A Routing and Network Dimensioning Strategy to reduce Wavelength Continuity Conflicts in All-Optical Networks

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Abstract

Due to the high computational complexity of exact methods (e.g., integer programming) for routing and wavelength assigment in optical networks, it is beneficial to decompose the problem into a routing task and a wavelength allocation task. However, by this decomposition it is not necessarily possible to obtain a valid wavelength assignment for a given routing because of wavelength continuity constraints in the network.

This paper proposes an extension of the routing and network dimensioning subproblem that facilitates a subsequent wavelength assignment in the absence of wavelength converters. The extension anticipates wavelength continuity conflicts by limiting the maximum number of traversing lightpaths at each node. Such a limitation is derived from a lower bound on the number of conflicts, given a routing and network configuration. Preliminary computational experiments indicate that our approach can reduce the number of lightpath blockings significantly.

Keywords: optical network design, routing, dimensioning, wavelength assignment

Introduction

First generation wavelength-division multiplexing (WDM) networks terminate the optical signals at each node using optical-electrical-optical conversion. The advent of all-optical networks that are capable of switching signals in the optical domain eliminates the need for electrical signal processing by expensive transponders. However, the lack of wavelength conversion functionality which was available by default in the previous case complicates the routing as a lightpath must be assigned the same wavelength from source to target node. Performing a routing without considering the wavelengths may result in configurations that cannot be established because the wavelength continuity cannot be guaranteed due to capacity limitations. If wavelength converters can be installed, one routing constellation can require a relatively high number of converters whereas another routing may be realizable with only few or even zero converters.

To avoid these effects, the routing and wavelength assignment (RWA) problem has been studied in various settings by many researchers from engineering, computer science, and operations research. Besides heuristics that normally do not allow to evaluate the quality of the solution, integer programming based approaches have been proposed. The major drawback is their computational complexity that limits the applicability for instances of realistic network scenarios. Recent approaches therefore decompose the problem in a routing subproblem (often combined with the dimensioning of the network) and a wavelength assignment subproblem. Within the routing task, wavelengths are ignored and lightpaths are treated as non-colored connections. In case wavelength conversion equipment is available, it is possible to guarantee that a valid wavelength assignment can be found for any valid routing [5]. If conversion is not an option, the installation of additional fibers and WDM systems must be considered in the wavelength assignment phase to avoid any lightpath blocking [4].

In this paper we propose a different approach to prevent network equipment upgrades in terms of extra capacity or wavelength converters within the wavelength assignment subproblem. For this we exploit a term that represents a lower limit for the number of lightpaths that cannot be routed without conversion via a node based on the number of incident fibers and the number of wavelengths per fiber [3]. We include this bound in a constraint that restricts the number of traversing lightpaths at each node. By considering these requirements in addition to the usual routing and dimensioning constraints, we obtain network configurations that can be established without any wavelength conflict more likely.

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Preliminaries

The optical network design problem consists of the routing of demands, the dimensioning of the network, and the assignment of wavelengths to the routing paths (lightpaths), such that the overall costs are minimized. To simplify matters and to focus on the real contribution, we neither consider node costs nor protection in this paper. The models can be extended appropriately in this regard, if needed.

Optical network design models Let $\mathcal{N} = (N, L)$ be a graph describing the topology of an optical network consisting of a set of nodes N and physical links L. By $L(n)$ we denote the set of links incident to node $n \in N$. The installation of one undirected fiber (i.e., a fiber pair for reverse unidirectional transmission) offers a WDM system with wavelength set Λ. A path p is characterized by its traversed nodes and links. For every demand q from the set $\mathcal{Q}$ we have to allocate $v(q)$ undirected lightpaths between source node $s(q) \in N$ and target node $t(q) \in N$.

Following the work in [4, 5], we decompose the overall problem in a combined routing and dimensioning subproblem and a wavelength assignment subproblem. The routing and dimensioning model calculates the paths for all demands and the number of fibers (with WDM system) at all links. Let $\mathcal{P}(q)$ denote the set of paths considered for the routing of commodity $q \in \mathcal{Q}$ and $\mathcal{P} = \cup_{q\in\mathcal{Q}}\mathcal{P}(q)$. With $\mathcal{P}(\ell)$ the subset of lightpaths on link $\ell \in L$ is specified. The variables $f_\ell \in \mathbb{Z}_0^+$ represent the number of fibers installed at link $\ell \in L$ and $x_{qp} \in \mathbb{Z}_0^+$ denote the number of lightpaths routed along path $p \in \mathcal{P}$ to satisfy demand $q \in \mathcal{Q}$. The cost of installing one fiber and a WDM system on link $\ell \in L$ is $c(\ell)$. The routing and dimensioning problem then reads:

$$\min \sum_{\ell\in L} c(\ell) f_\ell \tag{1}$$

$$s.t. \quad \sum_{p\in\mathcal{P}(q)} x_{qp} = v(q) \qquad \forall q \in \mathcal{Q}, \tag{2}$$

$$\sum_{q\in\mathcal{Q}} \sum_{p\in\mathcal{P}(\ell)} x_{qp} \le |\Lambda| f_\ell \;\; \forall \ell \in L, \tag{3}$$

$$f_\ell, x_{qp} \in \mathbb{Z}_0^+ \tag{4}$$

By fixing the f_ℓ variables (and selecting a different objective), we can determine the routing configuration for an already dimensioned network.

Let (f_ℓ^*, x^*) denote the (close-to) optimal solution of (1)–(4). The wavelength assignment subproblem is solved as a postprocessing step of the routing and dimensioning subproblem. It computes a feasible allocation of wavelengths for the previously selected routing paths. To guide the search, we maximize the number of lightpaths for which a valid wavelength assignment exists. If the objective value equals the total demand number, the entire wavelength routing is feasible, otherwise the difference between total demand and objective indicates the number of blocked lightpaths. For this subproblem we collect all positive x_{qp}^* variables into a set of routing paths P, where each routed path $\rho \in P$ has to be assigned $d(\rho) = \sum_{q\in\mathcal{Q}} x_{q\rho}^*$ wavelengths. The number of fibers available at a link is set to $f(\ell) = f_\ell^*$. The variables $y_{\rho\lambda} \in \mathbb{Z}_0^+$ determine how often a wavelength $\lambda \in \Lambda$ is assigned to the routing path $\rho \in P$. Now, the subproblem reads:

$$\max \sum_{\rho\in P} \sum_{\lambda\in\Lambda} y_{\rho\lambda} \tag{5}$$

$$s.t. \quad \sum_{\lambda\in\Lambda} y_{\rho\lambda} \le d(\rho) \quad \forall \rho \in P, \tag{6}$$

$$\sum_{\rho\in P(\ell)} y_{\rho\lambda} \le f(\ell) \;\; \forall \ell \in L, \;\; \forall \lambda \in \Lambda, \tag{7}$$

$$y_{\rho\lambda} \in \mathbb{Z}_0^+ \tag{8}$$

Wavelength continuity conflicts The major advantage of the above decomposition is the reduced computational complexity compared to the all-in-one problem. However, the drawback is that the wavelength continuity cannot be guaranteed for all routing paths as required. The wavelength assignment problem has been studied in many different settings. For the above scenario, the number of conflicts in a node can be bounded from below by applying the theory of f-edge-colorings [3]. For this, let $P_{em}(n)$ denote the subset of lightpaths that have node $n \in N$ as source or target, whereas $P_{tr}(n)$ denotes the set of lightpaths traversing $n \in N$ as intermediate node. Moreover, $P_{tr}(n, S)$ is the set of lightpaths traversing node $n \in N$ on two neighboring links of the subset $S \subseteq L(n)$.

Theorem 1 ([3]). *Given an optical network $\mathcal{N} = (N, L)$ with $f(\ell)$ fibers at link $\ell \in L$ each providing $|\Lambda|$ wavelengths as well as a set of routed lightpaths P, the number of wavelength continuity conflicts at node $n \in N$ is bounded from below by*

$$\Gamma(n) = \max\left\{0, \max_{S\subseteq L(n)} \Gamma(n,S)\right\} \text{ with } \Gamma(n,S) = \left\{|P_{tr}(n,S)| - |\Lambda| \left\lfloor \frac{1}{2}\sum_{\ell\in S} f(\ell)\right\rfloor\right\} \tag{9}$$

For subsets $S \subseteq L(n)$ with $\sum_{\ell\in S} f(\ell)$ even, the term within the second maximization cannot be positive, and thus we can restrict to link subsets with odd number of fibers in total.

In [3] it is shown that $\Gamma(n) \equiv 0$ is a very good indicator for the existence of feasible wavelength assignments. In fact, $\sum_{n\in N}\Gamma(n)$ provides a lower bound on the number of converters needed in a conflict-free assignment of all lightpaths. Computational experiments have shown that the bound equals the best known solution in all 80 instances considered. Moreover, this bound can be computed in a fraction of a second.

Wavelength continuity conflict constraints

The bound in (9) provides a very fast and good estimate for the existence of a feasible wavelength assignment, given a set of routing paths and a dimensioning of the network. It however cannot avoid the generation of routing configurations that turn out to be undesirable from a wavelength assignment point of view. Also, if a network is already dimensioned, but the routing has to be updated due to changing traffic patterns, it would be beneficial to solve the routing problem first and the wavelength assignment afterwards instead of solving the RWA problem as a whole. Again, an actual routing can turn out to be infeasible due to wavelength continuity conflicts.

Motivated by this drawback of the decomposition, it is our goal to introduce new constraints to the routing (and dimensioning) problem such that the resulting routing cannot be infeasible within the subsequent wavelength assignment phase anymore. Clearly, it is unlikely that a small set of constraints can guarantee that every routing constellation has a feasible wavelength assignment. Nevertheless, the experiments of [3] show that in many cases all conflicts are detected by $\Gamma(n) > 0$ for $n \in N$.

The key to obtain routing scenarios that have a higher probability of showing no lightpath blocking during the wavelength assignment process is therefore the restriction to routings that meet $\Gamma(n) = 0$ for all $n \in N$. In particular, $\Gamma(n, S) \leq 0$ should hold true for all $n \in N$ and $S \subseteq L(n)$. For the routing problem without dimensioning this limitation can be easily implemented as linear constraint:

$$\sum_{q\in\mathcal{Q}} \sum_{p\in\mathcal{P}_{tr}(n,S)} x_{qp} \leq |\Lambda| \left\lfloor \frac{1}{2}\sum_{\ell\in S} f(\ell)\right\rfloor \quad \forall n \in N,\ \forall S \subseteq L(n) \tag{10}$$

By the fact that $\Gamma(n) = 0$ if $\sum_{\ell\in S} f(\ell)$ is even, constraints (10) only have to be added for subsets of links that have an odd number of fibers. In particular, if $f(\ell)$ is even for all $\ell \in L(n)$, no routing can violate inequalities (10) for n and any $S \subseteq L(n)$, and thus the inequalities are not needed at all for such nodes.

For the routing and dimensioning problem (1)–(4), we need an additional variable stating the rounding down at the right hand side of (10). This number is in fact an upper bound on the number of lightpaths traversing a node that can be assigned the same wavelength. We restrict ourselves to $S = L(n)$ and introduce the integer variables $z_n \in \mathbb{Z}_0^+$. Now, the following constraints have to be added:

$$z_n \leq \frac{1}{2} \sum_{\ell \in L(n)} f_\ell \qquad \forall n \in N, \tag{11}$$

$$\sum_{q \in \mathcal{Q}} \sum_{p \in \mathcal{P}_{tr}(n)} x_{qp} \leq |\Lambda| z_n \quad \forall n \in N, \tag{12}$$

$$z_n \in \mathbb{Z}_0^+ \tag{13}$$

Inequalities (11) combined with the integrality of z_n calculate the upper bound for the number of lightpaths that can traverse a common node and be assigned the same wavelength. Constraints (12) limit the overall number of traversing lightpaths accordingly. Similar constraints can be formulated for all subsets $S \subset L(n)$ by introducing additional integer variables z_{nS}.

Figure 1(a) shows the well-known example of a star network with three links, two wavelengths per fiber, and one lightpath request between each pair of end nodes. Without constraints (11)–(13), a single fiber is installed on each link, but no feasible wavelength assignment exists. With constraints (11)–(13), two fibers are installed on the cheapest link and the wavelength allocation can be carried out without any problem. On the other hand, Figure 1(b) shows an example where constraints (11)–(13) are not enough to guarantee a feasible wavelength assignment.

(a) Constraints (11)–(13) avoid $f_\ell = 1$ for all $\ell \in L$

(b) Constraints (11)–(13) still allow $f_\ell = 1$ for all $\ell \in L$, but no conflict-free assignment exists

Figure 1: Example for the effect of wavelength continuity constraints.

The following lemma shows that the constraints (11)–(13) need not to be included in the dimensioning and routing subproblem for all nodes. Let $\mathcal{Q}(n) := \{q \in \mathcal{Q} : s(q) = n \text{ or } t(q) = n\}$ be the set of commodities starting or ending at node $n \in N$. Furthermore, define for a subset $\mathcal{Q}' \subseteq \mathcal{Q}$, $v(\mathcal{Q}') := \sum_{q \in \mathcal{Q}'} v(q)$. and hence

Lemma 1. *If $v(\mathcal{Q}(n)) > |\Lambda|$ for some $n \in N$, then constraints* (11)–(13) *(for n) cannot be violated by any solution of* (1)–(4)*.*

Proof. Given a vector $f_\ell \in \mathbb{Z}_0^+$, the channel capacity of the links incident to $n \in N$ is $|\Lambda| \sum_{\ell \in L(n)} f_\ell$. Each emanating lightpath requires a single channel on one of the incident links. Hence, $|\Lambda| \sum_{\ell \in L(n)} f_\ell - v(\mathcal{Q}(n))$ remain for the traversing lightpaths. As each of these requires two channels,

$$\sum_{q \in \mathcal{Q}} \sum_{p \in \mathcal{P}_{tr}(n)} x_{qp} \leq \left\lfloor \frac{1}{2} \left(|\Lambda| \sum_{\ell \in L(n)} f_\ell - v(\mathcal{Q}(n)) \right) \right\rfloor$$

holds in any (fractional) solution of (1)–(4). If the right hand side is less than $|\Lambda| \lfloor \frac{1}{2} \sum_{\ell \in L(n)} f_\ell \rfloor$, (11)–(13)

are implicitly satisfied, otherwise may be not. First consider the case that $\sum_{\ell \in L(n)} f_\ell$ is even. Then,

$$\left\lfloor \frac{1}{2}\left(|\Lambda| \sum_{\ell \in L(n)} f_\ell - v(\mathcal{Q}(n))\right)\right\rfloor \leq \frac{1}{2}|\Lambda| \sum_{\ell \in L(n)} f_\ell - \frac{1}{2}v(\mathcal{Q}(n)) \leq |\Lambda| \left\lfloor \frac{1}{2} \sum_{\ell \in L(n)} f_\ell \right\rfloor$$

and thus (11)–(13) always hold. Now, consider the case that $\sum_{\ell \in L(n)} f_\ell$ is odd. Then,

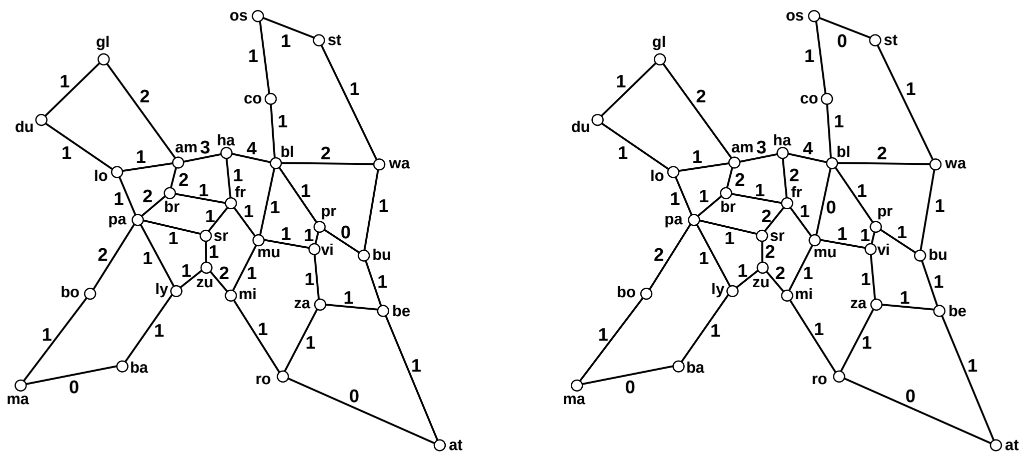
$$\left\lfloor \frac{1}{2}\left(|\Lambda| \sum_{\ell \in L(n)} f_\ell - v(\mathcal{Q}(n))\right)\right\rfloor \leq |\Lambda| \left\lfloor \frac{1}{2} \sum_{\ell \in L(n)} f_\ell \right\rfloor + \frac{1}{2}|\Lambda| - \frac{1}{2}v(\mathcal{Q}(n)) \leq |\Lambda| \left\lfloor \frac{1}{2} \sum_{\ell \in L(n)} f_\ell \right\rfloor$$

if $v(\mathcal{Q}(n)) \geq |\Lambda|$. Hence, the inequalities are always satisfied under this condition. □

So, for nodes with high emanating demand compared to the number of wavelengths available at a fiber, local wavelength continuity conflicts cannot occur. Note that for subsets $S \subset L(n)$, Lemma 1 cannot be applied, since we do not know beforehand how often the links-subset is used by emanating lightpaths.

Case Study

In a case study we investigate the performance of a subsequent wavelength assignment based on given routing and dimensioning for different networks. For this purpose we calculate the routing and capacity allocation according to the normal formulation (1)–(4) and additionally for the extended version with extra routing constraints (11)–(13). Based on the resulting fiber capacity and the routing configuration we determine the maximum number of paths for which wavelengths can be identified without any conflict. The results are obtained using ILOG CPLEX solver, version 9.1 [2]. Each problem instance was executed on a 64 bit AMD Opteron machine and computation time was limited to 10000 seconds for each of the subproblems.



(a) wihtout constraints (11)–(13)   (b) with constraints (11)–(13)

Figure 2: Capacity dimensioning represented by allocated fiber quantities for an European network.

The test scenarios comprise a German, European, and US network defined in [1]. We restrict the number of eligible paths between two nodes to the 100 shortest ones with respect to their physical length. Fiber capacity in the objective function is measured in terms of total fiber kilometers, accordingly. The number of wavelengths per fiber has been set to 40 and 80 in different test runs. For the German and US network it turns out that the normal routing and dimensioning strategy already yields no wavelength conflicts. In fact, in the US network $\min_{n \in N} \mathcal{Q}(n) = 152$ and thus by Lemma 1 the additional constraints could not sort any effect. Also, for the German network $\mathcal{Q}(n) > |\Lambda|$ for almost all nodes.

In the European network, $\min_{n \in N} \mathcal{Q}(n) = 28$ and for more than half the nodes, the emanating demand is less than 80. Indeed, in the case with $|\Lambda| = 80$ the additional routing limitations help to improve the wavelength assignment: with the normal routing, a feasible routing including all 1008 lightpaths does not exist. After the time limit is reached, it is clear that at least 21 lightpaths cannot be assigned a wavelength (i.e., the dual bound is 987) and a solution with 969 lightpaths has been found. For the routing with additional constraints, a feasible wavelength assignment for 986 lightpaths could be calculated within the time limit. Here, the dual bound shows the potential to assign wavelengths for all paths (i.e., the dual bound remains 1008 throughout the computation time). We note that the dimensioning in this case is even slightly cheaper than in the unrestricted case (28503 vs. 28527). In both cases the dimensioning and routing problem could not be solved to optimality within the time limit. A gap of 8.0% remains for the case without additional constraints and a gap of 11.2% for the case with constraints (11)–(13). Figure 2 shows the number of allocated fibers for both scenarios.

Concluding remarks

In this paper we introduced novel constraints that are intended for the consideration of wavelength continuity conflicts within the routing (and dimensioning) problem in all-optical networks without explicitly taking the wavelength assignment into account. The constraints limit the solution space of the routing problem to routings with $\Gamma(n) = 0$ for all nodes $n \in N$, where $\Gamma(n)$ bounds the number of wavelength continuity conflicts in a node of the network from below. In a subsequent step, wavelengths are allocated for the resulting routing paths.

Preliminary computational experiments indicate that indeed the number of lightpath blockings can be reduced this way for instances where the strategy without these additional requirements is not able to produce valid wavelength assignments for all paths. Surprisingly, within the same time limit a less expensive network configuration was found although the routing has to satisfy more constraints.

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