Inverse Stochastic Linear Programming

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Abstract

Inverse optimization perturbs objective function to make an initial feasible solution optimal with respect to perturbed objective function while minimizing cost of perturbation. We extend inverse optimization to two-stage stochastic linear programs. Since the resulting model grows with number of scenarios, we present two decomposition approaches for solving these problems.

Keywords: Inverse Optimization, Stochastic Programming, Decomposition Algorithms

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1 Introduction

An inverse optimization problem infers the values of the objective coefficients, given the values of optimal decision variables. The aim of inverse optimization is to perturb the objective vector from c to d so that an initial feasible solution $\hat{x}$ with respect to objective vector c becomes an optimal solution with respect to perturbed objective vector d and the "cost of perturbation" is minimized.

Inverse optimization has many application areas, and inverse problems have been studied extensively in the analysis of geophysical data [20, 21, 24, 25]. Recently, inverse optimization has extended into a variety of fields of study. Inverse optimization was applied in geophysical studies [5, 6], to predict the movements of earthquakes assuming that earthquakes move along shortest paths. Traffic equilibrium [11] is another application area where the minimum total toll is imposed to make the user equilibrium flow and system optimal flow equal. Inverse multicommodity flows were used in railroad scheduling to determine the arc costs based on a specific routing plan [10].

Another application of inverse optimization is the area of just-in-time scheduling. In this case the objective is to schedule the production so as to deviate, in each period, as little as possible from the target production quantity of each product [14]. In political gerrymandering the goal is to modify the current boundaries so as to achieve majority for a certain outcome while taking into account population numbers segmented as per various political options, and limitations on the geometry of the boundaries [7, 14].

Zhang and Liu [17] suggested a solution method for general inverse linear programs (LPs) including upper and lower bound constraints based on the optimality conditions for LPs. Their objective function was to minimize the cost of perturbation based on the L_1 norm. Ahuja and Orlin [1] studied inverse optimization for deterministic problems and showed that the inverse of a deterministic LP is also an LP. They attained the inverse feasible cost vectors using optimality conditions for LPs and minimized the cost of perturbation based on both the L_1 and L_∞ norm.

To consider the inverse optimization problem under the weighted L_1 norm involves solving the problem according to the objective Min $\sum_{j \in J} v_j \mid d_j - c_j \mid$, where J is the variable index set, d_j and c_j are the perturbed and original objective cost coefficients, respectively, and v_j is the weight coefficient. By introducing variables α_j and β_j for each variable $j \in J$, this objective is equivalent to the following problem:

$$\text{Min} \sum_{j \in J} v_j(\alpha_j + \beta_j)$$

$$\text{s.t. } d_j - c_j = \alpha_j - \beta_j, \; j \in J,$$

$$\alpha_j \geq 0, \; \beta_j \geq 0, \; j \in J.$$

Two-stage stochastic linear programming (2SSLP) [3, 4, 8] considers LPs in which some problem data are random. In this case, "first-stage" decisions are made without full information on the random events while "second-stage" decisions (or corrective actions) are taken after full information on the random variables becomes available. This paper extends deterministic inverse LP to 2SSLP and provides preliminary computational results. Although many of the applications of inverse optimization are stochastic in nature, to the best of our knowledge, deterministic versions of these problems have been considered so far. With this paper, we add this stochastic nature to inverse problems.

In the next section, we formally characterize feasible cost vectors for inverse 2SSLP. In Section 3, we outline two large-scale decomposition techniques for solving inverse 2SSLPs. We conclude with computational results in Section 4.

2 Inverse Stochastic Linear Programming

We consider the extensive form of 2SSLP with a finite number of scenarios. Let J^0 denote the index set of first-stage variables, I^0 denote the index set of first-stage constraints, $\mathcal{K}$ denote the set of scenarios, J^k denotes the index set of second-stage variables for scenario $k \in \mathcal{K}$, I^k denotes the index set of second-stage constraints for scenario $k \in \mathcal{K}$. The 2SSLP in extensive form (EF) can be given as follows:

$$\textbf{EF:} \ \text{Max} \sum_{j \in J^0} c_j x_j + \sum_{k \in \mathcal{K}} \sum_{j \in J^k} p^k q_j^k y_j^k$$

$$\text{s.t.} \ \sum_{j \in J^0} a_{ij} x_j \leq b_i, \ i \in I^0, \tag{1}$$

$$\sum_{j \in J^0} t_{ij}^k x_j + \sum_{j \in J^k} w_{ij}^k y_j^k \leq h_i^k, \ k \in \mathcal{K}, i \in I^k, \tag{2}$$

$$x_j \geq 0, \ \ j \in J^0; \ y_j^k \geq 0, \ k \in \mathcal{K}, \ j \in J^k. \tag{3}$$

We associate first-stage constraints (1) with the dual variables π_i^0, and second-stage constraints (2) with π_i^k. Then the dual of **EF** can be given as follows:

$$\text{Min} \sum_{i \in I^0} b_i \pi_i^0 + \sum_{k \in \mathcal{K}} \sum_{i \in I^k} h_i^k \pi_i^k$$

$$\text{s.t.} \ \sum_{i \in I^0} a_{ij} \pi_i^0 + \sum_{k \in K} \sum_{i \in I^k} t_{ij}^k \pi_i^k \geq c_j, \ j \in J^0, \tag{4}$$

$$\sum_{i \in I^k} w_{ij}^k \pi_i^k \geq p^k q_j^k, \ k \in \mathcal{K}, \ j \in J^k, \tag{5}$$

$$\pi_i^0 \geq 0, \ i \in I^0, \ \pi_i^k \geq 0, \ k \in \mathcal{K}, \ i \in I^k. \tag{6}$$

LP optimality conditions require that at optimality, a primal solution $(x, \{y^k\}_{\forall k \in \mathcal{K}})$ is feasible to (1)-(3), and a corresponding dual solution $(\pi^0, \{\pi^k\}_{\forall k \in \mathcal{K}})$ is feasible to (4)-(6), and the following complementary slackness (CS) conditions are satisfied:

- $\forall i \in I^0$, if $\sum_{j \in J^0} a_{ij} x_j < b_i$ then $\pi_i^0 = 0$.

- $\forall k \in \mathcal{K}, i \in I^k$, if $\sum_{j \in J^0} t_{ij}^k x_j + \sum_{j \in J^k} w_{ij}^k y_j^k < h_i^k$ then $\pi_i^k = 0$.

Let B^0 denote the set of binding constraints among the first-stage constraints (1) with respect to an initial primal feasible solution $(\hat{x}, \{\hat{y}^k\}_{\forall k \in \mathcal{K}})$, and let $B^k, k \in \mathcal{K}$ be the set of binding constraints among the second-stage constraints (2). Then we can now rewrite the CS conditions as follows:

- $\pi_i^0 = 0$ for all $i \in I^0 \backslash B^0$,
- For any $k \in \mathcal{K}$, $\pi_i^k = 0$ for $i \in I^k \backslash B^k$.

Let $\mathbf{EF}(d, q^{'})$ denote the 2SSLP where c_j's are replaced with d_j's and q_j^k's are replaced with $(q_j^k)^{'}$'s. It is worth noting that $(\hat{x}, \{\hat{y}^k\}_{\forall k \in \mathcal{K}})$ is an optimal solution to $\mathbf{EF}(d, q^{'})$ if and only if there exists a dual solution $(\pi^0, \{\pi^k\}_{\forall k \in \mathcal{K}})$ that satisfies (4)-(6) with c_j replaced with d_j, q_j^k replaced with $(q_j^k)^{'}$ and the primal-dual pair satisfies the CS conditions. Combining the dual feasibility condition with the newly constructed CS conditions gives the following characterization of inverse feasible cost vectors for 2SSLP:

$$\sum_{i \in B^0} a_{ij} \pi_i^0 + \sum_{k \in \mathcal{K}} \sum_{i \in B^k} t_{ij}^k \pi_i^k \geq d_j, \quad j \in J^0, \tag{7}$$

$$\sum_{i \in B^k} w_{ij}^k \pi_i^k \geq p^k (q_j^k)^{'}, \quad k \in \mathcal{K},\ j \in J^k, \tag{8}$$

$$\pi_i^0 \geq 0 \quad i \in I^0, \quad \pi_i^k \geq 0, \quad k \in \mathcal{K},\ i \in I^k.$$

Under the weighted L_1 norm, the problem is

$$\text{Min} \sum_{j \in J^0} v_j^0 |d_j - c_j| + \sum_{k \in \mathcal{K}} \sum_{j \in J^k} p^k v_j^k |(q_j^k)^{'} - q_j^k| \tag{9}$$

subject to the inverse feasible cost vectors (7)-(8). The coefficients $v_j^0,\ j \in J^0$ and $v_j^k,\ j \in J^k$, denote the weight vectors associated with the first and second stage, respectively. In order to linearize this nonlinear objective we define α_j^0, β_j^0 and set $d_j - c_j = \alpha_j^0 - \beta_j^0$, where $\alpha_j^0 \geq 0$ and $\beta_j^0 \geq 0,\ \forall j \in J^0$. In the same manner, we define α_j^k, β_j^k and set $(q_j^k)^{'} - q_j^k = \alpha_j^k - \beta_j^k$, where $\alpha_j^k \geq 0$ and $\beta_j^k \geq 0,\ \forall k \in \mathcal{K},\ \forall j \in J^k$.

The inverse 2SSLP under the weighted L_1 norm is to minimize the first-stage weighted absolute cost of perturbation plus the expected second-stage weighted absolute cost of perturbation. We formally state the inverse 2SSLP in EF as follows:

$$\text{Min} \sum_{j \in J^0} v_j^0(\alpha_j^0 + \beta_j^0) + \sum_{k \in \mathcal{K}} \sum_{j \in J^k} v_j^k p^k (\alpha_j^k + \beta_j^k)$$

$$\text{s.t.} \sum_{i \in B^0} a_{ij}\pi_i^0 + \sum_{k \in \mathcal{K}} \sum_{i \in B^k} t_{ij}^k \pi_i^k - \alpha_j^0 + \beta_j^0 \geq c_j,\ j \in J^0, \tag{10}$$

$$\sum_{i \in B^k} w_{ij}^k \pi_i^k - p^k \alpha_j^k + p^k \beta_j^k \geq p^k q_j^k,\ k \in \mathcal{K},\ j \in J^k, \tag{11}$$

$$\pi_i^0 \geq 0,\ i \in B^0,\ \pi_i^k \geq 0,\ k \in \mathcal{K},\ i \in B^k, \tag{12}$$

$$\alpha_j^0, \beta_j^0 \geq 0,\ j \in J^0,\ \alpha_j^k, \beta_j^k \geq 0,\ k \in \mathcal{K},\ j \in J^k. \tag{13}$$

By defining $c_j^{\pi^0} = c_j - \sum_{i \in B^0} a_{ij}\pi_i^0 - \sum_{i \in B^k} \sum_{k \in \mathcal{K}} t_{ij}^k \pi_i^k$ and $c_j^{\pi^k} = q_j^k - \frac{1}{p^k} \cdot \sum_{i \in B^k} w_{ij}^k \pi_i^k$, we can restate equations (10) and (11) as follows:

$$-\alpha_j^0 + \beta_j^0 \geq c_j^{\pi^0},\ j \in J^0, \tag{14}$$

$$-\alpha_j^k + \beta_j^k \geq c_j^{\pi^k},\ k \in \mathcal{K}, j \in J^k. \tag{15}$$

There are two sets of three mutually exclusive cases to consider:

Case 1. $c_j^{\pi^0} > 0$

- $\alpha_j^0 = 0, \quad \beta_j^0 = c_j^{\pi^0} \Rightarrow d_j = c_j - c_j^{\pi^0}$

Case 2. $c_j^{\pi^0} < 0$

- $\alpha_j^0 = \beta_j^0 = 0 \Rightarrow d_j = c_j$

Case 3. $c_j^{\pi^0} = 0$

- $c_j^{\pi^0} = 0 \Rightarrow \alpha_j^0 = \beta_j^0 = 0 \Rightarrow d_j = c_j$

Case 4. $c_j^{\pi^k} > 0$

- $\alpha_j^k = 0, \quad \beta_j^k = c_j^{\pi^k} \Rightarrow (q_j^k)' = q_j^k - c_j^{\pi^k}$

Case 5. $c_j^{\pi^k} < 0$

- $\alpha_j^k = \beta_j^k = 0 \Rightarrow (q_j^k)' = q_j^k$

Case 6. $c_j^{\pi^k} = 0$

- $c_j^{\pi^k} = 0 \Rightarrow \alpha_j^k = \beta_j^k = 0 \Rightarrow (q_j^k)' = q_j^k$

3 Decomposition Approaches for Solving Inverse Stochastic Linear Programs

Unfortunately, the inverse 2SSLP problem (10) - (13) grows with the number of scenarios $|\mathcal{K}|$. This leads us to consider decomposition approaches. Table 1 shows the rearranged constraints and variables in a matrix format where $\mathcal{K} = 1 \cdots K$ which demonstrates the idea behind how the division between the constraints and variables has been made. For each set of variables, a dot appears if the variables in the set have nonzero coefficients. As can be seen in the table, the constraint sets $J^k, k \in \mathcal{K}$ have a nice structure. So, we can set J^0 as the linking constraint set and a decomposition approach such as Dantzig-Wolfe decomposition [9] or Lagrangian relaxation [12] may be utilized. Furthermore, $(\{\alpha^k, \beta^k\}_{\forall k \in \mathcal{K}})$ do not appear in J^0 constraints and $(\pi^0, \alpha^0, \beta^0)$ do not appear in $J^k, k \in \mathcal{K}$ constraints. Therefore, the problem is relatively easy to solve when only these variables are present. So, $(\{\pi^k\}_{\forall k \in \mathcal{K}})$ are the linking variables for which Benders' decomposition [2] is appropriate.

3.1 Dantzig-Wolfe Decomposition of the Inverse Extensive Form

Dantzig-Wolfe decomposition [9] is an application of inverse projection to linear programs with special structure [18]. With Dantzig-Wolfe decomposition, the LP is decomposed into two sets of constraints as "easy" and "hard". Rather than solving the LP with all the variables present, the variables are added as needed.

Table 1: Structure of the inverse 2SSLP constraint matrix.

	$(\pi^0,\, \alpha^0,\, \beta^0)$	$(\pi^1,\, \ldots,\, \pi^K)$	$(\alpha^1,\, \ldots,\, \alpha^K)$	$(\beta^1,\, \ldots,\, \beta^K)$
J^0	• • •	• • • • • • •		
J^1		•	•	•
J^2		•	•	•
.		.	.	.
.		.	.	.
J^K		•	•	•

Observe that if one views the $(\pi^1, \cdots, \pi^K)$ variables as "first-stage" variables, the resulting inverse 2SSLP may be interpreted as a 2SSLP as well. Based on Table 1 $J^k, k \in \mathcal{K}$ decompose into a set of disjoint block constraints. So, for the inverse 2SSLP, $J^k, k \in \mathcal{K}$ are easy constraints and J^0 are hard constraints. Optimizing the subproblem by solving K independent LPs may be preferable to solving the entire system. Let $(\pi^k, \alpha^k, \beta^k)^1 \;\cdots\; (\pi^k, \alpha^k, \beta^k)^{q_k}$ be the extreme points and $(\pi^k, \alpha^k, \beta^k)^{q_k+1} \;\cdots\; (\pi^k, \alpha^k, \beta^k)^{r_k}$ be the extreme rays of P^k. We can rewrite the points in the easy polyhedron as a combination of their extreme points and extreme rays. Substituting these into the hard constraint set and into the objective function gives the following Dantzig-Wolfe master problem:

$$\text{Min} \sum_{j\in J^0} v_j^0(\alpha_j^0+\beta_j^0) + \sum_{k\in\mathcal{K}}\sum_{j\in J^k} v_j^k p^k \left[\sum_{s=1}^{r_k} z_s^k[(\alpha_j^k)^s+(\beta_j^k)^s]\right]$$

$$\text{s.t.} \sum_{i\in B^0} a_{ij}\pi_i^0 + \sum_{k\in\mathcal{K}}\sum_{i\in B^k} t_{ij}^k \sum_{s=1}^{r_k} z_s^k(\pi_i^k)^s - \alpha_j^0 + \beta_j^0 \geq c_j,\ j\in J^0, \tag{16}$$

$$\sum_{s=1}^{q_k} z_s^k = 1,\ k = 1\cdots K, \tag{17}$$

$$z_s^k \geq 0,\ k=1\cdots K,\ s=1\cdots r_k. \tag{18}$$

In the above problem, constraints (16) are coupling constraints while constraints (17) are convexity rows. Note that problem (16) - (18) has fewer constraints than the original problem (10) - (13). However, since the points in the easy polyhedra are rewritten in terms of extreme points and extreme rays, the number of variables in the Dantzig-Wolfe master problem is typically much larger than in the original problem. Therefore a restricted master problem can be constructed with a very small subset ($\Lambda(k)$) of the columns in the full master problem as follows:

$$\text{Min} \sum_{j\in J^0} v_j^0(\alpha_j^0+\beta_j^0) + \sum_{k\in\mathcal{K}}\sum_{j\in J^k} v_j^k p^k \left[\sum_{s\in\Lambda(k)} z_s^k[(\alpha_j^k)^s+(\beta_j^k)^s]\right]$$

$$\text{s.t.} \sum_{i\in B^0} a_{ij}\pi_i^0 + \sum_{k\in\mathcal{K}}\sum_{i\in B^k} t_{ij}^k \sum_{s\in\Lambda(k)} z_s^k(\pi_i^k)^s - \alpha_j^0 + \beta_j^0 \geq c_j, \quad j\in J^0, \qquad (u) \tag{19}$$

$$\sum_{s\in\Lambda(k), s\leq q_k} z_s^k = 1,\ \ k=1\cdots K, \qquad (u_0^k) \tag{20}$$

$$z_s^k \geq 0, \quad k=1\cdots K,\ \ s\in\Lambda(k). \tag{21}$$

If the reduced costs of all variables in the restricted master problem are nonnegative, the optimal solution to the restricted master is the optimal solution to the full master. Otherwise, the column with the minimum reduced cost is added to the restricted master. Finding the minimum reduced cost is to solve the Dantzig-Wolfe subproblem. In our case, there are K subproblems to solve instead of one. Let (u, u_0^k) are the optimal dual multipliers associated with the set of constraints of the restricted master problem, so that the $k^{th}(k \in \mathcal{K})$ subproblem takes the following form:

$$\text{Min} \left[\sum_{j \in J^k} v_j^k p^k (\alpha_j^k + \beta_j^k) - \sum_{i \in B^k} t_{ij}^k \pi_i^k \right] u_j - u_0^k \tag{22}$$

$$\text{s.t.} \sum_{i \in B^k} w_{ij}^k \pi_i^k - p^k \alpha_j^k + p^k \beta_j^k \geq p^k q_j^k, \; j \in J^k, \tag{23}$$

$$\pi_i^k, \alpha_j^k, \beta_j^k \geq 0, \; j \in J^k. \tag{24}$$

The Dantzig-Wolfe algorithm terminates when the optimum solution of the subproblem is greater than or equal to zero for all $k \in \mathcal{K}$. Otherwise, the variable with the minimum reduced cost is added to the restricted master problem.

3.2 Benders' Decomposition of the Inverse Extensive Form

In Benders' decomposition [2], variables are divided into two sets as "easy" and "complicating" (linking) variables. The problem with only easy variables is relatively easy to solve. Benders' decomposition projects out easy variables and then solves the remaining problem with linking variables. In this algorithm, easy variables are replaced with more constraints. The number of constraints is exponential in the number of easy variables. However, constraints are added on an as needed basis which overcomes the problem of an exponential number of constraints.

Based on Table 1, for the inverse extensive form, $[\pi^0, \alpha^0, \beta^0, (\alpha^1, \beta^1), \cdots, (\alpha^K, \beta^K)]$ are the "easy" variables and $[(\pi^1, \cdots, \pi^K)]$ are the "linking" or "complicating" variables. The original problem (10) - (13) is equivalent to:

$$\text{Min} \;\; z^0$$

$$\text{s.t.} \;\; z^0 - \sum_{k \in \mathcal{K}} \sum_{j \in J^k} v_j^k p^k (\alpha_j^k + \beta_j^k) - \sum_{j \in J^0} v_j^0 (\alpha_j^0 + \beta_j^0) \geq 0, \tag{25}$$

$$\sum_{i \in B^0} a_{ij} \pi_i^0 - \alpha_j^0 + \beta_j^0 \geq c_j - \sum_{k \in \mathcal{K}} \sum_{i \in B^k} t_{ij}^k \pi_i^k, \;\; j \in J^0, \tag{26}$$

$$-p^k \alpha_j^k + p^k \beta_j^k \geq p^k q_j^k - \sum_{i \in B^k} w_{ij}^k \pi_i^k, \;\; k \in \mathcal{K}, j \in J^k. \tag{27}$$

Having written the equivalent problem (25)-(27) and associated optimal dual variables (u_j^0, u_j^k) with constraints (26)-(27) respectively, we can project out the easy variables to come up with the following Benders' Master Problem (BMP):

$$\text{Min} \;\; z^0$$

$$\text{s.t.} \;\; z^0 \geq \sum_{j \in J^0} (u_j^{0i})^T (c_j - \sum_{k \in \mathcal{K}} \sum_{i \in B^k} t_{ij}^k \pi_i^k) + \sum_{k \in \mathcal{K}} \sum_{j \in J^k} (u_j^{ki})^T (p^k q_j^k - \sum_{i \in B^k} w_{ij}^k \pi_i^k) \;\; i = 0, \cdots, q, \tag{28}$$

$$0 \geq \sum_{j \in J^0} (u_j^{0i})^T (c_j - \sum_{k \in \mathcal{K}} \sum_{i \in B^k} t_{ij}^k \pi_i^k) + \sum_{k \in \mathcal{K}} \sum_{j \in J^k} (u_j^{ki})^T (p^k q_j^k - \sum_{i \in B^k} w_{ij}^k \pi_i^k) \;\; i = q+1, \cdots, r. \tag{29}$$

Since BMP has a lot of constraints to optimize directly, the basic idea behind Benders' decomposition is to solve a *relaxed master* problem with only a small subset of the constraints. If there is some constraint in the BMP that is violated by the solution to the relaxed master problem, the violated constraint is added to the master problem. To find the violated constraint the following

Benders' subproblem (BSP) for the inverse extensive form is solved:

$$\text{Max} \sum_{j\in J^0} (u_j^0)^T (c_j - \sum_{k\in\mathcal{K}} \sum_{i\in B^k} t_{ij}^k \bar{\pi}_i^k) + \sum_{k\in\mathcal{K}} \sum_{j\in J^k} (u_j^{ki})^T (p^k q_j^k - \sum_{i\in B^k} w_{ij}^k \bar{\pi}_i^k) \tag{30}$$

$$\text{s.t.} \sum_{j\in J^0} a_{ij} u_j^0 \leq 0,\ i \in B^0, \tag{31}$$

$$u_j^0 \leq v_j^0,\ j \in J^0, \tag{32}$$

$$u_j^k \leq p^k v_j^k,\ k \in \mathcal{K}, j \in J^k, \tag{33}$$

$$u_j^0 \geq 0,\ j \in J^0, \quad u_j^k \geq 0,\ k \in \mathcal{K}, j \in J^k. \tag{34}$$

If the solution u^i to BSP is an extreme point, then a constraint of type (28) is added to the relaxed master problem. If the solution is an extreme direction, then a constraint of type (29) is added to the relaxed master problem. Benders' decomposition algorithm iteratively generates upper and lower bounds on the optimal solution value to the original problem and is terminated when the difference between the bounds is less than or equal to a pre-specified value.

4 Computational Results

We formed and solved inverse problems on four 2SSLP instances from the literature, namely, *LandS*, *pltexp*, *stormg* and *pltexp*. The instance *LandS* is from the Slptestset [23] and is an electric investment planning problem based on [16]. This problem considers the challenge of planning investments in the electricity generation industry. The instances *stormg* and *pltexp* are posted at [22]. *Stormg* is a two period freight scheduling problem described in [19]. In this model, routes are scheduled to satisfy a set of demands at stage 1, demands occur, and unmet demands are delivered at higher costs in stage 2 to account for shortcomings [22]. *Pltexp* is a stochastic capacity expansion model inspired by manufacturing flexibility research in Jordan [13]. The model tries to allocate new production capacity across a set of plants so as to maximize profit subject to uncertain demand

[22].

Table 2 and 3 show the characteristics of the original instances and the corresponding inverse problems, respectively.

Table 2: Characteristics of the original instances.

Instance	Scenarios	Variables (1st,2nd)	Constraints(1st,2nd)
LandS	3	16(4,12)	9(2,7)
stormg2	2	1380(121,1259)	713(185,528)
pltexpA2	6	460(188,272)	166(62,104)
pltexpA2	16	460(188,272)	166(62,104)

Table 3: Characteristics of the inverse instances.

Instance	Scenarios	Variables (1st,2nd)	Constraints(1st,2nd)
LandS	3	103(10,93)	40(4,36)
stormg2	2	6519(427,6092)	2639(121,2518)
pltexpA2	6	4326(438,3888)	1820(188,1632)
pltexpA2	16	10806(438,10368)	4540(188,4352)

For each original 2SSLP instance we solved the EF with a zero objective and regarded the solution as the initial feasible solution for the inverse problem. The computational results for solving the extensive form of the inverse problem using CPLEX Interactive Optimizer 9.0 [15] are reported in Table 4. The second column 'obj (feasible)' gives the objective value at the initial feasible solution, while the third column 'obj (optimal)' gives the optimal objective value after perturbing the objective cost vector via solving the inverse problem. According to computational results, an interesting observation is that if the objective function coefficients change in order to make the initial feasible solution optimal, the change is due to second-stage objective function coefficients rather than the first-stage objective function coefficients. First-stage objective function coefficients stayed the same for all four instances. In all of the instances tested, c and d are the same, q and q' are the same in some of the instances and $q' < q$ in others which is an expectable result according to cases established in Section 2.

We leave the exploration of the decomposition algorithms for future work. We anticipate that as the size of the problem increases, decomposition will become essential.

Table 4: Computational Results.

Instance	Obj (feasible)	Obj (optimal)	CPLEX Time (sec.)
LandS	400	960	0.09
stormg2	55644718.41	68219577.82	0.06
pltexpA2	100	100	0.00
pltexpA2	100	100	0.05

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