Tubechallenges: Can OR help break records?

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Abstract

Several 'tubechallenges' have been attempted on the London Underground network, often gaining vast press coverage. The most famous of them consists of trying to travel to all 275 stations of the London Underground network (known as 'the tube') in the shortest possible time. In this article we examine how Operational Research can assist potential record-breakers, by formulating this complex optimisation problem as an integer programming model and trying to establish the optimal route that one has to follow, taking into consideration as many route constraints as possible.

1 Introduction

In this article we consider an Operational Research approach to the problem of finding the best route one has to travel on a network in order to visit all the network nodes at least once, in the shortest amount of time. This translates to the so called 'tubechallenge' and several of its variants, where individuals attempt to travel the entire London Underground (subway) network in a single day [4, 5]. The task is indeed a 'challenge', given that the optimal route requires travelling almost for the entire hours of operation of the Underground, and the amount of luck involved (even a small delay can jeopardise the whole attempt).

We present a simple integer programming model to formulate the above problem and propose a solution methodology based on the Branch and Cut technique. Computational results, including the theoretical optimal routes, are also presented.

2 An Integer Programming model

We consider the following problem: Given an undirected network (V, E) with N vertices, E edges and edge costs, we want to find the minimum cost walk (starting at one vertex and finishing at another) such that every vertex is visited at least once in the walk. This problem can easily be formulated as a tour problem by introducing an artificial vertex and joining it with all other vertices with edges of zero cost. Then a walk in the original network is equivalent to a tour in the extended network. In other words, the artificial vertex can be considered as a person's 'home' and the challenge becomes to visit all tube stations at least once, starting and finishing at the home station.

The problem can be formulated as an Integer Programme as follows:

$$\min\Big\{\sum_{i,j\in V} c_{ij}x_{ij} + c_{ji}x_{ji}\Big\} \tag{1}$$

such that

$$\sum_{j\in V} x_{ij} = \sum_{j\in V} x_{ji}, \text{ for all } 1 \le i \le N \tag{2}$$

$$\sum_{j\in V} x_{ij} \ge 1, \text{ for all } 1 \le i \le N \tag{3}$$

$$\sum_{j\in V} x_{0j} = 1 \tag{4}$$

$$\sum_{i\in S,\, j\notin S} x_{ij} \ge 1, \text{ for all } 0 \in S \subset V \tag{5}$$

$$x_{ij} \in \{0, 1, 2, \ldots\}, \text{ for all } 0 \le i, j \le N \tag{6}$$

The vertices are labelled $\{1, 2, \ldots, N\}$ with the artificial home vertex labelled 0. The decision variables x_{ij} represent the number of times the arc (i, j) is used in the solution. The costs c_{ij} represent the time taken to travel between vertices i and j. The costs are symmetric, i.e., $x_{ij} = x_{ji}$. The objective function (1) seeks to mininise the total cost which is the sum of the arc costs (the travelling time between two stations) over all the edges used. Constraints (2) stipulate that the route must be a tour, i.e., the number of times a station is 'visited' must equal the number of times it is 'exited'. Constraints (3) and (4) state that each vertex must be visited at least once, with the home vertex visited exactly once. Finally constraints (5) are the subtour elimination constraints, requiring that the solution must be a cycle and not a union of disjoint cycles.

The above problem is similar with the well-studied symmetric travelling salesman problem, with the difference that vertices may be visited more than once [1, 3].

3 Computational implementation

We started by considering the full tubechallenge, namely to find the optimal route to travel to all London Underground stations in the minimum time. All information about the network was obtained from the London Underground website. It consists of are 12 underground lines and 274 stations.[1] The first task was to obtain the location and the costs of the edges. This proved to be straightforward, as London Underground

[1] I discovered later that the 'official' (i.e., Guinness book of records) tubechallenge considers two distinct Paddington stations: one served by the Hammersmith & City line, and the other served by the remaining three lines that go to Paddington. In that sense, one may suggest that there are 275 stations in total. I only considered *one* Paddington, but this did not prove to be a limitation in the end, because the best solution, even this case, travels to both stations anyway.

publishes maps containing the time (to the nearest minute) taken between any two consecutive stations.

The next step was to reduce the complexity of the problem by removing unnecessary vertices of degree 2. For example, the part of the Northern line from Chalk Farm to Edgware (eight edges) was shrunk to a single edge (with a cost the sum of the eight costs) as the intermediate stations are irrelevant to the solution. Similar reductions can be performed to many parts of the network, not necessarily at the end of a line (e.g., between Park Royal and South Harrow). This reduced the size of the network by about a third, to 199 stations.

The third step was to translate the integer programme in a computer program and solve it. It was programmed using C++ on a Pentium III PC with CPLEX as the solver. There is an important difficulty using the Integer Programme described earlier. The subtour elimination constraints (5) are too many to be explicitly listed. Even with the shrunk network, there are $2^{198} - 1$ of them, an enormous number of them (approximately 4 followed by 59 zeros) so finding all of them even on a computer is practically impossible, let alone solving them. For this reason, as it is common procedure in many mixed integer programmes in a Branch and Cut framework, it was decided to include only a subset of them (those with $|S| = 1$) in the initial problem, and add them as necessary during the Branch and Cut solution procedure.

For this we need to devise an efficient algorithm to test if a given integer solution violates any of the subtour elimination constraints that we have chosen to exclude from the formulation. We chose the minimum cut procedure of Gomory and Hu [2] which can identify the minimum network cut separating any pair of vertices. If the minimum cut with input weights the incumbent values of x_{ij} is zero, then the solution is rejected as it contains subtours. If it has a value of 2 or more, then the solution is valid. (The value 1 is not achievable as it is implicitly not allowed from the constraints (3)–(4).) The Gomory-Hu algorithm runs in polynomial time.

4 Results

The problem was solved on a computer in a reasonable amount of time. All decision variables in the optimal solution had values 0 or 1, meaning that no section of track between two stations was travelled twice in the same direction. For the full challenge, a theoretical optimum of 930 minutes was achieved. This, however, does not consider the time for interchanges between tube lines. Given that the solution includes 75 such changes and the best record to date stands at 1115.7 minutes (18 hours 35 minutes and 43 seconds), this could be broken with an average time for each interchange of about 2.5 minutes.

The optimal routes and other characteristics of the best solution for the tubechallenge, the Zone 1 challenge and the Circle line bottle challenge are shown in tables 1–4.

5 Limitations and future work

The main limitation with the above simplistic approach is that the time taken for interchanges is not taken into consideration. This is particularly relevant to the smaller challenges (Zone 1 and Circle line) as in these cases the time to interchange is

N	*stations visited*	*optimal value*	*cuts added*	*branch and cut nodes*	*CPU* time
199	264	930	567	2013320	3874

OPTIMAL ROUTE				
HARROW & W'STONE	WEMBLEY PARK	BOND STREET	WANSTEAD	CHARING CROSS
PADDINGTON	NEASDEN	OXFORD CIRCUS	REDBRIDGE	EMBANKMENT
BAYSWATER	DOLLIS HILL	REGENTS PARK	GANTS HILL	WESTMINSTER
NOTTING HILL GATE	WILLESDEN GREEN	BAKER STREET	NEWBURY PARK	ST JAMES'S PARK
HOLLAND PARK	KILBURN	GREAT PORTLAND ST	BARKINGSIDE	WESTMINSTER
SHEP BUSH (CENT)	WEST HAMPSTEAD	EUSTON SQUARE	FAIRLOP	WATERLOO
WHITE CITY	FINCHLEY ROAD	KING'S X ST PANCRAS	HAINAULT	LAMBETH NORTH
EAST ACTON	SWISS COTTAGE	FARRINGDON	GRANGE HILL	WATERLOO
NORTH ACTON	ST JOHN'S WOOD	BARBICAN	CHIGWELL	SOUTHWARK
WEST RUISLIP	BAKER STREET	MOORGATE	RODING VALLEY	LONDON BRIDGE
NORTH ACTON	MARYLEBONE	OLD STREET	WOODFORD	BOROUGH
WEST ACTON	EDGWARE RD (B'LOO)	ANGEL	BUCKHURST HILL	ELEPHANT & CASTLE
EALING BROADWAY	MARYLEBONE	KING'S X ST PANCRAS	LOUGHTON	KENNINGTON
EALING COMMON	BAKER STREET	HIGHBURY & IS'TON	DEBDEN	OVAL
NORTH EALING	EDGWARE RD (CIRCLE)	FINSBURY PARK	THEYDON BOIS	STOCKWELL
PARK ROYAL	PADDINGTON	WALTHAMSTOW CENT	EPPING	CLAPHAM NORTH
ALPERTON	ROYAL OAK	FINSBURY PARK	THEYDON BOIS	MORDEN
SUDBURY TOWN	WESTBOURNE PARK	COCKFOSTERS	DEBDEN	CLAPHAM NORTH
SUDBURY HILL	LADBROKE GROVE	FINSBURY PARK	LOUGHTON	STOCKWELL
SOUTH HARROW	LATIMER ROAD	ARSENAL	BUCKHURST HILL	BRIXTON
RAYNERS LANE	SHEP BUSH (H&C)	HOLLOWAY ROAD	WOODFORD	STOCKWELL
UXBRIDGE	GOLDHAWK ROAD	CALEDONIAN ROAD	SOUTH WOODFORD	VAUXHALL
RAYNERS LANE	HAMMERSMITH	KING'S X ST PANCRAS	SNARESBROOK	PIMLICO
WEST HARROW	TURNHAM GREEN	RUSSELL SQUARE	LEYTONSTONE	VICTORIA
HARROW-ON-THE-HILL	CHISWICK PARK	HOLBORN	LEYTON	SLOANE SQUARE
NORTH HARROW	ACTON TOWN	CHANCERY LANE	STRATFORD	SOUTH KENSINGTON
PINNER	HATTON CROSS	ST PAUL'S	WEST HAM	GLOUCESTER ROAD
NORTHWOOD HILLS	HEATHROW TERM 4	BANK	CANNING TOWN	SOUTH KENSINGTON
NORTHWOOD	HEATHROW TERMS 123	LIVERPOOL STREET	NORTH GREENWICH	KNIGHTSBRIDGE
MOOR PARK	HATTON CROSS	ALDGATE	CANARY WHARF	HYDE PARK CORNER
RICKMANSWORTH	ACTON TOWN	LIVERPOOL STREET	CANADA WATER	GREEN PARK
CHORLEYWOOD	TURNHAM GREEN	BETHNAL GREEN	SURREY QUAYS	PICCADILLY CIRCUS
CHALFONT & LATIMER	GUNNERSBURY	MILE END	NEW CROSS	LEICESTER SQUARE
AMERSHAM	KEW GARDENS	STEPNEY GREEN	SURREY QUAYS	COVENT GARDEN
CHALFONT & LATIMER	RICHMOND	MILE END	NEW CROSS GATE	LEICESTER SQUARE
CHESHAM	KEW GARDENS	BOW ROAD	SURREY QUAYS	TOTTENHAM CT ROAD
CHALFONT & LATIMER	GUNNERSBURY	BROMLEY-BY-BOW	CANADA WATER	GOODGE STREET
CHORLEYWOOD	TURNHAM GREEN	WEST HAM	BERMONDSEY	WARREN STREET
RICKMANSWORTH	STAMFORD BROOK	PLAISTOW	CANADA WATER	EUSTON
MOOR PARK	RAVENSCOURT PARK	UPTON PARK	ROTHERHITHE	MORNINGTON CRES
CROXLEY	HAMMERSMITH	EAST HAM	WAPPING	CAMDEN TOWN
WATFORD	BARONS COURT	BARKING	SHADWELL	EDGWARE
CROXLEY	WEST KENSINGTON	UPNEY	WHITECHAPEL	CAMDEN TOWN
MOOR PARK	EARL'S COURT	UPMINSTER	SHOREDITCH	KENTISH TOWN
NORTHWOOD	KENSINGTON (OLYMPIA)	UPNEY	WHITECHAPEL	TUFNELL PARK
NORTHWOOD HILLS	EARL'S COURT	BARKING	ALDGATE EAST	ARCHWAY
PINNER	WIMBLEDON	EAST HAM	TOWER HILL	HIGHGATE
NORTH HARROW	EARL'S COURT	UPTON PARK	MONUMENT	EAST FINCHLEY
HARROW-ON-THE-HILL	HIGH ST KENSINGTON	PLAISTOW	CANNON STREET	FINCHLEY CENTRAL
NORTHWICK PARK	NOTTING HILL GATE	WEST HAM	MANSION HOUSE	MILL HILL EAST
PRESTON ROAD	QUEENSWAY	STRATFORD	BLACKFRIARS	FINCHLEY CENTRAL
WEMBLEY PARK	LANCASTER GATE	LEYTON	TEMPLE	HIGH BARNET
STANMORE	MARBLE ARCH	LEYTONSTONE	EMBANKMENT	

Table 1: Optimal route for the tubechallenge

N	*stations visited*	*optimal value*	*cuts added*	*branch and cut nodes*	*CPU* time
199	262	932	339	145006	171 sec

OPTIMAL ROUTE				
HEATHROW TERM 4	BOROUGH	WANSTEAD	ANGEL	NORTHWOOD
HEATHROW TERMS 123	LONDON BRIDGE	REDBRIDGE	KING'S X ST PANCRAS	NORTHWOOD HILLS
HATTON CROSS	SOUTHWARK	GANTS HILL	CALEDONIAN ROAD	PINNER
ACTON TOWN	WATERLOO	NEWBURY PARK	HOLLOWAY ROAD	NORTH HARROW
CHISWICK PARK	LAMBETH NORTH	BARKINGSIDE	ARSENAL	HARROW-ON-THE-HILL
TURNHAM GREEN	ELEPHANT & CASTLE	FAIRLOP	FINSBURY PARK	WEST HARROW
GUNNERSBURY	KENNINGTON	HAINAULT	COCKFOSTERS	RAYNERS LANE
KEW GARDENS	OVAL	GRANGE HILL	FINSBURY PARK	UXBRIDGE
RICHMOND	STOCKWELL	CHIGWELL	WALTHAMSTOW CENT	RAYNERS LANE
KEW GARDENS	CLAPHAM NORTH	RODING VALLEY	FINSBURY PARK	SOUTH HARROW
GUNNERSBURY	MORDEN	WOODFORD	HIGHBURY & IS'TON	SUDBURY HILL
TURNHAM GREEN	CLAPHAM NORTH	BUCKHURST HILL	KING'S X ST PANCRAS	SUDBURY TOWN
STAMFORD BROOK	STOCKWELL	LOUGHTON	EUSTON SQUARE	ALPERTON
RAVENSCOURT PARK	BRIXTON	DEBDEN	GREAT PORTLAND ST	PARK ROYAL
HAMMERSMITH	STOCKWELL	THEYDON BOIS	BAKER STREET	NORTH EALING
BARONS COURT	VAUXHALL	EPPING	EDGWARE RD (CIRCLE)	EALING COMMON
WEST KENSINGTON	PIMLICO	THEYDON BOIS	PADDINGTON	EALING BROADWAY
BARONS COURT	VICTORIA	DEBDEN	EDGWARE RD (B'LOO)	WEST ACTON
HAMMERSMITH	ST JAMES'S PARK	LOUGHTON	MARYLEBONE	NORTH ACTON
GOLDHAWK ROAD	WESTMINSTER	BUCKHURST HILL	BAKER STREET	WEST RUISLIP
SHEP BUSH (H&C)	EMBANKMENT	WOODFORD	ST JOHN'S WOOD	NORTH ACTON
LATIMER ROAD	CHARING CROSS	SOUTH WOODFORD	SWISS COTTAGE	EAST ACTON
LADBROKE GROVE	EMBANKMENT	SNARESBROOK	FINCHLEY ROAD	WHITE CITY
WESTBOURNE PARK	TEMPLE	LEYTONSTONE	WEST HAMPSTEAD	SHEP BUSH (CENT)
ROYAL OAK	BLACKFRIARS	LEYTON	KILBURN	HOLLAND PARK
PADDINGTON	MANSION HOUSE	STRATFORD	WILLESDEN GREEN	NOTTING HILL GATE
HARROW & W'STONE	CANNON STREET	WEST HAM	DOLLIS HILL	QUEENSWAY
PADDINGTON	MONUMENT	CANNING TOWN	NEASDEN	LANCASTER GATE
BAYSWATER	TOWER HILL	NORTH GREENWICH	WEMBLEY PARK	MARBLE ARCH
NOTTING HILL GATE	ALDGATE	CANARY WHARF	STANMORE	BOND STREET
HIGH ST KENSINGTON	LIVERPOOL STREET	CANADA WATER	WEMBLEY PARK	OXFORD CIRCUS
EARL'S COURT	BETHNAL GREEN	SURREY QUAYS	PRESTON ROAD	REGENTS PARK
KENSINGTON (OLYMPIA)	MILE END	NEW CROSS	NORTHWICK PARK	OXFORD CIRCUS
EARL'S COURT	STEPNEY GREEN	SURREY QUAYS	HARROW-ON-THE-HILL	TOTTENHAM CT ROAD
WIMBLEDON	MILE END	NEW CROSS GATE	NORTH HARROW	GOODGE STREET
EARL'S COURT	BOW ROAD	SURREY QUAYS	PINNER	WARREN STREET
GLOUCESTER ROAD	BROMLEY-BY-BOW	CANADA WATER	NORTHWOOD HILLS	EUSTON
SOUTH KENSINGTON	WEST HAM	BERMONDSEY	NORTHWOOD	MORNINGTON CRES
SLOANE SQUARE	PLAISTOW	CANADA WATER	MOOR PARK	CAMDEN TOWN
SOUTH KENSINGTON	UPTON PARK	ROTHERHITHE	RICKMANSWORTH	EDGWARE
KNIGHTSBRIDGE	EAST HAM	WAPPING	CHORLEYWOOD	CAMDEN TOWN
HYDE PARK CORNER	BARKING	SHADWELL	CHALFONT & LATIMER	KENTISH TOWN
GREEN PARK	UPNEY	WHITECHAPEL	AMERSHAM	TUFNELL PARK
PICCADILLY CIRCUS	UPMINSTER	SHOREDITCH	CHALFONT & LATIMER	ARCHWAY
LEICESTER SQUARE	UPNEY	WHITECHAPEL	CHESHAM	HIGHGATE
COVENT GARDEN	BARKING	ALDGATE EAST	CHALFONT & LATIMER	EAST FINCHLEY
HOLBORN	EAST HAM	LIVERPOOL STREET	CHORLEYWOOD	FINCHLEY CENTRAL
RUSSELL SQUARE	UPTON PARK	MOORGATE	RICKMANSWORTH	MILL HILL EAST
HOLBORN	PLAISTOW	BARBICAN	MOOR PARK	FINCHLEY CENTRAL
CHANCERY LANE	WEST HAM	FARRINGDON	CROXLEY	HIGH BARNET
ST PAUL'S	STRATFORD	BARBICAN	WATFORD	
BANK	LEYTON	MOORGATE	CROXLEY	
LONDON BRIDGE	LEYTONSTONE	OLD STREET	MOOR PARK	

Table 2: Optimal route for the tubechallenge, with a Heathrow start imposed.

N	*stations visited*	*optimal value*	*cuts added*	*branch and cut nodes*	*CPU* time
63	72	124	73	289	0.41 sec

OPTIMAL ROUTE			
VAUXHALL	REGENTS PARK	GOODGE STREET	WESTMINSTER
PIMLICO	BAKER STREET	TOTTENHAM CT ROAD	GREEN PARK
VICTORIA	MARYLEBONE	LEICESTER SQUARE	HYDE PARK CORNER
ST JAMES'S PARK	EDGWARE RD (B'LOO)	COVENT GARDEN	KNIGHTSBRIDGE
VICTORIA	PADDINGTON	HOLBORN	HYDE PARK CORNER
SLOANE SQUARE	EDGWARE RD (CIRCLE)	RUSSELL SQUARE	GREEN PARK
SOUTH KENSINGTON	BAKER STREET	HOLBORN	PICCADILLY CIRCUS
GLOUCESTER ROAD	GREAT PORTLAND ST	CHANCERY LANE	CHARING CROSS
EARL'S COURT	EUSTON SQUARE	ST PAUL'S	EMBANKMENT
HIGH ST KENSINGTON	KING'S X ST PANCRAS	BANK	TEMPLE
NOTTING HILL GATE	FARRINGDON	LONDON BRIDGE	BLACKFRIARS
BAYSWATER	BARBICAN	BOROUGH	MANSION HOUSE
NOTTING HILL GATE	MOORGATE	ELEPHANT & CASTLE	CANNON STREET
QUEENSWAY	OLD STREET	LAMBETH NORTH	MONUMENT
LANCASTER GATE	ANGEL	WATERLOO	TOWER HILL
MARBLE ARCH	KING'S X ST PANCRAS	SOUTHWARK	ALDGATE
BOND STREET	EUSTON	WATERLOO	LIVERPOOL STREET
OXFORD CIRCUS	WARREN STREET	EMBANKMENT	ALDGATE EAST

Table 3: Optimal route for the Zone 1 tubechallenge.

N	*stations visited*	*optimal value*	*cuts added*	*branch and cut nodes*	*CPU* time
50	55	91	39	142	0.20 sec

OPTIMAL ROUTE			
ST PAUL'S	EDGWARE RD (B'LOO)	ALDGATE	SOUTH KENSINGTON
CHANCERY LANE	PADDINGTON	TOWER HILL	GLOUCESTER ROAD
HOLBORN	BAYSWATER	MONUMENT	HIGH ST KENSINGTON
RUSSELL SQUARE	PADDINGTON	CANNON STREET	NOTTING HILL GATE
HOLBORN	EDGWARE RD (CIRCLE)	MANSION HOUSE	QUEENSWAY
COVENT GARDEN	BAKER STREET	BLACKFRIARS	LANCASTER GATE
LEICESTER SQUARE	GREAT PORTLAND ST	TEMPLE	MARBLE ARCH
TOTTENHAM CT ROAD	EUSTON SQUARE	EMBANKMENT	BOND STREET
GOODGE STREET	KING'S X ST PANCRAS	CHARING CROSS	GREEN PARK
WARREN STREET	FARRINGDON	EMBANKMENT	PICCADILLY CIRCUS
OXFORD CIRCUS	BARBICAN	WESTMINSTER	GREEN PARK
REGENTS PARK	MOORGATE	ST JAMES'S PARK	HYDE PARK CORNER
BAKER STREET	BANK	VICTORIA	KNIGHTSBRIDGE
MARYLEBONE	LIVERPOOL STREET	SLOANE SQUARE	

Table 4: Optimal route for the Circle line bottle tubechallenge.

a bigger chunk of the total time. Indeed, from the solution presented one may notice that the route seems to change between tube lines quite frequently; this is because the simple integer programming model presented in section 2. This limitation can be easily tackled as explained in the next section.

A similar limitation appears at platform changes at a terminal station to catch a train of the same line in the opposite direction. This is less important as it only happens in the larger problem (on the entire underground network) and it only happens a few times anyway, so the provision for such platform changes will probably will not affect the optimal solution route.

Another issue that may affect the quality of the solution is the quality of the input data. The distances between stations, as found on the London Underground website, are rounded to the nearest whole minute. Although we would expect that rounding errors may somehow cancel (some costs are slightly overestimates and others underestimates), more precision in the costs will help identify a better quality solution.

A final issue that we did not consider is station opening hours. Given that the challenge requires travelling on the tube from almost the moment it opens to the moment it shuts, station opening hours can become an issue. Other practical issues, such as how to get to the starting station very early in the morning may make one solution more attractive than another. I suspect that Heathrow airport can be an attractive starting station as it provides somewhere to sleep the night before and a quite early station opening time. Such practical constraints can be easily incorporated the network design model by assigning $x_{0k} = 1$ in the constraints, where k is the desired starting station. For this reason I also included the solution to the model with a Heathrow start imposed (table 2).

Finally it is worth mentioning that for the Zone 1 and the Circle line Challenges, the input network used contained only the stations in Zone 1 and within the Circle line boundary, respectively. This implies that the solution had to use these stations only (at least once), but no others. In other words, for the Zone 1 challenge one is not allowed to use a Zone 2 station (to transfer between two Zone 1 stations more quickly). This is not a problem in the Zone 1 challenge, as no such suitable stations in zone 2 seem to exist; for the Circle line however it can be possible that allowing the route to go through a station outside the Circle line boundary may improve the solution.

In the same lines, interchanges using other means (e.g., travelling between stations on foot) have not been considered here although incorporating such alternatives in the model is straightforward by adding extra edges in the input network.

5.1 Adding extra time for interchanges

The approach described in the previous section has some limitations. From the solution, we observe that the route tends to change tube lines quite frequently. This is expected as in the formulation there is no consideration on the time taken for interchanges between tube lines. This can be an important issue as the time required for an interchange is comparable to travelling 2-3 tube stops.

The model can be improved to take the time for interchanges as follows: Each interchange station (i.e., served by more than one line) is split into distinct stations (a *group*), one for each line, that are joined up by edges of cost equal to the cost of walking from one platform to the other. For instance Holborn was replaced by two vertices, Horborn (Pic) and Holborn (Cen), one for the Picadilly line and one for the Central. The two stations were joined by an edge of cost equal to the cost (timewise) of the interchange. The constraints (3) need to be adjusted accordingly to require that at least one station in each group is visited, i.e., $\sum_{i \in G} \sum_{j \notin G} x_{ij} \geq 1$.

6 Conclusion

In this article an approach was made to model the problem of travelling to all stations of the London Underground network using Operational Research. An Integer Programming model was formulated, which was solved to optimality on a computer. The model is flexible enough to allow further constraints and requirements to be added to the base model, as desired.

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