# A MIP Approach for some Practical Packing Problems: Balancing Constraints and Tetris-like Items (*)

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**Abstract.**

This paper considers packing problems with balancing conditions and items consisting of clusters of parallelepipeds (mutually orthogonal, i.e. *tetris*-like items). This issue is quite frequent in space engineering and a real-world application deals with the Automated Transfer Vehicle project (funded by the European Space Agency), at present under development.

A Mixed Integer Programming (MIP) approach is proposed. The three-dimensional single bin packing problem is considered. It consists of orthogonally placing, with possibility of rotation, the maximum number of parallelepipeds into a given parallelepiped. A MIP formulation of the problem is reported together with a MIP-based heuristic approach. Balancing conditions are furthermore examined, as well as the orthogonal placement (with rotation) of *tetris*-like items into a rectangular domain.
*Key words:* ***packing, balancing conditions,*** **tetris*-like items, mixed integer programming, MIP-based heuristic approach*}**
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## 1. Introduction

The literature concerning the optimization of multidimensional packing problems is widespread and advanced methods are available to look into efficient solutions of difficult instances (Coffman et al. 1997; Dyckhoff et al. 1997; Martello et al. 2000). Most of the work focuses on the orthogonal placement of rectangular items into rectangular domains (with no additional conditions). Non-standard packing issues, involving specific additional conditions, however, sometimes arise in practice. This occurs for instance in space engineering, when dealing with payload or cargo accommodation within satellites, space modules and space vehicles: an efficient exploitation of the available volume is necessary, in compliance with tight requirements on modularity, accessibility, operability, logistic, functional and operational conditions (Fasano 2003).

The work presented in this paper originates from the activity performed by Alenia Spazio S.p.A., within the context of the Automated Transfer Vehicle (ATV) project. The ATV is the European transportation system supporting the *International Space Station*. On the basis of the *Cargo Manifest* plan (*NASA*), defining the mass types and quantities to be transported to and from the *Space Station*, a detailed cargo accommodation analysis has to be performed, for each launch and for each carrier. From the cargo accommodation point of view, the ATV consists of an unpressurized module (i.e the external module) and a pressurized module (see Figure 1). Accommodation rules and constraints are

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given for the unpressurized, pressurized and overall cargo. The unpressurized cargo consists of fluids and the pressurized one of items. Complex geometrical and functional conditions have to be considered (Fasano et al. 2003). The ATV case presents several packing issues at different levels. Small items have to be accommodated into containers (bags); containers or 'large items' into racks and racks into locations inside the ATV Cargo Carrier, taking into account mass and volume capacity limitations (at container, rack and cargo carrier level), specific positioning rules, static and dynamic balancing conditions. Small items, drawers and bags are assumed to be parallelepipeds; 'large items' are assumed to be clusters of parallelepipeds; racks are convex domains subdivided into sectors by parallel planes.

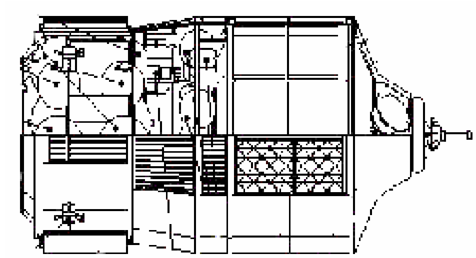

Figure 1 The Automated Transfer Vehicle

In this context, but not only, several two-dimensional or three-dimensional packing issues arise, consisting of accommodating items into both rectangular and non-rectangular domains, in the presence of additional conditions such as, for instance, those deriving from the static or dynamic balancing, as well as from specific accommodation requirements. Some items may have an assigned position or orientation and the domain may be partitioned into sectors, contain separation planes (with no prefixed position) or forbidden regions. In general, items are parallelepipeds of homogeneous density and orthogonal rotations are admitted. In some specific applications, however, items cannot be adequately represented by single parallelepipeds (for reasons of shape and dimension). Clusters of parallelepipeds have then to be considered giving rise to (two-dimensional or three-dimensional) *tetris*-like problems.

Specific non-standard packing problems are frequently tackled by dedicated approaches based on heuristics or meta-heuristics (Colaneri et al. 2003; Daughtrey et al. 1991). Mixed Integer Programming (MIP, see Nemhauser and Wolsey 1988) in some cases represents a valuable alternative (Fasano 1999; Mathur 1998; Padberg 1999). This approach seems quite suitable to solve some classes of non-standard packing problems with additional conditions (e.g. separation planes, fixed position or orientation of specific items, static balancing, non-rectangular domains) and *tetris*-like items (Fasano 2003).

This paper considers the three-dimensional single bin packing problem first. It consists of placing (orthogonally and with possibility of rotation) the maximum number of parallelepipeds (from a given set) into a parallelepiped. A MIP formulation of this problem is reported in this work and a heuristic approach to find satisfactory (sub-optimal) solutions for the MIP model is described. The paper points out the susceptibility of the adopted approach to straightforward extensions. Static balancing conditions are examined, as well as the orthogonal placement (with rotation) of *tetris*-like items into a rectangular domain. We show how to extend the MIP formulation to accommodate balancing constraints that are frequently encountered in practice. We also show how to deal with objects that are clusters of parallelepipeds, i.e. Tetris-like parallelepipeds. These extensions were motivated by the ATV project.

## 2. The basic problem

### 2.1 Problem definition

Given a set of $n$ parallelepipeds and a parallelepiped $D$ (domain), place the maximum number of items into $D$ with the following positioning rules:

- *each (picked) parallelepiped side has to be parallel to a side of the domain* (*orthogonality* conditions);
- *all (picked) parallelepipeds have to be contained within D (domain* conditions);
- *the (picked) parallelepipeds cannot overlap* (*non-intersection* conditions).

Different optimization criteria, such as volume or mass maximization, could be considered.

### 2.2 MIP model (basic formulation)

A possible MIP formulation of the basic problem is reported below.
For each parallelepiped $i$ denote by $L_{1i}, L_{2i}, L_{3i}$, with $L_{1i} \leq L_{2i} \leq L_{3i}$, its sides and by $w_{1i}, w_{2i}, w_{3i}$ the coordinates of its center with respect to a predefined orthonormal reference frame (with origin $O$ and axes $w_1, w_2, w_3$). The domain $D$ is a parallelepiped with sides $D_1, D_2, D_3$, parallel to the $w_1, w_2, w_3$ reference frame axes respectively. A vertex of $D$ is, moreover, supposed to be coincident with the reference frame origin $O$ and $D$ lies within the positive quadrant of the reference frame (see Figure 2).

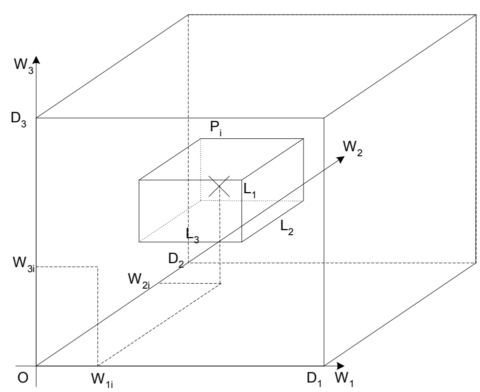


Figure 2 Orthogonal three-dimensional packing

By setting $\alpha \in \{1,2,3\}$, $\beta \in \{1,2,3\}$, $i \in \{1,\ldots,n\}$, the binary variables $\chi_i, \delta_{\alpha\beta i} \in \{0,1\}$ are introduced with the following meaning:
$\chi_i = 1$ if item $i$ is picked, $\chi_i = 0$ otherwise;
$\delta_{\alpha\beta i} = 1$ if $L_{\alpha i}$ is parallel to the $w_\beta$ axis, $\delta_{\alpha\beta i} = 0$ otherwise.
The *orthogonality* conditions are expressed by the following constraints:

$$\forall \alpha, \forall i \qquad \sum_{\beta=1}^{3} \delta_{\alpha\beta i} = \chi_i, \qquad (1)$$

$$\forall \beta, \forall i \qquad \sum_{\alpha=1}^{3} \delta_{\alpha\beta i} = \chi_i \, . \tag{2}$$

Equations 1-2 state that for each picked parallelepiped, each side must be parallel to one and only one reference frame axis. (For non-picked parallelepipeds, all $\delta$ variables are set to zero.)

The *domain* conditions are expressed by the following constraints:

$$\forall \beta, \forall i \quad 0 \le w_{\beta i} - \frac{1}{2}\sum_{\alpha=1}^{3} \delta_{\alpha\beta i} L_{\alpha i} \le w_{\beta i} + \frac{1}{2}\sum_{\alpha=1}^{3} \delta_{\alpha\beta i} L_{\alpha i} \le \chi_i D_\beta \, . \tag{3}$$

Inequalities (3) state that all vertexes of each picked parallelepiped must stay within the domain for each orthogonal orientation. (For non-picked parallelepipeds all *w* coordinates are set to zero).

The *non-intersection conditions* for each pair *(i,j)* of picked parallelepipeds are equivalent to the *logical* conditions below:

$$\forall i, \forall j, i < j$$

$$\left\{ \left| w_{1i} - w_{1j} \right| \ge \frac{1}{2}\sum_{\alpha=1}^{3} (\delta_{\alpha 1 i} L_{\alpha i} + \delta_{\alpha 1 j} L_{\alpha j}) \right\} \vee$$

$$\left\{ \left| w_{2i} - w_{2j} \right| \ge \frac{1}{2}\sum_{\alpha=1}^{3} (\delta_{\alpha 2 i} L_{\alpha i} + \delta_{\alpha 2 j} L_{\alpha j}) \right\} \vee$$

$$\left\{ \left| w_{3i} - w_{3j} \right| \ge \frac{1}{2}\sum_{\alpha=1}^{3} (\delta_{\alpha 3 i} L_{\alpha i} + \delta_{\alpha 3 j} L_{\alpha j}) \right\} \tag{4}$$

Conditions (4) state that for each pair of picked parallelepipeds, for each orthogonal orientation, their side projections must not overlap along at least one axis of the reference frame. The logical conditions 4 can be expressed in terms of MIP constraints. A straightforward formulation is attained by introducing, for each $\beta$ and for each *(i,j),* with $i<j$, the set of binary variables $\sigma^{+}_{\beta ij}, \sigma^{-}_{\beta ij} \in \{0,1\}$ and the following constraints:

$$\forall i, \forall j, i < j \qquad w_{\beta i} - w_{\beta j} \ge \frac{1}{2}\sum_{\alpha=1}^{3} \left( \delta_{\alpha\beta i} L_{\alpha i} + \delta_{\alpha\beta j} L_{\alpha j} \right) - \left(1 - \sigma^{+}_{\beta ij}\right) D_\beta \, , \tag{5}$$

$$\forall i, \forall j, i < j \qquad w_{\beta j} - w_{\beta i} \ge \frac{1}{2}\sum_{\alpha=1}^{3} \left( \delta_{\alpha\beta i} L_{\alpha i} + \delta_{\alpha\beta j} L_{\alpha j} \right) - \left(1 - \sigma^{-}_{\beta ij}\right) D_\beta \, , \tag{6}$$

$$\forall i, \forall j, i < j \qquad \sum_{\beta=1}^{3} \left( \sigma^{+}_{\beta ij} + \sigma^{-}_{\beta ij} \right) \ge \chi_i + \chi_i - 1 \, . \tag{7}$$

$\sigma^{+}_{\beta ij} = 1$ implies that the side's projections of *i* and *j* don't overlap along the $w_\beta$ axis and *j* precedes *i*. $\sigma^{+}_{\beta ij} = 0$ makes the corresponding *non-intersection* constraint redundant. Analogous considerations hold for $\sigma^{-}_{\beta ij}$. Equation (7) guarantees that if both *i* and *j* are picked at least one of the *non- intersection* constraints holds. It can be easily proved that for each *(i,j)* $D_\beta$ is the minimum value that guarantees the compatibility between the disjunctive constrains 5-6, for any position of *i* and *j* within *D*.

The following *target function* is introduced:

$$\max \sum_{i=1}^{n} \chi_i \,. \qquad (8)$$

(When the total volume or mass has to be maximized, the target function has the form: $\max \sum_{i=1}^{n} K_i \chi_i$ , where $K_i$ are constants.)

A 3-dimensional model relative to *n* items contains:

*3n(n-1)* $\sigma$ variables (0-1)
*9n* $\delta$ variables (0-1)
*n* $\chi$ variables (0-1)
*6n orthogonality* constraints
*3n domain* constraints
*3n(n-1)+n(n-1)/2 non-intersection* constraints.

## 2.2 Auxiliary constraints

The mathematical model reported in section 2.1 may be refined by considering some necessary conditions that are implicit in the basic formulation. It is well known, in fact, that a MIP model is generally made easier to solve by expanding the number of (valid) constraints. Some examples of auxiliary constraints are reported in this section.

***Pairs of items***

The pair of items *(i,j), i < j* is considered together with some possible implications.

The following one is quite evident (notice that for each item i, $L_{1i} \le L_{2i} \le L_{3i}$):

*if* $L_{1i}+L_{1j} \ge D_\beta$, *then items i and j cannot be aligned along the* $w_\beta$ *axis.*

This implication is expressed by:

$$\forall\beta, \forall i, \forall j, i < j \;\, st \;\, L_{1i} + L_{1j} \ge D_\beta \qquad \sigma^{+}_{\beta ij} = \sigma^{-}_{\beta ij} = 0 \,. \qquad (9)$$

A set of implications correlating *alignment* and *orientations* conditions could be considered. (They have been introduced by S. Gliozzi, IBM). An example of this kind of implication follows:

*if* $L_{1i}+L_{2j} \ge D_\beta$, *then items i and j cannot be aligned along the* $w_\beta$ *axis, with* $L_{2j}$ *parallel to the* $w_\beta$ *axis.*

This is expressed by:

$$\forall\beta, \forall i, \forall j, i < j \;\, st \;\, L_{1i} + L_{2j} \ge D_\beta \qquad \delta_{2\beta j} \le 1 - \sigma^{+}_{\beta ij} - \sigma^{-}_{\beta ij} \,. \qquad (10)$$

***Triplets of items***

The triplet of items *(i,j,k), i < j < k* is considered together with some possible implications. A first one is quite obvious:

*if i precedes j and j precedes k along the* $w_\beta$ *axis, then i precedes k along the same axis.* This implication is expressed by:

$$\forall\beta, \forall i, \forall j, \forall k, i < j < k \qquad \sigma^{-}_{\beta ik} \ge \sigma^{-}_{\beta ij} + \sigma^{-}_{\beta jk} - 1 \qquad (11)$$

and all possible permutations of *(i,j,k)* give rise to analogous auxiliary constraints (with the appropriate selection for the variables $\sigma^{+}, \sigma^{-}$).

A further implication for triplets holds:

*if* $L_{1i}+L_{1j}+L_{1k} \geq D_\beta$, *then the whole triplet cannot be aligned along the* $w_\beta$ *axis.* It is expressed by the following constraints:

$$\forall \beta, \forall i, \forall j, \forall k, i < j < k \,/\, L_{1i} + L_{1j} + L_{1k} \geq D_\beta$$

$$\sigma^+_{\beta ij} + \sigma^-_{\beta ij} + \sigma^+_{\beta jk} + \sigma^-_{\beta jk} + \sigma^+_{\beta ik} + \sigma^-_{\beta ik} \leq 2 \qquad (12)$$

The proof is quite immediate: notice that not more than 2 items may be aligned and that for each pair of items at least one of the corresponding variables $\sigma^+, \sigma^-$ must be null.

The implications correlating *alignment* and *orientations* conditions are not limited to pairs of items (they could involve triplets and so on). Analogous considerations hold for the implications relative to the triplets and further implications could be investigated to obtain new auxiliary constraints. (The scale of the resulting MIP model could however be very large and a *Branch and Cut* approach could be quite suitable, see Padberg 1999).

## 3. MIP-based heuristic approach

The MIP model described in section 2 is hard to solve using standard techniques. Difficulties are determined in particular by the presence of the *non-intersection* conditions, such that the search for optimal solutions by means of standard MIP algorithms is often a very difficult task. Since sub-optimal solutions are sufficient in most real-world cases, some MIP-based heuristics have been implemented to look into quick satisfactory (sub-optimal) solutions (Fasano 1999, 2003).

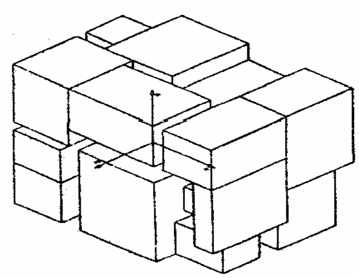

Figure 3 A case study (20 objects, occupied volume > 80%)

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## 4. Extensions

When dealing with standard packing problems, the modeling (*non-algorithmic*) approach proposed in this paper is generally less efficient than other methods, present in the literature. It seems, however, quite suitable to tackle some practical non-standard instances with additional conditions (e.g. fixed positions or orientations for some items, separation planes, non-rectangular domains, forbidden regions within the domain, see Fasano 1999, 2003). The cases of balancing conditions and *tetris*-like items are examined in this section. The MIP-based heuristic approach described in section 3 is directly applicable to the extensions.

## 4.1 Balancing conditions

In some practical cases, balancing conditions have to be considered. An extension of the basic problem (section 2) to include balancing conditions is reported here below.
Each item $i$ is assumed to have mass $M_i$ and its center of gravity coincident with its geometrical center (this assumption could be dropped quite easily). An *overall* center of gravity is then associated to each subset of picked parallelepipeds. Consider the following balancing rule:
the overall center of gravity must stay within a given convex domain C (contained in D). Denoting by $\boldsymbol{V_1}(V_{11},V_{21},V_{31}),\ldots,\boldsymbol{V_r}\,(V_{1r},V_{2r},V_{3r})$ the $r$ vertices of $C$, a point $\boldsymbol{P}(p_1,p_2,p_3)$ is inside $C$ if and only if the following (*convexity*) conditions hold (see e.g. Williams 1993):

$$\forall\beta \qquad p_\beta = \sum_{\gamma=1}^{r} V_{\beta\gamma}\psi_\gamma\ , \tag{14}$$

$$\sum_{\gamma=1}^{r} \psi_\gamma = 1\ , \tag{15}$$

where $\forall\gamma,\ \psi_\gamma \geq 0$.
The conditions below are introduced together with (15) to state that the *overall* center of gravity stay within the convex domain $C$:

$$\forall\beta \qquad \sum_{i=1}^{n} \frac{M_i}{m} w_{\beta i} = \sum_{r=1}^{r} V_{\beta r}\psi_r\ , \tag{16}$$

where $m = \sum_{i=1}^{n} M_i \chi_i$ .

It is supposed $m > 0$ and, by conditions 3, for each non-picked item $i$, all $w_{\beta i}$ variables are null.
The (15) and (16) are equivalent to the following linear conditions:

$$\sum_{r=1}^{r} \psi_\gamma{}^* = m\ , \tag{17}$$

$$\forall\beta \qquad \sum_{i=1}^{n} M_i w_{\beta i} = \sum_{\gamma=1}^{r} V_{\beta\gamma}\psi_\gamma{}^*. \tag{18}$$

where $\forall\gamma,\ \psi_\gamma{}^* = m\psi_\gamma$.

Notice that the nonlinear equations $\forall\gamma,\ \psi_\gamma{}^* = m\psi_\gamma$ don't need to be included in the model.
The balancing conditions are simplified when $C$ is rectangular. In this case they have the form: $\forall\beta\ \ C_{L\beta}m \leq \sum_{i=1}^{n} M_i w_{\beta i} \leq C_{U\beta}m$, where for each $\beta$, $[C_{L\beta}, C_{U\beta}]$ are the admitted intervals for the *overall* center of gravity.

## 4.2 *Tetris*-like items (cluster of parallelepipeds)

With the approach proposed in this paper, fixing items position and orientation is immediate. This characteristic can be profitably exploited to cope with items consisting of clusters of parallelepipeds (mutually orthogonal) i.e. *tetris*-like items.

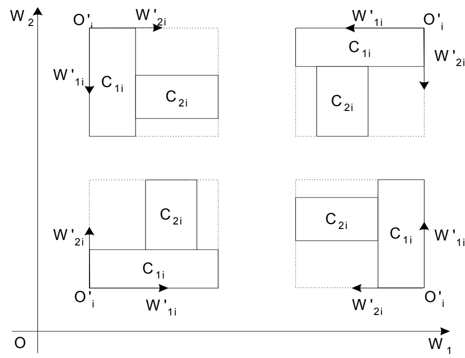

Figure 4 *Tetris*-like item (bidimensional representation)

Given the *tetris*-like item *i*, introduce a *local* (orthogonal) reference frame $(w'_1,w'_2,w'_3)_\mathrm{i}$ with origin $\boldsymbol{O'_i}$ (see Figure 4). The set of components (parallelepipeds) $C_i$ is associated to each item *i.*
Prefixed orientations are considered first and an extension to the case of orthogonal rotations is reported next. The balancing conditions of section 4.1 are directly applicable to the case of tetris-like items.

***Prefixed orientation***

It is supposed that each $w'_\beta$ axis (of the *local* reference frame) is parallel to the corresponding $w_\beta$ axis (of the *global* reference frame).
It is set:

$$\forall\beta,\forall h\in C_i,\forall i \qquad w_{\beta hi}=o'_{\beta i}+W'_{\beta hi}, \tag{19}$$

where, for each item *i,* $w_{\beta hi}$ and $o'_{\beta i}$ are the coordinates of the center of component *h* and the coordinates of $o'_i$ with respect to the *overall* reference frame $(w_1,w_2,w_3)$, $W'_{\beta hi}$ (constants) are the coordinates of the center of component *h* with respect to the *local* reference frame $(w'_1,w'_2,w'_3)_i$.
The extension of the *domain* constraints of section 2.2 is quite immediate and it is not reported: inequalities (3) can be applied to each single component or to the smallest parallelepipeds enveloping each *tetris*-like item.
The *non-intersection* constraints (5), (6), (7) are generalized as follows, for components *h, l* of items *i* and *j* respectively:

$$\forall\beta,\forall h\in C_i,\forall l\in C_j,\forall i,\forall j,i<j$$
$$w_{\beta hi}-w_{\beta lj}\geq\frac{1}{2}\left(L_{\beta hi}+L_{\beta lj}\right)-\left(1-\sigma^{+}_{\beta hlij}\right)D_\beta, \tag{20}$$

$$\forall\beta,\forall h\in C_i,\forall l\in C_j,\forall i,\forall j,i<j$$
$$w_{\beta lj}-w_{\beta hi}\geq\frac{1}{2}\left(L_{\beta hi}+L_{\beta lj}\right)-\left(1-\sigma^{-}_{\beta hlij}\right)D_\beta, \tag{21}$$

$$\forall h\in C_i,\forall l\in C_j,\forall i,\forall j,i<j$$
$$\sum_{\beta=1}^{3}\sigma^{+}_{\beta hkij}+\sigma^{-}_{\beta hkij}\geq\chi_i+\chi_j-1, \tag{22}$$

where $L_{\beta hi}$, $L_{\beta lj}$ are the sides of components *h* and *l*, respectively, parallel to the $w_\beta$ axis (of the *overall* reference frame).

***Orthogonal rotations***

When orthogonal rotations are admitted (19), (20), (21) can be further generalized. For this purpose to each item $i$, for each possible orientation $\omega \in \{1, \ldots s\}$, the binary variable $\theta_{\omega i} \in \{0,1\}$ is introduced, with the following meaning:
$\theta_{\omega i} = 1$ if item $i$ has the orientation ω, $\theta_{\omega i} = 0$ otherwise.
(In the bi-dimensional case there are 4 possible orientations, 24 in the three-dimensional one).
The following equation is introduced:

$$\forall i \qquad \sum_{\omega=1}^{s} \vartheta_{\omega i} = 1 \tag{23}$$

It states that each item $i$ has one (and only one) orientation.
Equation (19) is substituted by the following:

$$\forall \beta, \forall h \in C_i, \forall i \qquad w_{\beta hi} = o'_{\beta i} + \sum_{\omega=1}^{s} \vartheta_{\omega i} W'_{\omega \beta hi}, \tag{24}$$

where $W'_{\omega\beta hi}$ is the distance between the coordinates of the center of component $h$ and the origin of the *local* reference frame, along the $w_\beta$ axis (of the *overall* reference frame), corresponding to the orientation $\omega$. Inequalities (20) and (21) are substituted by the following:

$$\forall \beta, \forall h \in C_i, \forall l \in C_j, \forall i, \forall j, i < j$$
$$w_{\beta hi} - w_{\beta lj} \geq \frac{1}{2} \sum_{\omega}^{s} \left( \vartheta_{\omega i} L_{\omega \beta hi} + \vartheta_{\omega j} L_{\omega \beta lj} \right) - \left(1 - \sigma^{+}_{\beta hlij}\right) D_\beta, \tag{25}$$

$$\forall \beta, \forall h \in C_i, \forall l \in C_j, \forall i, \forall j, i < j$$
$$w_{\beta lj} - w_{\beta hi} \geq \frac{1}{2} \sum_{\omega}^{s} \left( \vartheta_{\omega i} L_{\omega \beta hi} + \vartheta_{\omega j} L_{\omega \beta lj} \right) - \left(1 - \sigma^{-}_{\beta hlij}\right) D_\beta, \tag{26}$$

where $L_{\omega\beta hi}$, $L_{\omega\beta lj}$ are the sides of components $h$ and $l$ respectively, parallel to the $w_\beta$ axis, corresponding to the orientation $\omega$ (for items $i$ and $j$).
4.3 Applications
The presence of balancing conditions generally makes the solution procedure lengthier. On the basis of the experimental analysis performed up to now, the computational time can increase by up to 25-30%. The presence of *tetris*-like items also makes the problem more difficult to solve. A rough evaluation of the effect due to the presence of *tetris*-like item can be obtained by considering the total number of single parallelepipeds and *tetris*-like items components. Balancing conditions have been considered for the ATV project to accommodate bags and large items into racks (Fasano et al. 2003). *Tetris*-like items have to be positioned on the rack front (rectangular) surface, so that bi-dimensional *tetris*-like issues have to be considered. This application is at present under development.

## 5. Conclusions

This paper originates from the activity performed to tackle the cargo accommodation problem for the European Automated Transfer Vehicle (ATV). In this context, difficult non-standard packing issues, involving additional conditions, have to be considered. In particular, in some cases, static balancing conditions are given and items, for shape and dimensions reasons, cannot be adequately represented by single parallelepipeds.

This paper focuses on packing problems characterized by the presence of balancing conditions and *tetris*-like items.

A Mixed Integer Programming approach to solve the basic problem (single bin packing), consisting of placing (orthogonally, with possibility of rotation) the maximum number of parallelepipeds into a given parallelepiped, is presented.
Even if advanced algorithms available in literature are generally more efficient to solve the basic problem, the MIP approach proposed seems quite suitable to tackle some non-standard packing issues, with additional conditions, that occur quite frequently in practice.
A MIP-based heuristic approach has been implemented and is described in this paper. A clear advantage of the MIP-based approach proposed in this paper is the capability to deal with several additional conditions, frequently encountered in the applications.
It has been adopted to solve successfully some practical packing issues originated from the ATV project, involving static balancing conditions and *tetris*-like items.
Quite difficult balancing conditions occurs indeed when accommodating large items and bags into racks, since the rack center of gravity must stay within a given convex (non-rectagular) domain. Bi-dimensional t*etris*-like issues moreover arise when dealing with the external configuration of the racks, since *tetris*-like items have often to be accommodated on the front surface of the racks. This application is at present under development.

Future research could be done to improve the heuristic procedure, to look into advanced branching strategies (to solve the MIP model), as well as to investigate a dedicated *Branch and Cut* approach.

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