Necessary optimality condition for Nonsmooth Switching Control problem.

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Abstract. This paper is concerned with a class optimal switching nonsmoth optimal control problem is considered. Both the switching instants and the control function are to the chosen such that the cost functional is minimized.The necessary optimality conditions are derived by means of normal cone and Dubovitskii Milyutin theory.

1.Introduction

A switching system consists of a number of subsystems and a switching law.The switching law is to define of the subsystem to be activated at certain specified switching instants during the planing horizon.Switching system arise in many real world aplications,such as the control of mechanical systems,the automative industry,aircraft and air traffic control,and switching power converts.Some details on these aplications can be found in [20]

For problem of the optimal control of switching systems,the objective is to seek a switching law and a control function such that some perfomance criterion is minimizing subject to some constraints on the state and the control variables.These optimal control problems have atractyed increasing attention (see [1,3,4,5,6,8,11,18,19,20] because of their practical significance and theoretical challenge.However,ther are many open issiues yet to be answered.For example,even for problems only linear subsystems and quadratic costs,a closed form solution of the optimal switching instants is still unavailable.

In the present paper, the author's main aim is to formulate necessary optimality conditions for nonsmooth case (cost functional is nonsmooth) by using nonsmooth analysis and the method which was suggested and formalized by Dubovitskii and Milyutin [7,2]

The Dubovitskii and Milyutin formalizm containts the followings three major companents:

a)To treat local minima via the empty intersection of certain sets in the primal space built upon the inital cost and constraints data

b)To approximate the above sets by convex cones with no intersections.

c)To arrive at dual necessary optimality conditions in the form of an abstract Euler equation by employing convex separation

To start our discussion, first we have describe certain points about functional analysis and nonsmooth analysis construction. For more information we refer the reader to [13].

2.Tools of nonsmooth analysis.

If φ_k is lower semicontinuous around x, then its basic subdifferential can be shown by: $\partial\varphi(x^0)=\underset{x\to x^0}{Lim\sup}\,\hat{\partial}\varphi(x)$.Here, $\hat{\partial}\varphi\ (x^0):=\left\{x^*\in R^n Li\min f\ \dfrac{\varphi(u)-\varphi(x)-\langle x^*,u-x\rangle}{|u-x|}\geq 0\right\}$

is the Frechet subdifferential. By using plus-minus symmetric constructions, we can write

$\partial^+\varphi(x^0):=-\partial(-\varphi)(x^0),\ \hat{\partial}^+\varphi(x^0):=-\partial(-\hat{\varphi})(x^0)$

which are called basic superdifferential and Frechet superdifferential, respectively. Here

$\hat{\partial}\varphi^{+}\left(x^{0}\right):=\left\{x^{*}\in R^{n} Lim\sup\dfrac{\varphi(x)-\varphi\left(x^{0}\right)-\left\langle x^{*},x-x^{0}\right\rangle}{\left|x-x^{0}\right|}\geq 0\right\}$.For a Locally Lipschitzian function subdifferential and superdifferential may be different. For example, if we take $\varphi(x)=|x|$ on R, then $\partial\varphi(0)=[-1,1]$ but $\hat{\partial}\varphi(0)=\{-1,1\}$.

If φ is Lipschitz continuous around point x^{0} then, the strictly differentiability of the function φ at x^{0} (see [2,13]) are equivalent to $\partial\varphi(x^{0})=\partial^{+}\varphi(x^{0})=\{\nabla\varphi(x^{0})\}$. If $\partial\varphi(x^{0})=\hat{\partial}\varphi(x^{0})$ then, this function lower regular at x^{0}. Symmetrically we can give upper regularity of the function at the point by using definitions of superdifferential and Frechet superdifferential. Also if the extended-real-valued function is Lipschitz continuous around the given point and upper regular at this point then the Freshet superdifferential is not empty. Furthermore it is equal to Clarke generalized subdifferential at this point (for proof, see [16]). By using all these nonsmooth analysis tools, we will try to find the Frechet superdifferential form of the necessary optimality condition for the step discrete system.

3.Problem formulation

$$\dot{x}_k = f_k(x_k,u_k,t), \qquad t_{k-1}\leq t\leq t_k \tag{3.1}$$

$$x_1(t_0)=x_0,$$

$$F_i[x_N(t_N),t_N]\leq 0 \quad i=1,2,...,l \tag{3.2}$$

$$F_i[x_N(t_N),t_N]=0 \quad i=l+1,l+2,..., m \tag{3.3}$$

$$x_{k+1}(t_k)=M_k(x_k(t_k),t_k), \ k=1,...,N-1 \quad (\text{ in here } t_1,t_2,...,t_{N-1} \text{ be unknown}) \tag{3.4}$$

$$G(x(t),t)\leq 0 \tag{3.5}$$

$$\mathbf{S}(x,u)=\sum_{i=1}^{n}\varphi_i(x_k,u_k,t) \tag{3.6}$$

In this problem, $f_i: R\times R^{n}\times R^{r}\to R^{n}$, G_k, M_k and F_k are given continuous, at least continuously partially differentiable vector-valued functions with respect to it's coordinates , $M_i: R^{n}\times R\to R$ and $G: R^{n}\times R\to R$, $\varphi_i(x_k,u_k,t)$ are given functions which satisfying Frechet subdifferential u_i (t): R → $U_i\subset R^r$ are controls . The set U_i, are assumed to be nonempty and bounded.

Theorem 1: *Assume that $\varphi_i: R^{n}\to R$ is finite at $x_i^{0}(t_i)$, then for the optimality of pairs $\left(u^{0}(t),x^{0}\right)$ in the problem described (3.1)-(3.6) it is necessary that for any $x_i^{*}\in\hat{\partial}\varphi\left(x^{0}(t_i)\right)$ the following conditions are true:*

Discrete maximum principle for the control

$$\sum_{t=t_{i-1}}^{t_i-1}\Delta_{u_{i(t)}}H_i[t]\leq 0, \text{ for all } u_i(t)\in U_i,\ i=1,2,3,\ t\in T_i \tag{3.7}$$

Discrete maximum principle at the switching points t_k $k=1,2,..,N-1$

$$T_{k+1}-T_k=\frac{\partial M_k}{\partial t}, k=1,...,N-1 \tag{3.8}$$

At the end of the point t_N

$$T_N = \sum_{\upsilon=1}^{p+g} \lambda_\upsilon \frac{\partial F_\upsilon}{\partial t} \qquad (3.9)$$

where $T_k = \max_{u_k \in U_k} H_k(x_k, u_k, \psi_k, t)$ *and* $\psi(.)$ *is adjoin trajectory and satisfying (3.11) systems. If the set* $f_i(t, x^0, U)$ *is convex, then the necessary optimality condition is global over all* $u_i \in U_i$.

Remark. *In this theorem it is necessary (3.8) and (3.9) conditionsBecause we dont know when structure of the system will be change we we will try find optimality conditions at the switching points.We can image this problem as a rocket with two types of engines that work consecutively. With work of the second engine depends on the first one. Moreover, the rocket moves from one controlling area to a second one that changes all the structure (controls, functions, conditions, etc.*

Proof. *Let* $(u^0(t), x^0)$ *be optimal process.As we already mentioned we will implement the Dubovitskii-Milyutin formalism and definition of Frechet subdifferential.We start analize the cost criterien.Let take any* $x_i^* \in \hat{\partial}\varphi(x^0(t_i))$ *and any pairs* (u_k, x_k) *which satisfying (3.1)-(3.6) conditions.Then by using of the Frechet subdifferential definiation we can write*

$$S(x,u) - S(x^0, u^0) = \sum_{i=1}^{n} \left(\varphi_i(x_k, u_k, t) - \varphi_i(x_k^0, u_k^0, t) \right) \geq \sum_{i=1}^{n} \left(\left\langle x_k^*, x_k - x_k^0 \right\rangle + o\left(\left| x_k - x_k^0 \right| \right) \right)$$

From this releation we can get

$\sum_{i=1}^{n} \left(\left\langle x_k^*, x_k - x_k^0 \right\rangle + o\left(\left| x_k - x_k^0 \right| \right) \right) \leq 0$. Applying theorem of Girsanov [2, theorem 7.4] we get that the cone of directions of decrease of the cost criterion is

$K_1 = \left\{ x_k : \sum_{i=1}^{n} \left(\left\langle x_k^*, \Delta \dot{x}_k \right\rangle \leq 0 \right) \right\}$. For the moment, assume that $K_1 \neq 0$. Then dual cone of K_1 is given by $K_1^* = \left\{ -\lambda \sum_{i=1}^{n} \left(\left\langle x_k^*, \Delta \dot{x}_k^0 \right\rangle \right) \right\} =$

Let us investigate admissble variation for the **(3.5).**

If $G_k(x_k(t), t) < 0$ satisfaing for any $t \in [t_{k-1}.t_k]$ then set of admissble variation is the all space.It is meanimg that we have to investigate equality condition $G_k(x_k(t), t) = 0$. In this case set of admissble variation corrosponding prohobited variation for the following minimizing function

$\Phi(x,u) = \max_t G_k(x_k(t), t)$ (see [2,7]). Then

$$\Delta_x \Phi(x,u) = \max_t \left[G_k(x^0{}_k(t), t) + \varepsilon \Delta G_k(x^0{}_k(t), t) \dot{x}_k + o(\varepsilon, t) \right] =$$

$\varepsilon \frac{\partial G_k}{\partial x} \Delta x + o(\varepsilon, t)$. From here set of admissble variation is $\max_t \frac{\partial G_k}{\partial x_k} \Delta x_k < 0$. It is clear that set of admissble variation is conus and let we denote this conus by $K_2 = \left\{ x_k : \max_t \frac{\partial G_k}{\partial x_k} \Delta x_k \right\}$. It is known that linear functional on the continous functions has a integral form.By using the [2] we can put dual conus form of $K_2^* = -\alpha_k \int_{t_{k-1}}^{t_k} \frac{\partial G_k}{\partial x_k} \Delta x_k$.

It is interesting to find admissible variation for the (3.2), (3.3) togather.If we will tray to find admissible variation for the (3.2) and (3.3) by using (2.4) condition we can that admissible conus for this relation will be following form.

$\sum_{k=1}^{N-1}\beta_k\left[T_{k+1}-T_k-\frac{\partial M_k}{\partial t}\right]_{t=t_k}-\beta_N\left[T_N-\sum_{\upsilon=1}^{p+g}\lambda_\upsilon\frac{\partial F_\upsilon}{\partial t}\right]_{t=t_N}$ (full proof can be found in the appendix).Now if we can formulate Euler equation for problem. Then we can write

$$\sum_{k=1}^{N}\Delta_u H_k[t]+\sum_{k=1}^{N-1}\beta_k\left[T_{k+1}-T_k-\frac{\partial M_k}{\partial t}\right]_{t=t_k}-\beta_N\left[T_N-\sum_{\upsilon=1}^{p+g}\lambda_\upsilon\frac{\partial F_\upsilon}{\partial t}\right]_{t=t_{N=}}=0 \ . \qquad (3.10)$$

Here $H_k=(f_k,\psi_k)-\lambda_0 x_k^*;\ \psi_k(t)=\omega_k(t)-\int_{t_{k-1}}^{t_k}\frac{\partial G_k}{\partial x_k}dt$,

$\omega_k(t)$-solution of following conjucate differential equation with the inital condition.

$$\dot{\omega}_k(t)=\left(-\frac{\partial f_k}{\partial x_k}\right)^*\omega_k+\lambda_0 x_k^* \quad (t_{k-1}\le t\le t_k)$$

$$\dot{\omega}(t_k)=\frac{\partial M_k}{\partial x_k}\omega_{k+1}(t_k)\ ,\ k=1,...,N-1, \qquad (3.11)$$

$$\omega_N(t_N)=-\sum_{\upsilon=1}^{p+g}\lambda_\upsilon\frac{\partial F_\upsilon[x_N(t_N),t_N]}{\partial x}$$

If we consider (3.10) ,consequensely we can get necessary optimality conditions (3.7),(3.8) and (3.9)

Corollary 3.1. Let $(u^0(t),x^0)$ *be an optimal process to the (3.1)-(3.6) problem ,where* $\varphi_i:R^n\to R$ *is assumed to be differentiable at* $x_i^0(t_i)$*.Then one has the maximum principle (3.7)-(3.9) with* $\omega_k(t)$ satisfying (3.11) and $x_i^*\in\hat{\partial}\varphi(x^0(t_i))=\nabla\varphi(x_k^0(t_i))$

Example:

Consider the following two step time optimal control problem.

$$\dot{x}_1=x_2,\ \ \dot{x}_2=u \qquad 0\le t\le\tau; \qquad (3.12)$$

$$\dot{y}=y_2\ ,\ \dot{y}_2=-y_2,\ \tau\le t\le T \qquad (3.13)$$

$y_1(\tau)=x_1(\tau),\ y_2(\tau)=x_2(\tau),\ y_3(\tau)=-1,\ y_4(\tau)=0$ -switching condition (3.14)

Problem is that arrive the shortest time from point $(x_1(0),x_2(0))=(c;0)$ to the point $(y_1(T),y_2(T))=(0;0)$, where $c>0$, controls satisfaying $|u|\le 1,$ and $|v|\le 1$ conditions.Thus Hamilton-Pontryagin functions along the optimal controls are following form.

$$H_1=H(\varphi,x,u)=x_2\varphi_1+u\varphi_2,\ \ H_2=H(\psi,y,v)=y_2\psi_1-y_2\psi_2+y_3\psi_3-y_4\psi_4+v\psi_4$$

Here $\varphi=(\varphi_1,\varphi_2)$ and $\psi=(\psi_1,\psi_2,\psi_3,\psi_3)$ is solution of folowing differential equation

$$\dot{\varphi}_1=0,\ \dot{\varphi}_2=-\varphi_1\ ,\ 0\le t\le\tau \qquad (3.15)$$

$$\dot{\psi}=0,\ \dot{\psi}=-\psi_1+\psi_2,\ \dot{\psi}=0,\ \dot{\psi}=-\psi_3+\psi_4 \qquad (3.16)$$

with inital conditions

$$\varphi_1(\tau)=\psi_1(\tau),\varphi_2(\tau)=\psi_2(\tau),\psi_1(t)=a,\psi_2(t)=0,\psi_3(t)=b,\psi_4(t)=0, \qquad (3.17)$$

here a and b unknown numbers.

Let us justify following two conditions.

1.If $a\ne 0,$ then a) $a<0$ b) $b\ne 0$.

Let us prove these releations.

a) From optimality condition (1) we can get
$u(t) = sign\varphi_2(t) = signa\left[\tau - t + 1 - e^{\tau - T}\right] = signa$,because for the $t < \tau$ it is imply $\tau - t + 1 - e^{\tau - T} > 0$ .Then optimal control for above requarement problem is $u = \pm 1$ and corresponding solutions will be

$$x_1(t) = \frac{t^2}{2} + c \text{ and } x_2(t) = t \text{ ,for } u=1 \quad (3.18)$$

$$x_1(t) = -\frac{t^2}{2} + c \text{ and } x_2(t) = -t \text{ ,for } u=-1. \quad (3.19)$$

If we consider (3.18) in the (3.1),(3.2) and (3.3) we can find easly that for the $u = 1$,
$y_1(t) = x_1(\tau) + x_2(\tau)\left[1 - e^{\tau - t}\right] > 0$ for all $t \ge \tau$ .But it is contradiction of the inital condition $y(T) = 0$ It is means that u=-1 and a<0.

b) Assume contradiction : $b = 0$,then $\psi_3 \equiv 0$ and $\psi_4 \equiv 0$ for all the $t \in [\tau, T]$.
If we consider optimality condition (3.8) it is implys that
$[x_2\varphi_1 - x_2\varphi_2]_{t=\tau} = [y_2\psi_1 - y_2\psi_2]_{t=\tau}$,but from (3.4) and first relation of the (3.3) we can get $x_2(\tau) = 1$, which it is contradiction of the $x_2(\tau) = -\tau$

2. If $b \neq 0$ then a)b>0 and b) $a \neq 0$ we get
Let us prove these reieations.
a) From optimality condition1,for the control v ,
$v(t) = sign\psi_4(t) = signb\left(1 - e^{\tau - T}\right) = signb \,.\, \left(1 - e^{\tau - T}\right) > 0$ for any t<T,consecuently v=± 1
But if we get v=-1 then corresponding solution $y_3(t) = \tau - t - e^{\tau - T} \le 0$ for all the $t > \tau$ .But is it contrary for the $y(T) = 0$ .Then v=1 and b>0.
b) Assume a=0 Then $\psi_1(t) \equiv 0$ $\psi_2(t) \equiv 0$ for all the $t \in [\tau, T]$ and $\varphi_1 \equiv 0, \varphi_2 \equiv 0$ for all the $t \in [0, \tau]$.If we consider necessary optimality condition (3.8) and switching condition $y(\tau) = 0$,we can get $\psi_4(\tau) \equiv 0$ .It is means that $b\left(1 - e^{\tau - T}\right) = 0$,which contrary in the case of $\tau < T$ This can be prove as 1(b).We have to use necesary conditions (3.8) and switching conditions .
At least we get that our optimal control is : $u(t) = -1$ for the $0 \le t \le \tau$ and $v(t) = 1$ for the $\tau \le t \le T$.
Indeeed, if we take $y_1(t) = x_1(\tau) + x_2(\tau)\left[1 - e^{\tau - t}\right]$, $y(T) = 0$ consider (3.19) we can get
$\tau^2 + 2\tau\left[1 - e^{\tau - T}\right] - 2c = 0$
If we take $y_3(t) = \tau - t - e^{\tau - T}$ and $y(T) = 0$ we can get $\tau - T - e^{\tau - T} = 0$.
Solwing following two nonliner equation we can find switching point τ and minimal finishing time T for the control prossess .
$\tau^2 + 2\tau\left[1 - e^{\tau - T}\right] - 2c = 0$, $\tau - T - e^{\tau - T} = 0$.

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4.Appendix

If $\left(x_k(t), u_k(t)\right)$ admissible trajectory for the system (3.4) then

$x_{k+1}(t_k) - x_{k+1}^0(t_k) = M_k(x_k(t_k), t) - M_k(x_k^0(t_k), t)$ .From here

$\dot{x}_{k+1}(t_k) = \frac{\partial M_k(x_k(t_k),t)}{\partial x_k}\dot{x}_k(t) + \frac{\partial M_k(x_k(t_k),t)}{\partial t}$ m=1,2,3,...,N-1.If we multiply both sides of this equation by the ψ_k (t) and summing up from 1 up to N-1 ,we can get

$\sum_{k=1}^{N-1}\psi_k(t)\dot{x}_{k+1}(t_k) = \sum_{k=1}^{N-1}\psi_k(t)\left(\frac{\partial M_k(x_k(t_k),t)}{\partial x_k}\Delta x_k(t)\right)$ .Theis sum we can write

$$\sum_{k=1}^{N-1}\psi_{k-1}(t)\dot{x}_k(t_{k-1}) + \psi_{N-1}(t)\dot{x}_N(t_{N-1}) - \psi_1(t)\dot{x}_2(t_1) = \sum_{k=1}^{N-1}\psi_k(t)\left(\frac{\partial M_k(x_k(t_k),t)}{\partial x_k}\Delta x_k(t) + \frac{\partial M_k(x_k(t_k),t)}{\partial t}\right)$$

.If consider the equation of (3.1),then

$$\sum_{k=1}^{N-1}\psi_k(t)\dot{x}_k(t_{k-1}) + \psi_N(t)\dot{x}_N(t_{N-1}) - \psi_1(t)\dot{x}_1(t_0) = \sum_{k=1}^{N-1}\psi_k(t)\left(\frac{\partial M_k(x_k(t_k),t)}{\partial x_k}\Delta x_k(t) + \frac{\partial M_k(x_k(t_k),t)}{\partial t}\right)$$

h$\sum_{k=2}^{N-1}\left(\psi_{k-1}(t) - \psi_k(t)\frac{\partial M_k(x_k(t_k),t)}{\partial x_k}\right)\dot{x}_k(t_{k-1}) + \psi_N(t)\dot{x}_N(t_{N-1}) - \psi_1(t)\dot{x}_1(t_0) =$
$\sum_{k=1}^{N-1}\psi_k\frac{\partial M_k(x_k(t_k),t)}{\partial t}$

If we consider conjugate system we can get and $H_k(x_k,u_k,\psi_k,t) = \psi_k(t)f(x_k,u_k,t))$ then
$H_N(x_N,u_N,\psi_N,t) - H_1(x_1,u_1,\psi_1,t_0) = 0$

From here we can rewrite last relation follawing form

$\sum_{k=1}^{N-1}(H_{k+1}(x_{k+1},u_{k+1},\psi_{k+1},t_k) - H_k(x_k,u_k,\psi_k,t_{k-1})) = \sum_{k=1}^{N-1}\psi_k\frac{\partial M_k(x_k(t_k),t))}{\partial t}$. It is means that admissible control for switching equation satisfying this equation.