the total project throughput duration or the other objective functions,such as cost related and resource leveling etc., is referred to as the resource constrained project scheduling problem (RCPSP) in the literature. This problem has attracted considerable attention in the last 30 years owing to its importance of practical applications in the construction,manufacture and other industries. The literature contains various solution procedures for the RCPSP to facilitate the allocation of resources to project activities, satisfying resource limitations. For reviews or survey of the RCPSP, the reader can be referred to [3,13-17,32]. These optimal (or near-optimal) techniques can mainly be categorized into three areas: heuristic methods, stochastic optimization strategies and exact solution techniques. First area, heuristic methods, were focused on branch-and-bound and its several efficient improved, e.g. heuristics, based on the normal precedence [1, 11, 20]; objective function based on cash (or

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crash flows) [9, 18]; based on the generalized (or preemption) [19] etc. Second solution techniques, stochastic optimization methods, were focused on genetic and tabu search algorithms [18, 21, 32] etc. Thirdly, exact algorithms, based on mathematical programming, [24] etc.

The above mentionedall these algorithms concerning the resource-constrained project scheduling assume fixed activity durations and do consider uncertain projects of random or fuzzy duration. Since many uncertain variables, such as weather, on site construction, productivity level, extremely expensive and rare resource etc. affect activity duration during the project implementation, it thus is necessary and meaningful to consider the project scheduling with uncertain activity duration. As a matter of fact, a number of papers present several mathematical models and algorithms on resource-constrained project scheduling with random activity duration [8, 31], as well as with fuzzy activity duration [22]. A common feature in these papers is that assumed each type of resource is limited supply with a resource limited that is fixed at the same level throughout the project duration. However, in the market fierce competition and bad environment conditions, the available resource supply is also an uncertain factor, and results in the uncertain activity duration of the project scheduling [11]. It follows that activities require resources of various types with variable capacities should be considered, at the same time, the uncertainty of resources supplying is also taken into account. Thus the need for proper resource-constrained project scheduling models under the uncertain resource requirements is very great.

The problem addressed in this paper is one where a project scheduling problem in the uncertain resource environments which have not been published elsewhere. For simplicity, we assume that partial activity duration is uncertainty (which is partitioned two parts:random and fuzzy). The objective function to be minimized, without loss of generality, is taken as the total completion duration of project. Here we mainly construct a proper mathematical model for pre-given confidence level and the important degree of some activities, and then the original uncertain mathematical programming will be converted into an equivalent deterministic nonlinear programming problem by means of some efficient measures. The evaluation and adjustment measures may be applied to obtain a modifying solution according as solutions to deterministic problem.

2 Some assumptions and notations

The following initial assumptions are made firstly:

1. A project consists of different activities, which are represented, in the activity-on-the-node format. Without loss of generally, the activity 1 denotes the start of the project and is a predecessor of every other activity in the project, activity $N + 1$ denotes the end activity of the project and is a successor of every other activity in the project;

2. The activities are related by a set of finish-start precedence relations with a time lag of 0,implying that no activity can be started before all its predecessors have completed;

3. The activities are to be performed without preemption;

4. The available resource amounts assigned to each activity are known and fixed constants over the processing interval of the activity. Once a resource is occupied by an activity, the resource will not be released until the activity is done;

5. The duration of partial activities are a random or fuzzy variable with given density and membership function, respectively;

6. There are K renewable resource types for the sake of simplicity, in which the total amounts available resource requirements is deterministic, random and fuzzy throughout the project duration interval;

7. The objective is to complete the project as early as possible without violating any deterministic resource and precedence, as well as uncertain resource constraints for the pre-given confidence level.

In addition, those fuzzy and random variables are near independent in the uncertain environment.

Now let us introduce the following notations: N: the number of activities in the project. K: the number of resource types. f_j the completion time (duration) of activity j. H: the set of pairs of activities indicating finish-start precedence relation. S_t: the set of activities in progress during time interval$(t-1,t] = \{i|f_i - d_i < t \leq f_i\}$, i.e. the set of on-going activities at time t. r_{ikt}:the amount of resource type k that is required by activity i at time t. R_{kt}: the total availability of deterministic resource type k at time t. $\tilde{a}_{kt}$: the total availability of fuzzy resource type$k$ at time t, and assume that $\tilde{a}_k$ follows the trapezoidal fuzzy numbers which are fuzzy quantities fully determined by quadruples $(a_{1kt}, a_{2kt}, a_{3kt}, a_{4kt})$ of crisp numbers such that $a_{1kt} \leq a_{2kt} \leq a_{3kt} \leq a_{4kt}$. $b_{kt}(\xi)$: the total availability of random resource type k at time t. UD: he set of activity with random duration UD_1 or fuzzy duration UD_2 such that $UD = UD_1 + UD_2$. d_j: he duration of activity j, if $j \in UD_1$ then d_j is a random variable, it is assumed that they depend linearly on resource capacities r_{jkt}, e.g. $d_j = \sum_k r_{jkt} a_{jkt}(\xi)$, where $a_{jkt}(\xi)$ be random variable with pre-given density function. $\underline{a}_{jkt}$: the lower bound of the random variable $a_{jkt}(\xi)$. $\overline{a}_{jkt}$: the upper bound of the random variable$a_{jkt}(\xi)$. d_j: the duration of activity j,$j \in UD_2$ or d_j is a triangle or trapezoidal fuzzy numbers. DR: the set of the total available resource requiremints at the disposal of the project management is pre-given and fixed (i.e.R_{kt}) at time t. SR: the set of total available resource requirements at the disposal of the project management is a random variable(process)with pre-given density function throughout the scheduling horizon. FR: the set of total available resource requirements at the disposal of the project management is a fuzzy number (be related to time) with pre-given membership function throughout the scheduling horizon. It follows that $|DR\bigcup FR\bigcup SR| = K$ where $|A|$ denotes the cardinality of set A. $Pr\{\cdot\}$:he probability of random events $\{\bullet\}$. $Pos\{\cdot\}$ the possibility of fuzzy events $\{\bullet\}$.

That the each activity duration is assumed, in general, follows a beta or normal probability density function in the bound interval $[a, b]$ [11, 33]. Thus, for a normal distribution random variable in the bound interval, its mean is $\mu = 0.5(a + b)$ and the variance is $\sigma^2 = [(b - a)/6]^2$, where σ is standard deviation.

3 Mathematical programming model with uncertain duration and resource parameters

The chance-constrained(or probabilistic constrained) programming models stochastic decision systems with the assumptions that the constraints will hold at least α of the time, where α is referred to as the confidence level. Analogous to chance-constrained programming with random variables, in a fuzzy environment we assume that the constraints will hold with at least possibility α, and the chance is represented by the possibility that constraints are satisfied[23] .

Under the available resource requirements are uncertainty, We shall adopt chance-constrained method in dealing with the uncertain resource-constrained problems, assume that the duration of each activity is restricted within the some range will hold at least a certain level, so-called the important degree of activity, this differs from papers [11] to determine starting time of each activity.

Provided that the duration d_j of activity j ($j \in UD_1 \,\text{and}\, UD_2$) be random variable or fuzzy number, and their chance are no less than the probability or possibility for pre-given β_j, Henceforth we have the following unifying form:

$$Ch\{\underline{d}_j \leq d_j \leq \overline{d}_j\} \geq \beta_j \tag{1}$$

Consequently, the problem with chance-constrained programming is to minimize the project completion time can be formulated as follows:

$$\begin{aligned} Minminize \quad & f_{N+1} && (2)\\ s.t. \quad & f_j - d_j \geq f_i \,\text{and}\, f_1 = 0 \,\forall (i,\, j) \in H, && (3)\\ & Ch\{\underline{d}_j \leq d_j \leq \overline{d}_j\} \geq \beta_j \, j \in UD && (4)\\ & \sum_{i\in S_t} r_{ikt} \leq R_{kt} \, k \in DR, && (5)\\ & Pos\{\sum_{i\in S_t} r_{ikt} \leq \breve{a}_{kt}\} \geq \alpha_{kt} \, k \in FR, && (6)\\ & Pr\{\sum_{i\in S_t} r_{ikt} \leq b_{kt}(\xi)\} \geq p_{kt} \, k \in SR, && (7) \end{aligned}$$

$\text{for} t = 1,\, 2,\, \cdots,\, f_N.$

Note: For simplicity, d_j in constrained conditions (3), it is any selected from the set of satisfying the conditions (4) above. It follows that d_j be a deterministic (or crisp) as long as it is selected.

In this mathematical model, Eq. (2) is the objective function in which minimizes the project completion time defined by minimizing the finish time of the unique dummy end activity $N+1$. Constraints (3) ensure that no activity can be started until all its predecessors have been completed, and the dummy start activity 1 is assigned a value of 0. During any time interval $(t-1,\, t]$, there-into the constraints (5) represent that the resource utilization does not exceed the resource availability levels for any of the deterministic resource types, constraint set (6) and (7) denote that chance-constrained of resource utilization do not exceed the resource availability level for any of the fuzzy and random resource types respectively.

4 Equivalent deterministic programming problems

We now discuss that the constraint set will be converted into deterministic constraints. **Case 1**. d_j, the duration of activity j, is a random variable. Assumed that, for any $j \in UD_1$ and $k = 1,\, 2,\, \cdots,\, K$, the independent random variable $a_{jkt}(\xi)$in the uncertain duration of activity (1) follows a normal distribution with the mean and variance are

$$\mu_{jkt} = 0.5(\underline{a}_{jkt} + \overline{a}_{jkt})$$

and $\sigma^2_{jkt} = (\underline{a}_{jkt} + \overline{a}_{jkt})^2/36$, respectively. Provided that there exist some numbers $D^{(1)}_{\beta_j}$ and $D^{(2)}_{\beta_j}$ ($D^{(1)}_{\beta_j} \le D^{(2)}_{\beta_j}$) such that for $j \in UD_1$

$$\Phi(\frac{D^{(2)}_{\beta_j} - \mu(d_j)}{\sigma(d_j)}) - \Phi(\frac{D^{(1)}_{\beta_j} - \mu(d_j)}{\sigma(d_j)}) \ge \beta_j, \tag{8}$$

where $\Phi(\cdot)$ represents the cumulative distribution function of the standard normal random variable, then we can let that $d_j = (D^{(1)}_{\beta_j} + D^{(2)}_{\beta_j})/2$.

Case 2. d_j, the duration of activity j, $j \in UD_2$ is a fuzzy number with membership function $\delta_j(x)$.

It is clear to see that, for any given the important degree level β_j, based on the concepts and techniques of fuzzy mathematical theory [30], there exist some values $K^{(1)}_{\beta_j}$ and $K^{(2)}_{\beta_j}$ ($K^{(1)}_{\beta_j} < K^{(2)}_{\beta_j}$) such that

$$Pos\{K^{(1)}_{\beta_j} \le d_j \le K^{(2)}_{\beta_j}\} \ge \beta_j,\, j \in UD_2, \tag{9}$$

that is $\delta_j(K^{(2)}_{\beta_j}) - \delta(K^{(1)}_{\beta_j}) \ge \beta_j$. Similarly, put $d_j = (K^{(1)}_{\beta_j}) + K^{(2)}_{\beta_j})/2$, $j \in UD_2$.

In the light of the discussion of the chance-constrained programming with fuzzy parameters being converted to crisp equivalent programming problem[23], it is obvious that, for any given confidence level α_{kt}, $k \in FR$ and $t = 1,\, 2,\, \cdots,\, f_N$, there exist some values $K_{\alpha_{kt}}(t)$ such that

$$K_{\alpha_{kt}}(t) = \sup\{K_t | K_t = \zeta^{-1}_{kt}(\alpha_{kt})\},\, k \in FR. \tag{10}$$

Thus, the crisp equivalents of chance constraints (6) are obtained and shown by the following forms:

$$\sum_{i\in S_t} r_{ikt} \le K_{\alpha_{kt}(t)},\ K_{\alpha_{kt}(t)} = \zeta_{kt}^{-1}(\alpha_{kt}) \tag{11}$$

Assumed that $b_{kt}(\xi)$, ($k \in SR$, $t = 1, \cdots, f_N$), are random variables having the independent beta distribution within finite interval $[\underline{b}_{kt}, \overline{b}_{kt}]$, let their distribution function are F_{kt}. The inverse of function F_{kt} is defined by $F_{kt}^{-1}(x) = \inf\{y : F_{kt}(y) \ge x\}$. Analogous to the fuzzy constraints (6) dealing with method) for$k \in SR$ and $t = 1, \cdots, f_N$

$$\sum_{i\in S_t} r_{ikt} \le K_{p_{kt}}(t), \tag{12}$$

where $K_{p_{kt}} = F_{kt}^{-1}(p_{kt})$. Both $K_{\alpha_{kt}}$ and $K_{p_{kt}}$ satisfying (11) and (12) are called as "quantiles".

Hence an equivalent deterministic programming of chance-constrained programming problem with uncertain duration and resource parameters in the construction project scheduling (2)-(7) can be presented as

$$\begin{aligned}
&Minminize && f_{N+1} && (13)\\
&s.t. && f_j - d_j \ge f_i \text{ and } f_1 = 0\,\forall(i,\,j) \in H, && (14)\\
& && \Phi\Big(\frac{D_{\beta_j}^{(2)} - \mu(d_j)}{\sigma(d_j)}\Big) - \Phi\Big(\frac{D_{\beta_j}^{(1)} - \mu(d_j)}{\sigma(d_j)}\Big) \ge \beta_j,\ j \in UD_1 && (15)\\
& && Pos\{K_{\beta_j}^{(1)} \le d_j \le K_{\beta_j}^{(2)}\} \ge \beta_j,\ j \in UD_2, && (16)\\
& && \sum_{i\in S_t} r_{ikt} \le R_{kt}\, k \in DR, && (17)\\
& && \sum_{i\in S_t} r_{ikt} \le K_{\alpha_{kt}}(t)\, k \in FR, && (18)\\
& && \sum_{i\in S_t} r_{ikt} \le K_{p_{kt}}(t)\, k \in SR, && (19)
\end{aligned}$$

for $t = 1, 2, \cdots, f_N$.

5 Two-stage modified algorithm

Owing to special uncertainty considered here, thus we could use the following simple iterative method to capture a suitable duration d_j of activity j ($j \in UD$).

Step 1. Initialize set the initial point $d_j^{(0)}(= \mu(d_j)$, and step-inscrement $h > 0$, set $d_j^{(1)} = d_j^{(2)} = d_j^{(0)}$;

Step 2. Do until the probability (or possibility) larger than or equal to the important degree level β_j: $Ch_j = Ch\{d_j^{(1)} \le d_j \le d_j^{(2)}\}$, where $d_j^{(1)} = d_j^{(1)} - h$, $d_j^{(2)} = d_j^{(2)} + h$; Set $h = h + h$ and $d_j^{(1)} = d_j^{(1)} - h$, $d_j^{(2)} = d_j^{(2)} + h$ and control the interval $[d_j^{(1)}, d_j^{(2)}] \subseteq [\underline{d}_j, \overline{d}_j]$.

We now present our algorithm to solve uncertain programming problem using the following two-stage approach.

Initialization. Determine initial datum of the project, special the important degree of the related to activity with uncertain duration and the confidence level for uncertain resource constraints.

Stage One. Conversion.

Step 1: Convert uncertain programming problem (2)-(7) into an equivalent deterministic programming problem using fuzzy and probabilistic chance constrained programming techniques.

Step 2: Compute the duration d_j of activity j $(j \in UD)$.

Step 3: Select some different "quantiles" $K^n_{\alpha_{kt}}(t)$ and corresponding $K^m_{p_{kt}}(t)$ $(n,\, m = 1, \cdots, M_1$ at least$M_1 = 1)$ by using of inequalities (11) and (12), respectively.

Step 4: Solve the deterministic project scheduling programming problem obtained from Step1 and Step2 by using of some efficient algorithms existed, e.g. efficient branch-and-bound algorithms [7]. Repeat the process M_1 times for different $K^n_{\alpha_{kt}}(t)$ and $K^m_{p_{kt}}(t)$. Let $f^1_{N+1},\, f^2_{N+1}, \cdots,\, f^{M_1}_{N+1}$ be the M_1 objective function value.

Stage Two. Evaluate and adjust. According to the above M_1 objective function values, finds a minimum $f^q_{N+1} + \min_m f^m_{N+1}$. Compute, for $t = 1,\, 2, \cdots,\, f_{N+1}$,

$$A^m(t) = K^m_k(t) - \sum_{i \in S_t} r_{ikt} > 0,\; k \in SR \cup FR. \tag{20}$$

Provided that $A^m(t) > 0\,(t = 1,\, 2, \cdots,\, f_{N+1})$ in all parallel scheduling of activity at some different time intervals, $K^m_k(t)$ will be replaced by a certain $K^s_k(t)$ which is less than $K^m_k(t)$ $(s < m)$ such that $K^m_k(t) - K^s_k(t) = \min A^m(t)$. In this way, the availability of resource type k is determined efficiently. Optimal project duration is obtained f^q_{N+1} , as well as the availability of resources with minimum limited.

Here, we mainly considered the project scheduling problem with maximal available uncertain resource constraints, generally, these resources are only derived from previous comparable experiments (costly value) and market estimates. In practical applications, the range of possible values of an estimated parameter can be restricted by a confidence level pre-given, that is the probability of the available resource utilization does not exceed the resource availability level.

6 An illustrational example

To illustrate that this paper proposed model and solving method, a simple project with uncertain resource requirements and duration of activities is planned with the network shown in Fig. 1 and activity datum and resource requirements in Table 1. Their maximal available resource capacity as follows: $R_1 = 2$ is fixed resource; R_2 is random variable follows beta distribution in the interval [8, 11]; R_3 is fuzzy variable with the trapezoidal number $\check{R}_3 = (4,\, 5,\, 6,\, 8)$. The duration of random(Normal distribution) activity $C \sim N(14,\, (\frac{2}{3})^2)$, the duration of fuzzy(with trapezoidal number) activity $I = (12, 13, 14, 16)$. Assumed that the important degree of the activity C and I is $\beta_C = 0.3$ and $\beta_I = 0.25$ respectively; the confidence levels are $p_2 = 0.96$, $\alpha_3 = 0.98$.

Through simple computation, there are the following cases:

(1). $d_C = 14$, $d_I = 13$, $R_2 = 10$, $R_3 = 5$, the resulting project scheduling duration is 109 unit time. The detail results are summarized in Table2.

(2). $d_C = 15$, $d_I = 14$, $R_2 = 10$, $R_3 = 5$, we obtain that the project scheduling duration is 110 unit time(detail results is omitted).

(3). $R_2 = 11$, $R_3 = 6$, for $d_C = 14or15$, $d_I = 13or14$, respectively. The project scheduling duration is 92 or 93 unit time(detail results is omitted), etc.

Table 1. Activity information

Activity	A	B	C	D	E	F	G	H	I	J	K
Duration	5	10	random	18	14	16	19	12	fuzzy	20	6
R_1	1	1	0	1	0	1	2	1	1	1	2
R_2	9	3	5	4	3	4	5	4	5	5	8
R_3	5	0	2	3	2	4	5	3	2	3	5

In the following Table 2, AS denotes Activities for Scheduling. CSA denotes Completion time of Scheduled Activity. $R_1/R_2/R_3$ denotes resource requirement in the process of activities scheduling.

Table 2. Summary schedule results with $d_C = 14$, $d_I = 13$, $R_2 = 10$, $R_3 = 5$

Time	1	6	18	20	31	34	51	67	85	104
AS	A	B, E, H	E^a, I	I^a, C	C^a, J	J^a	F	D	G	K
CSA	5	15, 19, 17	19, 30	33, 30	33, 50	50	66	84	103	109
$R_1/R_2/R_3$	1/9/5	2/10/5	1/8/4	1/10/4	1/10/5	1/5/3	1/4/4	1/4/3	2/5/5	2/8/5

Note: ^{a}denotes on-going activities.

7 Conclusions and future work

A chance-constrained programming model and a simple two-stage algorithm was developed in this paper, for solving the construction project scheduling problem in the uncertain resource environments. This project scheduling with uncertain resource constraints were mainly consisted of which the maximal available resource is random and fuzzy variables. For the sake of simplicity, it is only discussed that the duration of partial activities are uncertainty. A proper uncertain mathematical model, to minimize the total completion duration of project scheduling problem with uncertain resource-constrained, was constructed for pre-given confidence level and the important degree of the activity by using of chance-constrained programming method. The original uncertain problem was converted into an equivalent usual deterministic project scheduling problem with resource-constrained by means of some efficient measures. A simple two-stage algorithm and numerical example was also proposed.

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Appendix:

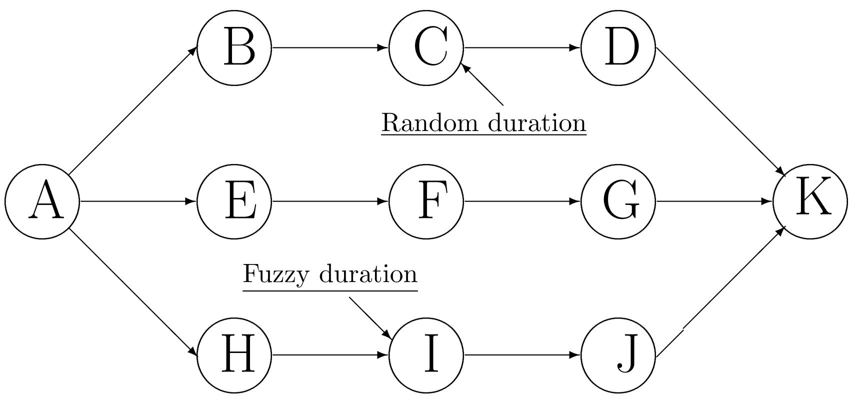

Fig. 1 Project network