Operations Risk Management by Planning Optimally the Qualified Workforce Capacity

Emmanuel Fragnière* Jacek Gondzio † Xi Yang ‡

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Abstract

Operational risks are defined as risks of human origin. Unlike financial risks that can be handled in a financial manner (e.g. insurances, savings, derivatives), the treatment of operational risks calls for a "managerial approach". Consequently, we propose a new way of dealing with operational risk, which relies on the well known aggregate planning model. To illustrate this idea, we have adapted this model to the case of a back office of a bank specializing in the trading of derivative products. Our contribution corresponds to several improvements applied to stochastic programming techniques. First, the model is transformed into a multistage stochastic program in order to take into account the randomness associated with the volume of transaction demand and with the capacity of work provided by qualified and non-qualified employees. Second, as advocated by Basel II, we calculate the probability distribution based on Bayesian Network to circumvent the difficulty to obtain data in operations. Third, we go a step further by relaxing the traditional assumption in stochastic programming that imposes a strict independence between the decision variables and the random elements. Comparative results show that these improved stochastic programming models tend generally to allocate more human expertise in order to hedge operational risks. Finally, we employ the dual solutions of the stochastic programs to detect periods and nodes that are at risk in terms of expertise availability.

1 Introduction

Back offices of banks typically deal with the transactions of financial contracts and with all the related paperwork and database management. For example, a trader (front office) "writes" an over the counter option with a counterparty. The back office will prepare the contracts, conduct all the exchange of information in due time and comply at the same time with the very stringent financial regulations. In the more and more frequent cases where the back office deals with derivative products, workers have to understand complicated pricing systems because it is part of their duties to conduct some price settlements, "reconciliations" and verifications. Several surveys (see [1, 15]) also indicates that operations of back offices are becoming far more complicated than it used to be. The reasons for this evolution are multiple: IT harmonization due to banks consolidation, new stringent norms and regulations affecting the operations (e.g. IAS-IFRS, Sarbannes-Oxley, Basel II, new taxation system like Qualified Intermediary), significant increase of service productivity

*Haute Ecole de Gestion of Geneva and University of Bath, 7 route de Drize, 1227 Carouge, Switzerland, Emmanuel.Fragniere@hesge.ch
†University of Edinburgh, JCMB, Kings Building, West Mains Road, UK, J.Gondzio@ed.ac.uk
‡University of Edinburgh, JCMB, Kings Building, West Mains Road, UK, X.Yang@ed.ac.uk

over the last years (it is said that productivity has almost doubled over the last 15 years), boom of new sophisticated financial products (in particular structured products). So more and more complex risks are attached to the activities of back offices and most of them require above all knowledge, experience and expertise to be addressed correctly and in an efficient manner. For example, these risks can be mistakes, inefficiencies of processes.

Unfortunately, the risk management of back offices in the banking sector has not benefited from the modeling advances in financial risks (market, credit and liquidity risks) promulgated by the Basel Committee (www.bis.org). Basel II classifies risks of a bank in four categories: strategic, financial, non-financial/operational, reputation and compliance risks. Regarding the categories of non-financial/operational risks four subcategories are used: fraud, political, IT, operations (transactions mistakes, inefficiencies of processes) risks. In operations, risks have two facets. They can be internal and external. Internal risk is typically linked to operations (so controllable). External risks are the consequence of external causes (for instance a change of some US GAAP accounting rules for derivatives which will affect the back office procedures as well as the training of the staff).

The logic underlying risk management in a Basel II context is always the same for each category of banking risks. Basel II proposes two methods: the standardized and the advanced ones. When the bank invests in qualified staff, software, and develops an advanced model, the bank is able to “economize” some capital assuming that it contributes actively to risk management. When the bank is not up to develop its own advanced model then the capital that is to be set aside is calculated from a standardized approach. So far the modeling of operational risk has not been very developed. This task is rather subjective regarding its qualitative nature, related to managerial issues.

A notion arising from service science makes the distinction between explicit (information) and implicit (or tacit) knowledge. Typically in a back office, tasks are very standardized and documented in working procedures. This corresponds to the explicit knowledge. Tacit knowledge on the other hand as defined by [23] corresponds to information that is difficult to express, formalize or share, in contrast to explicit knowledge, which is conscious and can be put into words. When something unexpected happens that might affect the service production, which is not included in the procedures, solely the expertise of qualified workers will permit to correct the problem to go back to normal. This statement is confirmed for instance in [24] that qualitative skills like information search style, level of education and training on risk influence the capability of risk manager to identify risks. In the case of operations the risk can “materialize” under different states (see [7]) according to the TEID model: Threat, Event, Ignorance, Damage. Typically, the qualified worker knows how to act on these different states of risks through, prevention, identification, and protective approaches. Consequently we think that the aggregate planning model adapted to services (basically containing no inventory of goods), since the production is intangible (see [27] for the characteristics of a service), could be the model of choice to assess the level of expertise necessary to deal with operations risks in the back offices of banks. This is what we intend to demonstrate through different extensions of the stochastic programming version of the aggregate planning model. The Aggregate Planning Model (APM) was developed in the middle of last century and has been successfully applied in production planning problems, see [18, 17, 12], and manpower planning problems, see [22, 10, 2]. The most important feature of the aggregate planning model is the aggregation, either of products or manpower or both, which are presented as inventory constraints.

In a real context of enterprise risk management, many decisions must be made that will have an impact in the future as well as on many interrelated activities. Hence, when future events are to be considered in business activity planning, it may be pertinent to take into account uncertain parameters within the planning model. The corresponding branch of Mathematical Programming is known as Stochastic Programming [4, 21]. Initiated in the late fifties by Dantzig and Madansky, Stochastic Programming is a growing field of mathematical programming. In particular, a multistage stochastic program with recourse is a multi-period mathematical program where parameters are assumed to be uncertain along the time path. The term recourse means that the decision variables adapt to the different outcomes of random parameters at each time period. Stochastic programming model allows one to handle simultaneously several scenarios. It provides an adaptive policy that is much closer in spirit to the way decision-makers have to deal with uncertain futures in real life.

To model precisely the different risks that can be encountered in the operations of a back office, we transform our aggregate planning model into a stochastic programming model to include different types of random variables schemes as well as innovative links with decision variables. From a classical multistage stochastic linear programming version of the aggregate planning model, we will develop several types of extensions that will enable us to capture the true nature of operations risks.

Our basic multistage linear programming problem takes into account uncertainties on the demand parameters. In this context it corresponds to a situation where the risk is external (like a market risk) and we consider that the quality of work has no influence whatsoever on it (uncontrollable). The solution simply adapts to the evolution of the different scenarios described by the event tree.

A first improvement explored to better manage operational risk is to assume that capacities as well as demand are uncertain as will be shown in Section 3. This time, we assume again that the available capacity of production (manpower in our banking example) is an external risk, which means that it is not controllable through managerial activities.

A second improvement, as advocated by Basel II, is to calculate probability distribution based on Bayesian network. This comes from the fact that data related to operations risks are rarely available. We thus apply this scheme to our basic model where the demand parameters are assumed to be random. This scheme has already been applied before by Morton and Popova [26] on a similar model to evaluate capacity strategies of a manufacturing problem.

A third improvement to deal with operational risks, and to our best knowledge implemented for the first time in a business case, is to establish a relation between the random variables and decision variables. Demand is set to be dependent on decisions. The model becomes then far more complicated (nonlinear). In the standard form of stochastic programming, the decision variables and uncertainties are independent, see [4, 9, 29, 21]. The other category of stochastic programs is the model with so called endogenous uncertainty, in which decision variables can influence the uncertainties, firstly addressed by Pflug [28]. While most work in non-standard problems is about the decision dependent probability distributions, in this paper we are dealing with other uncertain elements in the model (e.g. demand) depending on decisions. The reader interested in further discussions of non-standard problems is referred to [16, 8, 20] and the references therein. In terms of risk management, another type of philosophy is applied. This one corresponds to COSO II (or ERM COSO, www.coso.org) which is certainly the most influential text in operational risk. Indeed the COSO approach assumes that the crude risk is reduced in function of the level of quality of the ICS (Internal control system, see www.theIIA.org, the Institute of Internal Auditing, for more details). As an example, we can imagine that a bank main activity is in the development and trading of structured products (the crude risk is thus huge in terms of financial and operational risks). However, if highly qualified people are dealing with these activities (front and back offices), the ICS presents a high level of quality. Consequently, the residual risk is minimized. The managerial treatment of risk becomes thus crucial in regard of the COSO philosophy. Another element that has to be taken into account in the management of back offices is the notion of explicit and tacit knowledge (expertise) as explained above. In standardized operations, procedures enable the employees to solve most known problems. On the other hand when a problem occurs which is not part of explicit knowledge (codified knowledge), only the tacit knowledge (i.e. the expertise) can help to solve the problem. In our model we will thus assume that if more capacity of qualified workers is available this should lead to better service because mistakes in the operations are reduced, moreover this builds up the reputation and as a consequence the demand for such service increases. For this reason, the assumption of variables and parameters independencies needs to be relaxed in our stochastic programming problems. However this improvement leads to complicated models that are often non-convex.

Finally, for each developed model we use dual solutions to identify in the plan which scenarios are under stress regarding the availability of qualified workers. This analysis complements well the one provided by the primal solutions. Indeed it represents a way to price the risks of lacking some qualified resources. We focus on the main inventory constraints of the aggregate planning model, which are equality constraints with variables on both sides of the equations. In consequence, the dual solutions do not correspond to marginal values of resources. Nevertheless, dual variables (Lagrange multipliers associated with the constraints) give a relevant indication of the constraints that would need to be relaxed. For instance, it can show when liquidity should be available in supplement to be able to hire additional qualified workforce capacity.

This paper is organized as follows. In Section 2 we discuss the construction of the aggregate planning model adapted to our case study. In Section 3, we transform this model into a multistage stochastic programming problem with first random demand functions and second random capacities. In Section 4, an approach is applied to revise probability distributions with Bayesian networks. In Section 5, we relax the usual assumption of stochastic programming that establishes an independence between decision variables and random variables. The results of the model's application are discussed in Section 6. In Section 7, we show how to employ dual solutions to identify situations that are at risk in the workforce plan, followed with conclusions and further research directions in Section 8.

2 Presentation of Aggregate Planning Model

Suppose we know with certainty all the parameters that are essential to make our planning decisions, as a consequence we have a deterministic model with its following simple optimization form:

$$\min c^T x \tag{1a}$$

s.t.

$$Ax = b, \tag{1b}$$

$$g_i^T x_i = d_i, \quad i = 1, \cdots N \tag{1c}$$

$$x \geq 0. \tag{1d}$$

Equation (1a) is the cost objective function. Equation (1b) corresponds to the dynamic constraints and Equation (1c) represents the local constraints. Each equation is explained in more details in the following part.

In our specific context, the main task is to complete the transactions that are required. There are three categories of professionals employed: qualified people, non-qualified people and temporary employees. Our aggregate planning model must indicate how many of each group of people should be employed to satisfy the demand for transactions.

The objective is to minimize the costs and losses while maximizing the profit.

The Cost function includes three elements, i.e., hiring cost, firing cost and salaries paid to employees. Qualified people have higher firing cost and salaries than non-qualified people while hiring cost are assumed to be the same for both groups. We only pay salaries to temporary people without hiring or firing costs.

Profit is related with the volume of transactions successfully completed and hence defined as multiplication of this volume. As we impose satisfaction of all demands on time and there is no inventory of transactions, this volume is actually equivalent to the demand. Finally, profit is proportional to demand. It is worth mentioning that when demand does not change in the model, i.e., it is a parameter not a variable, profit is constant and does not need to be optimized in the objective. We have the following objective function corresponding to Equation (1a):

$$\begin{aligned} \min \quad & \sum_t (hQ^t + hNQ^t) \times Hirecost + \\ & + fQ^t \times FirecostQ + fNQ^t \times FirecostNQ + \\ & + wageQ \times eQ^t + wageNQ \times eNQ^t + wageX \times Extra^t \end{aligned} \tag{2}$$

where $t = 1, \cdots, T$ is the time stage (a stage corresponds to a year), $Hirecost-$, $Firecost-$, $wage-$ stand for hiring cost, firing cost and salaries per person, respectively, and $h-$, $f-$, $e-$ are the numbers of people hired, fired and kept. $-Q$, $-NQ$ and $-X$ stand for the quantities of $Qualified$ people, $Non-Qualified$ people and temporary employees. $Extra$ is the number of temporary person-day, e.g. $Extra = 1$ means a temporary employee working one day. In this model, we assume that both qualified and non-qualified people are kept at work at least a full year. Hence, $wageQ$ and $wageNQ$ are both yearly salaries. Temporary people are hired daily, which means $wageX$ is the payment to one temporary employee to work one day.

While we try to achieve the optimal value, there are two categories of constraints to be satisfied. The first group of constraints corresponds to the inventory of employees. Except for the first stage, we can hire and fire at each stage. Thus the number of employees we hold in a given period represents the number in previous stage subtracted by the number fired and increased by the number of newly hired. This is presented as follows:

$$\begin{aligned} eQ^{t-1} + hQ^t - fQ^t &= eQ^t, \quad \forall t \\ eNQ^{t-1} + hNQ^t - fNQ^t &= eNQ^t, \quad \forall t \end{aligned} \tag{3}$$

which corresponds to Equation (1b).

The second one, i.e., Equation (1c), is the capacity constraint, which requires each demand to be completed on time at that stage. The capacities are calculated in the following way:

$$eQ^t \times \alpha Q \times 260 + eNQ^t \times \alpha NQ \times 260 + Extra^t \times \alpha X = Capacity^t, \quad \forall t \tag{4}$$

where αQ, αNQ and αX are work capacities for qualified, non-qualified and temporary employees respectively. These are numbers of transactions one person in the corresponding group can complete per day. In addition, we impose the following restrictions:

$$\alpha Q > \alpha X > \alpha NQ. \tag{5}$$

We suppose that there are 260 working days a year. The capacity represents the sum of transactions processed by qualified people, non-qualified people and temporary employees in a year. The capacity must be larger than or equal to the demand in a given year $Demand^t$.

To sum up the full mathematical programming model can be written in the following way:

$$\begin{aligned} \min \quad & \sum_t (hQ^t + hNQ^t) \times Hirecost + \\ & + fQ^t \times FirecostQ + fNQ^t \times FirecostNQ + \\ & + wageQ \times eQ^t + wageNQ \times eNQ^t + wageX \times Extra^t \end{aligned} \tag{6a}$$

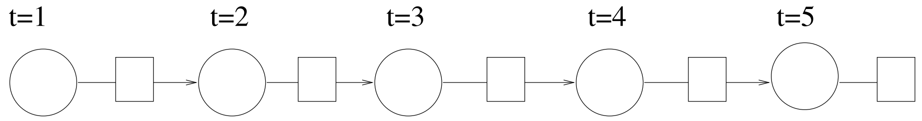

Figure 1: A deterministic multistage model representation: circles stand for chance nodes and rectangles stand for decisions.

subject to

$$\begin{aligned} eQ^{t-1} + hQ^t - fQ^t &= eQ^t, \quad \forall t = 2, \cdots, T \\ eNQ^{t-1} + hNQ^t - fNQ^t &= eNQ^t, \quad \forall t = 2, \cdots, T \end{aligned} \tag{6b}$$

$$eQ^t \times \alpha Q \times 260 + eNQ^t \times \alpha NQ \times 260 + Extra^t \times \alpha X = Capacity^t, \quad \forall t = 1, \cdots, T \tag{6c}$$

$$Capacity^t \geq Demand^t, \quad \forall t = 1, \cdots, T \tag{6d}$$

$$hQ^t, hNQ^t, fQ^t, fNQ, eQ^t, eNQ^t, Extra^t \geq 0. \tag{6e}$$

Figure 1 reflects a 5-period instance (T=5) [13]. Decision variables are the number of people to be employed. Consequently they are defined as integers, which makes that the model is an integer programming problem.

However, in reality, it is not always true that we know everything before making decisions. We will develop the stochastic aggregate planning model in the following section.

3 Multistage Stochastic Programming

In this section, we develop the stochastic programs with randomness in demand and capacities.

3.1 Random Demand Parameters

We can assume that the demands are random parameters and can thus take several possible values. This is usually modeled as an event tree (see for instance Figure 2). As a result, we obtain a multistage stochastic program.

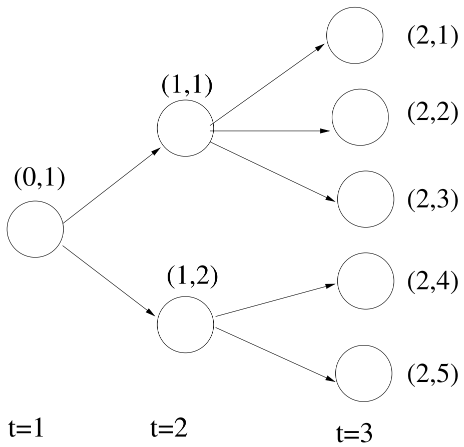

Figure 2: An example of event tree describing different demand states of nature

The deterministic equivalent formulation of our aggregate planning model with uncertain demands becomes:

$$\begin{aligned} \min \quad & \textstyle\sum_t \sum_j Prob_j^t \times ((hQ_j^t + hNQ_j^t) \times Hirecost+ \\ & +fQ_j^t \times FirecostQ + fNQ_j^t \times FirecostNQ+ \\ & +wageQ \times eQ_j^t + wageNQ \times eNQ_j^t + wageX \times Extra_j^t) \end{aligned} \tag{7a}$$

subject to

$$eQ^{t-1}_{\pi_j} + hQ^t_j - fQ^t_j = eQ^t_j, \quad \forall j, \quad \forall t = 2, \cdots, T \tag{7b}$$

$$eNQ^{t-1}_{\pi_j} + hNQ^t_j - fNQ^t_j = eNQ^t_j, \quad \forall j, \quad \forall t = 2, \cdots, T \tag{7c}$$

$$eQ^t_j \times \alpha Q \times 260 + eNQ^t_j \times \alpha NQ \times 260 + Extra^t_j \times \alpha X = Capacity^t_j, \quad \forall j, \quad \forall t = 1, \cdots, T \tag{7d}$$

$$Capacity^t_j \geq Demand^t_j, \quad \forall j, \quad \forall t = 1, \cdots, T \tag{7e}$$

$$j = 1, \cdots, J_t,$$

where J_t is the set of demand state at time t and $Prob$ is the probability distribution of demand, which defines (partial) path probabilities: $Prob^t_j$ is the probability (at the start) that a path goes through node j at time t and π_j denotes the ancestor of node j in the event tree. This formulation remains an integer programming problem.

3.2 Random Capacity Parameters

Due to some unpredictable events, like illness, holiday, or some unexpected accidents, constant capacity of employees cannot be guaranteed. Hence, we introduce uncertain capacity parameters into the model, $\widetilde{\alpha Q}$ and $\widetilde{\alpha NQ}$. Temporary employees are just employed in cases we need more people. Consequently, we assume that they only take a small part of the total demand and thus we neglect the variation of their capacities. The corresponding capacity constraints become:

$$eQ^t_{j,l} \times \alpha Q_l \times 260 + eNQ^t_{j,l} \times \alpha NQ_l \times 260 + Extra^t_{j,l} \times \alpha X = Capacity^t_{j,l}, \tag{8}$$

$$Capacity^t_{j,l} \geq Demand^t_j, \tag{9}$$

where $l = 1, \cdots, L_t$ is the set of all possible capacity values, $t = 1, \cdots, T$, $j = 1, \cdots, J_t$.

The other concern about capacity is new employees' capability. There is always a certain period for a person who is newly employed to get used to the work environment and become familiar with the responsibility. We cannot expect a new employee to be as efficient as an experienced one. Hence, their capabilities have to be valued separately. This is reflected in the following capacity constraint:

$$\begin{aligned} &(eQ^t_{j,l,v} - (1 - iniabt_v) \times hQ^t_{j,l,v}) \times \alpha Q_l \times 260 + \\ &+(eNQ^t_{j,l,v} - (1 - iniabt_v) \times hNQ^t_{j,l,v}) \times \alpha NQ_l \times 260 + \\ &+Extra^t_{j,l,v} \times \alpha X = Capacity^t_{j,l,v}, \end{aligned} \tag{10}$$

where $t = 1, \cdots, T$, $j = 1, \cdots, J_t$, $l = 1, \cdots, L_t$, $v = 1, \cdots, V_t$ is the set of all possible initial capability values (*iniabt*) and *iniabt* is the ratio of a new employee's capability by an experienced employee's capability, i.e., when an actually experienced employee completes one transaction, the new employee can do *iniabt* transaction. For a new hired employee, we have to subtract the lack capacity $(1 - iniabt_v) \times hQ^t_{j,l}$ and $(1 - iniabt_v) \times hNQ^t_{j,l}$ from the total capacity. $\pi_{j,v}$ is the parent node of node (j, v) and π_v are the corresponding parent nodes of all nodes v. After one year, the new employee can work as efficiently as other employees. Obviously,

$$0 < iniabt < 1. \tag{11}$$

The randomness in demand and capacity in the present model are both external risks which are not controlled by the decisions in the model. In the later section, we will exploit the internal risk from the uncertain demand, which depends on the decisions.

4 Revision of Operation Efficiency Probability Distributions

Operation efficiency is a way to measure the work done by employees, which is how many transactions an employee completes per unit time (labour cost) and how many errors an employee makes per transaction (error rate). In banking, a mistake in operations could bring big losses to the company and also decrease the demand in subsequent stages. Limiting the number of mistakes to the strict minimum is essential. This notion of operational efficiency is thus intimately linked to operational risk management [14, 6]. It is related with people's knowledge and skills. Even if we know that people are qualified or not, errors can still happen unexpectedly. Hence it is necessary to consider randomness of operation efficiency in the model. And we need the probability distribution describing the behavior of the random variable. However, in operational risk management, this kind of data is hard to collect. Firstly, long time data is lacking, which means the data of only recent few years is available. Secondly, a company never publishes their errors and operational losses and this makes the data unavailable. In such a case, we can rely on Bayesian Network.

Bayesian Networks (BN) have emerged as a method of choice to deal with operational risks, especially in the banking sector, since BN does not necessitate to gather huge amount of past data. It is indeed extremely difficult to collect sufficient years of statistics describing operational risks to be then able to assume any theoretical behaviors. BN is in fact grounded on classical decision theory and also adopts computing schemes of Artificial Intelligence [19]. Typically with a Bayesian approach we start with a subjective probability associated with a particular event. The a priori probability (also called subjective probability as opposed to objective probability) is assessed by the manager and corresponds to their own intuition and expertise. Along the way, obtaining new imperfect information and depending on the quality of past imperfect information provided by the issuer, the manager will be more or less inclined to modify his initial judgement. In a formal model this would be called the a posteriori analysis where a priori probabilities are modified using the Bayes formula. The Bayesian Network can be presented as a quantitative approach to handle qualitative dynamic choices. It does not help to make a better prediction. On the other hand, through the modeling of decision trees this approach should enable the manager to structure the dynamic dimension of the decision process.

Bayesian Network (BN) as a successful description of causalities has been widely applied in several areas like diagnosis, heuristic search, ecology, data mining and intelligent trouble shooting systems. It is defined as follows [19]:

Definition A Bayesian network consists of the following:

- A set of variables and a set of directed edges between states.
- Each variable has a finite set of mutually exclusive states.
- The variables together with the directed edges form a directed acyclic graph(DAG). (A directed graph is acyclic if there is no directed path $A_1 \to \cdots \to A_n \quad s.t. \quad A_1 = A_n$.)
- To each variable A with parents $B_1, \ldots, B_n$, there is attached the potential table $P(A|B_1, \ldots, B_n)$. B is a child of A and A is a parent of B, if there is a link directed from A to B. And, always, the parent(s) are set to be cause(s) of child.

Causal relations also have a quantitative side, namely their strength. This is expressed by attaching numbers to the links. In BN, it is natural to set the conditional probability to be a strength of the link. Let A be a parent of B, $P(B|A)$ is the strength of their link.

The two fundamental rules for probability calculus are

$$P(A|B)P(B) = P(A,B), \tag{12}$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \tag{13}$$

where $P(A, B)$ is the probability of the joint event that both A and B happen. Sometimes $P(A|B)$ is called the likelihood of B given A. Assume A has n outcomes $a_1 \ldots a_n$, with an effect on the event B, and we know B. Then, $P(a_i|B)$ is a measure of how likely it is that a_i is the cause. In particular, if all a_i's have prior probabilities, Bayes' rule yields

$$P(a_i|B) = \frac{P(B|a_i)P(a_i)}{P(B)}. \tag{14}$$

In operational risk, unexpected variability in operation efficiency is the cause and has effect on losses. Applying BN in this context means using the information of loss we have, to revise the probability distribution of operation efficiency. We illustrate the use of BN in an example below.

Let $ErrQ$ be the operation efficiency of qualified people which is the number of errors made by qualified people in one transaction and similarly, let $ErrNQ$ be the operation efficiency of non-qualified employees. One error can definitely lead to a failure of the current transaction. We assume that $0 \leq ErrQ \leq ErrNQ \leq 1$. Firstly we need to know an a priori probability of the operation efficiency from expert's knowledge. For example, if we denote A as $ErrQ \geq \lambda$ which means qualified employees have shown average poor efficiency and $\overline{A}$ as $ErrQ < \lambda$, where λ is a benchmark set to judge the operation efficiency. Suppose that the experts give the following estimates:

$$P(A) = 0.2 \quad P(\overline{A}) = 0.8. \tag{15}$$

In addition, denote B as $Loss > 0$ which is resulted by qualified people and $\overline{B}$ as $Loss = 0$. Again, experts also determine for instance that

$$P(B|A) = 0.7 \quad P(B|\overline{A}) = 0.1. \tag{16}$$

We can calculate

$$P(B) = P(B|A)P(A) + P(B|\overline{A})P(\overline{A}) = 0.7 \times 0.2 + 0.1 \times 0.8 = 0.22. \tag{17}$$

Now suppose that after one year, we know that loss happened in this year, which is the event B. We get new confidence about the operation efficiency by BN as

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{0.7 \times 0.2}{0.22} = 0.64. \tag{18}$$

In this simple example we notice that the probability of poor operation efficiency increases from 0.2 to 0.64.

The following relationships describe the probability revision process:

$$P(A|\overline{B}) = \frac{P(\overline{B}|A)P(A)}{P(\overline{B})} = P(A)\frac{1 - P(B|A)}{1 - P(B)} \tag{19}$$

$$P(\overline{A}|B) = 1 - P(A|B) = 1 - \frac{P(B|A)P(A)}{P(B)} \tag{20}$$

$$P(\overline{A}|\overline{B}) = 1 - P(A|\overline{B}) = 1 - P(A)\frac{1 - P(B|A)}{1 - P(B)}. \tag{21}$$

We can do the above operations similarly for non-qualified people.

All information needed to revise the above probability is the same as in Equation (18). For the random variables with more than 2 possible values, we still need to revise all the probabilities in the same way with Equation (18). In addition, after revising the probabilities of A, we also need to update $B's$ probabilities following the same way as Equation (17).

We can check that the revision process corresponds to our intuition. The conditional probability of positive loss given poor operation efficiency is higher than the unconditional probability when we know nothing about the operation efficiency, which is

$$P(B|A) \geq P(B). \tag{22}$$

Then, when loss happens, we revise $A's$ probability as follows:

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)} \geq P(A). \tag{23}$$

The inequality (23) follows easily from (22). This revision tells us that incurring loss increases the probability of poor operation efficiency, which agrees with people's intuitive judgement.

Finally, integrating the BN framework in our aggregate planning model, we obtain the following equations:

$$\begin{aligned} \min \quad & \textstyle\sum_{t,j,l,v,s} Prob^t_{j,l,v} \times Prob\_OE^t_s \times ((hQ^t_{j,l,v,s} + hNQ^t_{j,l,v,s}) \times Hirecost \\ & +fQ^t_{j,l,v,s} \times FirecostQ + fNQ^t_{j,l,v,s} \times FirecostNQ \\ & +wageQ \times eQ^t_{j,l,v,s} + wageNQ \times eNQ^t_{j,l,v,s} + wageX \times Extra^t_{j,l,v,s} \\ & +(eQ^t_{j,l,v,s} \times \alpha Q_l \times ErrQ^t_s + eNQ^t_{j,l,v,s} \times \alpha NQ_l \times ErrNQ^t_s) \\ & \times 260 \times lossrate) \end{aligned} \tag{24a}$$

subject to

$$\begin{aligned} eQ^{t-1}_{\pi_{j,l,v,s}} + hQ_{j,l,v,s} - fQ^t_{j,l,v,s} &= eQ^t_{j,l,v,s}, \quad \forall j, \quad \forall l, \quad \forall v, \quad \forall s, \quad \forall t = 2, \cdots, T, \\ eNQ^{t-1}_{\pi_{j,l,v,s}} + hNQ^t_{j,l,v,s} - fNQ^t_{j,l,v,s} &= eNQ^t_{j,l,v,s}, \quad \forall j, \quad \forall l, \quad \forall v, \quad \forall s, \quad \forall t = 2, \cdots, T, \end{aligned} \tag{24b}$$

$$\begin{aligned} (eQ^t_{j,l,v,s} - (1 - iniabt_v) \times hQ^t_{j,l,v}) \times \alpha Q_l \times 260 \quad & + \\ +(eNQ^t_{j,l,v,s} - (1 - iniabt_v) \times hNQ^t_{j,l,v}) \times \alpha NQ_l \times 260 \quad & + \\ +Extra^t_{j,l,v,s} \times \alpha X \quad & = Capacity^t_{j,l,v,s}, \\ & \forall j, \quad \forall l, \quad \forall v, \quad \forall s, \quad \forall t, \end{aligned} \tag{24c}$$

$$Capacity^t_{j,l,v,s} \geq Demand^t_{j,l,v,s}, \quad \forall j, \quad \forall l, \quad \forall v, \quad \forall s, \quad \forall t, \tag{24d}$$

$$\begin{aligned} Prob\_OE^t_s &= \frac{Prob\_OE^{t-1}_{\pi_s} \times Prob_{loss}(Loss > 0|s)}{Prob^{t-1}_{loss}(Loss > 0)}, \quad \forall s, \quad \forall t = 2, \cdots, T, \\ & \quad if \quad loss \quad happens, \end{aligned} \tag{24ea}$$

$$\begin{aligned} Prob\_OE^t_s &= \frac{Prob\_OE^{t-1}_{\pi_s} \times (1 - Prob_{loss}(Loss > 0|s))}{1 - Prob^{t-1}_{loss}(Loss > 0)}, \quad \forall s, \quad \forall t = 2, \cdots, T, \\ & \quad if \quad no \quad loss \quad happens, \end{aligned} \tag{24eb}$$

$$Prob^t_{loss}(Loss > 0) = \sum_s Prob^{t-1}_{loss}(Loss > 0|s) \times Prob\_OE(s), \quad \forall s, \quad \forall t = 2, \cdots, T, \tag{24ec}$$

where $j = 1, \cdots, J_t$, $l = 1, \cdots, L_t$, $v = 1, \cdots, V_t$, $s = 1, \cdots, S_t$ is the set of operation efficiency and π_s is the parent node of s. *Lossrate* means loss per error. Equations (24ea) and (24eb) are the probability revisions of operation efficiency. At each stage t, we revise the probabilities of previous stage $t-1$ by collecting the *loss* information. The probability distribution revised at each stage is essential in the objective and also to the decision making accordingly. Equation (24ec) calculates the new probability of loss after operation efficiency probability revision at each stage. As time goes by, we continuously collect information to update the probability distributions, see Figure 3. The optimization problem (24) is an integer linear problem.

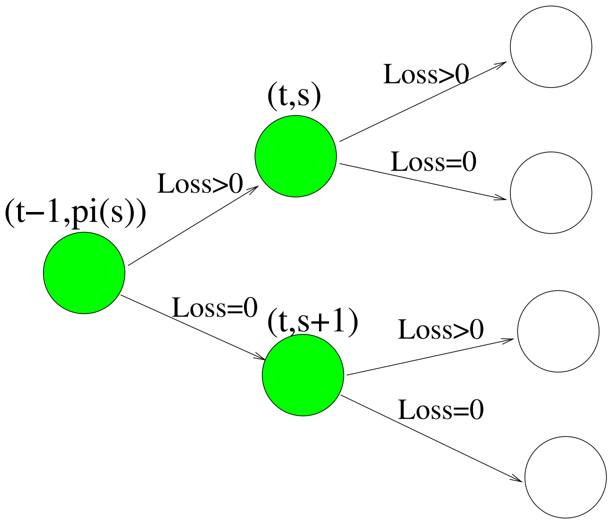

Figure 3: The process of probability distribution revision, where $pi(s)$ is the ancestor node of s.

As said earlier, in operational risk management, the data of probability distribution is difficult to access. BN is applied to build up the probability distribution of operational efficiency. It attempts to collect as much information as possible and simulates the process of the probability revision.

5 Models with Random Parameters Dependent on Decisions

Demand can be dependent upon several factors, like the trend of the market or the management of the company. One of the most important factors is the reputation acquired based on the quality of transactions processed so far, which is essentially related with the individual employee expertise. Let us see how to consider this issue in the model.

Random demand parameters in our aggregate planning model (7) do not depend on decisions. Indeed, the demand, as well as its probability distributions, are set as input data of models. However, in reality, they should not be treated as such. If we consider for example the reputation of companies–the demand could be dependent upon the success of previous decisions; when customers receive products or services of high quality, they are more likely to continue the business and even increase the volume or bring more business to the company, which will increase the demand at next stage. Conversely, if customers are not satisfied with what they have got, they may change to other companies. Meanwhile, the quality of service or products mostly depends on staff and what kind of people to employ is the decision that our modified aggregate planning model is going to optimize.

Operation efficiency is used to measure the work done by employees. Qualified people have additional professional knowledge and skills enabling them to achieve a higher throughput with a lower rate of error than their non-qualified colleagues. However, on the other hand, non-qualified employees are much cheaper to employ in both salary and firing terms. We attempt to determine the number of qualified people required to minimize losses due to employee error, hence not only directly impacting profits, but also growing demand in subsequent stages due to a better customer experience. We are therefore trading off the additional cost of qualified employees against the reduction in error based losses and the growth in demand they produce.

To measure employees' work in a macro view, we use the average number of errors per transaction that employees make. It is denoted as β:

$$\beta = Ave.Errors = \frac{ErrQ \times No.Trans_Q + ErrNQ \times No.Trans_{NQ}}{No.Trans_Q + No.Trans_{NQ}}. \tag{25}$$

A *standard* number is used as a reference to determine whether this β is acceptable. If $\beta \leq standard$, that means people have done a good job and demand will increase above what we expected for next stage, otherwise the demand will decrease. This is reflected in the following constraint:

$$Demand' < (1 + (standard - \beta)) \times Demand. \tag{26}$$

where *Demand* is initiated in the event tree and $Demand'$ is the value of demand updated by the decisions in the same node. Now, demand is changing with decisions, which makes the profit dependent on decisions too and therefore we take it into account in the objective function. Hence, the model becomes:

$$\begin{aligned} \min \quad & \textstyle\sum_{t,j,l,v,s} Prob^t_{j,l,v} \times Prob\_OE^t_s \times ((hQ^t_{j,l,v,s} + hNQ^t_{j,l,v,s}) \times Hirecost \\ & + fQ^t_{j,l,v,s} \times FirecostQ + fNQ^t_{j,l,v,s} \times FirecostNQ \\ & + wageQ \times eQ^t_{j,l,v,s} + wageNQ \times eNQ^t_{j,l,v,s} + wageX \times Extra^t_{j,l,v,s} \\ & + (eQ^t_{j,l,v,s} \times \alpha Q_l \times ErrQ^t_s + eNQ^t_{j,l,v,s} \times \alpha NQ_l \times ErrNQ^t_s) \\ & \times 260 \times lossrate - Demand'^t_{j,l,v,s} \times Profit) \end{aligned} \tag{27a}$$

subject to

$$\begin{aligned} eQ^{t-1}_{\pi_{j,l,v,s}} + hQ_{j,l,v,s} - fQ^t_{j,l,v,s} &= eQ^t_{j,l,v,s}, \quad \forall j, \quad \forall l, \quad \forall v, \quad \forall s, \quad \forall t = 2, \cdots, T, \\ eNQ^{t-1}_{\pi_{j,l,v,s}} + hNQ^t_{j,l,v,s} - fNQ^t_{j,l,v,s} &= eNQ^t_{j,l,v,s}, \quad \forall j, \quad \forall l, \quad \forall v, \quad \forall s, \quad \forall t = 2, \cdots, T, \end{aligned} \tag{27b}$$

$$\begin{aligned} (eQ^t_{j,l,v,s} - (1 - iniabt_v) \times hQ^t_{j,l,v}) \times \alpha Q_l \times 260 \quad & + \\ +(eNQ^t_{j,l,v,s} - (1 - iniabt_v) \times hNQ^t_{j,l,v}) \times \alpha NQ_l \times 260 \quad & + \\ +Extra^t_{j,l,v,s} \times \alpha X \quad & = Capacity^t_{j,l,v,s}, \\ & \forall j, \quad \forall l, \quad \forall v, \quad \forall s, \quad \forall t, \end{aligned} \tag{27c}$$

$$Capacity^t_{j,l,v,s} \geq Demand'^t_{j,l,v,s}, \quad \forall j, \quad \forall l, \quad \forall v, \quad \forall s, \quad \forall t, \tag{27d}$$

$$\beta^t_{j,l,v,s} \geq \frac{eQ^t_{j,l,v,s} \times \alpha Q_l \times ErrQ_s + eNQ^t_{j,l,v,s} \times \alpha NQ_l \times ErrNQ_s}{eQ^t_{j,l,v,s} \times \alpha Q_l + eNQ^t_{j,l,v,s} \times \alpha NQ_l}, \quad \forall j, \quad \forall l, \quad \forall v, \quad \forall s, \quad \forall t, \tag{27e}$$

$$Demand'^{t+1}_{j,l,v,s} = (1 + (standard - \beta^t_{\pi_{j,l,v,s}})) \times Demand^{t+1}_{j,l,v,s}, \quad \forall j, \quad \forall l, \quad \forall v, \quad \forall s, \quad \forall t, \tag{27f}$$

$$Prob\_OE_s^t = \frac{Prob\_OE_{\pi_s}^{t-1} \times Prob_{loss}(Loss > 0|s)}{Prob_{loss}^{t-1}(Loss > 0)}, \quad \forall s, \quad \forall t = 2, \cdots, T, \quad if \quad loss \quad happens, \tag{27ga}$$

$$Prob\_OE_s^t = \frac{Prob\_OE_{\pi_s}^{t-1} \times (1 - Prob_{loss}(Loss > 0|s))}{1 - Prob_{loss}^{t-1}(Loss > 0)}, \quad \forall s, \quad \forall t = 2, \cdots, T, \quad if \quad no \quad loss \quad happens, \tag{27gb}$$

$$Prob_{loss}^t(Loss > 0) = \sum_s Prob_{loss}^{t-1}(Loss > 0|s) \times Prob\_OE(s), \quad \forall s, \quad \forall t = 2, \cdots, T, \tag{27gc}$$

where $j = 1, \cdots, J_t$, $l = 1, \cdots, L_t$, $v = 1, \cdots, V_t$, $s = 1, \cdots, S_t$. $Profit$ is the income per transaction completed. This is a nonlinear integer model.

It is worth mentioning that while demand is influenced by previous decision, there exists also an influence in the opposite direction. $Demand_t$ is one of the main factors changing decision at stage t. Conversely decision at stage t affects those at stage $t-1$. Hence the influence of $Demand_t$ on t-stage-decisions is spread to $t-1$-stage-decisions, which is shown in Figure 4.

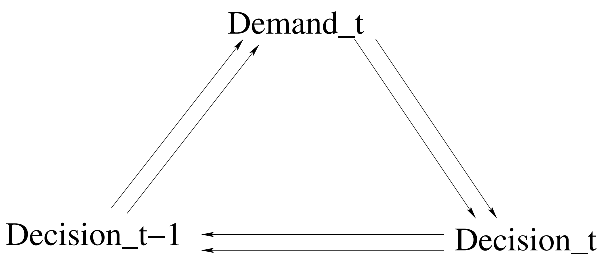

Figure 4: Influence Chart

6 Implementations of the Models

All models discussed in this paper are written in AMPL. When demand function is not applied in the model, it is an integer linear problem and can be solved by CPLEX in AMPL. The model with demand function is difficult to solve. We only manage to solve small instances. Consider a 3-stage-problem, suppose each random variable has 2 possible values and there are 4 random variable sets, i.e., demand, capacities, *iniabt* and operation efficiency. Hence, in the first stage there are $2^4 = 16$ nodes and 256 in the second stage, 4096 in the third stage. There are 4369 nodes in total for this 3-stage-problem. At each node there are 7 integer decision variables and one continuous demand variable. Overall, there are 30583 decision variables in the model. In addition, 2 inventory constraints and one capacity constraint and 2 demand constraints at each node sum up to 21845 constraints.

This aggregated planning model is trying to help decision makers to get the optimal decisions while satisfying the demand and controlling the risk. Our effort is focused on risk management. As discussed above, qualified people armed with better knowledge and skills are considered to be safer to the company than non-qualified people. We test four different models, i.e., basic model, model with random capacity, model with BN revision and model with demand function dependent on decisions. The basic model is referred to the stochastic programming model assuming the randomness only in demand and no dependence between decisions and random variables, as given by Equations (7) in Section 3. The model with random capacity is corresponding to Equations (8) to (10), BN revision is governed by Equation (27g). Demand function depends on decisions through Equations (27e) and (27f). The solution of the basic model excluding risk factors suggests employing more non-qualified people than the more elaborate models to go for highest profit. The decisions provided by the basic model are questionable since they expose the company to the risk of demand satisfying failure and operation errors. In the other models, we can see an average increase of employment of qualified people. Tables 1, 2 and 3 show the summary of results of each model. Since we have 4369 nodes in the event tree, we take average numbers of both qualified and non-qualified people employed among these nodes to illustrate the difference in the tables. In results presented in Tables 1 and 2, the risk is from random demand, random capacities and probability distributions, which are all principally resulting from market risks and consequently cannot be controlled. The analysis of results collected in Table 3, by considering the dependence between the decisions and variables, suggests that the controllable risks require more qualified people. By taking into account the operational risk in random capacity and the demand depending on decisions and constructing more reliable probability distribution, the model makes the decision to employ more qualified people than the basic model, so that the operation is more secure. Meanwhile, the cost, profit and loss are well balanced. In addition, the model with demand function depending on decisions reflects an average increase of demand by 3.9%.

Models	Basic Model	Random Capacity
Qualified People	5	13
Non-Qualified People	53	5

Table 1: Comparison between the basic model and the model with random capacity

Models	Basic Model	BN Revision
Qualified People	5	12
Non-Qualified People	53	20

Table 2: Comparison between the basic model and the model with BN revision

Models	Basic Model	Demand Function
Qualified People	3	17
Non-Qualified People	53	3

Table 3: Comparison between the basic model and the model with demand function depending on decisions

It is natural that the number of employees decreases when the corresponding cost (e.g. hiring, firing cost or salaries) increases. On the other hand, if people improve their skills, which means they can deal with more transactions or make less errors than before, they are more valuable for their employers. In our case study parameter $Lossrate$ has more influence on decisions made than $Profit$, i.e., the decrease of $Lossrate$ pushed down the number of qualified people employed. In the process, number of non-qualified people does not change a lot. When demand varies from stage to stage, qualified people are more frequently fired or hired.

In the Appendix, we present some details of an approximated solution associated with the nonlinear model described by Equation (27).

7 The Pricing of Operational Risk

In the case of a convex nonlinear programming problem with inequality constraints, the dual prices correspond to the Lagrange multipliers. Their interpretation is similar to that of the shadow prices in linear programming. For an additional unit of the right hand side parameter of a given constraint the associated Lagrange multiplier indicates by how many units the objective function will vary, while other quantities remain the same.

In our context, the shadow price of the constraint describing the availability of qualified workers gives the value of an additional hour of expertise provided by a qualified employees. In terms of risk management, we obtain pricing information to set up a kind of “strategic reserve” (terms from the military science designing a supplementary force available and ready to act just in case of urgent needs).

Nowadays, in business this notion of strategic reserve for dealing with operational risk is not accepted. Generating a significant cost to hire expertise just to be able to solve difficult operations problems in case they would arise is not considered to be viable. However we believe that our model enables the risk budget planner to address the necessity to plan sufficient expertise in order to deal with unexpected operational problems. Moreover the dual solutions represents a relevant way to quantify the expertise dedicated to risk management. The shadow price in the context of stochastic programming to produce a uniform $CO2$ tax was first applied in a result analysis by Bahn et al. in [3].

7.1 The Shadow Price in Optimization

The *Lagrangian* associated with the nonlinear programming problem of the follows:

$$\min f(x) \tag{28a}$$

subject to

$$g(x) < b, \tag{28b}$$

$$h(x) = 0, \tag{28c}$$

has the following form

$$L(x, y, z) = f(x) + y^T (g(x) - b) + z^T h(x), \tag{29}$$

where $y \geq 0$ and z are called *Lagrange multipliers*. We have

$$\min_x \max_{y,z} L(x, y, z) = \min f(x). \tag{30}$$

Hence, instead of solving the original model, the optimal value can be obtained by the *Lagrange* function. This is the basic tool in solving nonlinear model. We know that the optimal value of $L(x, y, z)$ must be achieved at the stationary point. By taking the first derivative, we have at the optimal point,

$$\frac{df(x)}{dx_j} + \sum y_i \frac{dg_i(x)}{dx_j} + \sum z_k \frac{dh_k(j)}{dx_j} = 0. \tag{31}$$

The multiplier y_i of inequality constraint can be (locally) interpreted as one unit increase of resource g_i will lead to y_i unit decrease of objective function $f(x)$. y_i are also defined as the shadow prices or dual prices. If y_i is a relative large value compared to others, to achieve the optimal objective value, the corresponding resource will be less prior to be increased than others. The Lagrange multiplier z_k associated with the equality constraint indeed measures the “force” of this equality constraint. However observe that Lagrange multipliers depend on the scaling of constraints. Hence, the same problem after scaling has a different Lagrange multiplier.

We have developed a fourth category of model with some approximations to make it convex and smooth (though nonlinear). Keeping the continuity property enables us to produce shadow prices (Lagrange multipliers in the case of convex non linear models) which gives the implicit value of resources. In that case we obtain information related to value of expertise of qualified workers. To our knowledge this is the first time that shadow prices are used to value the expertise of workers in an operations risk management context.

7.2 Exploiting the Shadow Pricing Approach

In model (27), we have two groups of constraints: inventory constraints (27b) correspond to Equation (28c) and capacity constraints (27d) correspond to Equation (28b). Let us focus on the inventory constraints that express the balance between employees hired, fired and held at each node of both qualified and non-qualified people. Since Lagrange multipliers associated with equality constraints such as inventory equations (27b) depend on the scaling of these constraints, we need to be careful and keep their original scaling. Then we can compare the magnitude of the absolute values of Lagrange multipliers associated with these constraints and deduce from such a comparison which constraints are tight.

Tables 4, 5, 6 and 7 show the shadow prices of constraints in the four different models explained in Section 6. In each table, the absolute values of Lagrange multipliers associated with qualified people inventory constraints are in general greater than those of non-qualified people which means these constraints are tight. In terms of risk management it means that our attention relative to the availability of qualified workers should be especially dedicated. For each node in every time period we identify the greatest danger to have a shortage of qualified people. In other words, employing more qualified people in the improved models, i.e. models with random capacity, demand function, etc, will decrease the operational risk. This can also be illustrated in the following shadow prices comparison tables. Tables 5, 6 and 7 show the shadow prices of the inventory constraints in the model with random capacity, model with BN and the model with demand function depending on decisions, respectively. There is a general decrease in all absolute values of Lagrange multipliers in these three models, while the risk also shrinks.

Typically we see that for capacity constraints, which are inequalities, associated with some small absolute values at certain nodes, e.g. the prices in nodes 3, 4, 5 and 6 which are in the stage 2 are smaller than those in nodes 1 and 2 which are in the stage 1 in Table 4. That means the resources (employees) at nodes 1 and 2 are more expensive, i.e. the objective is more sensitive to the constraints at these nodes. Because the number of employees at this stage does not only have to satisfy the demand but also influences the decision of either hiring or firing at next stage. Hence, it relates to more costs than that of stage 2, which is the last stage.

Inventory Constraints	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Qualified People	7500	-2000	-4500	1200	-3000	800
Non-Qualified People	570	-1445	-1080	660	-645	440
Capacity Constraints	30.5769	27.1154	14.5385	15	9.9808	10

Table 4: Shadow prices of inventory constraints and capacity constraints in basic model.

Capacity Constraints	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Qualified People	2700	-3771.2	-583.2	829.44	-388.3	552.96
Non-Qualified People	-125.605	-1302.18	-75.816	254.016	-50.544	169.344

Table 5: Shadow prices of inventory constraints in model with random capacity.

Capacity Constraints	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Qualified People	2700	-720	641.52	-171.072	427.68	-114.048
Non-Qualified People	165.48	-499.68	103.413	-80.746	68.942	-53.8307

Table 6: Shadow prices of inventory constraints in model with BN.

Capacity Constraints	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Qualified People	7500	-2000	-695.809	1200	-18.2857	800
Non-Qualified People	825	-1700	365.798	660	355.429	440

Table 7: Shadow prices of inventory constraints in model with demand function depending on decisions.

8 Conclusion

Aggregate planning models as a category of mathematical programming models deal with the basic production or operation management problems. The multistage aggregate planning model mainly focuses on the labor allocation management problem. By satisfying the demand constraints at each stage, optimal staff allocation is determined while minimizing costs (including salaries, hiring and firing costs) and losses, resulting from erroneous operations.

In the context of real enterprise risk management, decisions must be made that will affect future choices and outcomes. Hence when considering future events in business activity planning, it is pertinent to take into account uncertain parameters within the planning model. This is often done using a multistage stochastic programming model.

Although stochastic programming is a planning tool that simultaneously takes into account cause-and-effect relations and random variables, most applications in financial risks have been limited to the case where random variables are assumed to follow some theoretical probability distribution functions. In order to add more relevance to the risk planning process of banking operations, we have combined the methodology of Bayesian networks with aggregate planning models.

In general, the demand (a parameter of the model) is assumed to be independent of decisions. However, in reality, this is often not the case. If we consider for example the reputation of companies–the demand could be dependent upon the success of previous decisions; when customers receive products or services of high quality, they are more likely to continue the business and even increase its volume, which will increase the demand at next stage. Conversely, if customers are not satisfied with the service provided, they may change to other companies. This problem has been addressed in our stochastic aggregate planning model by establishing a link between the random parameters and the decision variables. In particular, our model is in line with the COSO risk management philosophy assuming that the quality of the Internal Control Systems affects the residual risks. Simply said operations risks are controllable through good management and this is what we take into account in our model for the first time.

This latter model results in a mixed integer nonlinear problem that we have solved with the commercial solver. Finally, we interpret interesting results obtained with this methodology that confirms that our modeling concept is relevant. Additionally, shadow prices of inventory constraints are used to price the risks of operations. Our model indicates at which period money should be set aside to be able to hire sufficient qualified workforce if needed.

To further deal with the work efficiency, we intend to apply learning curves to the model that describe the employees' learning process more precisely. In addition, the notion of service delay will also be considered in the model, which means we relax the assumption that we always have enough temporary employees as back up. Finally, as the models we have at present are difficult to solve, we intend to develop specific algorithmic techniques to solve them efficiently.

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A Appendix

Solutions to the model given by Equation (27), with integer constraints relaxed.

Parameters set:

$$\begin{aligned} wageQ &= 60000, \\ wageNQ &= 12000, \\ wageX &= 250, \\ FirecostQ &= 12500, \\ FirecostNQ &= 3000, \\ Hirecost &= 5000, \\ Standard &= 0.04. \end{aligned}$$

Employees	Work Capacities		Operation Efficiency		Initial Capabilities	
Qualified People	4.0	3.5	0.001	0.0015	0.75	0.55
Non-Qualified People	1.0	0.8	0.002	0.0025	0.75	0.55
Temporary	3.0		–		–	

Table 8: Work ability parameters.

The results are as follows:

Employees	Qualified	Non-Qualified	Temporary
No. Held	1	50	0

Table 9: Decisions at first stage.

node j	node l	node v	node s	Qualified	Non-qualified	Temp
1	1	1	1	1	46	0
1	1	1	2	1	46	0
1	1	2	1	1	46	0
1	1	2	2	1	46	0
1	2	1	1	2.89231	49.8462	0
1	2	1	2	2.89231	49.8462	0
1	2	2	1	2.29672	49.8462	180.662
1	2	2	2	2.89231	49.8462	0
2	1	1	1	3.33376	55.8957	0
2	1	1	2	4.84615	49.8462	0
2	1	2	1	4.84615	49.8462	0
2	1	2	2	4.84615	49.8462	0
2	2	1	1	7.50764	49.8462	266.683
2	2	1	2	8.38681	49.8462	0
2	2	2	1	5.72199	49.8462	808.328
2	2	2	2	8.38681	49.8462	0

Table 10: Decisions at second stage. Node j corresponds to demand state, l is the work capability, v is the initial capability and s is the operation efficiency.

node j	node l	node v	node s	Qualified	Non-qualified	Temp	Demand
1	1	1	1	1	43.9114	0	12457
1	2	1	1	3.17468	46	0	12457
1	3	1	1	1	43.917	0	12458.4
1	4	1	1	2.29718	49.8462	0	12458.4
2	1	1	1	4.47046	46	0	16609.3
2	2	1	1	7.73767	46	0	16609.3
2	3	1	1	3.51081	49.8462	0	16611.2
2	4	1	1	6.8607	49.8462	0	16611.2
3	1	1	1	1	55.8957	0	15572.9
3	1	1	3	2.50644	49.8462	0	15566.7
3	1	2	1	1	55.8957	0	15572.9
3	2	1	1	4.3369	55.8957	0	15572.9
3	2	1	3	5.71286	49.8462	0	15566.7

Table 11: Part results at third stage. Node j corresponds to demand state, l is the work capability, v is the initial capability and s is the operation efficiency.