A polyhedral approach to reroute sequence planning in MPLS networks * †

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Abstract

This paper is devoted to the study of the reroute sequence planning problem in multi-protocol label switching networks from the polyhedral viewpoint. The reroute sequence plan polytope, defined as the convex hull of the incidence vectors of the reroute sequences which do not violate the network link capacities, is introduced and some of its properties are investigated. Drawing heavily on previous theoretical work on a seemingly unrelated distributed system reconfiguration problem, pseudopolynomially separable classes of facet-defining inequalities are introduced. These results are then embedded in a branch-and-cut algorithm which practical relevance is empirically assessed.

Keywords: Polyhedral combinatorics, scheduling, branch-and-cut, MPLS networks, OR in telecommunications.

Introduction

Multi-Protocol Label Switching (MPLS) is a new switching technology allowing fast packet forwarding on large public Internet backbone networks as well as an improvement in terms of traffic engineering methodologies [1]. In

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MPLS networks, traffic is routed along Label Switched Paths (LSP), each such LSP having certain bandwidth requirements.

Due to evolutions in terms of bandwidth or service demand, some LSP may be dynamically added, modified or removed potentially leading to unsactisfactory resource utilization in the underlying network (e.g., congestion, long routing paths, etc.). This issue is usually adressed by first obtaining a better network configuration using a central traffic engineering tool and by subsequently reconfiguring the network via LSP reroutings, on a periodic basis [2].

In order to minimize service disruption, the rerouting of a LSP consists in first establishing the new path while the traffic is still carried by the old path, then in switching the traffic to the new path and, finally, in torning the old path down.

Due to the finite capacity of the links, a feasible sequence of LSP reroutings i.e., a sequence of LSP reroutings which does not induce any capacity violations may not exist. A popular approach to deal with this issue is to allow for some LSP to be temporarily interrupted [2]. The network reconfiguration then consists in interrupting some LSP, in subsequently rerouting the others and in finally recreating the interrupted LSP. The reroute sequence planning problem asks for the least impacting way of performing such a reconfiguration.

In this paper, we study the reroute sequence planning problem from the viewpoint of polyhedral combinatorics, building on previous work by Sirdey and Kerivin [8, 3, 7] on a distributed system reconfiguration problem, the *process move programming problem*, equivalent to the special case of a network with two nodes and an arbitrary number of links.

1 An integer linear program

In this section, we formulate the reroute sequence planning problem as an integer linear program using linear ordering variables [4].

Let V and L respectively denote the nodes and links of a MPLS network and let P denotes the set of LSP established in that network. Given $l \in L$ and $p \in P$, C_l and w_p respectively denote the bandwidth offered by link l and the bandwidth requirement of path p.

Additionally, given $p \in P$, $S_p \subseteq L$ and $T_p \subseteq L$ respectively denote the set of links used by LSP p in the initial and in the final network configurations.

Conversely, given $l \in L$, $S_l \subseteq P$ and $T_l \subseteq P$ respectively denote the set of paths which use link l in the initial and in the final configurations. Lastly, $S_p^\Delta = S_p \setminus T_p$ and $T_p^\Delta = T_p \setminus S_p$ denote the set of links used by LSP p in the initial (respectively final) configuration but not in the final (respectively initial) one and $S_l^\Delta = S_l \setminus T_l$ and $T_l^\Delta = T_l \setminus S_l$ denote the set of LSP which use link l in the initial (respectively final) configuration but not in the final (respectively initial) one.

We then define $\Delta = \{p \in P : S_p \neq T_p\}$, the set of LSP which are affected by the reconfiguration and, for each $p, p' \in \Delta$ $(p \neq p')$, the binary-valued linear ordering variable $\delta_{pp'}$ which takes value one if and only if p is rerouted before p'. Then, on top of satisfying constraints [4]

$$\begin{cases} \delta_{pp'} + \delta_{p'p} = 1 & \forall \{p, p'\} \subseteq \Delta, \quad (1) \\ \delta_{pp'} + \delta_{p'p''} - \delta_{pp''} \leq 1 & p \neq p' \neq p'' \neq p \in \Delta \text{ (transitivity)}, \quad (2) \end{cases}$$

an admissible reroute sequence plan must also satisfy, for each $p \in \Delta$ and each $l \in T_p^\Delta$[1],

$$w_p \leq K_l + \sum_{p' \in S_l^\Delta} w_{p'} \delta_{p'p} - \sum_{p' \in T_l^\Delta \setminus \{p\}} w_{p'} \delta_{p'p}, \quad (3)$$

where $K_l = C_l - \sum_{p' \in S_l} w_{p'}$.

Since it may happen that no admissible plan exists, we additionally introduce, for each $p \in \Delta$, the binary-valued variable δ_{pp} which takes value 1 if and only if LSP p is interrupted. Because the interruptions are performed at the begining of the network reconfiguration, the linear ordering variables are then required to order only the reroutings of non interrupted LSP. As shown in [8], this is achieved by replacing constraints (1) by constraints

$$\begin{cases} \delta_{pp'} + \delta_{p'p} + \delta_{pp} + \delta_{p'p'} \geq 1 & \forall \{p, p'\} \subseteq \Delta, \quad (4) \\ \delta_{pp'} + \delta_{p'p} + \delta_{pp} \leq 1 & p \neq p' \in \Delta \quad (5) \end{cases}$$

(constraints (2) being left unchanged). Capacity-wise, constraints (3) translate into the following constraints, for each $p \in \Delta$ and each $l \in T_p^\Delta$:

$$(1 - \delta_{pp}) w_p \leq K_l + \sum_{p' \in S_l^\Delta} w_{p'} (\delta_{p'p'} + \delta_{p'p}) - \sum_{p' \in T_l^\Delta \setminus \{p\}} w_{p'} \delta_{p'p}.$$

[1]Hence, we consider that, during the rerouting of LSP p, there is no need to reserv twice the bandwidth of p on the links in $S_p \cap T_p$.

Letting, for each $p \in \Delta$, γ_p denote the cost of interrupting LSP p (which may be related to bandwidth consumptions, QoS requirements, etc.) leads to the integer linear program given in Figure 1. The polytope associated to this program is hereafter referred to as the *reroute sequence plan polytope*.

2 The reroute sequence plan polytope

2.1 Basic properties

To be completed with a study of the basic properties of the reroute sequence plan polytope (dimension, simple facets) drawing heavily on the theoretical work of Sirdey and Kerivin on the partial linear ordering and process move program polyopes [8, 3].

2.2 Cover inequalities

This section is devoted to the adaptation of the source and target cover inequalities, introduced by Kerivin and Sirdey [3] in the context of the process move programming problem, to the reroute sequence planning problem.

Let $p_0 \in \Delta$ and $l_0 \in S^{\Delta}_{p_0}$. Let $\emptyset \subset A \subseteq T^{\Delta}_{l_0}$ and $B \subseteq S^{\Delta}_{l_0} \setminus \{p_0\}$ be such that

$$\sum_{p \in A} w_p > K_{l_0} + \sum_{p \in \bar{B}} w_p, \tag{6}$$

where $\bar{B} = S^{\Delta}_{l_0} \setminus (B \cup p_0)$. Constraint (6) expresses the fact that rerouting all the paths not in $B \cup \{p_0\}$ does not free enough resources on l_0 to be able to reroute all the paths in A. Hence the *source cover inequality* generated by p_0, l_0, A and B,

$$\sum_{A} \delta_{pp_0} + \sum_{m \in B} \delta_{p_0 p} \leq (|A| + |B| - 1)(1 - \delta_{p_0 p_0}).$$

Similarly, let $p_0 \in \Delta$ and $l_0 \in T^{\Delta}_{p_0}$. Let $A \subseteq T^{\Delta}_{l_0} \setminus \{p_0\}$ and $\emptyset \subset B \subseteq S_{l_0}$ be such that

$$w_{p_0} + \sum_{p \in A} w_p > K_{l_0} + \sum_{p \in \bar{B}} w_p, \tag{7}$$

where $\bar{B} = S_{l_0} \setminus B$. Constraint (7) expresses the fact that rerouting all the paths not in B does not free enough resources on l_0 so as to reroute all the

$$
\underbrace{
\begin{array}{ll}
\text{Minimize } \sum_{p\in\Delta} \gamma_p \delta_{pp} & \\
\text{s. t.} & \\
\delta_{pp'} + \delta_{p'p} + \delta_{pp} + \delta_{p'p'} \geq 1 & \forall \{p, p'\} \subseteq \Delta, \\
\delta_{pp'} + \delta_{p'p} + \delta_{pp} \leq 1 & p \neq p' \in \Delta, \\
\delta_{pp'} + \delta_{p'p''} - \delta_{pp''} \leq 1 & p \neq p' \neq p'' \neq p \in \Delta, \\
(1-\delta_{pp}) w_p \leq K_l + \sum_{p' \in S_l^{\Delta}} w_{p'} (\delta_{p'p'} + \delta_{p'p}) - \sum_{p' \in T_l^{\Delta} \setminus \{p\}} w_{p'} \delta_{p'p} & \forall p \in \Delta, \forall l \in T_p^{\Delta}, \\
\delta_{pp'} \in \{0, 1\} & p, p' \in \Delta.
\end{array}
}
$$

Figure 1: Formulation of the reroute sequence planning problem as an integer linear program using linear ordering variables.

paths in $A \cup \{p_0\}$. Hence the *target cover inequality* generated by p_0, l_0, A and B,

$$\sum_{A} \delta_{pp_0} + \sum_{m \in B} \delta_{p_0 p} \leq (|A| + |B| - 1)(1 - \delta_{p_0 p_0}).$$

It turns out that for fixed p_0 and l_0, both the source and target cover inequalities can be separated in pseudopolynomial time by solving a knapsack problem [3, 7].

To be completed by an extension of the results in [3] so as to determine conditions under which the source and target cover inequalities turn out to be facet-defining for the reroute sequence plan polytope.

3 A branch-and-cut algorithm

To be completed with a description regarding how the branch-and-cut algorithm of Sirdey and Kerivin [7] (including the simulated annealing algorithm of Sirdey, Carlier and Nace [6]) can be adapted to solve the reroute sequence planning problem.

4 Computational experiments

To be completed with an empirical study of the practical usefullness of the algorithm on tight instances generated following Józsa and Makai [2].

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