Security-constrained transmission planning: A mixed-integer disjunctive approach

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Abstract— **We extend a static mixed integer disjunctive (MID) transmission expansion planning model so as to deal with circuit contingency criterion. The model simultaneously represents the network constraints for base case and each selected circuit contingency. The MID approach allows a commercial optimization solver to achieve and prove solution optimality. The proposed approach is applied to a regional Venezuelan network, and results are discussed.**

Index Terms-- **Optimization methods; Power transmission planning.**

I. Nomenclature

n	number of buses (nodes)
m	number of circuits (branches)
κ	number of circuit contingencies
Ω_i	set of circuits connected to bus i
f^0	vector of circuit flows (base case)
f^k	vector of circuit flows (k-th contingency)
$\overline{f}$	vector of normal circuit capacities
$\overline{f}^c$	vector of emergency circuit capacities
g	vector of bus generations
d	vector of active bus loads
θ^0	vector of bus voltage angles (base case)
θ^k	vector of voltage angles (k-th contingency)
x	decision vector for building/not circuits
c	vector of circuit construction costs
S^0	node-branch incidence matrix (base case)
S^k	incidence matrix (k-th contingency);
$[\gamma^0]$	circuit susceptance matrix (base case)
$[\gamma^k]$	circuit susceptance matrix (k-th contingency);
M	penalty matrix ("big" M)

II. Introduction

As the 2003 East Coast blackout in the US and the 1999 countrywide blackout in Brazil illustrate, the transmission network is a critical component for the adequate functioning of electricity sectors all over the world. With the reestructuring of those sectors in many countries, which includes the separation of transmission and generation, the planning activity has become even more complex, because the networks must now be able to accomodate a wider range of generation dispatches (due, for example, to power exchanges among regions or countries) and are operated much closer to the limits (which makes security constraints very important).

As discussed in [1], one of the major challenges in solving the transmission planning problem is the joint occurrence of integer variables (investment decisions) and nonlinear constraints (product of circuit suspectances – related to the investment decisions – and voltage angle differences). In [2], we proposed a mixed-integer disjunctive (MID) scheme, composed of two steps: (i) transformation of nonlinearities into "disjoint" mixed integer (MILP) constraints, which allowed the used of commercial software; (ii) calculation of well-adjusted "big M" (penalty) terms to these disjuntive constraints, which avoided the numerical problems which usually affect the computational efficiency of this type of approach.

The MID approach was found to be computationally efficient for realistic-sized systems, with several hundred buses and circuits. In this paper, we extend the MID scheme to represent security constraints such as the N-1 criterion (no overloads when any single circuit is removed). It is shown that, although the number of linear variables and constraints increases linearly with the number of security constraints, the number of integer variables remains the same as in the "base case" problem, which still allows the efficient use of MILP techniques.

The paper is organized as follows. In section III we present the MID model formulation; Section IV presents and discusses results for a real-world case study; in section V we present the main conclusions and section IV discusses ongoing and future research.

III. The MID expansion model

A. *"Base case" formulation*

We use a linearized active power flow model, which is felt to be adequate for mid- to long-term transmission planning studies. We also use a "static" model, that is, we do not consider the time evolution of load and generation.

The "base case" transmission expansion problem (no security constraints) is formulated as the following non-linear mixed integer mathematical programming problem [1]

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$$\mathrm{Min}_{\{x,f,\theta\}}\ c'x$$

subject to (1)

$$\mathrm{S}f = d - g \quad (1a)$$

$$f - [\gamma x]\mathrm{S}'\theta = 0 \quad (1b)$$

$$-\bar{f}x \le f \le \bar{f}x \quad (1c)$$

$$x \in \{0,1\}^{m}$$

where x represents the binary (build/not build) investment decisions and c is the investment cost. For notational simplicity, we suppose that all circuits are candidates for expansion (the x variables for existing circuits are made = 1). The notation $[\gamma x]$ indicates pairwise products between diagonal components and decision variables; $\bar{f}x$ is also a pairwise product on the vector elements.

Equations (1a) and (1b) represent respectively the first and second Kirchhoff laws; constraints (1c) represent the circuit flow limits.

B. The MID approach

As mentioned in the Introduction, the main difficulty in solving (1) lies in the product of the binary decision variable x and the voltage angle variables θ in (1b). As shown in [2], we replace those constraints by the following linear disjunctive form:

$$-\mathrm{M}(1-x) \le f - [\gamma]\mathrm{S}'\theta \le \mathrm{M}(1-x) \quad (2)$$

Constraint (2) is interpreted as follows: when the circuit is not built (x=0), the flow limit constraint (1c) makes f equal to zero; and the disjunctive constraint (2) is relaxed (no second law). Conversely, when the circuit is built (x =1), constraint (1c) enforces the flow limits and the disjunctive constraint (2) enforces the second law.

As mentioned in the Introduction, a separate "big M" penalty term in (2) is calculated for each candidate, so as to minimize ill conditioning [3]. As shown in [2], the actual disjunctive inequalities can be made "tighter" on the right hand side by dropping the penalty term.

C. Security Constraints

We now introduce the MID modeling of security constraints, which is the main objective of this paper. For example, the widely used "N-1" planning criterion states that load supply should be ensured not only under base-case conditions, but also in the case of single circuit failures. This criterion is modeled by repeating all network constraints for each circuit contingency, indexed by k=1,…,K (note that the penalty, incidence and susceptance matrices for the k-th contingency are different from the base case matrices, because the circuit has been removed).

The security-constrained MID model is formulated as follows:

$$\mathrm{Min}_{\{x,f,\theta\}}\ c'x$$

subject to (3)

$$\mathrm{S}^0 f^0 = d - g$$

$$-\mathrm{M}^0(1-x) \le f^0 - [\gamma^0]\mathrm{S}^{0\prime}\theta^0 \le \mathrm{M}^0(1-x)$$

$$-\bar{f}x \le f^0 \le \bar{f}x$$

$$\mathrm{S}^k f^k = d - g$$

$$-\mathrm{M}^k(1-x) \le f^k - [\gamma^k]\mathrm{S}^{k\prime}\theta^k \le \mathrm{M}^k(1-x)$$

$$-\bar{f}^k x \le f^k \le \bar{f}^k x$$

$$x \in \{0,1\}^m,\ k=1,\ldots,\mathrm{K}$$

Although the size of problem (3) increases linearly with the number of contingencies K (continuous variables f and θ, as well as network constraints), the number of binary variables x remains the same; as a consequence, the combinatorial complexity of the MID model is not affected by the incorporation of security constraints.

D. Additional solution features

Logical precedence constraints are used to avoid duplication of work in the case of multiple candidate circuits in the same right-of-way.

If all the investment variables are fixed (a trial expansion plan), (3) becomes a linear program. Since the plan may be infeasible (power flows violate some circuit limits), the solution algorithm performance can be enhanced by summing to the objective function a linear term reflecting the minimum total load curtailment times a high penalty. The load curtailment slack variables are added to each power balance equation, and eventually used to meet network constraints for both the base case and contingencies. With this formulation MID always has a solution and provides an alternative measure of infeasibility to total circuit overload calculated by solving linearized power flow equations, for base case and contingencies. Further details can be found in [4].

Problem (3) is solved by a commercial "branch-and-bound" (B&B) solver. All feasible solutions obtained during the search while optimality is not proved are saved for further detailed network performance analysis.

For large scale networks, and when solving long term expansion planning problems, the B&B search tree can become large, so instead of seeking for the optimal solution we can control the optimality gap and stop with the best feasible solution when a pre-specified tolerance is reached. Also, we can stop the search after finding a certain number of feasible solutions, without caring for the optimality gap.

IV. Case Study

Figure 1 illustrates the Venezuelan network topology (colours for voltage levels). The expansion is for northeast region, considering the loads and generations five years ahead. The rest of the network has been already expanded.

The existing system has 134 buses and 271 circuits. There are 125 candidate circuits in 72 rights-of-way. The security criterion is applied to 37 critical circuit contingencies.

The resulting MID problem (3) considering security constraints has on the order of 43 thousand rows, 31 thousand variables (of which 125 are binary) and 150 thousand non-zero elements, and is solved in about two minutes on a 2 GHz Pentium IV computer with 512MB.

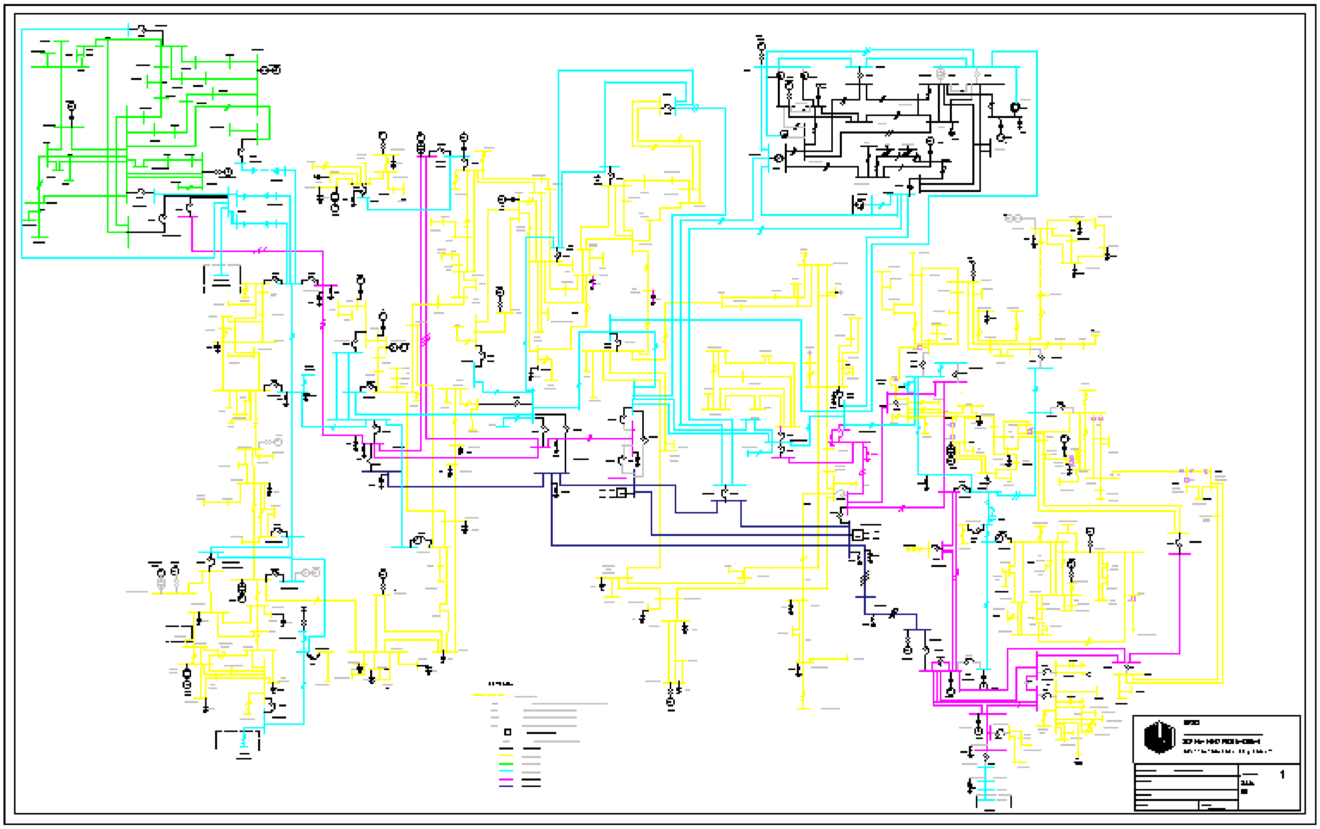

Fig. 1 – Venezuelan network topology (colors are voltage levels)

The optimal expansion plan for the region requires 6 reinforcements. The optimal solution of the “base case” problem (without security constraints) requires only one reinforcement. This example indicates that the security constraints should be considered as an integral part of the planning problem.

V. Conclusions

In this work, the consideration of the contingency criterion in static transmission planning was shown to be handled by a natural extension of the mixed integer disjunctive model. The combinatorial complexity of the model is unaltered (the number of discrete variables related to investment decisions remain the same if one considers contingencies or not), since only the number of constraints and the number of continuous variables grow linearly with the number of contingencies.

The case study results show that there is a great impact of the security criterion on the expansion plan, which cannot be dealt with usual planning models that only deal with the “base case” network configuration. Although the computational effort of the LP relaxation problem solved at each node of the B&B search tree increases due its size, on the other hand the feasible set of binary investment decisions is greatly reduced, so the total computational effort to achieve and prove optimality of the security constrained expansion model depends on both the (increased) solution time of each node LP and the number of (possibly decreased) visited nodes of the search tree.

VI. Future Work

The authors intend to strengthen the multiple contingency mixed integer disjunctive formulation by means of additional constraints based on relaxed network flow duality cuts devised for each contingency case, and therefore reduce the search tree of the B&B algorithm. Using modern commercial solvers, these cuts can be automatically generated, pooled and incorporated when needed to the LP relaxations solved for the nodes of the B&B search.

The proposed approach extending the MID formulation for security constraints can also be applied to represent uncertainties in dispatch, which are important for hydrothermal systems where generation dispatch varies due to hydro plant inflow and load scenarios. The planned network must therefore be robust, meaning that network constraints must be met for all relevant scenarios. In the same way as for security constraints, we can create for each scenario continuous variables for circuit flows and bus voltage angles, as well as network constraints. Since the binary investment variables are the same for all scenarios, the robustness criterion is guaranteed.

The authors also intend to use the MID formulation for multi-stage transmission expansion planning. Contrarily to the increase in problem size incurred in the extensions for security constraints and uncertainties in dispatch scenarios, in dynamic planning, besides the number of continuous variables and network constraints, the number of binary investment also grow linearly with the number of time stages. Nevertheless, the MID static expansion model can be used as a building block for pseudo-dynamic forward and backward heuristic approaches [5].

VII. References

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VIII. Biographies

Gerson Oliveira has a BSc degree in EE from PUC/Rio and a DSc degree in Systems Engineering from COPPE/UFRJ. He joined PSR as an associate consultant in 1999, where he is currently involved with transmission planning models. Previously he worked at CEPEL, the Brazilian power research center, where he was a project manager and senior researcher in the areas of optimization and statistical techniques applied to PSPO.

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Laura Bahiense has a BSc degree in Computer Science, and MSc and DSc degrees in Systems Engineering (optimization), all from the Federal University of Rio de Janeiro. She joined PSRI in 1999, where she is working on the development of software related to Power Systems Optimization. Previously she worked at the Laboratory of Formal Methods at the Catholic University of Rio de Janeiro in an LMF-Equitel-Siemens joint project, where she developed a logical methodology and an optimization tool to control and optimize the production stage of new products. She is member of the Mathematical Programming Society.

Luiz M. Thomé has a BSc in EE from UFRJ, and was awarded post-graduate credits in power system engineering from COPPE/UFRJ. He joined PSR in 1997, where he directs network analysis software development and research on transmission tariffs and transmission planning issues. Previously Mr. Thomé managed the development of Cepel's network analysis software, used by most Brazilian utilities (AC power flow, contingency analysis, equivalent network, voltage sensitivity, electromechanical transient analysis).

Mario Veiga Pereira has a BSc degree in EE from PUC/Rio and DSc degree in Optimization from COPPE/UFRJ. He is currently engaged in regulatory studies and the development of new methodologies and tools for risk management in competitive markets. Previously he was a project manager at EPRI's PSPO program and research coordinator at Cepel, where he developed methodologies and software for expansion planning, reliability evaluation and hydrothermal scheduling tools. He was one of the recipients of the Franz Edelman Award for Management Science Achievement, granted by ORSA/TIMS for his work on stochastic optimization applied to hydro scheduling.