Test problems of circles in circle packing with constraints and known the optimal solutions*

Hong-fei Teng [1, 2**], Chao Che [2], Yu Chen [1], Yi-shou Wang [1]

1. Department of Mechanical Engineering, 2. Institute of Computing Technology,
Dalian University of Technology, Dalian, 116024, P.R. China

Abstract:
Test problems are generally used to effectively evaluate the algorithms. Based on the engineering background of the layout optimization for a retrievable satellite module, this paper describes the test problems for circles packing problem with the optimal solutions known. That is, N(≦217) circles with different size are packed in a circular container recursively, fulfilling the requirements of both inner and outer tangency. This test problem involves the packing optimization with or without constraints (such as equilibrium constraints). But the above test problems are not the problem with all the optimal solutions known. On the basis of above work, we construct 7 groups of test problems with all the optimal solutions containing N=4,9,17,25,41,33,49 circles. The test problems can be extended to 3-D packing optimization for spherical or cylindrical objects with dynamic equilibrium constraints, more similar to the case of layout optimization model for a satellite module.

Keyword: *test problem, circle packing, constraint, layout of satellite module*

1. Introduction

The packing problems belong to NP-hard problem. We always expect some more effective algorithms to tackle this type of problem, and some evaluation works should be done to see they're better or not. In the appraising process, computational accuracy（or convergence）and efficiency（or convergence rate）are of primary parameters of evaluation. In general, we often use three methods for evaluation:

a. Algorithm theory analysis.

b.Test problem verification, whose optimum solution had been known.

c. Verification in the engineering applications and practices.

However, algorithm analysis is a complicated work (such as complexity analysis, convergence proof and so on), and method c also is an arduous task and difficult to get the result. Therefore, method b is employed frequently.

Many scholars and researchers have developed a number of test problems to evaluate algorithms' performance. De Jong [1] built up the famous DJ-Function Platform to make particular experiments analyzing the performance and mechanism of six schemes for genetic algorithm. Keane [2] advanced a test function to find out how the number of dimensions affect genetic algorithm. Some of 2-D packing and cutting stock examples are given, in [3-4], in order to draw distinction on the performance of different heuristics algorithms. Bischoff [5] proposed many test problems to prove that, the fitted field of existing algorithms is very narrow. More than 1000

---

* Project Sponsored by NSFC of P. R .China (Grant No.60073036, 50275019, 50575031) & Project Sponsored by Doctoral Specialty Foundation of High School of P. R .China (Grant No.20010141005),

** Corresponding author. Email Address: tenghf@dlut.edu.cn

test problems are employed to evaluate the ability of different algorithms, which are employed to search the optimum solution in [6]. Schwerin and Wäscher[7] presented a problem generator for the Bin-Packing Problem (BPP), which generated the test problems to examine MTP algorithm, and some disadvantages of MTP were found .

To construct a good test problem, we have to take account of three key points:

- The optimum should be known in advance.
- The test problem should definite complexity, the optimal solutions.
- The scale of the problem could be modified recursively.

Against the background of satellite module layout design[10] (shown in Figure 1), we construct the circle packing test problem with the optimal solutions known. On the basis of above test problem, we proposed the test problems with all the optimal solutions known and they satisfy above three conditions.

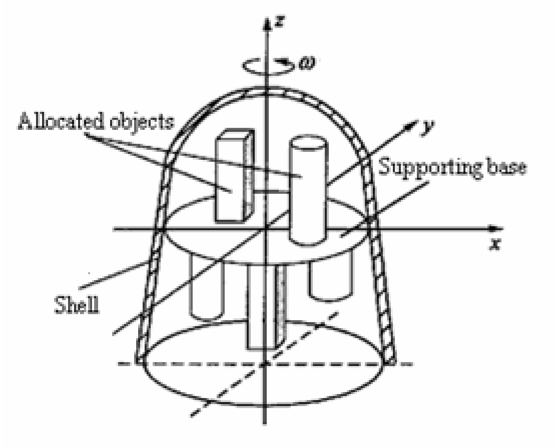

Fig. 1 The objects located in the satellite module

2. The construction of the test problem for *N* circles packing

In the Cartesian coordinate system xOy, we assume that there are a number of circles with different size packed in a circular container D. Let R be the radius of D, and its centroid coordinate is (0,0). Thus， $D(R)$ denoted the container $D(R,0,0)$. A_i denotes circle $A_i(r_i,x_i,y_i)$, in which i is the serial number of the circles. $C_i(x_i,y_i)$ and r_i are its centroid and radius respectively, i=1,2,⋯,N (N≤217). Any adjacent circles in the container are required to be tangent. Fig. 2 illustrates the optimum layout (N = 217). Fig. 3 and Fig. 4 illustrate the enlarge version of part P and part Q of Fig. 2. （The number in Fig.2 denote the sequence number of the circle）

2.1 Notation

$A_{i,j,k}$ denotes a circle in Fig.2. Here, i is the serial number decided by the packing order, i=1,2,…,217 (same as the meaning stated above), and j, k denote that the circle is the k^{th} one (counted from the positive direction of the x-axis, counter-clockwise)on the j^{th} orbit, j=1, 2, ⋯, 32. For instance, in Fig. 3, $A_{10,4,1}$ denotes the first circle in 4th orbit, whose serial number is 10. The meaning of orbit, which is also decided by the order of packing, would be declared in following section.

2.2 Test problem construction

Based on the solution of classical circles in the circle packing problem, firstly, pack four uniform circles in the container, and keep them tangent to adjacent circles and the container. Here, it’s all right to pack 3 or other amount of circles in first step, but the solution is different. After that, 5 feasible areas arise, which are the areas have not been covered by the circles in the container. Then, pack a circle in each feasible area each time. Continue to pack a circle in each feasible area generating from last step until the amount of the packed circles up to certain scale. The dimensions of the circles to be packed are unknown, but tangency and centripetal requirement should be met in the packing process, which guaranteed to obtain the optimum solution. Fortunately, the radiuses and center coordinates of the packed circles’, can be calculated in terms of the geometric relation.

In fact, we have developed a test problem until now, in which the radius values of circles and container are considered to be the known conditions, and the centre coordinates of packed circles is the solution should be solved out. In the following sections, we introduce the packing procedures in detail, and then prove that the solutions are optimum.

The test problem, constructed with the strategy above, is the one without behavior constraint. However, benefiting from the symmetry of obtained layout solution, the value of static equilibrium equals to zero. Hence, the test problem also could be regarded as the ones with the behavior constraints.

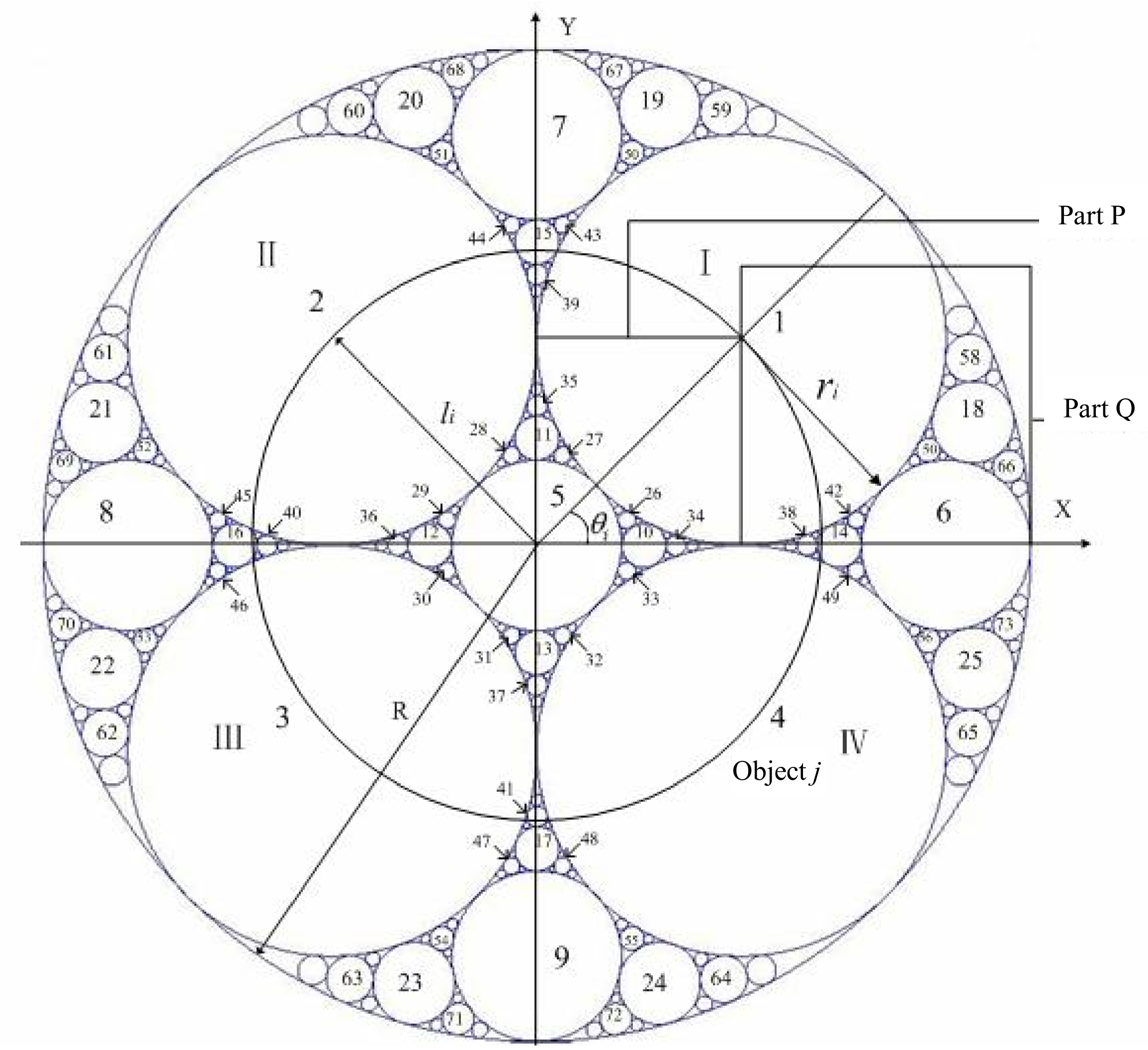

Fig.2 The optimum layout solution

Notation: The sequence number of unlabeled circles illustrated in Fig.3 and Fig. 4

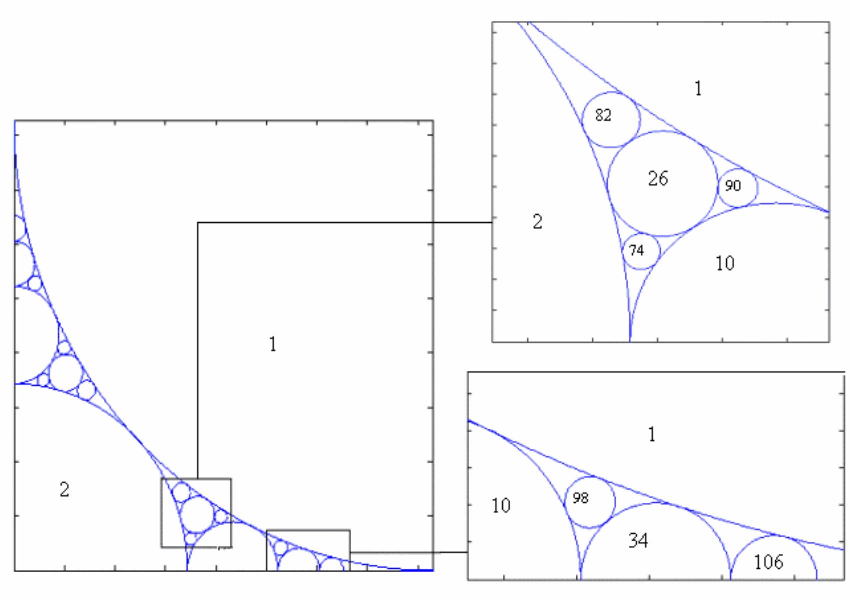

Fig.3 Part P in Fig.2

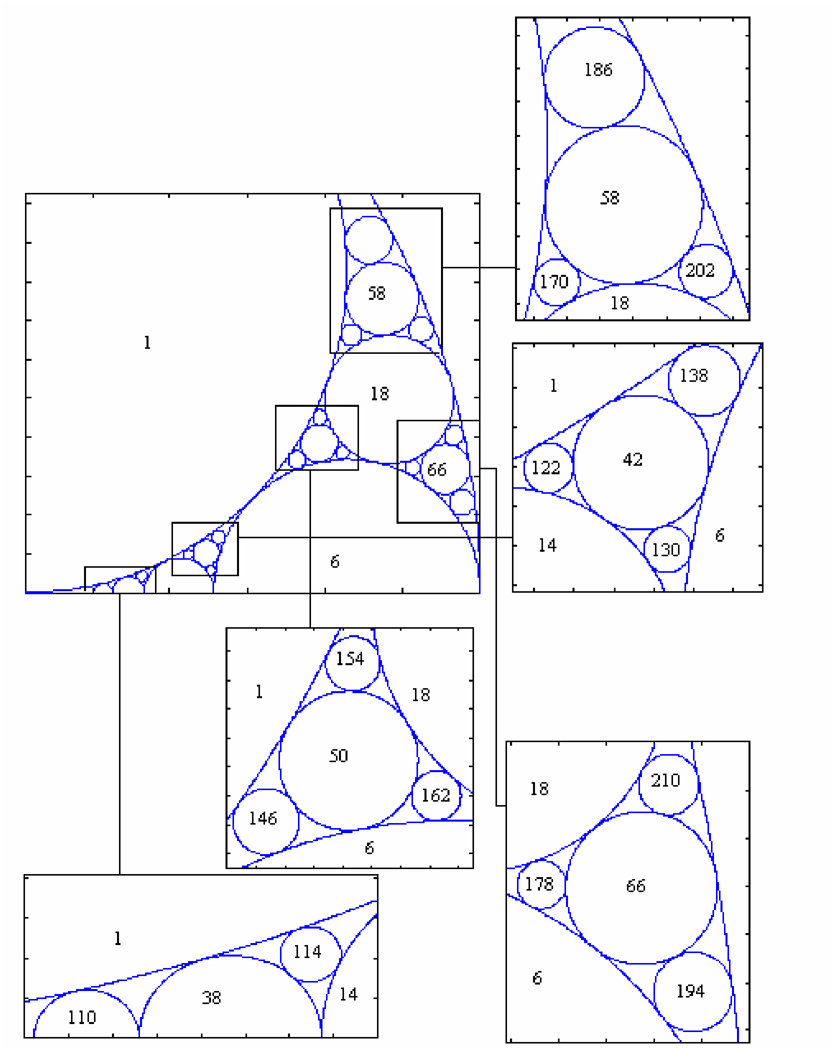

Fig.4 Part Q in Fig. 2

2.3 Process

The packing process includes following steps.

Step 1

Pack four identical circles $A_{i,j,k}$ (i=1,2,3,4, j=1, k=1,2,3,4)in the container D, make each circle tangent to the ones around it and internally tangent to D. Here, we employ the solution of 4 unit circles packing inside the smallest known circle (see [8]), which had been extensively known. For the sake of convenient expression of the final solution, position a circle in each quadrant and make them tangent to x and y axis.

Step 2

In every feasible area, pack only one circle $A_{i,j,k}$ tangent to the adjacent circles(including the container), counter-clockwise, from the centre of the container outwards.

In order to mark the circles packed in the feasible areas generated in the last step, we define B as the batch number of the packed circles, which denote the packing order and indicate whether the circles is in a identical batch. For example, the batch of 4 circles, packed in step 1, is equal to1 (B =1). More information about it can be obtained from Table 1.

Step 3

If the amount of packing circles reaches 217, stop packing. Otherwise return to Step 2.

2.4 How to calculate the radii and center coordinates of the circles

There are two kinds of methods, geometry proving and solving the system of equations. The first one is direct-view and simple, but only operative to the first 10 circles A_i($0<i<10$) and the ones, whose center locate on the coordinate axis. The second one, little more complicated comparing with the first one, is universal. Thereafter, considering of universality, we introduce the second one to get the solution.

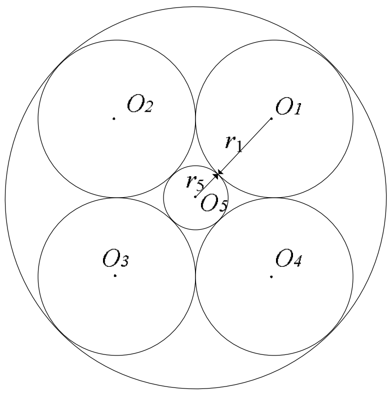

Fig. 5 The feasible area enclosed by 4 uniform packing circles

2. 4. 1 The case in centre feasible area

The packed circle should be tangent to the 4 uniform circles, shown in Fig. 5. $O_{1\text{-}4}$is the center of A_1 -A_4,.

∵ Circles $O_{1\text{-}4}$ are central symmetry

Their radiuses and centre coordinate can be determined, at first. Then, position the circle O_5 at the origin, which is tangent to $O_{1\text{-}4}$ at the same time, and $x_5 = y_5 = 0$.

Since O_1 and O_5 are tangent,

$$\therefore \quad x_1^2 + y_1^2 = (r_1 + r_5)^2 \qquad (1)$$

r_5 can be solved from equation (1).

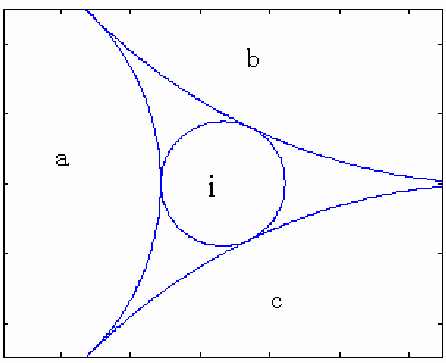

Fig. 6 The feasible area enclosed by 3 arcs of packing circles

2. 4. 2 The feasible areas enclosed by 3 arcs of packed circles

As shown in Fig.6, the packed circle should be tangent to three known arcs, which belong to $A_a(r_a,x_a,y_a)$,

$A_b(r_b,x_b,y_b)$, $A_c(r_c,x_c,y_c)$ respectively. Since the distance between the two tangent circles' center is equal to the sum of their radiuses, we get equation (2):

$$\begin{cases}(x_a-x_i)^2+(y_a-y_i)^2=(r_a+r_i)^2\\(x_b-x_i)^2+(y_b-y_i)^2=(r_b+r_i)^2\\(x_c-x_i)^2+(y_c-y_i)^2=(r_c+r_i)^2\end{cases}\qquad(2)$$

$A_i(r_i,x_i,y_i)$ can be solved out from solved the equation above.

2.4.3 The feasible areas enclosed by 3 arcs of packed circles and container

As shown in Fig.7 the circle to be packed in this type of area should be tangent to the packed circles and internally tangent to the container. $A_a(r_a,x_a,y_a)$, $A_b(r_b,x_b,y_b)$ and $D(R,0,0)$ are known, and equation (3) is given according to the geometric relation between them.

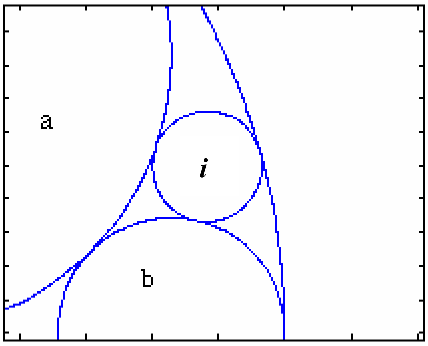

Fig.7 The feasible area enclosed by arcs of packing circles and the container

$$\begin{cases}(x_a-x_i)^2+(y_a-y_i)^2=(r_a+r_i)^2 & (3a)\\(x_b-x_i)^2+(y_b-y_i)^2=(r_b+r_i)^2 & (3b)\\x_i^2+y_i^2=(R-r_i)^2 & (3c)\end{cases}$$

$A_i(r_i,x_i,y_i)$ can be get from the solution of equation (3a)-(3c).

2.5 Different expressions of the solution

With the method above, a special optimum solution has been obtained as shown in Fig.2 and Appendix Table 1. However, the solution of this problem, in principle, is a set of relative positions of the circles. As a result, the solution can be expressed in different ways, depending on the actual location of the circles. Suppose that hold the relative position of all the circles, rotate a angle θ, counter-clockwise. Then C_i (x_i,y_i), the centre coordinate of A_i , would transformed to C_i (x_i', y_i'), see Fig. 8. Thus:

$$\begin{cases}x_i'=x_i\cos\theta-y_i\sin\theta\\y_i'=x_i\sin\theta+y_i\cos\theta\end{cases},\quad\theta\in[0,2\pi]$$

(4)

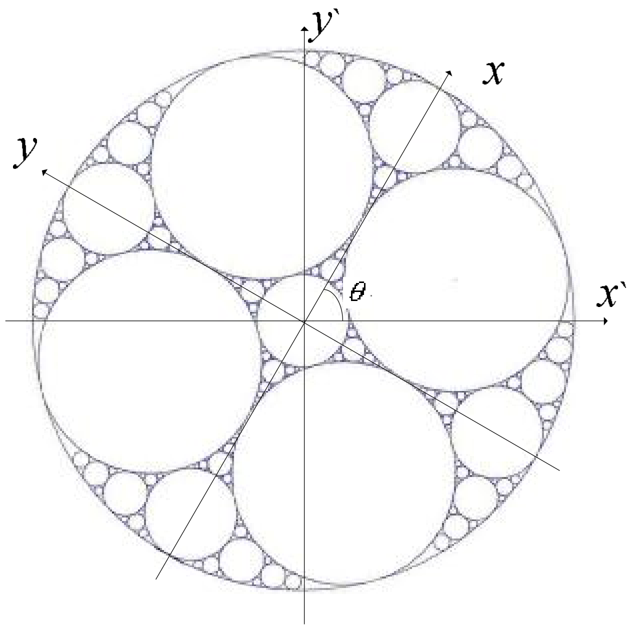

Fig. 8 The solution rotated by θ

The optimal solutions, obtained by algorithms, are usually not identical with the optimal solutions shown in Appendix Table 1, but can be transformed to the similar ones using the method above. In practice, we compare the transformed version of the optimal solutions obtained by every algorithm, and know which one is superior obviously.

2.6 The optimal layout solution analysis of *N*=217 circles packing

1) Tangency

A_i and $A_j (i \neq j)$ are tangent, or tangent to D from the inside.

2) Symmetry

a. Axial symmetry: In a Cansetian coordinate system, the circles in quadrant Ⅰ、Ⅱ、Ⅲ、Ⅳ (see Fig.2), are symmetrical about the x-axis or the y-axis.

b. Central symmetry: To any A_i , a central symmetric circle A_j can be found，i, j=1,3,4,⋯,217. Special case: A_5 is central symmetric to itself。

3) Recursiveness

After pack the four original circles in the container, the circles are packed in the remaining feasible area recursively satisfying the tangency requirement until the number of the packed circles up to the given amount.

4) Uniform radius in same orbit

As shown in Fig.2, it's imaged that there are many concentric circles, position at the container center and with the center of A_i on its arc, called orbit. The circles, whose centers on the same orbit, have the same radius.

5) Unified expression

The radius r_i of A_i, the distance l_i from the centroid of A_i to (0,0) and the radius of enveloped circle $R_{si} = max\{l_i + r_i\}$, can be denoted by R, the radius of the container.

3 Test Problem with the optimal solutions known

3.1 The circle packing test problems (N_{max}≤217) without constraints

In Cartesian coordinates system Oxy, pack N circles $A_i(r_i, c_i)$ within the container D, where $c_i(x_i,y_i) \in R^2$ is the centroid coordinate of the circle A_i, r_i is the radius (shown in Appendix Table 1), $i \in I = \{1, 2, \cdots, 217\}$. The optimization objective is to minimize the radius R_s of the container, subject to the following constraints: (1) Non-overlapping between any two circles; (2) Each circle should be located within the circle representing the container. Then the mathematical model of above test problem can be formulated as follows[8]:

Find $\boldsymbol{X} = \{(x_i, y_i) \mid i \in I\}, I = \{1, 2, \cdots, 217\}$

$$\text{Minimize } f(\boldsymbol{X}) = \min R_s = \min(\max\{\sqrt{x_i^2 + y_i^2} + r_i\}) \tag{5}$$

Subject to:

(a) non-overlapping between any two circles:

$$g_1(\boldsymbol{X}) = r_i + r_j - \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \le 0, (i \neq j, i, j \in I) \tag{6}$$

(b) Each circle should be located within the circle representing the container:

$$g_2(\boldsymbol{X}) = x_i^2 + y_i^2 + r_i - R \le 0, i \in I \tag{7}$$

3. 2 The circles packing test problems (N_{max}≤217) with constraints

In Cartesian coordinates system Oxy, pack N circles $A_i(r_i, c_i)$ within the circular container D rotating at the angular velocity ω , where $c_i(x_i,y_i) \in R^2$ is the centroid coordinate of the circle A_i, r_i is the radius (shown in Appendix Table 1), $i \in \boldsymbol{I}=\{1,2,\cdots N\}$. Suppose that the mass of A_i circle with uniform mass distribution is $m_i = r_i^2$ .Minimize the radius R_s of the container subject to the constraint of above test problems, besides the constraint of static non-equilibrium given by as follows:

(c) When a rigid body of circular shape rotates around an axis and the centroid is not on the rotating axis, the inertial force of rotation will come into being, which is called as static non-equilibrium given by formula (8):

$$g_3(X) = \sqrt{(\sum_{i=1}^{N} m_i x_i)^2 + (\sum_{i=1}^{N} m_i y_i)^2} - [\delta_J] \le 0 \tag{8}$$

Where $[\delta_J]$ is the allowable value of static non-equilibrium.

3.3 Instructions:

(1) As shown in Appendix Table 1. We can attain 5 test problems by the packing batch *B* and attain 7 test problems by *p* groups.

(2) Given N_{max}=217, B_{max}=5, The center of A_i in the optimal layout of the test problem is shown as Appendix Table 1.

(3) The above test problems can be regarded as the packing test problems with the optimal solution known. However, it is not sure whether all the optimal solutions of the test problem are known. In the later section, we will discuss why it is not the test problem with all the optimal solutions known and how to construct the test problems with all the optimal solutions known.

4. Not all the test problems constructed by "*B* packing" are with all the optimal solutions known

In an attempt to provide context for the remaining discussion, the following terms are defined.

DEFINITION 1. ***B* packing**: *Presuppose the number N of the circle A_i to be packed is given, but the radius r_i and the centroid $c_i(x_i, y_i)$ are unbeknown, i∈I={1,2….,N}. Suppose S new feasible areas Ω_s^{B+1} are generated (s=1, 2, ··· , S), after four identical circles are firstly packed into D (B=1). Pack a maximal tangent circle in each existing feasible area of Ω_s^{B+1}. The packing process is called a **packing batch**. In this procedure, all the current feasible areas Ω_s^{B+1} are packed with a circle. Packing in the similar way until the packing batch B reaches the given batch B_{max} (=5). We can get the optimal solution according to above packing rule, which is called as "packing with a tangent circle" or "B packing" for short.*

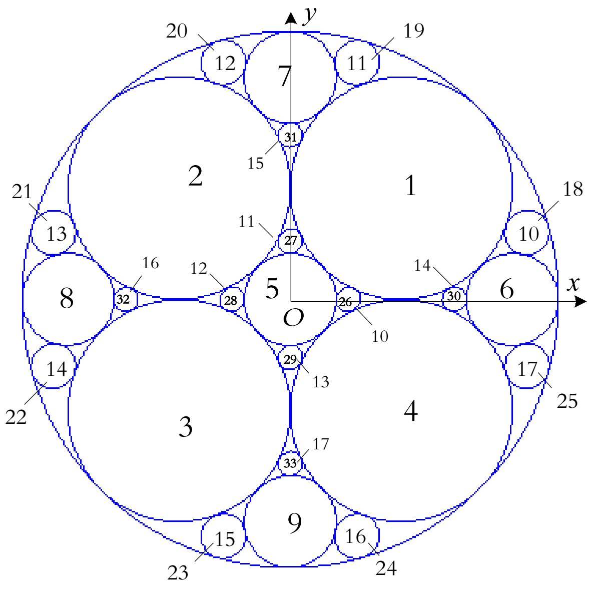

Fig.9 One optimal layout of the test problem (*N*=25) formed by "*B* packing" (the numbers inside the circle correspond with $\{A_i\}^k$, the numbers outside the circle correspond with $\{A_i\}^B$)

DEFINITION 2. ***p* packing**: *Divide the A_i in $\{A_i\}^B$ into different groups according to the radius,* *where the circles in the same group have the same radius. There are p groups,* $p=[1,p_{\max}]$ *(When* B_{max}*=5,* $p_{\max}$ *=23). Thus, pack* A_i *orderly by group according to the rule of "packing with a tangent circle".*

DEFINITION 3. ***k* packing**: *The sequence of the* A_i *in* $\{A_i\}^B$ *is rearranged into the new sequence set* $\{A_i\}^k$ *in the descending order of the radius* r_i*. Thus, pack* A_i *in the order of* $\{A_i\}^k$ *by the rule of "packing with a tangent circle".*

DEFINITION 4. ***q* packing**: *Similarly, divide the* A_i *in* $\{A_i\}^k$ *into different q groups according to the same radius. There are q group* $q=[1,q_{\max}]$ *(*$q_{\max}$ *=23, actually* $q_{\max}=p_{\max}$*). Thus, pack* A_i *orderly by q group according to the rule of "packing with a tangent circle"*

The sequence set of A_i *acquired* by *"B Packing"* can be denoted as $\{A_i\}^B=\{\overbrace{A_1^B\sim A_4^B}^{B=1},\overbrace{A_5^B\sim A_9^B}^{B=2},\overbrace{A_{10}^B\sim A_{25}^B}^{B=3},\overbrace{A_{26}^B\sim A_{73}^B}^{B=4},\overbrace{A_{74}^B\sim A_{217}^B}^{B=5}\}$, The subscript of A_i refers to the sequence of the items. The superscript B denotes the items are packed by "B packing". From the Appendix Table 1, we know that the sequence $\{A_i\}^B$ is not completely sorted in descending order of radius, namely, there exists $A_i(r_i^B)<A_j(r_j^{B+1})(i\neq j,i,j\in I)$. When the scale of the test problem is up to the given N_{max}, $\forall A_i(r_i)$ has at least two feasible areas of all the current feasible areas Ω_s^B to pack, one is Ω_{si}^B occupied by A_i already, the other is the current empty feasible areas Ω_{sj}^B not being occupied (yet will be packed by A_j at the $(B+1)^{\text{th}}$ packing batch). Thus, the packing location of A_i is not uniquely determined, which means the optimal solution of the test problem is not unique. Similarly, this case will appear in the packing approach "*p* packing". For example, given the candidate test problem of N_{max}=25, $A_{10}^B\sim A_{17}^B$ in the optimal configuration shown in figure 5, have two or more than two feasible areas packed at the current packing batch. In other words, they can be packed into position of $A_{18}^{B+1}\sim A_{25}^{B+1}$ besides the positions they are occupied now.

5. The circle packing test problems with all the optimal solutions known

From above analysis, we know that the test problems constructed by "***B*** packing" are not with all the optimal solutions known. How are the test problems with all the optimal solutions constructed? The following methods should be adopted: Arrange the A_i in $\{A_i\}^B$ in the descending order of radius and get the sequence set $\{A_i\}^k$. Divide the A_i of $\{A_i\}^k$ with same radius into one group and obtain the new sequence set $\{A_i\}^q = \{\underbrace{A_1 \sim A_4}_{q=1}, \underbrace{A_5 \sim A_9}_{q=2}, \cdots, \underbrace{A_{34} \sim A_{41}}_{q=6}, \underbrace{A_{42} \sim A_{49}}_{q=7}, \cdots, \underbrace{A_{202} \sim A_{217}}_{q=23}\}$. The subscript of A_i is arranged in the order of i in $\{A_i\}^k$. The data rearranged is seen in Appendix Table 1. Select the of group as the test problems with all the optimal solutions known. The amount of the items of the test problem N=4,9,17,25,33,41,49. The optimal layout of N=49 test problem is shown in Figure 10. It has the beauty of symmetry! Note that the layouts of the optimal solution of the test problems without constraint (N=9, 17) are the same with those of Huang[11].

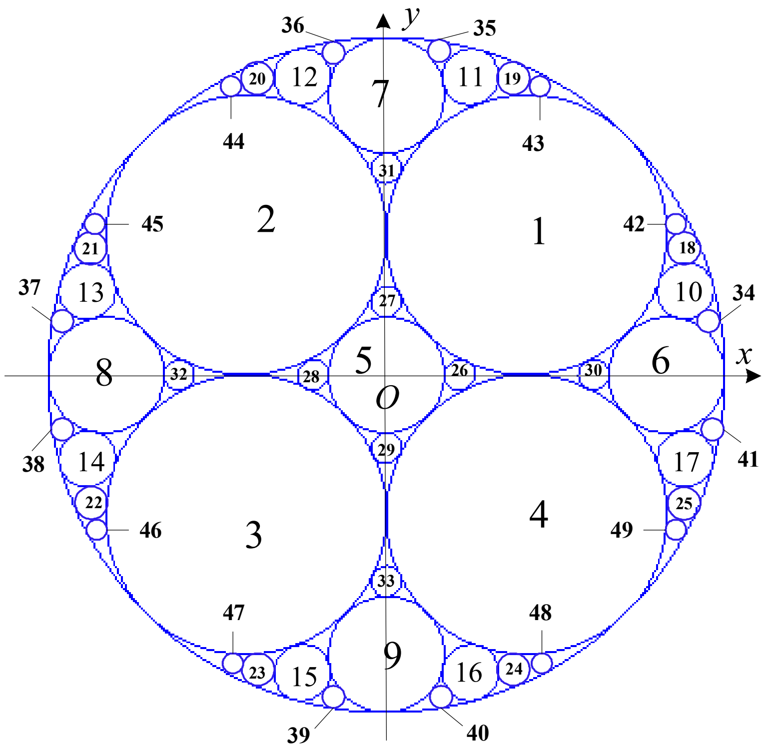

Fig. 10 The optimal layout of the test problem (N=49) with all the optimal solutions known (the number in figure correspond with $\{A_i\}^k$)

6. Conclusion

In this study, we construct 2-D (circle set) packing test problems with the optimal solutions known by "*B Packing*" first. But they are not the test problems with all the optimal solutions known. We obtain the radius r_i, the centroid coordinate c_i and the amount N of the circle A_i through "*B* packing". Meanwhile, r_i and c_i can be expressed by the radius R of the container. On the basis of above data, we get the sequence set $\{A_i\}^k$ by sorting r_i in descending order, then divide the circles into q(=23) groups by the same radius, which constitute of the descending sequence set $\{A_i\}^q$. We select group q=7 of circles A_i as the 2-D packing test problems with all the optimal solutions known, as well as with or without performance constraint. In the end, we get q=7 test problems, whose scale is N=4, 9, 17, 25, 33, 41, 49, respectively (shown in Appendix Table 1).

The test problems are mainly used to test the computation ability of the evolutionary algorithms and have some special characters as follows:

(1) The test problems can be classified as two kinds: test problems with constraints and without performance constraint. These two kinds of test problems can be used to verify the performance of different evolutionary algorithms.

(2) Because all the optimal solutions are known in advance, the variance between the solution gained by algorithms and the optimal solution can be analyzed quantitatively, and be compared qualitatively in the form of visual layout, which is beneficial to visibly compare the computational capability (computational accuracy, efficiency and success ratio) between different algorithms.

(3) The test problems with much larger scale（N>49, q>7） can be constructed by the proposed approach.

(4) The test problems are considerably difficult to solve, and the optimal configuration has the beauty of mathematical symmetry.

The details of the proof and construction method of the test problems with all the optimal solutions known are discussed in another paper.

References

[1] K.A. De Jong. An Analysis of the Behavior of a Class of Genetic Adaptive Systems [Ph.D. Dissertation], Michigan: University of Michigan, 1975.

[2] Andy Keane. Bump A Hard Problem, http://www.soton.ac.uk/~ajk/bump.html,1994,6

[3] E. Hopper. Two-dimensional packing utilising evolutionary algorithms and other meta-heuristic methods，PhD Thesis, Cardiff University, 2000.

[4] Edmund Burke, Graham Kendall. Comparison of Meta-Heuristic Algorithms for Clustering Rectangles. Computers & Industrial Engineering, 1999, 37(1-2): 383-386.

[5] EE Bischoff, MSW. Ratcliff. Issues in the Development of Approaches to Container Loading. Omega, 1995, 23(4): 377-390.

[6] Arnold Neumaier, Oleg Shcherbina, Waltraud Huyer, *et al*. A comparison of complete global optimization solvers. http://www.optimization-online.org/DB_HTML/2004/04/861.html,2004.4

[7] Petra Schwerin, Gerhard Wäscher. The Bin-Packing Problem: A Problem Generator and Some Nmuerical Experiments with FFD Packing and MTP. International Transactions in Operational Research,1997,4(5-6):377-389

[8] Hong-fei Teng, Shou-lin Sun, Wen-hai Ge, *et al*. Layout optimization for the objects installed on a rotating table—the packing problem with equilibrium behavior constrains. Science in China(Series A), 1994, **37**(10): 1272~1280

[9] Sidney Kravitz. Packing Cylinders into Cylindrical Containers. Mathematics Magazine. 1967,40 (2): 65-70.

[10] Zhi-guo Sun, Hong-fei Teng. Optimal layout design of a satellite module. Engineering Optimization, 2003, 35(5): 513-529.

[11] Wen-qi Huang, Ru-chu Xu. Two personification strategies for solving circles packing problem. Science of China (Series E), 1999, 42(6): 595-600

Appendix:

Table 1 Centre coordinate and radius of the circles in the test problems

Number i of $\{A_i\}^k$	Centroid (x_i, y_i)	*Radius* r_i	Number i of $\{A_i\}^B$	Group number q	Group number p	*Batch* B
1	$(\sqrt{2}-1,\ \sqrt{2}-1)$	$\sqrt{2}-1$	1	1^*	1	1
2	$(-\sqrt{2}+1,\ \sqrt{2}-1)$	$\sqrt{2}-1$	2			
3	$(-\sqrt{2}+1,\ -\sqrt{2}+1)$	$\sqrt{2}-1$	3			
4	$(\sqrt{2}-1,\ -\sqrt{2}+1)$	$\sqrt{2}-1$	4			
5	$(0, 0)$	$3-2\sqrt{2}$	5	2^*	2	2
6	$(2\sqrt{2}-2, 0)$	$3-2\sqrt{2}$	6			
7	$(0,\ 2\sqrt{2}-2)$	$3-2\sqrt{2}$	7			
8	$(2\sqrt{2}-2, 0)$	$3-2\sqrt{2}$	8			
9	$(0,\ 2\sqrt{2}-2)$	$3-2\sqrt{2}$	9			
10	$((3+\sqrt{2})/5, 3/5(\sqrt{2}-1))$	$(\sqrt{2}-1)/5$	18	3^*	4	3
11	$(3/5(\sqrt{2}-1), (3+\sqrt{2})/5)$	$(\sqrt{2}-1)/5$	19			
12	$(-3/5(\sqrt{2}-1), (3+\sqrt{2})/5)$	$(\sqrt{2}-1)/5$	20			
13	$(-(3+\sqrt{2})/5, 3/5(\sqrt{2}-1))$	$(\sqrt{2}-1)/5$	21			
14	$(-(3+\sqrt{2})/5, -3/5(\sqrt{2}-1))$	$(\sqrt{2}-1)/5$	22			
15	$(-3/5(\sqrt{2}-1), -(3+\sqrt{2})/5)$	$(\sqrt{2}-1)/5$	23			
16	$(3/5(\sqrt{2}-1), -(3+\sqrt{2})/5)$	$(\sqrt{2}-1)/5$	24			
17	$((3+\sqrt{2})/5, -\ 3/5(\sqrt{2}-1))$	$(\sqrt{2}-1)/5$	25			
18	$(2(25+29\sqrt{2})/151, 8(10\sqrt{2}-7)/151)$	$(10\sqrt{2}-7)/151$	58	4^*	8	4
19	$(8(10\sqrt{2}-7)/151, 2(25+29\sqrt{2})/151)$	$(10\sqrt{2}-7)/151$	59			
20	$(-8(10\sqrt{2}-7)/151, 2(25+29\sqrt{2})/151)$	$(10\sqrt{2}-7)/151$	60			
21	$(-2(25+29\sqrt{2})/151, 8(10\sqrt{2}-7)/151)$	$(10\sqrt{2}-7)/151$	61			
22	$(-2(25+29\sqrt{2})/151, -8(10\sqrt{2}-7)/151)$	$(10\sqrt{2}-7)/151$	62			
23	$(-8(10\sqrt{2}-7)/151, -2(25+29\sqrt{2})/151)$	$(10\sqrt{2}-7)/151$	63			
24	$(8(10\sqrt{2}-7)/151, -2(25+29\sqrt{2})/151)$	$(10\sqrt{2}-7)/151$	64			
25	$(2(25+29\sqrt{2})/151, -8(10\sqrt{2}-7)/151)$	$(10\sqrt{2}-7)/151$	65			
26	$((10-6\sqrt{2})/7, 0)$	$(8\sqrt{2}-11)/7$	10	5^*	3	3
27	$(0,\ (10-6\sqrt{2})/7)$	$(8\sqrt{2}-11)/7$	11			
28	$(-(10-6\sqrt{2})/7, 0)$	$(8\sqrt{2}-11)/7$	12			
29	$(0,\ -(10-6\sqrt{2})/7)$	$(8\sqrt{2}-11)/7$	13			
30	$((20\sqrt{2}-24)/7, 0)$	$(8\sqrt{2}-11)/7$	14			
31	$(0,\ (20\sqrt{2}-24)/7)$	$(8\sqrt{2}-11)/7$	15			
32	$(-(20\sqrt{2}-24)/7, 0)$	$(8\sqrt{2}-11)/7$	16			
33	$(0, -(20\sqrt{2}-24)/7)$	$(8\sqrt{2}-11)/7$	17			

34	$((\sqrt{2}+11)/13, 5(\sqrt{2}-1)/13)$	$(\sqrt{2}-1)/13$	66			
35	$(5(\sqrt{2}-1)/13, (\sqrt{2}+11)/13)$	$(\sqrt{2}-1)/13$	67			
36	$(-5(\sqrt{2}-1)/13, (\sqrt{2}+11)/13)$	$(\sqrt{2}-1)/13$	68			
37	$(-(\sqrt{2}+11)/13, 5(\sqrt{2}-1)/13)$	$(\sqrt{2}-1)/13$	69	6*	9	4
38	$(-(\sqrt{2}+11)/13, -5(\sqrt{2}-1)/13)$	$(\sqrt{2}-1)/13$	70			
39	$(-5(\sqrt{2}-1)/13, -(\sqrt{2}+11)/13)$	$(\sqrt{2}-1)/13$	71			
40	$(5(\sqrt{2}-1)/13, (-\sqrt{2}+11)/13)$	$(\sqrt{2}-1)/13$	72			
41	$((\sqrt{2}+11)/13, -5(\sqrt{2}-1)/13)$	$(\sqrt{2}-1)/13$	73			
42	$((17+31\sqrt{2})/71, 15(17\sqrt{2}-9)/497)$	$(17\sqrt{2}-9)/497$	186			
43	$(15(17\sqrt{2}-9)/497, (17+31\sqrt{2})/71)$	$(17\sqrt{2}-9)/497$	187			
44	$(-15(17\sqrt{2}-9)/497, (17+31\sqrt{2})/71)$	$(17\sqrt{2}-9)/497$	188			
45	$(-(17+31\sqrt{2})/71, 15(17\sqrt{2}-9)/497)$	$(17\sqrt{2}-9)/497$	189	7*	20	5
46	$(-(17+31\sqrt{2})/71, -15(17\sqrt{2}-9)/497)$	$(17\sqrt{2}-9)/497$	190			
47	$(-15(17\sqrt{2}-9)/497, -(17+31\sqrt{2})/71)$	$(17\sqrt{2}-9)/497$	191			
48	$(15(17\sqrt{2}-9)/497, -(17+31\sqrt{2})/71)$	$(17\sqrt{2}-9)/497$	192			
49	$((17+31\sqrt{2})/71, -15(17\sqrt{2}-9)/497)$	$(17\sqrt{2}-9)/497$	193			
50	$(4(20+7\sqrt{2})/151, 8(16\sqrt{2}-19)/151)$	$(16\sqrt{2}-19)/151$	50			
51	$(8(16\sqrt{2}-19)/151, 4(20+7\sqrt{2})/151)$	$(16\sqrt{2}-19)/151$	51			
52	$(-8(16\sqrt{2}-19)/151, 4(20+7\sqrt{2})/151)$	$(16\sqrt{2}-19)/151$	52			
53	$(-4(20+7\sqrt{2})/151, 8(16\sqrt{2}-19)/151)$	$(16\sqrt{2}-19)/151$	53	8	7	
54	$(-4(20+7\sqrt{2})/151, -8(16\sqrt{2}-19)/151)$	$(16\sqrt{2}-19)/151$	54			
55	$(-8(16\sqrt{2}-19)/151, -4(20+7\sqrt{2})/151)$	$(16\sqrt{2}-19)/151$	55			
56	$(8(16\sqrt{2}-19)/151, -4(20+7\sqrt{2})/151)$	$(16\sqrt{2}-19)/151$	56			
57	$(4(20+7\sqrt{2})/151, -8(16\sqrt{2}-19)/151)$	$(16\sqrt{2}-19)/151$	57			
58	$((-40+52\sqrt{2})/119, 0)$	$(18\sqrt{2}-23)/119$	34			
59	$(0, (-40+52\sqrt{2})/119)$	$(18\sqrt{2}-23)/119$	35			
60	$(-(-40+52\sqrt{2})/119, 0)$	$(18\sqrt{2}-23)/119$	36			4
61	$(0, (-40+52\sqrt{2})/119)$	$(18\sqrt{2}-23)/119$	37	9	6	
62	$((-198+186\sqrt{2})/119, 0)$	$(18\sqrt{2}-23)/119$	38			
63	$(0, (-198+186\sqrt{2})/119)$	$(18\sqrt{2}-23)/119$	39			
64	$(-(-198+186\sqrt{2})/119, 0)$	$(18\sqrt{2}-23)/119$	40			
65	$(0, -(-198+186\sqrt{2})/119)$	$(18\sqrt{2}-23)/119$	41			
66	$((23-11\sqrt{2})/41, 3(21\sqrt{2}-29)/41)$	$(21\sqrt{2}-29)/41$	26			
67	$(3(21\sqrt{2}-29)/41, (23-11\sqrt{2})/41)$	$(21\sqrt{2}-29)/41$	27	10	5	
68	$(-3(21\sqrt{2}-29)/41, (23-11\sqrt{2})/41)$	$(21\sqrt{2}-29)/41$	28			
69	$(-(23-11\sqrt{2})/41, 3(21\sqrt{2}-29)/41)$	$(21\sqrt{2}-29)/41$	29			

70	$(-(23-11\sqrt{2})/41, -3(21\sqrt{2}-29)/41)$	$(21\sqrt{2}-29)/41$	30			
71	$(-3(21\sqrt{2}-29)/41, -(23-11\sqrt{2})/41)$	$(21\sqrt{2}-29)/41$	31			4
72	$(3(21\sqrt{2}-29)/41, -(23-11\sqrt{2})/41)$	$(21\sqrt{2}-29)/41$	32			
73	$((23-11\sqrt{2})/41, -3(21\sqrt{2}-29)/41)$	$(21\sqrt{2}-29)/41$	33			
74	$((93\sqrt{2}-105)/41, 3(21\sqrt{2}-29)/41)$	$(21\sqrt{2}-29)/41$	42			
75	$(3(21\sqrt{2}-29)/41, (93\sqrt{2}-105)/41)$	$(21\sqrt{2}-29)/41$	43	10	5	
76	$(-3(21\sqrt{2}-29)/41, (93\sqrt{2}-105)/41)$	$(21\sqrt{2}-29)/41$	44			
77	$(-(93\sqrt{2}-105)/41, 3(21\sqrt{2}-29)/41)$	$(21\sqrt{2}-29)/41$	45			
78	$(-(93\sqrt{2}-105)/41, -3(21\sqrt{2}-29)/41)$	$(21\sqrt{2}-29)/41$	46			
79	$(-3(21\sqrt{2}-29)/41, -(93\sqrt{2}-105)/41)$	$(21\sqrt{2}-29)/41$	47			
80	$(3(21\sqrt{2}-29)/41, -(93\sqrt{2}-105)/41)$	$(21\sqrt{2}-29)/41$	48			
81	$(3(21\sqrt{2}-29)/41, -(93\sqrt{2}-105)/41)$	$(21\sqrt{2}-29)/41$	49			
82	$((23+\sqrt{2})/25, 7(\sqrt{2}-1)/25)$	$(\sqrt{2}-1)/25$	194			
83	$(7(\sqrt{2}-1)/25, (23+\sqrt{2})/25)$	$(\sqrt{2}-1)/25$	195			
84	$(-7(\sqrt{2}-1)/25, (23+\sqrt{2})/25)$	$(\sqrt{2}-1)/25$	196			
85	$(-(23+\sqrt{2})/25, 7(\sqrt{2}-1)/25)$	$(\sqrt{2}-1)/25$	197	11	21	
86	$(-(23+\sqrt{2})/25, -7(\sqrt{2}-1)/25)$	$(\sqrt{2}-1)/25$	198			
87	$(-7(\sqrt{2}-1)/25, -(23+\sqrt{2})/25)$	$(\sqrt{2}-1)/25$	199			
88	$(7(\sqrt{2}-1)/25, -(23+\sqrt{2})/25)$	$(\sqrt{2}-1)/25$	200			
89	$((23+\sqrt{2})/25, -7(\sqrt{2}-1)/25)$	$(\sqrt{2}-1)/25$	201			
90	$((551+421\sqrt{2})/1241, 21(-21+29\sqrt{2})/1241)$	$(29\sqrt{2}-21)/1241$	202			
91	$(21(-21+29\sqrt{2})/1241, (551+421\sqrt{2})/1241)$	$(29\sqrt{2}-21)/1241$	203			5
92	$(-21(-21+29\sqrt{2})/1241, (551+421\sqrt{2})/1241)$	$(29\sqrt{2}-21)/1241$	204			
93	$(-(551+421\sqrt{2})/1241, 21(-21+29\sqrt{2})/1241)$	$(29\sqrt{2}-21)/1241$	205	12	22	
94	$(-(551+421\sqrt{2})/1241, -21(-21+29\sqrt{2})/1241)$	$(29\sqrt{2}-21)/1241$	206			
95	$(-21(-21+29\sqrt{2})/1241, -(551+421\sqrt{2})/1241)$	$(29\sqrt{2}-21)/1241$	207			
96	$(21(-21+29\sqrt{2})/1241, -(551+421\sqrt{2})/1241)$	$(29\sqrt{2}-21)/1241$	208			
97	$((551+421\sqrt{2})/1241, -21(-21+29\sqrt{2})/1241)$	$(29\sqrt{2}-21)/1241$	209			
98	$(4(104+121\sqrt{2})/1319, 24(-27+32\sqrt{2})/1319)$	$(-27+32\sqrt{2})/1319$	170			
99	$(24(-27+32\sqrt{2})/1319, 4(104+121\sqrt{2})/1319)$	$(-27+32\sqrt{2})/1319$	171			
100	$(-24(-27+32\sqrt{2})/1319, 4(104+121\sqrt{2})/1319)$	$(-27+32\sqrt{2})/1319$	172			
101	$(-4(104+121\sqrt{2})/1319, 24(-27+32\sqrt{2})/1319)$	$(-27+32\sqrt{2})/1319$	173	13	18	
102	$(-4(104+121\sqrt{2})/1319, -24(-27+32\sqrt{2})/1319)$	$(-27+32\sqrt{2})/1319$	174			
103	$(-24(-27+32\sqrt{2})/1319, -4(104+121\sqrt{2})/1319)$	$(-27+32\sqrt{2})/1319$	175			
104	$(24(-27+32\sqrt{2})/1319, -4(104+121\sqrt{2})/1319)$	$(-27+32\sqrt{2})/1319$	176			
105	$(4(104+121\sqrt{2})/1319, -24(-27+32\sqrt{2})/1319)$	$(-27+32\sqrt{2})/1319$	177			

106	$(2(493+113\sqrt{2})/1351, 16(34\sqrt{2}-31)/1351)$	$(34\sqrt{2}-31)/1351$	210	14	23	5
107	$(16(34\sqrt{2}-31)/1351, 2(493+113\sqrt{2})/1351)$	$(34\sqrt{2}-31)/1351$	211			
108	$(-16(34\sqrt{2}-31)/1351, 2(493+113\sqrt{2})/1351)$	$(34\sqrt{2}-31)/1351$	212			
109	$(-2(493+113\sqrt{2})/1351, 16(34\sqrt{2}-31)/1351)$	$(34\sqrt{2}-31)/1351$	213			
110	$(-2(493+113\sqrt{2})/1351, -16(34\sqrt{2}-31)/1351)$	$(34\sqrt{2}-31)/1351$	214			
111	$(-16(34\sqrt{2}-31)/1351, -2(493+113\sqrt{2})/1351)$	$(34\sqrt{2}-31)/1351$	215			
112	$(16(34\sqrt{2}-31)/1351, -2(493+113\sqrt{2})/1351)$	$(34\sqrt{2}-31)/1351$	216			
113	$(2(493+113\sqrt{2})/1351, -16(34\sqrt{2}-31)/1351)$	$(34\sqrt{2}-31)/1351$	217			
114	$(6(57\sqrt{2}-53)/527, 0)$	$(32\sqrt{2}-39)/527$	106	15	14	
115	$(0, 6(57\sqrt{2}-53)/527)$	$(32\sqrt{2}-39)/527$	107			
116	$(-6(57\sqrt{2}-53)/527, 0)$	$(32\sqrt{2}-39)/527$	108			
117	$(0, -6(57\sqrt{2}-53)/527)$	$(32\sqrt{2}-39)/527$	109			
118	$((712\sqrt{2}-736)/527, 0)$	$(32\sqrt{2}-39)/527$	110			
119	$(0, (712\sqrt{2}-736)/527)$	$(32\sqrt{2}-39)/527$	111			
120	$(-(712\sqrt{2}-736)/527, 0)$	$(32\sqrt{2}-39)/527$	112			
121	$(0, -(712\sqrt{2}-736)/527)$	$(32\sqrt{2}-39)/527$	113			
122	$(3(11+5\sqrt{2})/71, 15(33\sqrt{2}-41)/497)$	$(33\sqrt{2}-41)/497$	146	16	15	
123	$(15(33\sqrt{2}-41)/497, 3(11+5\sqrt{2})/71)$	$(33\sqrt{2}-41)/497$	147			
124	$(-15(33\sqrt{2}-41)/497, 3(11+5\sqrt{2})/71)$	$(33\sqrt{2}-41)/497$	148			
125	$(-3(11+5\sqrt{2})/71, 15(33\sqrt{2}-41)/497)$	$(33\sqrt{2}-41)/497$	149			
126	$(-3(11+5\sqrt{2})/71, -15(33\sqrt{2}-41)/497)$	$(33\sqrt{2}-41)/497$	150			
127	$(-15(33\sqrt{2}-41)/497, -3(11+5\sqrt{2})/71)$	$(33\sqrt{2}-41)/497$	151			
128	$(15(33\sqrt{2}-41)/497, -3(11+5\sqrt{2})/71)$	$(33\sqrt{2}-41)/497$	152			
129	$(3(11+5\sqrt{2})/71, -15(33\sqrt{2}-41)/497)$	$(33\sqrt{2}-41)/497$	153			
130	$(4(290+13\sqrt{2})/1351, 16(40\sqrt{2}-43)/1351)$	$(40\sqrt{2}-43)/1351$	178	17	19	
131	$(16(40\sqrt{2}-43)/1351, 4(290+13\sqrt{2})/1351)$	$(40\sqrt{2}-43)/1351$	179			
132	$(-16(40\sqrt{2}-43)/1351, 4(290+13\sqrt{2})/1351)$	$(40\sqrt{2}-43)/1351$	180			
133	$(-4(290+13\sqrt{2})/1351, 16(40\sqrt{2}-43)/1351)$	$(40\sqrt{2}-43)/1351$	181			
134	$(-4(290+13\sqrt{2})/1351, -16(40\sqrt{2}-43)/1351)$	$(40\sqrt{2}-43)/1351$	182			
135	$(-16(40\sqrt{2}-43)/1351, -4(290+13\sqrt{2})/1351)$	$(40\sqrt{2}-43)/1351$	183			
136	$(16(40\sqrt{2}-43)/1351, -4(290+13\sqrt{2})/1351)$	$(40\sqrt{2}-43)/1351$	184			
137	$(4(290+13\sqrt{2})/1351, -16(40\sqrt{2}-43)/1351)$	$(40\sqrt{2}-43)/1351$	185			
138	$(6(91+59\sqrt{2})/1319, 24(42\sqrt{2}-47)/1319)$	$(42\sqrt{2}-47)/1319$	154	18	16	
139	$(24(42\sqrt{2}-47)/1319, 6(91+59\sqrt{2})/1319)$	$(42\sqrt{2}-47)/1319$	155			
140	$(-24(42\sqrt{2}-47)/1319, 6(91+59\sqrt{2})/1319)$	$(42\sqrt{2}-47)/1319$	156			
141	$(-6(91+59\sqrt{2})/1319, 24(42\sqrt{2}-47)/1319)$	$(42\sqrt{2}-47)/1319$	157			

142	$(-6(91+59\sqrt{2})/1319, -24(42\sqrt{2}-47)/1319)$	$(42\sqrt{2}-47)/1319$	158	18	16	5
143	$(-24(42\sqrt{2}-47)/1319, -6(91+59\sqrt{2})/1319)$	$(42\sqrt{2}-47)/1319$	159			
144	$(24(42\sqrt{2}-47)/1319, -6(91+59\sqrt{2})/1319)$	$(42\sqrt{2}-47)/1319$	160			
145	$(6(91+59\sqrt{2})/1319, -24(42\sqrt{2}-47)/1319)$	$(42\sqrt{2}-47)/1319$	161			
146	$(2(-1+\sqrt{2})/5, 8(-11+8\sqrt{2})/35)$	$(-11+8\sqrt{2})/35$	82	19	11	
147	$(8(-11+8\sqrt{2})/35, 2(-1+\sqrt{2})/5)$	$(-11+8\sqrt{2})/35$	83			
148	$(-8(-11+8\sqrt{2})/35, 2(-1+\sqrt{2})/5)$	$(-11+8\sqrt{2})/35$	84			
149	$(-2(-1+\sqrt{2})/5, 8(-11+8\sqrt{2})/35)$	$(-11+8\sqrt{2})/35$	85			
150	$(-2(-1+\sqrt{2})/5, -8(-11+8\sqrt{2})/35)$	$(-11+8\sqrt{2})/35$	86			
151	$(-8(-11+8\sqrt{2})/35, -2(-1+\sqrt{2})/5)$	$(-11+8\sqrt{2})/35$	87			
152	$(8(-11+8\sqrt{2})/35, -2(-1+\sqrt{2})/5)$	$(-11+8\sqrt{2})/35$	88			
153	$(2(-1+\sqrt{2})/5, -8(-11+8\sqrt{2})/35)$	$(-11+8\sqrt{2})/35$	89			
154	$(8(-1+\sqrt{2})/5, 8(-11+8\sqrt{2})/35)$	$(-11+8\sqrt{2})/35$	138			
155	$(8(-11+8\sqrt{2})/35, 8(-1+\sqrt{2})/5)$	$(-11+8\sqrt{2})/35$	139			
156	$(-8(-11+8\sqrt{2})/35, 8(-1+\sqrt{2})/5)$	$(-11+8\sqrt{2})/35$	140			
157	$(-8(-1+\sqrt{2})/5, 8(-11+8\sqrt{2})/35)$	$(-11+8\sqrt{2})/35$	141			
158	$(-8(-1+\sqrt{2})/5, -8(-11+8\sqrt{2})/35)$	$(-11+8\sqrt{2})/35$	142			
159	$(-8(-11+8\sqrt{2})/35, -8(-1+\sqrt{2})/5)$	$(-11+8\sqrt{2})/35$	143			
160	$(8(-11+8\sqrt{2})/35, -8(-1+\sqrt{2})/5)$	$(-11+8\sqrt{2})/35$	144			
161	$(8(-1+\sqrt{2})/5, -8(-11+8\sqrt{2})/35)$	$(-11+8\sqrt{2})/35$	145			
162	$(9(95+13\sqrt{2})/1241, 21(-53+45\sqrt{2})/1241)$	$(-53+45\sqrt{2})/1241$	162	20	17	
163	$(21(-53+45\sqrt{2})/1241, 9(95+13\sqrt{2})/1241)$	$(-53+45\sqrt{2})/1241$	163			
164	$(-21(-53+45\sqrt{2})/1241, 9(95+13\sqrt{2})/1241)$	$(-53+45\sqrt{2})/1241$	164			
165	$(-9(95+13\sqrt{2})/1241, 21(-53+45\sqrt{2})/1241)$	$(-53+45\sqrt{2})/1241$	165			
166	$(-9(95+13\sqrt{2})/1241, -21(-53+45\sqrt{2})/1241)$	$(-53+45\sqrt{2})/1241$	166			
167	$(-21(-53+45\sqrt{2})/1241, -9(95+13\sqrt{2})/1241)$	$(-53+45\sqrt{2})/1241$	167			
168	$(21(-53+45\sqrt{2})/1241, -9(95+13\sqrt{2})/1241)$	$(-53+45\sqrt{2})/1241$	168			
169	$(9(95+13\sqrt{2})/1241, -21(-53+45\sqrt{2})/1241)$	$(-53+45\sqrt{2})/1241$	169			
170	$(3(-59+95\sqrt{2})/857, 3(-69+53\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	98	21	13	
171	$(3(-69+53\sqrt{2})/857, 3(-59+95\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	99			
172	$(-3(-69+53\sqrt{2})/857, 3(-59+95\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	100			
173	$(-3(-59+95\sqrt{2})/857, 3(-69+53\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	101			
174	$(-3(-59+95\sqrt{2})/857, -3(-69+53\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	102			
175	$(-3(-69+53\sqrt{2})/857, -3(-59+95\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	103			
176	$(3(-69+53\sqrt{2})/857, -3(-59+95\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	104			
177	$(3(-59+95\sqrt{2})/857, -3(-69+53\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	105			

178	$((-1537+1429\sqrt{2})/857, 3(-69+53\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	114			
179	$(3(-69+53\sqrt{2})/857, (-1537+1429\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	115			
180	$(-3(-69+53\sqrt{2})/857, (-1537+1429\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	116			
181	$(-(-1537+1429\sqrt{2})/857, 3(-69+53\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	117	21	13	
182	$(-(-1537+1429\sqrt{2})/857, -3(-69+53\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	118			
183	$(-3(-69+53\sqrt{2})/857, -(-1537+1429\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	119			
184	$(3(-69+53\sqrt{2})/857, -(-1537+1429\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	120			
185	$((-1537+1429\sqrt{2})/857, -3(-69+53\sqrt{2})/857)$	$(-69+53\sqrt{2})/857$	121			
186	$(4(32-5\sqrt{2})/487, 8(-79+58\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	90			
187	$(8(-79+58\sqrt{2})/487, 4(32-5\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	91			
188	$(-8(-79+58\sqrt{2})/487, 4(32-5\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	92			
189	$(-4(32-5\sqrt{2})/487, 8(-79+58\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	93			
190	$(-4(32-5\sqrt{2})/487, -8(-79+58\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	94			
191	$(-8(-79+58\sqrt{2})/487, -4(32-5\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	95			
192	$(8(-79+58\sqrt{2})/487, -4(32-5\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	96			5
193	$(4(32-5\sqrt{2})/487, -8(-79+58\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	97			
194	$((-1102+994\sqrt{2})/487, 8(-79+58\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	122	22	12	
195	$(8(-79+58\sqrt{2})/487, (-1102+994\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	123			
196	$(-8(-79+58\sqrt{2})/487, (-1102+994\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	124			
197	$(-(-1102+994\sqrt{2})/487, 8(-79+58\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	125			
198	$(-(-1102+994\sqrt{2})/487, -8(-79+58\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	126			
199	$(-8(-79+58\sqrt{2})/487, -(-1102+994\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	127			
200	$(8(-79+58\sqrt{2})/487, -(-1102+994\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	128			
201	$((-1102+994\sqrt{2})/487, -8(-79+58\sqrt{2})/487)$	$(-79+58\sqrt{2})/487$	129			
202	$((359-227\sqrt{2})/217, 5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	74			
203	$(5(61\sqrt{2}-85)/217, 5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	75			
204	$(-5(61\sqrt{2}-85)/217, 5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	76			
205	$(-(359-227\sqrt{2})/217, 5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	77			
206	$(-(359-227\sqrt{2})/217, -5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	78			
207	$(-5(61\sqrt{2}-85)/217, -5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	79			
208	$(5(61\sqrt{2}-85)/217, -5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	80	23	10	
209	$((359-227\sqrt{2})/217, -5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	81			
210	$((-793+661\sqrt{2})/217, 5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	130			
211	$(5(61\sqrt{2}-85)/217, (-793+661\sqrt{2})/217)$	$(61\sqrt{2}-85)/217$	131			
212	$(-5(61\sqrt{2}-85)/217, (-793+661\sqrt{2})/217)$	$(61\sqrt{2}-85)/217$	132			
213	$(-(-793+661\sqrt{2})/217, 5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	133			

214	$(-(-793+661\sqrt{2})/217, -5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	134	23	10	5
215	$(-5(61\sqrt{2}-85)/217, -(-793+661\sqrt{2})/217)$	$(61\sqrt{2}-85)/217$	135			
216	$(5(61\sqrt{2}-85)/217, -(-793+661\sqrt{2})/217)$	$(61\sqrt{2}-85)/217$	136			
217	$((-793+661\sqrt{2})/217, -5(61\sqrt{2}-85)/217)$	$(61\sqrt{2}-85)/217$	137			

Note: the group number with * indicate they are the test problems with all the solutions known.