Efficient Formulations for the Multi-Floor Facility Layout Problem with Elevators

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Abstract

The block layout problem for a multi-floor facility is an important sub class of the facility layout problem with practical applications when the price of land is high or when a compact building allows for more efficient environmental control. Several alternative formulations for the block layout problem of a multi-floor facility are presented, where the material handling transportation between floors is executed through elevators. Both elevator types that either service all floors or a limited subset of the floors are considered. The department and floors are modeled as rectangular shapes. The floors may have different dimensions provided the floor projections on the ground floor are nested starting from the ground floor to the top floor. The formulations yield large mixed-integer programming problems. We will report on the results of the numerical experiments to solve these problems with a MIP solver for the base formulation and using various acceleration techniques such as symmetry-breaking constraints and valid inequalities. Insights on the relative difficulty of the problems based on the numerical experiment will also be shared.

Keywords

Facilities Design, Block Layout, Multi-Floor Layout, Mixed-Integer Programming, Valid Inequalities.

1 Introduction

Facilities design is a methodology for the design of the physical enclosure of a set of operations. Industrial engineering focuses on the design of a conceptual block diagram of the facility. The block diagram shows the location and dimensions of the building and departments, without details related to structure and utility networks. A good block diagram must facilitate the operations executed in the facility. Typically, the total distance traveled by the "materials" in the facility is used as a proxy for the cost of the facility layout and as indicator of the quality of the facility design. In countries or areas with high land cost, facilities are often constructed with multiple floors. In addition, a compact building shape may allow for more efficient environmental control. Typical examples are the construction of electronics manufacturing facilities with clean room requirements and the energy costs for air conditioning in locations with hot climates. The focus of this paper is on facilities with multiple floors where the vertical material transport between floors occurs through elevators. Elevators that service all floors in a building are the most commonly used. In manufacturing operations, often a type of elevator is used that services only two or a very few floors and that transports only materials. This type of elevator is sometimes called a "dumb waiter" because of its historical application in restaurants and mansions to connect the kitchen with the dining room. This type of elevator is significantly less expensive to install than a full-service elevator. The tradeoff between more and inexpensive elevators of the dumb waiter variety versus fewer and expensive full service elevators is an important design consideration for facilities with multiple floors.

2 Literature Review

The single-floor layout problem has been studied extensively in the past. To our knowledge, the most recent review of the research is given in Meller and Gau (1996). In the discrete version of the problem the building floor and departments are collections of equal-sized unit squares. In the continuous version the departments are polygons with orthogonal sides and vertices that can be located anywhere inside the building floor. Usually the shape of departments is restricted to rectangles. Both the continuous and discrete versions of the problem are known to be NP-Hard, so much of the research has focused on developing a wide variety of heuristics. Formulations for the single-floor continuous version were originally developed by Montreuil (1987). Optimal solutions algorithms were developed by Meller et al. (1999), Al-Khayyal et al. (2001), Sherali et al. (2003). The layouts generated by all of the above algorithms can typically not directly be implemented in practice. In order to make the layouts more implementable, the configuration of the material flow infrastructure was added to the layout problem. Early work in this area was done by Montreuil (1987). Goetschalckx and Kim (2006) developed an algorithm to regularize the department boundaries so that the material flow network consists of aisles without an excessive number of corners.

In contrast the research on the multiple-floor facility layout problem is limited. The material handling network to handle transportation between the different floors is modeled as set of elevators. Bozer et al. (1994) used spacefilling curves and pairwise interchanges to quickly create floor layouts for the discrete version of the problem. Meller and Bozer (1997) proposed a hierarchical decomposition approach, where the departments are assigned to floors in the first stage and each of the floor layouts is determined in the second stage. The number and location of elevators is given and elevators are located around the perimeter of the floors. Abdinnour-Helm and Hadley (2000) developed a GRASP greedy randomized adaptive search procedure and a non-linear optimization algorithm to create an initial layout. The initial layout is then improved with interchange and shift operations in a tabu search framework.

In a multi-floor layout problem, there are four classes of decisions to be made in addition to the decisions for a single-floor layout: 1) on which floor to locate each of the departments; 2) how many elevators to use; 3) to which elevator each material transportation operation between floors is assigned; 4) where to locate the elevators. Clearly, these decisions are interrelated. The formulations in prior research fix one or more of these types of decisions in advance.

The main contribution of this research is the development of two formulations for the continuous facility layout problem with elevators that include all four decisions identified above as decision variables. One formulation allows only full-service elevators and the second formulation allows for partial-service elevators. The resulting formulations are large mixed-integer programming problems. Even the single floor problem requires a substantial amount of computation time. Given the additional complexity of the multiple-floor problem, the computation times for this class are even larger. This research further investigates the use of traditional acceleration techniques such as valid inequalities, symmetry-breaking constraints, and ordering constraints through a numerical experiment. The most efficient acceleration techniques based on a number of test cases are identified. Finally, the numerical experiments illustrates that is more efficient to use more partial-service elevators rather than fewer full-service elevators, even when the cost differential in elevator construction is ignored.

3 Integrated Formulations

The following two formulations are developed for the layout problem of a multi-floor facility where the material handling transportation between floors is executed through elevators. This problem will be denoted as the **M**ulti-**F**loor **F**acility **L**ayout **P**roblem with **E**levators (MFFLPE). The MFFLPE can be divided into two sub classes depending on the fact if all elevators service all floors or not. The two classes have either only full-service elevators or allow also partial-service elevators and are indicated by MFFLPE-A and MFFLPE-B, respectively. For MFFLPE-B the design algorithm will decide which floors are serviced by which elevators. This problem is significantly more complex than the single-floor facility layout problem and a number of simplifying assumptions has be to made to allow for the solution of the formulations.

3.1 Assumptions

- The target of the design model is the gross conceptual layout, i.e., the required areas for the departments include the areas for the elevators and travel aisle space. The elevators do not consume any area, and the travel aisles do not consume any area and are not explicitly modeled.
- The maximum number of elevators is given. In the solution, elevators may be used or not. In version A each elevator services all floors, in version B each elevator services a contiguous set of floors, i.e. an elevator cannot skip an intermediate floor between the bottom and top floor serviced by it. Elevators do not have to serve all floors, simple elevators, which are also called “dumb waiters”, serve only two floors and are possible.
- The location of an elevator is a decision variable. In version B the top and bottom floor of service for each elevator are also decision variables.
- An elevator is a vertical structure, i.e. it has the same x and y coordinates on any floor it services.

- Elevators cannot be located inside any department; they must be located on the boundary of departments on all the floors they service.
- Negative rejection relationships between departments are not allowed. If rejection or incompatibility is present, it will be modeled as a constraint.
- No relationships of departments with the outside are present.
- A department cannot be split among multiple floors, i.e. its complete area must be located on a single floor.
- Floors have rectangular shape. Building floors do not have to be equal, but the projection of higher floors on the ground falls inside or on top of the projection of lower floors. All floors have the same height.
- The objective is to minimize the sum of horizontal and vertical travel costs. The horizontal and vertical components of the distance may be given a different cost factor. The horizontal travel is computed as the weighted centroid-to-centroid or centroid-to-elevator rectilinear distance. The vertical travel is computed as the weighted vertical distance (height).
- Vertical travel between floors can only occur through elevators. All elevators move bi-directionally and elevator capacity is not modeled.
- All departments have a rectangular shape. All departments have the same height equal to the common height of the floors.
- The shape and area of the departments is given. The decision variables for each department are the location of the department and its horizontal or "vertical" orientation. A vertical orientation means that the long side of a department is perpendicular to the width of the building and floors.

3.2 Notation

Parameters

F	the number of floors, by convention the ground (bottom) floor is indicated by 1 and the floor indices increase by increasing vertical location

N	the number of departments
E	the maximum number of elevators that can be used
h	common floor height for all floors
$X_k^L, X_k^R, Y_k^B, Y_k^T$	x and y boundary coordinates of floor *k*, $X_k^L \le X_{k+1}^L, X_k^R \ge X_{k+1}^R, Y_k^B \le Y_{k+1}^B, Y_k^T \ge Y_{k+1}^T$
W_k, L_k, A_k	width, length, and area of floor *k*, $W_k = X_k^R - X_k^L, L_k = Y_k^T - Y_k^B, A_k = W_k L_k$
l_i^l, l_i^s, a_i	length of the longer and shorter side and the area of department *i*, $a_i = l_i^l \cdot l_i^s$
c_{ij}^H, c_{ij}^V	transportation cost factor per unit of material flow between departments *i* and *j* in the horizontal and vertical direction
f_{ij}	material handling flow between departments *i* and *j*. It is assumed that $f_{ij} \ge 0$, i.e. no rejection relationships are allowed.
φ_{ij}	indicator of material handling flow between departments *i* and *j*, equal to one if $f_{ij} > 0$, equal to zero if $f_{ij} = 0$

Notation Simplification

$\forall i < j$	$i = 1..N-1, j = i+1..N$, i.e. all elements of the upper triangular matrix with a row and column for each department. The number of elements is equal to $N(N-1)/2$.

Variables

z_{ik}	equal to one if department *i* is assigned to floor *k*, zero otherwise.
p_i	index of the floor to which department *i* is assigned, $p_i \in \{1, 2, ...F\}$.
p_{ij}^+	floor difference between departments *i* and *j*, $p_{ij}^+ = \lvert p_i - p_j \rvert$.
q_{ij}	equal to one if departments *i* and *j* are assigned to different floors, zero if they are located on the same floor.
t_{ije}	equal to one if the flow between departments *i* and *j* is transported using elevator *e*, zero otherwise.
r_i, l_i, w_i	orientation (1 = long side vertical, 0 = long side horizontal), length, and width of departments *i*.

$u_{ij}^{L}, u_{ij}^{R}, u_{ij}^{A}, u_{ij}^{B}$	equal to one if departments *i* and *j* are located on the same floor and department *i* is located to the left (L), right (R), above (A), or below (B) of department *j*, zero otherwise.
$v_{ei}^{L}, v_{ei}^{R}, v_{ei}^{A}, v_{ei}^{B}$	equal to one if elevator *e* is located to the left (L), right (R), above (A), or below (B) of department *i*, zero otherwise.
x_i, y_i, x_i^c, y_i^c	x and y coordinates of the left-bottom corner and the centroid of department *i*
x_{ij}^{+}, y_{ij}^{+}	distance between departments *i* and *j* in the x and y directions if the departments are located on the same floor
X_e, Y_e	x and y coordinates of elevator *e*
X_{ei}^{+}, Y_{ei}^{+}	distance between elevator *e* and department *i* in the x and y directions
d_{ij}^{HS}, d_{ij}^{HD}	horizontal distance between departments *i* and *j* if they are assigned to the same floor or to a different floor, respectively
d_{ij}^{V}, d_{ij}^{H}	vertical and horizontal distance between departments *i* and *j*

3.3 Formulation A

Minimize $$MHC = \sum_{i=1}^{N}\sum_{j=1}^{N} f_{ij}\left(c_{ij}^{H} d_{ij}^{H} + c_{ij}^{V} d_{ij}^{V}\right) \quad (1.1)$$

Subject to

$$x_i + W_1\left(1 - z_{ik}\right) \geq X_k^L \qquad \forall i,k \qquad (1.2)$$

$$y_i + L_1\left(1 - z_{ik}\right) \geq Y_k^B \qquad \forall i,k \qquad (1.3)$$

$$x_i + w_i \leq X_k^R + W_1\left(1 - z_{ik}\right) \qquad \forall i,k \qquad (1.4)$$

$$y_i + l_i \leq Y_k^T + L_1\left(1 - z_{ik}\right) \qquad \forall i,k \qquad (1.5)$$

$$w_i = r_i l_i^s + \left(1 - r_i\right) l_i^l \qquad \forall i \qquad (1.6)$$

$$l_i = \left(1 - r_i\right) l_i^s + r_i l_i^l \qquad \forall i \qquad (1.7)$$

$$p_i = \sum_{k=1}^{F} k z_{ik} \qquad \forall i \qquad (1.8)$$

$$p_{ij}^{+} \geq p_i - p_j \qquad \forall i < j \qquad (1.9)$$

$$p_{ij}^{+} \geq -(p_i - p_j) \qquad \forall i < j \qquad (1.10)$$

$$p_{ij}^{+} \geq q_{ij} \qquad \forall i < j \qquad (1.11)$$

$$p_{ij}^{+} \leq F q_{ij} \qquad \forall i < j \qquad (1.12)$$

$$\sum_{k=1}^{F} z_{ik} = 1 \qquad \forall i \qquad (1.13)$$

$$z_{ik} + z_{jk} \leq 2 - q_{ij} \qquad \forall i < j \qquad (1.14)$$

$$\sum_{e=1}^{E} t_{ije} = \varphi_{ij} q_{ij} \qquad \forall i < j \qquad (1.15)$$

$$d_{ij}^{v} \geq h \cdot p_{ij}^{+} \qquad \forall i < j \qquad (1.16)$$

$$u_{ij}^{L} + u_{ij}^{R} + u_{ij}^{A} + u_{ij}^{B} + q_{ij} \geq 1 \qquad \forall i < j \qquad (1.17)$$

$$\sum_{i}^{N} a_i \cdot z_{ik} \leq A_k \qquad \forall k \qquad (1.18)$$

$$x_i + w_i \leq x_j + W_1(1 - u_{ij}^{L}) + W_1 q_{ij} \qquad \forall i < j \qquad (1.19)$$

$$x_j + w_j \leq x_i + W_1(1 - u_{ij}^{R}) + W_1 q_{ij} \qquad \forall i < j \qquad (1.20)$$

$$y_i + l_i \leq y_j + L_1(1 - u_{ij}^{B}) + L_1 q_{ij} \qquad \forall i < j \qquad (1.21)$$

$$y_j + l_j \leq y_i + L_1(1 - u_{ij}^{A}) + L_1 q_{ij} \qquad \forall i < j \qquad (1.22)$$

$$X_e \leq x_i + W_1(1 - v_{ei}^{L}) \qquad \forall e, i \qquad (1.23)$$

$$X_e + W_1(1 - v_{ei}^{R}) \geq x_i + w_i \qquad \forall e, i \qquad (1.24)$$

$$Y_e \leq y_i + L_1(1 - v_{ei}^{B}) \qquad \forall e, i \qquad (1.25)$$

$$Y_e + L_1(1 - v_{ei}^{A}) \geq y_i + l_i \qquad \forall e, i \qquad (1.26)$$

$$v_{ei}^{L} + v_{ei}^{R} + v_{ei}^{B} + v_{ei}^{A} \geq 1 \qquad \forall e, i \qquad (1.27)$$

$$x_i^C = x_i + w_i/2 \qquad \forall i \qquad (1.28)$$

$$y_i^C = y_i + l_i/2 \qquad \forall i \qquad (1.29)$$

$$x_{ij}^{+} \geq x_i^C - x_j^C \qquad \forall i < j \qquad (1.30)$$

$$x_{ij}^{+} \geq -\left(x_i^C - x_j^C\right) \qquad \forall i < j \qquad (1.31)$$

$$y_{ij}^{+} \geq y_i^C - y_j^C \qquad \forall i < j \qquad (1.32)$$

$$y_{ij}^{+} \geq -\left(y_i^C - y_j^C\right) \qquad \forall i < j \qquad (1.33)$$

$$X_{ie}^{+} \geq x_i^C - X_e \qquad \forall i, e \qquad (1.34)$$

$$X_{ie}^{+} \geq -\left(x_i^C - X_e\right) \qquad \forall i, e \qquad (1.35)$$

$$Y_{ie}^{+} \geq y_i^C - Y_e \qquad \forall i, e \qquad (1.36)$$

$$Y_{ie}^{+} \geq -\left(y_i^C - Y_e\right) \qquad \forall i, e \qquad (1.37)$$

$$d_{ij}^{H} = d_{ij}^{HS} + d_{ij}^{HD} \qquad \forall i < j \qquad (1.38)$$

$$d_{ij}^{HS} + \left(W_1 + L_1\right) q_{ij} \geq x_{ij}^{+} + y_{ij}^{+} \qquad \forall i < j \qquad (1.39)$$

$$d_{ij}^{HD} + \left(W_1 + L_1\right)(2 - t_{ije} - q_{ij}) \geq X_{ie}^{+} + Y_{ie}^{+} + X_{je}^{+} + Y_{je}^{+} \qquad \forall e, i < j \qquad (1.40)$$

$$X_e \geq X_k^L \qquad \forall e, k \qquad (1.41)$$

$$Y_e \geq Y_k^B \qquad \forall e, k \qquad (1.42)$$

$$X_e \leq X_k^R \qquad \forall e, k \qquad (1.43)$$

$$Y_e \leq Y_k^T \qquad \forall e, k \qquad (1.44)$$

The objective (1.1) computes the sum of the horizontal and vertical distances between each pair of departments, weighted by the material flow between those departments and multiplied by the horizontal and vertical transportation cost rate. Constraints (1.2) through (1.5) ensure that a department is located inside the floor coordinates of the floor to which it is assigned. Constraints (1.6) and (1.7) set the length and width of a department to the long or short side of the department depending on its orientation. Constraint (1.8) computes the index of the floor to which a department is assigned and constraints (1.9) and (1.10) compute the floor difference between two departments. Constraints (1.11) and (1.12) assure that departments can only have a floor difference if they are located on different floors. Constraint (1.13) assures that each department is assigned exactly to one floor. Constraint (1.14) indicates that if two departments are assigned to the same floor then they cannot be on different floors. Constraint (1.15) indicates that if two departments are assigned to a different floor and if they have material flow between them then they must use an elevator. Constraint (1.16) computes the vertical distance between two departments. Constraint (1.17) ensures that two departments must either not overlap on the same floor or be located on different floors. Constraint (1.18) ensures that sum of the areas of departments assigned to a floor does not exceed the floor area. Constraints (1.19) to (1.22) model the non-overlap between pairs of departments. Constraints (1.23) to (1.27) model the non-overlap between a department and an elevator. Constraints (1.28) to (1.40) are used to compute the horizontal and vertical distance between departments and between departments and elevators. Finally, constraints (1.41) to (1.44) ensure that the elevators are located inside the boundaries of all floors.

3.4 Formulation B

The second version of the formulation allows an elevator to service a contiguous set of floors but not necessarily all floors. This variant is motivated by existence of freight elevators in manufacturing plants that service only two or a very few adjacent floors and that are called "dumb waiters". Additional variables and constraints need to be introduced.

Variables

Z_{ek}	equal to one if elevator *e* services floor *k*, zero otherwise.
P_{ek}	equal to one if elevator *e* services any floor below floor *k*, zero otherwise, by definition $P_{e1} = 0$.
T_{ie}	equal to one if department *i* transports flow to any other department using elevator *e*, zero otherwise.

Constraints

$$X_e \geq X_k^L \cdot Z_{ek} \qquad \forall e,k \qquad (1.45)$$

$$Y_e \geq Y_k^B \cdot Z_{ek} \qquad \forall e,k \qquad (1.46)$$

$$X_e \leq X_k^R \cdot Z_{ek} + \left(1 - Z_{ek}\right) W_1 \qquad \forall e,k \qquad (1.47)$$

$$Y_e \leq Y_k^T \cdot Z_{ek} + \left(1 - Z_{ek}\right) L_1 \qquad \forall e,k \qquad (1.48)$$

$$Z_{e(k+1)} + \left(1 - Z_{ek}\right) + P_{ek} \leq 2 \qquad e = 1..E, k = 1..F-1 \qquad (1.49)$$

$$P_{ek} = \max\left\{Z_{e(k-1)}, P_{e(k-1)}\right\} \qquad e = 1..E, k = 2..F \qquad (1.50)$$

$$T_{ie} \geq t_{ije} \qquad i = 1..N, j = 1..N, e = 1..E \qquad (1.51)$$

$$Z_{ek} \geq T_{ie} + z_{ik} - 1 \qquad i = 1..N, k = 1..K, e = 1..E \qquad (1.52)$$

Note that this implies that

$$P_{e2} = \max\left\{Z_{e1}, P_{e1}\right\} = Z_{e1} \qquad e = 1..E \qquad (1.53)$$

Equations (1.49) and (1.50) are equivalent to the following set of linear equations

$$Z_{e(k+1)} - Z_{ek} + P_{ek} \leq 1 \qquad e = 1..E, k = 2..F-1 \qquad (1.54)$$

$$P_{ek} \geq Z_{e(k-1)} \qquad e = 1..E, k = 3..F \qquad (1.55)$$

$$P_{ek} \geq P_{e(k-1)} \qquad e = 1..E, k = 3..F \qquad (1.56)$$

$$P_{e2} = Z_{e1} \qquad e = 1..E \qquad (1.57)$$

The equations (1.41) through (1.44) in model A are substituted by equations (1.45) through (1.48) in model B, since in model B an elevator only has to be located within the rectangle boundaries of the floor if the elevator services this floor. Equation (1.54) eliminates the case when an elevator does not service the current floor, but services the floor above it and one or more floors below it. This case is not feasible because of the contiguous floors served assumption.

3.5 Symmetry-Breaking Constraints and Valid Inequalities

Both model A and model B are large mixed-integer programming formulations. Previous computational experience by Sherali et al. (2003) has shown that extensive run times are required. To reduce the solution effort, Sherali et al. (2003) proposed symmetry-breaking techniques and valid inequalities for the single-floor facility layout problem. We extend the same properties to the multi-floor facility layout problem with elevators. We propose the following symmetry-breaking techniques which are constraints (1.59) through (1.67) and valid inequalities which are constraints (1.68) through (1.75) under the assumption that the width and the length of all floors are same, as shown in equation (1.58). If the building floors do not have identical dimensions, the problem exhibits much less symmetry. It is expected that the performance enhancement by symmetry-breaking constraints is less in this case and this case was not tested numerically.

$$W_1 = W_2 = \cdots = W_F, L_1 = L_2 = \cdots = L_F \tag{1.58}$$

Four symmetry-breaking methods were investigated.

1) Position q method for department q

$$z_{qF} = 0 \tag{1.59}$$

$$x_q^c \le 0.5 X_1^R \tag{1.60}$$

$$y_q^c \le 0.5 Y_1^T \tag{1.61}$$

2) Position q method for an elevator

$$X_1 \le 0.5 X_1^R \tag{1.62}$$

$$Y_1 \le 0.5 Y_1^T \tag{1.63}$$

3) Position p-q method for departments p and q

$$x_p^c \le x_q^c + W_1 q_{pq} \tag{1.64}$$

$$y_p^c \le y_q^c + W_1 q_{pq} \tag{1.65}$$

$$p_p + 1 \le p_q + F\left(1 - q_{pq}\right) \tag{1.66}$$

4) Position p-q method for elevators

$$X_p \le X_q \tag{1.67}$$

Constraints (1.59) through (1.61) are used to model symmetry-breaking constraints for department *q*, where *q* is the department which has the largest sum of flows with other departments. This acceleration method will be denoted by "qD" in the numerical experiment. Constraint (1.59) ensures that department *q* is not assigned to the floor with highest index *F*, which eliminates the symmetry for the floors. Constraints (1.60) and (1.61) ensure that the centroid of department *q* is assigned only in the left-bottom quadrant of the floor, which eliminates the symmetry around the x and y axes in each floor. Constraints (1.62) and (1.63) are used to model the symmetry-breaking constraints for each elevator and this method will be denoted by "qE" in the numerical experiment. The constraints assure that location of elevator q is in the left-bottom quadrant of the floor. This method can only be applied to model A where there is no difference among the elevators. An elevator is chosen at random to function as the *q* elevator. In model B, the elevators may be different, and the qE method cannot be used.

Constraints (1.64) through (1.66) are used to model the symmetry-breaking constraints for the pair of departments that has the largest flow between them. This method will be referred to as "p-qD" in the numerical experiment. Constraints (1.64) and (1.65) become significant only when the departments *p* and *q* are assigned to the same floor ($q_{pq} = 0$). On the other hand, constraint (1.66) becomes significant only when the departments *p* and *q* are assigned to different floors ($q_{pq} = 1$). Constraint (1.67) is used to model the symmetry-breaking constraints for a pair of elevators. It will be denoted as method "p-qE" in the numerical experiment. Again this method can only be applied to model A. In this case, the elevators p and q are selected randomly from all elevators, because there is no difference between elevators. If the building shape is relatively longer in the y-direction than in the x-direction, then it might be more effective to change constraint (1.67) to the constraint $Y_p \le Y_q$, but this was not tested numerically.

Valid lower bounds for the distance between a pair of departments of between a department and an elevator can be derived from the location, dimensions, and orientation of the departments and the location of the elevators. Two sets of valid inequalities were investigated.

1) Valid inequalities for departments

$$d_{ij}^{H} + \left(W_1 + L_1\right) q_{ij} \geq 0.5\left(l_i^s + l_j^s\right) \tag{1.68}$$

$$d_{ij}^{HS} + \left(W_1 + L_1\right) q_{ij} \geq 0.5\left(l_i^s + l_j^s\right) \tag{1.69}$$

$$d_{ij}^{HD} + \left(W_1 + L_1\right)\left(1 - q_{ij}\right) \geq 0.5\left(l_i^s + l_j^s\right) \tag{1.70}$$

$$x_{ij}^{+} \geq 0.5\left(w_i + w_j\right) - \left(l_i^l + l_j^l\right)\left(1 - u_{ij}^L - u_{ij}^R\right) \tag{1.71}$$

$$y_{ij}^{+} \geq 0.5\left(l_i + l_j\right) - \left(l_i^l + l_j^l\right)\left(1 - u_{ij}^B - u_{ij}^A\right) \tag{1.72}$$

$$d_{ij}^{V} + h\left(1 - q_{ij}\right) \geq h \tag{1.73}$$

2) Valid inequalities for elevators

$$X_{ie}^{+} \geq 0.5 w_i - l_i^l \left(1 - v_{ei}^L - v_{ei}^R\right) \tag{1.74}$$

$$Y_{ie}^{+} \geq 0.5 l_i - l_i^l \left(1 - v_{ei}^B - v_{ei}^A\right) \tag{1.75}$$

Constraints (1.68), (1.69) and (1.70) are valid lower bounds, because the horizontal distance between department *i* and *j* is longer than sum of the half of shorter side of each department to avoid overlapping each other. Constraints (1.71) and (1.72) ensure that the distance between departments *i* and *j* in a direction is larger than the sum of half the side of each department in this direction, provided the department locations do not overlap in that direction. Constraint (1.73) assures that vertical distance between department *i* and *j* is larger than *h*, if they are assigned to different floors. Constraints (1.74) and (1.75) ensure that the distance between department *i* and elevator *e* in a direction is larger than half the side of the department *i* in this directions, provided the department location and the elevator location do no overlap in this direction.

4 Numerical Experiment

The performance of the proposed formulations and the different acceleration methods was evaluated using AMPL combined with CPLEX, version 9.1.0, on a SUN workstation with16 GB memory and with a 900 MHz Ultra SPARC III 64-bit processor. We conducted two numerical experiments. In the first experiment, we used only one data set. In this problem instance, the number of department is 12, the number of elevators is 2, and the number of floors is 3. The flow matrix is generated randomly and the flow density is 21%. In the second experiment, the number of departments was set to 9, 10, 11 and 12, the number of elevator was set to 1, 2 and 3, and the number of floors was kept fixed at 3. The flow matrix is generated randomly and the flow density ranges from 20 to 30%. The execution time limit was set to 300,000 seconds for the first experiment and 86,400 seconds for the second experiment, respectively. The results of the first experiment are shown in Table 1. Results of the second experiment are shown in Table 2. If a gap rather than the number of nodes is shown in the table, then this is the gap when the algorithm is terminated after 300,000 or 86,400 seconds, respectively. If the table shows seconds, then the algorithm reached the optimal solution in the indicated time.

The Table 1 shows that the qD method and valid inequalities for elevators perform the best and significantly better than the base case. The combination of these methods has the lowest execution time and the smallest number of nodes enumerated during the search process. The CPU time to reach the optimal solution with this combination of methods was 16,843 seconds, which is only 6% of the time (261,827seconds) required by the basic formulation without any symmetry-breaking strategies and valid inequalities.

Table 1. Execution Time Statistics for Acceleration Techniques

Valid Inequalities			Symmetry-Breaking Techniques: None	Position q: Dept.	Position q: Elev.	Position p-q: Dept.	Position p-q: Elev.
Valid Inequalities	None	Time	261,827	109,310	97,369	300,000	300,000
		Nodes	30,041,864	14,796,654	12,170,907	gap=3.72%	gap=0.10%
	Dept.	Time	280,276	17,213	288,541	72,704	78,263
		Nodes	25,929,170	1,295,205	18,311,978	5,952,687	7,266,159
	Elev.	Time	227,477	16,843	99,322	29,089	53,420
		Nodes	15,992,621	1,148,353	7,366,930	1,996,468	3,181,286
	D & E	Time	300,000	29,905	300,000	120,394	101,829
		Nodes	gap=0.32%	2,116,870	gap=1.12%	9,598,640	8,495,162

N=12, E=2, F=3

Execution time limit = 300,000 seconds

In addition, we tested also the addition of elevator-ordering constraints for model A. Since in model A all elevators are identical, alternative solutions can be found by just exchanging the indices of the elevators. However, adding constraints that prevented these alternative solutions increased the solution times significantly, even up to 20%. Elevator-ordering constraints were not used in the following experiment.

Table 2 shows the comparison between basic formulation and the best combination of acceleration techniques (qD-E), i.e. the combination with the symmetry-breaking techniques for department q and valid inequalities for elevators for a number of cases with different sizes and characteristics. The proposed enhanced formulation performs better than the basic formulation for all the cases. In the cases of N={10, 11} with E=2 and N=12 with E=1, the proposed formulation can find the optimal solution even though the basic formulation cannot within the execution time limit of 86,400 seconds. In the cases of N={10, 11} with E=3, and N=12 with E={2, 3}, both formulations cannot find the optimal solution. However, the enhanced qD-E formulation does find a feasible solution with smaller gap.

In addition, it can be observed that the objective function value is reduced by increasing the maximum number of elevators used for all the cases, except N=12 with E=3. In this last case, only the incumbent feasible solution and gap information is available after 86,400 seconds. Increasing the number of elevators increases the computation time exponentially. In the case of N=12, at the end of the allowed execution time, the objective function value of the best feasible solution found for three elevators is larger than for two elevators. However, since the optimization process is incomplete, no conclusions on the final quality of the respective optimal solutions can be made.

Table 2. Solution Statistics for Model A

F=3		Model A			
		OFV of the Best Feasible Layout		Time or Gap	
N	E	Basic	Valid(qD-E)	Basic	Valid(qD-E)
9	1	86,730.55	86,730.55	7	3
	2	86,071.75	86,071.75	116	14
	3	86,071.75	86,071.75	21,295	294
10	1	83,179.20	83,179.20	2,923	620
	2	82,826.75	82,826.75	gap=3.09%	3,125
	3	82,826.75	82,826.75	gap=3.59%	gap=3.13%
11	1	124,177.45	124,177.45	961	166
	2	123,350.05	123,319.55	gap=1.61%	10,908
	3	123,319.55	123,319.55	gap=3.65%	gap=0.57%
12	1	135,814.25	134,468.65	gap=7.12%	2,677
	2	133,381.35	133,375.45	gap=5.16%	gap=2.53%
	3	133,808.05	133,760.05	gap=11.39%	gap=9.68%

Execution time limit = 86,400 sec. (1day)

We further compared the performance of the model A and B by changing the dimensions of the building floors as shown in Table 3. All floors are identical in group A of the experiment, but in group B and C the uppers floors may have smaller dimensions and a smaller area. All the solution values for the optimal layouts computed by using model A and B are exactly same for the experiments in group A. However, as the results in group B with E=3 and group C with E={2, 3} indicate, the solution value of optimal layout by using model B is better than model A. This is because model B has more flexibility to assign elevators than model A. It is expected that including the different costs for the different types of elevators in the formulations will widen the performance gap between model A and model B. We show the optimal layouts for the case in group B with three elevators in Figure 1. In this case, the location (x,y)=(5.3, 1.0) of elevator 2 in model B is infeasible as the location of an elevator in model A since it falls outside the projection of the third floor, which has dimensions 4 by 2. It also should be observed that even though the maximum number of elevators is equal to three, the solution for model A only uses two elevators.

Table 3. Solution Statistics for Irohara9F3

N=9, F=3			OFV of Optimal Layout		Time to find optimal (sec.)	
Floor type		E	Model A	Model B	Model A	Model B
A	1F:6*2=12	1	86,730.55	86,730.55	7	6
	2F:6*2=12	2	86,071.75	86,071.75	116	56
	3F:6*2=12	3	86,071.75	86,071.75	21,295	550
B	1F:8*2=16	1	82,578.45	82,578.45	7	6
	2F:6*2=12	2	81,745.25	81,745.25	40	92
	3F:4*2=8	3	81,745.25	81,660.15	1,236	1,106
C	1F:8*2=16	1	76,602.65	76,602.65	637	821
	2F:8*2=16	2	76,602.65	76,167.05	2,154	23,205
	3F:2*2=4	3	76,602.65	76,150.95	9,440	1,563

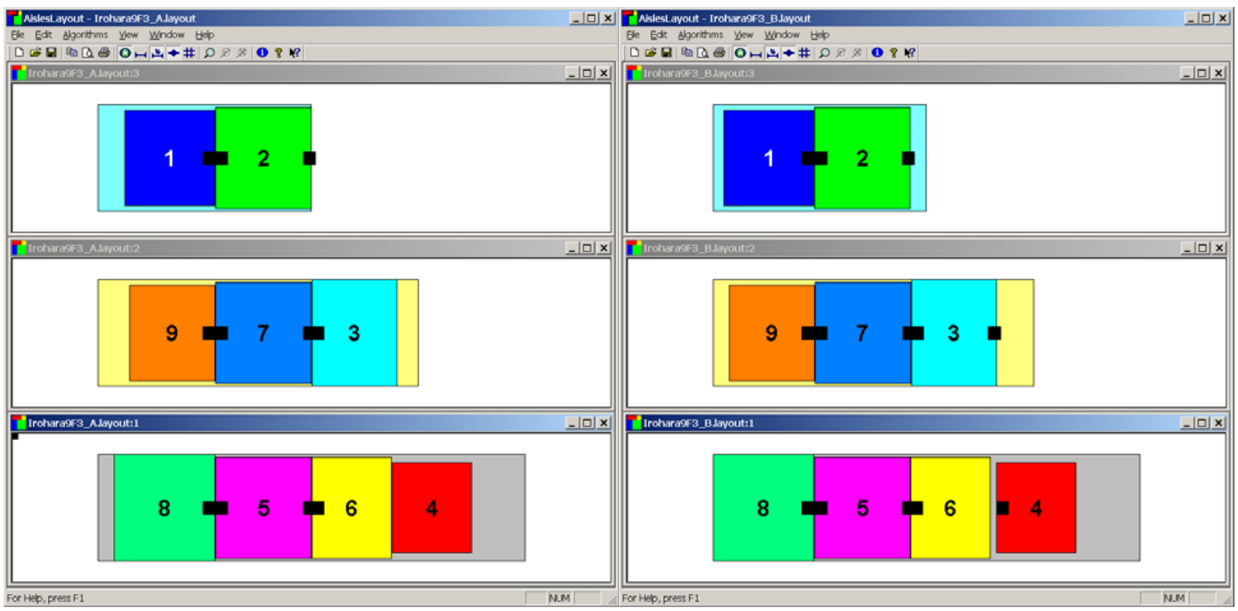

Figure 1. Optimal Layouts for Irohara9F3 by Model A (left) and Model B (right)

Case Irohara11F3

In this section, the details of one data set and its optimal layout are given. It is the problem instance with 11 departments in the second numerical experiment. It has two elevators and three equal floors measuring 4 by 4. This case will be denoted by Irohara11F3. The department dimensions and flow data are given in the table below. The flow density is 29% and the area fill ratio is 72%. For simplicity, the horizontal and vertical cost to travel one distance unit is assumed to be $1.00 and $5.00, respectively, for all flows between pairs of departments. The optimal layout had a distance score of 123,319.55 and is shown in Figure 1. The optimal layout was found in 10,998 seconds.

Table 4. Department Flow Data for Irohara11F3

	1	2	3	4	5	6	7	8	9	10	11
1	0	342	0	0	322	0	0	0	0	0	0
2	0	0	838	0	220	0	0	0	0	0	461
3	0	0	0	829	0	0	0	0	0	0	0
4	0	0	0	0	461	0	0	0	0	0	0
5	0	0	0	0	0	439	0	259	532	680	438
6	0	0	0	0	0	0	474	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	27	0	0
9	0	0	0	0	0	0	0	0	0	231	0
10	0	0	0	0	0	0	0	0	0	0	658
11	0	0	0	0	0	0	0	0	0	0	0

Table 5. Department Data, Optimal Locations, and Department Orientations for Irohara11F3

Data				Optimal Locations and Orientations				
#	long side	short side	area	x	y	w	l	floor
1	1.8	1.7	3.06	0.10	0.25	1.8	1.7	1
2	1.9	1.8	3.42	0.05	1.95	1.9	1.8	1
3	2.0	1.6	3.20	1.95	1.85	1.6	2.0	1
4	1.7	1.5	2.55	1.90	0.35	1.7	1.5	1
5	1.9	1.8	3.42	0.10	0.05	1.8	1.9	2
6	1.9	1.5	2.85	0.00	1.95	1.9	1.5	3
7	1.9	1.8	3.42	0.00	0.15	1.9	1.8	3
8	2.0	1.9	3.80	1.90	0.00	1.9	2.0	3
9	1.8	1.6	2.88	1.90	0.00	1.6	1.8	2
10	2.0	1.5	3.00	1.90	1.80	1.5	2.0	2
11	1.8	1.7	3.06	0.10	1.95	1.8	1.7	2
E1				1.00	1.95			
E2				1.90	1.00			

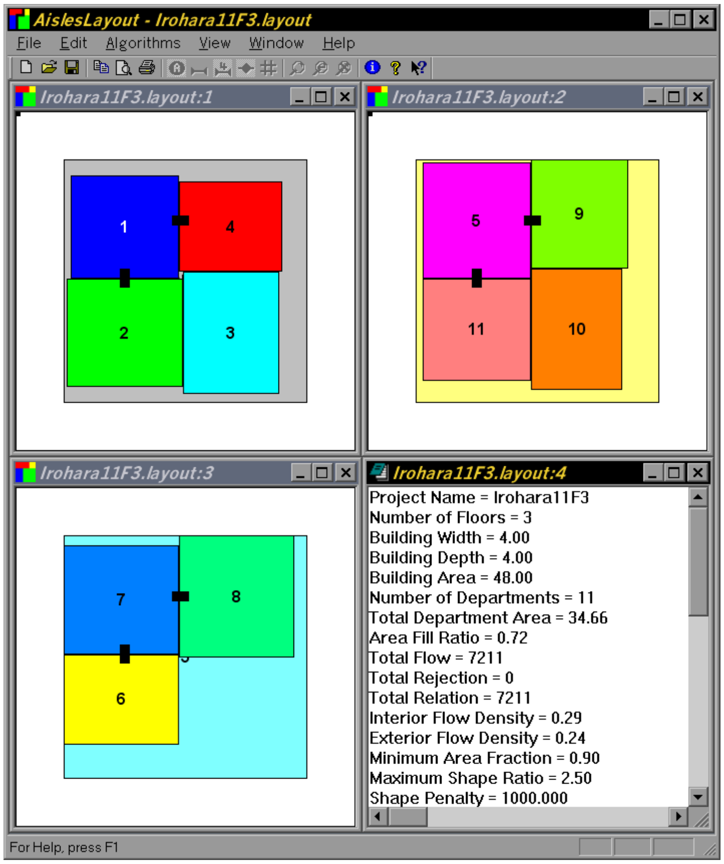

Figure 2. Optimal Layouts for Irohara11F3

Case Bozer15F3

The algorithm described above was also executed for the case presented by Bozer et al. (1994), which has 15 departments, three equal floors with dimensions of 15 by 5, and six elevators with fixed locations around the perimeter of the building. This case will be denoted by Bozer15F3. We used the final layout shown in that paper to determine the long and short side of each department. The non-rectangular department shapes in the layout by Bozer et al. (1994) were converted to rectangular shapes, which are required by our formulation, while the department areas were kept unchanged. These department dimensions are given in the table below and are used as input parameters to our algorithm. This layout had a distance score of 167,308 and is shown in Figure 3. The sum of department areas is equal to the sum of the floor ratios, i.e. the area fill ratio is 100%. Exercising our algorithm created the optimal layout for the given elevator locations, which is shown in Figure 4 and has a distance score of 121,419 or an improvement of 27%. The maximum shape ratio is equal to 2.94 and the maximum perimeter shape ratio is equal to 1.15, both of which are realized for department 13. The optimal layout was found in 3,911 seconds.

Table 6. Department Locations and Dimensions for Bozer15F3 Case

Dept	Corner		Dimensions		
	X-Coord.	Y-Coord.	Width	Depth	Floor
1	8.000	2.000	5.000	3.000	1
2	13.000	0.000	2.000	5.000	1
3	8.500	0.000	4.500	2.000	1
4	5.000	0.000	3.500	2.000	1
5	0.000	0.000	2.000	5.000	2
6	0.000	0.000	5.000	5.000	3
7	8.000	0.000	5.000	5.000	3
8	5.000	0.000	3.000	5.000	3
9	5.000	2.000	3.000	3.000	1
10	0.000	0.000	5.000	5.000	1
11	13.000	0.000	2.000	5.000	3
12	5.800	0.000	4.200	3.571	2
13	5.800	3.571	4.200	1.429	2
14	2.000	0.000	3.800	5.000	2
15	10.000	0.000	5.000	5.000	2

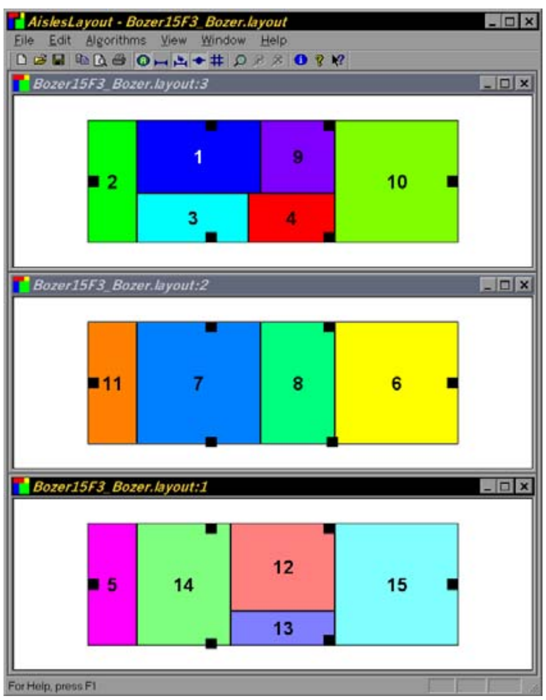

Figure 3. Bozer15F3 Floor Layouts Based Bozer and Meller Figure 6a

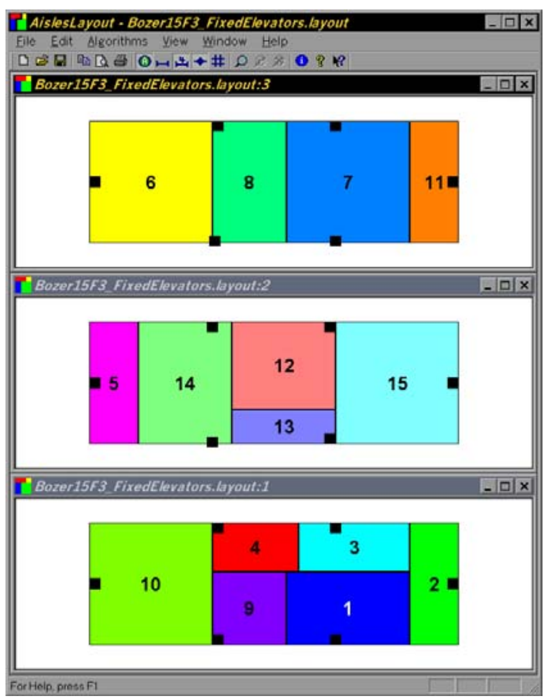

Figure 4. Bozer15F3 Optimal Floor Layouts for Fixed Elevator Locations

5 Conclusions

The facility layout problem with multiple floors and elevators is an important sub class of the facility layout problem. Practical applications occur when the land costs are high or when a compact building configuration allows for easier or more efficient environmental control. Two types of elevators are commonly used: the first type services all the floors; the second type services only two or three adjacent floors. Two formulations were presented that incorporate for the first time all the decision variables for this multi-floor layout problem with elevators. In order to reduce the significant computation times, enhanced formulations were tested that eliminated symmetry and used valid lower bounds on distances. The combination of department position and elevator valid inequalities was shown to be most effective for obtaining the optimal solution in a case with 12 departments. It reduced the computation times by a factor of 15.5 compared to the base formulation. The computation times for the base formulation are so much larger than for the enhance formulations, that only the enhanced formulation should be used. While the required processing times of the algorithms with the best combination of acceleration techniques are large, they are still acceptable for this design problem.

Experimental comparison between the solutions of model A with full-service elevators and model B with partial-service elevators showed that model B generated more efficient solutions, even without model the cost differential of the various elevator types. This indicates that model B should be used, even though it is more complex.

Future research will focus on decomposition techniques for the solution algorithms so that realistic problem sizes can be solved to optimality or to an acceptable gap. Furthermore, two extensions to the formulations are of practical interest. One direct extension indicated above is the inclusion of the fixed construction costs of the elevators. It is anticipated that this will make the solutions to formulation B much more cost efficient than those of model A. A second extension is to model other vertical material handling devices such as drop chutes and spiraling roller conveyors. These devices are often used in manufacturing and warehouses and are relatively inexpensive compared to elevators or even dumb waiter elevators. However, they have the disadvantage they can only handle material flow in the downward direction, which will require modifications to the formulations presented in this paper. Finally, more realistic layouts can be obtained if the material flow network is modeled in each of the floors and if the layouts in each of the floors are regularized. Both these extensions have made the single-floor layout problem much more complex and are expected to complicate the multi-floor layout problem even more.

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