E-model for Transportation Problem of Linear Stochastic Fractional Programming

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Abstract: This paper deals with the so-called transportation problem of linear stochastic fractional programming, and emphasizes the wide applicability of LSFP. The transportation problem, received this name because many of its applications involve in determining how to optimally transport goods. However, some of its applications (e.g., production scheduling) actually have nothing to do with transportation. The said special class of transportation problem has two distinct costs matrix in which costs involved in the problem are random in nature, and the demand vector under study is also random. The proposed E-model attempts to maximize the profit gained per unit of shipping cost, subject to regular supply constraints along with stochastic demand constraints. Solution procedure has been provided to optimize the said problem.

Keywords: Transportation Problem, Fractional Programming, Stochastic Programming, Sign Technique.

1. Introduction

Business decision-making involves uncertainty. Courtney et al.[1] refers to this phenomenon in their classical Harvard piece. The authors divide future scenarios into four categories – (1) Clear-Enough Future – with known and knowable data, (2) Discrete Alternatives Futures –where uncertainty is categorized into clear defined scenarios, (3) A Range of Futures – where the uncertainty cannot be fitted into discrete scenarios and (4) True Ambiguity where the future is totally uncertain and there is no basis for forecast.

[1] Hugh Courtney, Jane Kirland, Patrick Viguerie, Strategy Under Uncertainty, Harvard Business Review, Nov. 1., 1997.

Traditional analysts treat uncertainty as involving only (1) or (4) of the above. Since a number of uncertainties in real life fall within (2) or (3), strategy formulation calls for sophisticated analysis. Stochastic programming provides various sophisticated ways to deal uncertainties, which addresses (2) and (3). Wide range of applications of stochastic programming can be seen in Jeeva et al. (2002, 2004), Charles (2005a, 2005b, 2005e, 2005g).

When the market demands for a commodity are not known with certainty, the problem of scheduling shipments to a number of demand points from several supply points is a stochastic transportation problem, Williams (1963). Kurt Jörnsten et al. (1984, 1989) studied stochastic transportation model for petroleum transport as well proposed a cross decomposition algorithm to solve said problem. The stochastic transportation problem can be formulated as a convex transportation problem with nonlinear objective function and linear constraints. Holmberg (1995) compared several different methods based on decomposition techniques and linearization techniques for this problem; he had tried to find the most efficient method or combination of methods. He had also discussed and tested a separable programming approach, the Frank-Wolfe method with and without modifications, the new technique of mean value cross decomposition and the more well known Lagrangean relaxation with sub-gradient optimization, as well as combinations of these approaches.

Ratio optimization problems are commonly called fractional programs. One of the earliest fractional programs is an equilibrium model for an expanding economy introduced by Von Neumann in 1937, at a time when linear programming hardly existed. The linear and nonlinear models of fractional programming problems have been initially studied by Charnes et al. (1962), and Dinkelbach (1967). The fractional programming problems have been studied extensively by many researchers. Mjelde (1978) maximized the ratio of the return and the cost in resource allocation problems, Kydland (1969) on the other hand maximized the profit per unit time in a cargo-loading problem. Arora et al. (2001) discussed a fractional bulk transportation problem in which the numerator is quadratic in nature and denominator is linear.

Stochastic Fractional Programming (SFP) offers a way to deal with planning in situations where the problem data is not known with certainty. Such situations arise where technological aspects of the system under study may be highly complicated or incapable of being observed completely. Stochastic Programming and Fractional Programming constitute two of the more vibrant areas of research in optimization. Both areas have blossomed into fields that have solid mathematical foundations, reliable algorithms and software, and a plethora of applications that continue to challenge current state-of-the-art computing resources. For various reasons, these areas have matured independently. Many of the existing procedures that are of practical importance for solving stochastic programming and fractional programming problems rely mostly on simplified assumptions.

A linear stochastic fractional programming (LSFP) problem involves optimizing the ratio of two linear functions subject to some constraints in which atleast one of the problem data is random in nature with non-negative constraints on the variables. In addition, some of the constraints may be deterministic.

The LSFP framework attempts to model uncertainty in the data by assuming that the input or a part thereof is specified by a probability distribution, rather than being deterministic. Gupta (1981) described a model on capacitated stochastic transportation problem, which maximizes profitability. LSFP has been extensively studied by Gupta et al. (1979, 1981) and Charles et al. (2001-2006), basic concepts of LSFP is available in Charles (2001a, 2001b), various algorithms to solve LSFP has been discussed in Charles et al. (2002, 2004a, 2005c, 2005d), financial derivatives applications of non linear SFP are studied in Charles et al. (2004b, 2005f), multi-objective LSFP problem is discussed in Charles et al. (2003); Charles (2006a), discusses an application to assembled printed circuit board of multi-objective LSFP, algorithm to identify redundant fractional objective function in multi-objective LSFP is clearly discussed in Charles et al. (2006b). In this paper, special class of transportation problem has been considered wherein the LSFP be the handy technique to optimize the transportation problem. The said special class of un-capacitated transportation problem has two distinct costs matrix in which costs involved in the problem are random in nature that are assumed to follow normal distribution, and the demand vector under study is also random wherein the demand vector is assumed to follow probability distributions like normal and uniform. The proposed E-model attempts to maximize the profit gained per unit of shipping cost, subject to regular supply constraints along with stochastic demand constraints.

The remainder of this paper is organized as follows. Section 2 discusses the un-capacitated transportation problem of LSFP along with some basic assumptions. Deterministic equivalents of probabilistic demand constraints are described in Section 3 and also this section explains some of the preliminary properties of transportation problem of LSFP and expectation model for the un-capacitated transportation problem of LSFP problem is established. A numerical example is provided in Section 4 to demonstrate the proposed E-model, and Section 5 concludes this research paper with a summary and recommendations for future research.

2. The Un-capacitated Transportation Problem of LSFP

This section deals with the un-capacitated TP of LSFP for the distribution of a single homogenous commodity from *m* sources to *n* of destinations, where the demand for the commodity at each of the *n* destinations is a random variable. An un-capacitated TP of LSFP in a criterion space is defined as follows:

$$\text{Optimize } R(X) = \frac{N(X)+\alpha}{D(X)+\beta} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}x_{ij}+\alpha}{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}+\beta} \tag{1}$$

$$\text{Subject to, } \quad \sum_{j=1}^{n} x_{ij} \le a_i \qquad i = 1,2,..., m, \tag{2}$$

$$\Pr\left[\sum_{i=1}^{m} x_{ij} \ge r_j\right] \ge 1 - l_j \qquad j = 1,2, ..., n, \tag{3}$$

where, $0 \le X_{mxn} = \| x_{ij} \| \subset \mathbb{R}^{mxn}$ is a feasible set, $S = \{X \mid \text{Eqs. (2) - (3)}, X \ge 0, X \subset \mathbb{R}^{mxn}\}$ is non-empty, convex and compact set in $\mathbb{R}^{mxn}$, x_{ij} is an unknown quantity of the good shipped from supply point *i* to demand point *j*, profit matrix $N_{mxn} = \| p_{ij} \|$ which determines the profit $p_{ij} \sim$ N($u_{p_{ij}}, s^2_{p_{ij}}$) gained from shipment from i to j, cost matrix $D_{mxn} = \|c_{ij}\|$ which determines the cost $c_{ij} \sim$ N($u_{c_{ij}}, s^2_{c_{ij}}$) per unit of shipment from i to j, the denominator function $D(\mathrm{X}) + \beta$ is assumed to be positive throughout the constraint set, scalars α, β, which determines some constant profit and cost respectively, supply point i must have atmost a_i units, stochastic demand point j must obtain atleast r_j units, $1-l_j$ ($0<l_j<1$) is the least probability with which j^{th} stochastic demand constraint is satisfied.

Assumption 1a: Every point of supply and demand is positive.
Assumption 1b: Total supply is not less than total demand.
Assumption 1c: Non-integer solutions are acceptable.

3. Deterministic Equivalents of Probabilistic Demand Constraints and E-model

Let r_j be a random variable in Eqn. (3) and it follows N($u_{r_j}, s^2_{r_j}$), i = 1, 2, ..., m, j = 1, 2, ..., n, where u_{r_j} be the j^{th} mean and $s^2_{r_j}$ be the j^{th} variance. The j^{th} deterministic demand constraint for Eqn.(3) is obtained from Charles and Dutta (2001a) as follows:

$\Pr[\sum_{i=1}^{m} x_{ij} \geq r_j] \geq 1-l_j$ (or) $\Pr[r_j \leq \sum_{i=1}^{m} x_{ij}] \geq 1-l_j$ (or) $\Pr(Z_j \leq z_j) \geq 1-l_j$, where $Z_j = (r_j - u_{r_j})/s_{r_j}$ follows standard normal distribution and $z_j = (\sum_{i=1}^{m} x_{ij} - u_{r_j})/s_{r_j}$. Thus, $\phi(z_j) \geq \phi(K_{1-l_j})$, where $1-l_j = \phi(K_{1-l_j})$, is the cumulative distribution function of standard normal distribution. Clearly, $\phi(.)$ is a non-decreasing continuous function, hence $z_j \geq K_{1-l_j}$. The j^{th} deterministic demand constraint for Eqn. (3) is as follows:

$$\sum_{i=1}^{m} x_{ij} \geq u_{r_j} + K_{1-l_j}\, s_{r_j} \tag{4}$$

Let r_j be the uniform random variable which range from u_j^{low} to u_j^{up} i.e., $r_j \sim$ U(u_j^{low}, u_j^{up}), the probabilistic demand constraint in system (1) is equivalent to $\sum_{i=1}^{m} x_{ij} \geq \tau_j$, where $l_j' = 1 - l_j$, $\int_{\tau_j}^{u_j^{up}} \left(\frac{dx}{u_j^{up} - u_j^{low}} \right) = l_j'$, i.e., $\tau_j = l_j u_j^{up} + l_j' u_j^{low}$. Hence, the deterministic equivalent of j^{th} probabilistic demand constraint Eqn. (3) is:

$$\sum_{i=1}^{m} x_{ij} \geq l_j u_j^{up} + l_j' u_j^{low} \tag{5}$$

Definition1: If the total deterministic demand equals to total supply, i.e., $\sum_{j=1}^{n} u_{r_j} + K_{1-l_j}\ s_{r_j} = \sum_{i=1}^{m} a_i$.

Definition2: If the total deterministic demand is strictly more than total supply, then the transportation problem of LSFP has no feasible solution, i.e., $\sum_{j=1}^{n} u_{r_j} + K_{1-l_j}\ s_{r_j} > \sum_{i=1}^{m} a_i$.

Property1: The transportation problem of LSFP always has a feasible solution, i.e., feasible set S is non-empty.

Property 2: The set of feasible solution is bounded.

Property 3: The transportation problem of LSFP is solvable.

The proof of the above said properties are as follows:

Let x_{ij}^* be defined as

$$x_{ij}^* = \frac{a_i (u_{r_j} + K_{1-l_j}\ s_{r_j})}{\mathrm{T}} \ , \ i = 1,2,\ldots,m;\ j = 1,2,..,n, \tag{6}$$

where $0 < T = \sum_{j=1}^{n} (u_{r_j} + K_{1-l_j}\ s_{r_j})$.

Substituting x_{ij}^* for the supply and demand constraints, i.e., Eqs. (2) and (4) one can obtain the following:

$$\sum_{j=1}^{n} x_{ij}^{*} = \sum_{j=1}^{n} \frac{a_i (u_{r_j} + K_{1-l_j}\ s_{r_j})}{T} = \frac{a_i}{T} \sum_{j=1}^{n} (u_{r_j} + K_{1-l_j}\ s_{r_j}) = a_i \quad i = 1,2,\ldots,m, \quad \text{and}$$

$$\sum_{i=1}^{m} x_{ij}^{*} = \sum_{i=1}^{m} \frac{a_i (u_{r_j} + K_{1-l_j}\ s_{r_j})}{T} = \frac{(u_{r_j} + K_{1-l_j}\ s_{r_j})}{T} \sum_{j=1}^{m} a_i \geq \frac{(u_{r_j} + K_{1-l_j}\ s_{r_j})}{T} \sum_{j=1}^{m} (u_{r_j} + K_{1-l_j}\ s_{r_j}) = u_{r_j} + K_{1-l_j}\ s_{r_j}$$

$$j = 1,2,\ldots,m.$$

Hence, contraints Eqs. (2) and (4) are satisfied by x_{ij}^{*}. Since from assumptions 1.a and 1.b and Eqn.(6) it follows that $x_{ij}^{*} > 0$, $i = 1,2,\ldots,m$; $j = 1,2,\ldots,n$, it becomes obvious that $x^{*} = (x_{ij}^{*})$ is a feasible solution of the transportation problem of linear stochastic fractional programming. Thus it has been clearly shown that the feasible set S is not empty.

Further, from Eqs. (2) and (4) along with non negativity constraints it is clear that
$0 \leq x_{ij}^{*} \leq a_i$, $i = 1,2,\ldots,m$; $j = 1,2,..,n$.

Expectation of the profit and cost function of the probabilistic fractional objective function is defined as follows:

$$E(N(X)) = \sum_{i=1}^{m}\sum_{j=1}^{n} E(p_{ij}) x_{ij} + \alpha = \sum_{i=1}^{m}\sum_{j=1}^{n} u_{pij} x_{ij} + \alpha \quad (7)$$

$$E(D(X)) = \sum_{i=1}^{m}\sum_{j=1}^{n} E(c_{ij}) x_{ij} + \beta = \sum_{i=1}^{m}\sum_{j=1}^{n} u_{cij} x_{ij} + \beta \quad (8)$$

Hence the deterministic fractional objective function is as follows:

$$R^{*}(X) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} u_{pij} x_{ij} + \alpha}{\sum_{i=1}^{m}\sum_{j=1}^{n} u_{cij} x_{ij} + \beta} \quad (9)$$

Since the numerator function Eqn. (7) and denominator function Eqn. (8) of the fractional objective function Eqn. (9) are linear and the denominator function is assumed to be positive over the bounded feasible set S, it means that fractional objective function $R^{*}(X)$ is also bounded over the same feasible set S, and hence it can be concluded that transportation problem of LSFP is solvable. □

The E-model for un-capacitated TP of LSFP is as follows:

$$\text{Optimize } R^*(X) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} u_{pij} x_{ij} + \alpha}{\sum_{i=1}^{m}\sum_{j=1}^{n} u_{cij} x_{ij} + \beta} \qquad (10)$$

Subject to, $\sum_{j=1}^{n} x_{ij} \le a_i \qquad i = 1,2,..., m,$

$\sum_{i=1}^{m} x_{ij} \ge u_{r_j} + K_{1-l_j}\, s_{r_j} \qquad j = 1,2\ldots,n.$

where, $0 \le X_{mxn} = \| x_{ij} \| \subset \mathbb{R}^{mxn}$ is a feasible set, S = {X │ Eqs. (2) and (4), $X \ge 0$, $X \subset \mathbb{R}^{mxn}$} is non-empty, convex and compact set in $\mathbb{R}^{mxn}$, x_{ij} is an unknown quantity of the good shipped from supply point *i* to demand point *j*, $R^*(X)$ is the fractional objective function defined as ratio of expectation of the profit function over expectation of the cost function, the cost function is assumed to be positive throughout the constraint set, scalars α, β, which determines some constant profit and cost respectively, supply point *i* must have atmost a_i units, deterministic demand point *j* must obtain atleast $u_{r_j} + K_{1-l_j}\, s_{r_j}$ units. Similarly one can define E-model of system (1), when demand follows uniform distribution or/and normal distribution.

The E-model for the un-capacitated TP of LSFP can be solved using Charles et al. (2005d), which provides a sign technique based algorithm to solve the LSFP problem. One can also use the very famous Dinkelbach algorithm [Dinkelbach (1967)] to solve the system (10), or using any existing stochastic programming solver.

4. Numerical Example

This section describes the numerical example for the proposed E-model. A petroleum company has three refineries 1, 2, and 3 which produce 150, 200, and 250 units of petrol, respectively. The company is expected to supply to four of its outlets say 1-4. The demand varies from outlet to outlet, with the past experience the logistics department has made the following statement: the demand of outlet 1, and 2 are normally distributed with mean 100, and 150 with standard deviation 10, and 8 respectively; outlet 3, and 4 demands are uniformly distributed in the intervals (100, 150), and (150, 200), respectively. Preselected probabilities for the demand constraints of system (10) are atleast 0.90, 0.80, 0.05, 0.10.

The average transportation cost per unit and the average profit per unit for each transaction from i^{th} refinery to j^{th} outlet are given in Table 1.

Table 1: Profit and cost estimate of unit transaction (Rs)

$u_{p_{ij}}$	**1**	**2**	**3**	**4**	$u_{c_{ij}}$	**1**	**2**	**3**	**4**
1	2	7	12	5	**1**	10	9	11	4
2	8	14	9	12	**2**	8	12	3	4
3	23	15	5	4	**3**	25	16	6	7

The petroleum company would like to maximize the profit over the cost in a way to meet the stochastic demands at each of its outlets.

$$\text{Maximize } R^*(X) = \frac{\sum_{i=1}^{3}\sum_{j=1}^{4} u_{pij} x_{ij}}{\sum_{i=1}^{3}\sum_{j=1}^{4} u_{cij} x_{ij}} \qquad (11)$$

Subject to, $\sum_{j=1}^{n} x_{ij} \le a_i \qquad i = 1,2,3,$

$\sum_{i=1}^{m} x_{ij} \ge u_{r_j} + K_{1-l_j} s_{r_j} \qquad j = 1,2,$

$\sum_{i=1}^{m} x_{ij} \ge l_j u_j^{up} + l'_j u_j^{low} \qquad j = 3,4,$

$x_{ij} \ge 0 \qquad i = 1,2,3; j = 1,2,3,4,$

where $K_{1-l_1} = 1.28$, $K_{1-l_2} = 0.84$, $l_1 = 0.05$, $l_2 = 0.10$.

Since the total supply / availability of 600 units of petrol at three refineries not equals to the total demand 527.02 of units at four outlets, it is an unbalanced transportation problem of LSFP. System (11) has been solved using the sign technique Charles et al. (2005d), the result is shown in the last column of Table 2.

Table 2: Comparison of linear objective versus fractional objective

	Max $\sum_{i=1}^{3}\sum_{j=1}^{4} u_{p_{ij}} x_{ij}$	Min $\sum_{i=1}^{3}\sum_{j=1}^{4} u_{c_{ij}} x_{ij}$	Max $\dfrac{\sum_{i=1}^{3}\sum_{j=1}^{4} u_{p_{ij}} x_{ij}}{\sum_{i=1}^{3}\sum_{j=1}^{4} u_{c_{ij}} x_{ij}}$
	Subject to constraints of system (11)		
Profit (Rs.)	**9288.7**	3791.6	8195.2
Cost (Rs.)	8465.5	**3676.9**	6922.5
Ratio - Profit/Cost	1.0972	1.0312	**1.1838**

Table 2, gives the clear comparison of linear objective function versus fractional objective function subject to constraints of system (11), in which the last column provides the better ratio i.e., profit/cost compared to second and third columns. Second column of Table 2, depicts the profit maximization model, whereas third column of same table shows the cost minimization model, the last column maximizes the profit over the cost per shipment which is nothing but the TP of LSFP. The optimal solution for the TP of LSFP is $\{ x_{ij}$; i =1,2,3; j = 1,2,3,4} = {0, 45.75, 102.5, 0; 0, 0, 0, 200; 112.8, 109.22, 0, 0}, R*(X) = 1.1838, i.e., maximum profit over the cost per unit of shipment is 1.1838.

5. Conclusion

The proposed model would provide useful solution under those circumstances when the company likes to optimize the profit over the cost per unit of shipment in a way to meet the stochastic demands. This paper can be extended to an integer solution using branch and bound technique. V-model for TP of LSFP and Stochastic fractional recourse programming may be the interest of future research.

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