Formulation of Oligopolistic Competition in AC Power Networks: An NLP Approach

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Abstract—**In this paper, oligopolistic competition in a centralized power market is characterized by a multi-leader single-follower game, and formulated as a nonlinear programming (NLP) problem. An ac network is used to represent the transmission system and is modeled using rectangular coordinates. The follower is composed of a set of competitive suppliers, demands, and the system operator, while the leaders are the dominant suppliers. The ac approach allows one to capture the strategic behavior of suppliers regarding not only active but also reactive power. In addition, the impact of voltage and apparent power flow constraints can be analyzed. Different case studies are presented using a three-node system to highlight the features of the formulation. Results on a 14-node system are also presented.**

Index Terms—**Competition, complementarity, Cournot, mathematical problem with equilibrium constraints (MPEC), Nash equilibrium, oligopoly.**

I. Introduction

EARLY studies of market power in electricity markets either neglect the transmission network or consider very basic representations of it [1]–[3]. Since the transmission system can have a great impact on market outcomes, recent models incorporate the transmission system by using linear dc approximations [4]–[13]. Typically, this is done by means of the well-known power transfer distribution factors (PTDFs). Thus, the complexity of an ac transmission system is reduced to a set of convex constraints. Because of this simplification, reactive power and voltage constraints are neglected, even though they are inherent issues in the operation of power systems.

Due to difficulties with an ac approach, such as nonlinearity and nonconvexity, few models of competition have dealt with an ac transmission system. In [14], the sensitivities of an ac optimal power flow (OPF) are used to determine the changes in the players' bids. A diagonalization approach is used to solve the multi-player problem. Although an ac approach is taken, the study focuses only on the active power side, as it is usually done with the dc approach. The aim in using an ac approach is to capture the reactive power issues.

In [15], a model for a market with one dominant supplier and a set of followers is presented. The model is a Stackelberg approach, formulated as a bilevel problem, and the authors use a merit function for smoothing the complementarity constraints. However, this approach relies on a monopolist rationality, which reduces the problem to dealing with a sole player, and leads to the formulation of a mathematical problem with equilibrium constraints (MPEC) [4], [16].

Considering a multi-leader formulation makes the problem more realistic but also much harder to solve. If more than one leader is taken into account, then the game problem becomes an equilibrium problem with equilibrium constraints (EPEC) [17]–[19]. A straightforward approach to solve MPECs is to express the equilibrium conditions as nonlinear constraints and then apply nonlinear programming (NLP) solvers. The resulting NLP problems have nonconvex constraints and do not satisfy any of the classical constraint qualifications. They are therefore hard to solve using standard solvers [16], [19], [20]. In [4] and [18], an NLP formulation is used to solve an MPEC. A smoothing factor is included in the reformulation of the complementarity constraints, and it is driven to zero to find the solution of the original problem. In [21], a penalty interior-point algorithm is proposed. Alternatively, the complementarity constraints can be appended into the objective function of the MPEC by means of a penalization factor [22].

Multi-leader problems are typically solved by using diagonalization schemes [4], [14], [18], [21]. These iterative procedures resemble Gauss–Seidel or Jacobi methods. The main advantage of these schemes is that one MPEC is solved at a time, which reduces the difficulty to that of a one-leader problem. Recent studies show that this kind of approach may have difficulties, such as cycling [12], [18], [19]. In addition, as MPECs (and hence EPECs) are inherently nonconvex due to the complementarity constraints, formulations relying on this kind of approach cannot guarantee existence and/or uniqueness of local equilibria. This can happen even in small systems of three nodes where the ISO problem is convex (see, for instance, [17], [23]). To overcome this problem, another approach is to assume that leaders are price-takers with respect to transmission prices (or, in contrast, take smooth response functions), and under mild assumptions, existence and uniqueness of equilibrium may be determined [9].

Recently, EPECs arising from multi-leader-follower games have been reformulated as NLP problems, under the assumption that convexity holds in the follower problem [19]. Unlike formulations that rely on specialized solvers to solve MPECs, in this paper, the problem resulting from modeling oligopolistic competition is formulated as a single NLP problem that can be solved with standard and widely available software.

The contribution of this paper to the analysis of imperfect competition is the inclusion of a full ac network to represent

Manuscript received April 24, 2006; revised September 12, 2006. This work was supported in part by NSERC of Canada under Discovery Grant 15296 and Discovery Grant 312125 and in part by MITACS. Paper no. TPWRS-00243-2006.

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Digital Object Identifier 10.1109/TPWRS.2006.888986

the transmission system with the formulation of the game as an NLP problem. The formulation is based on one of the NLP approaches proposed in [19]. The ac transmission network is modeled using rectangular coordinates. By using an ac approach, the impact of strategic behavior on reactive power and voltage constraints can be analyzed. Also, a linear piecewise capability curve is included to capture the link between active and reactive power that generators have. To the authors' knowledge, this is the first time that a game-based formulation is used for modeling market power for both active and reactive power with a detailed nonlinear formulation of the system. Another aim of this paper is to illustrate the formulation of imperfect competition using a closer approach to real-world markets and to show how this formulation can assist the study of further issues of market power in electricity markets.

This paper is organized as follows. In Section II, the notation used throughout this paper is listed. The models taken to represent the market elements are described in Section III. The setting of the game is presented in Section IV, while the formulation of the game as an NLP problem is developed in Section V. Computational results are reported in Section VI. Conclusions are given in Section VII.

II. Notation

The following notation is introduced as preliminary to the description of the model presented in this paper.

A. Indexes and Sets

$h, \hbar$ Indexes for generation units.
i, j Indexes for nodes in the system.
ij Composed index for a transmission line between nodes i and j.
ν Index for generation companies (GenCos).
$\mathcal{H}_{\nu,i}$ Set of generation units owned by ν and placed at node i.
$\mathcal{N}$ Set of nodes.
$\mathcal{L}$ Set of transmission lines.

B. Constants

δ_i Slope for demand function at i (\$/MW^{2}h).
$\beta_{\nu,h,i}$ Parameter of the linear energy cost function of unit ν, h, i (\$/MWh).
$\gamma_{\nu,h,i}$ Parameter of the quadratic energy cost function of unit ν, h, i (\$/MW^{2}h).
ρ_{oi} Price intercept for demand at i (\$/MWh).
b_{ij} Series susceptance of transmission line ij (p.u.).
b_{ij}^{sh} Shunt susceptance of transmission line ij (p.u.).
g_{ij} Series conductance of transmission line ij (p.u.).
B_{ij} Element ij of the susceptance matrix (p.u.).
G_{ij} Element ij of the conductance matrix (p.u.).
Q_{Li} Fixed demand of reactive power at i (MVAR).

C. Variables

d_i Variable demand of active power at i (MW).
e_i, f_i Real and imaginary coordinates of voltage at i (p.u.).
$g_{\nu,i,h}$ Active power produced by unit ν, h, i (MW).
$q_{\nu,i,h}$ Reactive power produced by unit ν, h, i (MW).
p_i Active power injection at i (MW).
q_i Reactive power injection at i (MVAR).
P_i, Q_i Active and reactive powers at i (MW, MVAR).
V_i Voltage magnitude at i (p.u.).
λ_i Energy price at i (\$/MWh).
θ_i Voltage angle at i (p.u.).

D. Symbols

$(\bar{\cdot}), (\underline{\cdot})$ Maximum and minimum values for $(\cdot)$.
$(\cdot)^*$ Value for $(\cdot)$ at equilibrium.
$|(\cdot)|$ Magnitude of $(\cdot)$.
$a \perp b$ Complementarity condition between a and b.
$a \circ b$ Hadamard product of a and b.
$\boldsymbol{x}_{-\nu}$ means $\boldsymbol{x}_\ell \; \forall \ell \neq \nu$.

Vectors are denoted by dropping subscripts and using the same symbols in bold. Unless otherwise specified, sums apply over all the elements. For each constraint in a primal problem, the associated dual variable is placed at its right-hand side.

III. Market Elements

A. Transmission Network

Based on a rectangular-coordinates system, an ac model is used to represent the transmission network. The use of rectangular coordinates, instead of the classical polar coordinates, eases the formulation and computation of the first- and second-order OPF information. In particular, the second-order elements are constants, and therefore, they are inexpensive to compute, which is not the case when using polar coordinates. The reader is referred to [24] and [25] for a comparison of both kinds of coordinates.

For an ac power network, the finite set of nodes is denoted by $\mathcal{N}$, and $\mathcal{N}_i$ stands for the set of all nodes directly connected to node i. The set $\mathcal{L} := \{(i,j) \,|\, i \in \mathcal{N} \text{ and } j \in \mathcal{N}_i, \text{ and } j > i\}$ stands for sending i and receiving j nodes of all transmission lines in the system.

The voltage at node i can be stated in rectangular coordinates as

$$V_i = e_i + \jmath f_i, \quad \forall i \tag{1}$$

where $\jmath = \sqrt{-1}$ is the imaginary unit, and

$$e_i = |V_i| \cos\theta_i, \quad f_i = |V_i| \sin\theta_i, \quad \forall i \tag{2}$$

are the real and imaginary components, respectively.

The voltage magnitude and angle can be, respectively, defined as

$$|V_i|^2 = e_i^2 + f_i^2, \quad \text{and} \quad \theta_i = \arctan \frac{f_i}{e_i}, \quad \forall i. \tag{3}$$

To account for the system angular reference, the rectangular components of the slack node are set to be $e_s = V_s$ and $f_s = 0$.

The power at node i is

$$S_i = P_i + \jmath Q_i, \quad \forall i \tag{4}$$

where P_i and Q_i are the active and reactive components of power at node i. Recalling the rectangular components, the net power can be decomposed into its real (active) and imaginary (reactive) coordinates; this yields the following power flow equations:

$$P_i = \sum_j \{e_i(G_{ij}e_j - B_{ij}f_j) + f_i(B_{ij}e_j + G_{ij}f_j)\}, \quad \forall i \tag{5}$$

and

$$Q_i = \sum_j \{f_i(G_{ij}e_j - B_{ij}f_j) - e_i(B_{ij}e_j + G_{ij}f_j)\}, \quad \forall i \tag{6}$$

where G_{ij} is the ijth element of the bus conductance matrix $\boldsymbol{G} \in \mathbb{R}^{|\mathcal{N}|\times|\mathcal{N}|}$, and B_{ij} is the ijth element of the susceptance matrix $\boldsymbol{B} \in \mathbb{R}^{|\mathcal{N}|\times|\mathcal{N}|}$.

For both active and reactive, the power entering a line differs from that going out; the difference consists of the losses. To include the constraints of power flows in the transmission lines, the active and reactive power flows are taken as the average of both directions. Hence, the magnitude of the average flow of apparent power is

$$|\hat{S}_{ij}|^2 = |\hat{P}_{ij}|^2 + |\hat{Q}_{ij}|^2, \quad \forall (i,j) \in \mathcal{L} \tag{7}$$

where

$$\hat{P}_{ij} = \frac{1}{2} g_{ij} \left(e_i^2 + f_i^2 - e_j^2 - f_j^2\right) + b_{ij}(e_i f_j - e_j e_i) \tag{8}$$

$$\hat{Q}_{ij} = -\frac{1}{2}\left(b_{ij} + b_{ij}^{\text{sh}}\right)\left(e_i^2 + f_i^2 - e_j^2 - f_j^2\right) + g_{ij}(e_i f_j - e_j e_i). \tag{9}$$

The parameters g_{ij}, b_{ij}, and b_{ij}^{sh} are the series conductance, the series susceptance, and the shunt susceptance of the transmission line $(i,j) \in \mathcal{L}$.

B. Generation Companies

A GenCo ν is composed of a set $\mathcal{H}_{\nu,i}$ of generation units placed at node i. The power output of its generation unit $h \in \mathcal{H}_{\nu,i}$ is represented by $g_{\nu,h,i}$. The variable generation cost of GenCo ν is defined by the nondecreasing convex function $c_{\nu,h,i}(g_{\nu,h,i}) = \beta_{\nu,h,i} g_{\nu,h,i} + \gamma_{\nu,h,i} g_{\nu,h,i}^2$, where $\beta_{\nu,h,i}$ and $\gamma_{\nu,h,i}$ are nonnegative parameters.

The operation of generation units is subject to different physical constraints, such as active and reactive generation limits. The capacity of a generator to absorb or produce reactive power (MVAR) is limited by the generator heating considerations. As generation of active power increases, the capacity to absorb or produce reactive power decreases. This trade-off is defined by the generators' capability curve, known as the *D-curve*. A generator's D-curve is composed by the field, armature, and underexcitation limits [26]. These limits can be defined by quadratic expressions. Although the computational burden to include the explicit capacibility limits in the proposed model is inexpensive (the proposed model in this paper is already nonlinear and nonconvex), linear approximations have been used to include the D-curve. The reason for this approximation is twofold: 1) from the game point of view, including the actual D-curve would mean that the actual generator data are perfectly known; and 2) even in some real markets, generators just provide a set of points to built a linear piecewise D-curve. For instance, in PJM [27] generators can provide between two and eight points of the kind (MW point, MVAR min, MVAR max). Without loss of generality, in this paper, we consider two points to build the generators D-curve. One point sets the values of reactive power for the minimum generation limit of active power, and the second point sets the values of reactive power for the maximum limit of active power. Beside the minimum and maximum limits of active power, there is one linear approximation for the field heating limit $(\cdot^f)$ and one linear approximation for the underexcitation limit $(\cdot^u)$, i.e.,

$$\underline{g}_{\nu,h,i} \le g_{\nu,h,i} \le \bar{g}_{\nu,h,i}, \quad \forall i,h,\nu \tag{10}$$

$$q_{\nu,h,i} \le b_{\nu,h,i}^f g_{\nu,h,i} + a_{\nu,h,i}^f, \quad \forall i,h,\nu \tag{11}$$

$$q_{\nu,h,i} \ge b_{\nu,h,i}^u g_{\nu,h,i} + a_{\nu,h,i}^u, \quad \forall i,h,\nu. \tag{12}$$

When modeling all generation units that compose a competitive fringe, the supplier index ν will be dropped.

C. Demand

Let the benefit of the net consumer placed at node i be defined by the nondecreasing concave function $b_i(d_i) = \rho_{oi} d_i - \delta_i d_i^2$, where d_i stands for the demand level. The marginal benefit function $\rho_i(d_i) = \rho_{oi} - 2\delta_i d_i$, known as the *inverse demand function*, defines the price in terms of demand level. The terms ρ_{oi} and δ_i are parameters to denote the price intersection and demand slope, respectively. This affine function implies a price-responsive demand.

Although it is straightforward to adopt a price-responsive demand for reactive power, due to the incipient sophistication of reactive power markets, such a reactive-power demand is modeled as nonresponsive to price and denoted by a fixed demand Q_{Li}.

IV. Game-Based Model

In a pool-like centralized market, an independent entity is in charge of implementing the market; this entity is commonly referred to as an independent system operator (ISO). Suppliers and consumers submit their respective bids, and the ISO computes a market equilibrium. For a competitive power market, the main assumption is that suppliers cannot affect the market price, and consequently, they will bid their true marginal costs. In an oligopoly, however, there are a few dominant suppliers that can profitably manipulate the market outcome. In such a setting, the market price is actually influenced by the dominant supplier decisions, i.e., the market price is a function of the output levels of the dominant suppliers. Thus, these dominant suppliers set not only the power level but also the market price at which they sell power. Based on these facts, a natural setting for modeling competition is to consider a competitive fringe of suppliers together with a few dominant suppliers. On one hand, competitive generators, demands, and the ISO can be comprised into a *follower*; on the other hand, the dominant suppliers become the *leaders* and behave *à la* Cournot. Hence, this formulation gives rise to a multi-leader single-follower game. This game setting resembles the one initially proposed by Hogan [4] for a dc model. Each player problem is next described.

A. ISO Problem—Follower

The objective of the ISO is to minimize the social cost given by the difference between the total generation cost incurred from the competitive fringe and the total demand benefit. Moreover, the ISO acts as a follower, taking as given the dominant supplier decisions in order to find the minimum cost dispatch. Therefore, the ISO problem in rectangular coordinates can be stated mathematically as

$$\min \sum_{h,i} c_{h,i}(g_{h,i}) - \sum_i b_i(d_i) \tag{13}$$

s.t.

$$\sum_{\nu,\hbar} g^*_{\nu,\hbar,i} + \sum_h g_{h,i} - d_i - P_i(\boldsymbol{e},\boldsymbol{f}) = 0, \quad : \lambda_i, \forall i \tag{14}$$

$$\sum_{\nu,\hbar} q^*_{\nu,\hbar,i} + \sum_h q_{h,i} - Q_{Li} - Q_i(\boldsymbol{e},\boldsymbol{f}) = 0, \quad : \lambda_i^q, \forall i \tag{15}$$

$$|\hat{S}_{ij}(\boldsymbol{e},\boldsymbol{f})|^2 \le \bar{S}_{ij}^2, \quad : \bar{\mu}_{ij}, \quad \forall (i,j) \in \mathcal{L} \tag{16}$$

$$\underline{V}_i^2 \le V_i^2(\boldsymbol{e},\boldsymbol{f}) \le \bar{V}_i^2, \quad : \underline{\mu}_i^v, \bar{\mu}_i^v, \quad \forall i \tag{17}$$

$$\underline{g}_{h,i} \le g_{h,i} \le \bar{g}_{h,i}, \quad : \underline{\mu}_{h,i}^g, \bar{\mu}_{h,i}^g, \quad \forall i,h \tag{18}$$

$$q_{h,i} \le b_{h,i}^f g_{h,i} + a_{h,i}^f, \quad : \bar{\mu}_{h,i}^q, \quad \forall i,h \tag{19}$$

$$q_{h,i} \ge b_{h,i}^u g_{h,i} + a_{h,i}^u, \quad : \underline{\mu}_{h,i}^q, \quad \forall i,h \tag{20}$$

$$d_i \ge 0, \quad \forall i. \tag{21}$$

The objective function (13) stands for the social cost. Expressions (14) and (15) are the power flow balances of the system (supply meets demand) for both active and reactive power, respectively. Vectors $\boldsymbol{e} \in \mathbb{R}^{|\mathcal{N}|}$ and $\boldsymbol{f} \in \mathbb{R}^{|\mathcal{N}|}$ are the voltage rectangular components. On the left-hand side of these expressions, the first sum stands for the contribution of dominant suppliers, while the second sum accounts for the competitive suppliers. The dominant supplier's decision variables are denoted as $(\cdot)^*$ because they are exogenous to the ISO problem. However, such terms are the GenCo decision variables within each dominant GenCo problem. Notice that this holds for both active and reactive power balances. Expression (16) accounts for the transmission line limits in either direction; expression (17) stands for the lower and upper nodal voltage limits; and expressions (18)–(20) stand for the generation capacity limits.

Problem (13)–(21) is a classical OPF, except for the parametrization over the leaders' decision variables. In the economical sense, the Lagrange multipliers λ_i (also known as *shadow prices*) associated with the active power balances at the ith node can be interpreted as the optimal prices because they quantify the cost (or value, from the demand side) for supplying (or consuming) an additional MW at the ith node of the network. Furthermore, the Lagrange multipliers $\bar{\mu}_{ij}$ and μ_i^v, associated with the power flow limit in the ijth transmission line and the voltage limits at the ith node, are *congestion multipliers* and can be interpreted as the variation in social cost if such limits are relaxed.

The (first-order optimality) KKT conditions for the ISO problem are

$$0 = \lambda_i \frac{\partial P_i}{\partial e_i} + \lambda_i^q \frac{\partial Q_i}{\partial e_i} + \sum_{j \in \mathcal{N}_i} \left\{ \lambda_j \frac{\partial P_j}{\partial e_i} + \lambda_j^q \frac{\partial Q_j}{\partial e_i} + \bar{\mu}_{ij} \frac{\partial |\hat{S}_{ij}|^2}{\partial e_i} \right\} + \frac{\partial V_i^2}{\partial e_i} \left\{ \bar{\mu}_i^v - \underline{\mu}_i^v \right\}, \quad \forall i \tag{22}$$

$$0 = \lambda_i \frac{\partial P_i}{\partial f_i} + \lambda_i^q \frac{\partial Q_i}{\partial f_i} + \sum_{j \in \mathcal{N}_i} \left\{ \lambda_j \frac{\partial P_j}{\partial f_i} + \lambda_j^q \frac{\partial Q_j}{\partial f_i} + \bar{\mu}_{ij} \frac{\partial |\hat{S}_{ij}|^2}{\partial f_i} \right\} + \frac{\partial V_i^2}{\partial f_i} \left\{ \bar{\mu}_i^v - \underline{\mu}_i^v \right\}, \quad \forall i \tag{23}$$

$$0 = \sum_{\nu,\hbar} g^*_{\nu,\hbar,i} + \sum_h g_{h,i} - d_i - P_i(\boldsymbol{e},\boldsymbol{f}), \quad \forall i \tag{24}$$

$$0 = \sum_{\nu,\hbar} q^*_{\nu,\hbar,i} + \sum_h q_{h,i} - Q_{Li} - Q_i(\boldsymbol{e},\boldsymbol{f}), \quad \forall i \tag{25}$$

$$0 = \beta_{h,i} + 2\gamma_{h,i} g_{h,i} - \lambda_i + \bar{\mu}_{h,i}^g - \underline{\mu}_{h,i}^g - \bar{\mu}_{h,i}^q b_{h,i}^f + \underline{\mu}_{h,i}^q b_{h,i}^u, \quad \forall i,h \tag{26}$$

$$0 = -\lambda_i^q + \bar{\mu}_{h,i}^q - \underline{\mu}_{h,i}^q, \quad \forall i,h \tag{27}$$

$$0 \le -\rho_{oi} + 2\delta_i d_i + \lambda_i \quad \perp \quad d_i \ge 0, \quad \forall i \tag{28}$$

$$0 \le \bar{S}_{ij}^2 - |\hat{S}_{ij}(\boldsymbol{e},\boldsymbol{f})|^2 \quad \perp \quad \bar{\mu}_{ij} \ge 0, \quad \forall (i,j) \in \mathcal{L} \tag{29}$$

$$0 \le \bar{V}_i^2 - V_i^2(\boldsymbol{e},\boldsymbol{f}) \quad \perp \quad \bar{\mu}_i^v \ge 0, \quad \forall i \tag{30}$$

$$0 \le V_i^2(\boldsymbol{e},\boldsymbol{f}) - \underline{V}_i^2 \quad \perp \quad \underline{\mu}_i^v \ge 0, \quad \forall i \tag{31}$$

$$0 \le \bar{g}_{h,i} - g_{h,i} \quad \perp \quad \bar{\mu}_{h,i}^g \ge 0, \quad \forall i,h \tag{32}$$

$$0 \le g_{h,i} - \underline{g}_{h,i} \quad \perp \quad \underline{\mu}_{h,i}^g \ge 0, \quad \forall i,h \tag{33}$$

$$0 \le -q_{h,i} + b_{h,i}^f g_{h,i} + a_{h,i}^f \quad \perp \quad \bar{\mu}_{h,i}^q \ge 0, \quad \forall i,h \tag{34}$$

$$0 \le q_{h,i} - b_{h,i}^u g_{h,i} - a_{h,i}^u \quad \perp \quad \underline{\mu}_{h,i}^q \ge 0, \quad \forall i,h \tag{35}$$

$$\lambda_i, \lambda_i^q \quad \text{free.} \tag{36}$$

The symbol $\perp$ is henceforth used to compactly denote a complementarity condition. The KKT conditions are necessary for optimality in the ISO. However, because the ISO problem is not convex, a solution of the KKT conditions may be not only a minimum but also a maximum or a saddle point. This difficulty is inherent to nonconvex problems. A practical way to partially overcome this problem is to test different starting points.

The ISO problem (13)–(21) can be written compactly as

$$\min b(\boldsymbol{w}_0, \boldsymbol{w}_1) \tag{37}$$
$$\text{s.t. } \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{x}, \boldsymbol{w}_0, \boldsymbol{w}_1) = \boldsymbol{0}, \quad : \boldsymbol{\lambda}, \tag{38}$$
$$\boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_1) \geq \boldsymbol{0}, \quad : \boldsymbol{\mu} \tag{39}$$
$$\boldsymbol{w}_0 \geq \boldsymbol{0} \tag{40}$$

where equality and inequality constraints are comprised into $\boldsymbol{c}_{\mathcal{E}}(\boldsymbol{x}, \boldsymbol{w}_0, \boldsymbol{w}_1)$, and $\boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_1)$, respectively, while their corresponding dual variables are comprised into vectors $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. The vectors $\boldsymbol{w}_0$ and $\boldsymbol{w}_1$ contain the control and state variables of the ISO problem such that $\boldsymbol{w}_0 = (\boldsymbol{d})$ and $\boldsymbol{w}_1 = (\boldsymbol{g}, \boldsymbol{q}, \boldsymbol{e}, \boldsymbol{f})$. The vector $\boldsymbol{x}$ contains all the decisions variables of dominant GenCos. For the sake of simplicity, all the function arguments are omitted. This formulation yields the following compact form of the KKT conditions of the ISO:

$$\boldsymbol{0} = \nabla_{\boldsymbol{w}_1} b - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{E}} \boldsymbol{\lambda} - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{I}} \boldsymbol{\mu} \tag{41}$$
$$\boldsymbol{0} = \boldsymbol{c}_{\mathcal{E}} \tag{42}$$
$$\boldsymbol{0} \leq \nabla_{\boldsymbol{w}_0} b - \nabla^T_{\boldsymbol{w}_0} \boldsymbol{c}_{\mathcal{E}} \boldsymbol{\lambda} - \nabla^T_{\boldsymbol{w}_0} \boldsymbol{c}_{\mathcal{I}} \boldsymbol{\mu} \quad \perp \quad \boldsymbol{w}_0 \geq \boldsymbol{0} \tag{43}$$
$$\boldsymbol{0} \leq \boldsymbol{c}_{\mathcal{I}} \quad \perp \quad \boldsymbol{\mu} \geq \boldsymbol{0} \tag{44}$$
$$\boldsymbol{\lambda} \quad \text{free.} \tag{45}$$

Let us define the variable vectors $\boldsymbol{y}_0 = (\boldsymbol{w}_0, \boldsymbol{\mu})$ and $\boldsymbol{y}_1 = (\boldsymbol{w}_1, \boldsymbol{\lambda})$ and define the following functions:

$$\boldsymbol{0} = \boldsymbol{h}_{\mathcal{E}}(\boldsymbol{x}, \boldsymbol{y}_0, \boldsymbol{y}_1) = \begin{pmatrix} \nabla_{\boldsymbol{w}_1} b - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{E}} \boldsymbol{\lambda} - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{I}} \boldsymbol{\mu} \\ \boldsymbol{c}_{\mathcal{E}} \end{pmatrix} \tag{46}$$

and

$$\boldsymbol{0} \leq \boldsymbol{h}_{\mathcal{I}}(\boldsymbol{y}_0, \boldsymbol{y}_1) = \begin{pmatrix} \nabla_{\boldsymbol{w}_0} b - \nabla^T_{\boldsymbol{w}_0} \boldsymbol{c}_{\mathcal{E}} \boldsymbol{\lambda} - \nabla^T_{\boldsymbol{w}_0} \boldsymbol{c}_{\mathcal{I}} \boldsymbol{\mu} \\ \boldsymbol{c}_{\mathcal{I}} \end{pmatrix}. \tag{47}$$

By introducing slack variables $\boldsymbol{s}$ into (47), the KKT conditions of the ISO can be stated as

$$\boldsymbol{0} = \boldsymbol{h}_{\mathcal{E}}(\boldsymbol{x}, \boldsymbol{y}_0, \boldsymbol{y}_1) \tag{48}$$
$$\boldsymbol{0} = \boldsymbol{h}_{\mathcal{I}}(\boldsymbol{y}_0, \boldsymbol{y}_1) - \boldsymbol{s} \tag{49}$$
$$\boldsymbol{0} \leq \boldsymbol{s} \quad \perp \quad \boldsymbol{y}_0 \geq \boldsymbol{0}. \tag{50}$$

B. Dominant GenCos—Leaders

The set of leaders is given by $\mathcal{V}$, and their decision variables form a vector $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_{|\mathcal{V}|})$. Thus, given leader ν, its rivals' decisions $\{\boldsymbol{x}_\ell \,|\, \ell \neq \nu\}$ are denoted as $\boldsymbol{x}_{-\nu}$; this vector is considered to be fixed within each leader problem. This means that each leader problem is parametrized by $\boldsymbol{x}_{-\nu}$. Because nodal prices come from the ISO problem and are common to all leaders, such prices can be seen as shared decision variables. Since the leaders' profit depends on both its own decisions and the nodal prices, the profit function is denoted as $f(\boldsymbol{x}_\nu, \boldsymbol{y}_1)$.

The objective of each leader is to maximize its own profit[1] (revenues minus costs), which is stated in (51) as a minimization of negative profits. The revenues are defined by the power it sells at nodal prices, $\lambda_i \sum_h g_{\nu,h,i}$, while the costs for providing such power is $c_{\nu,h,i}(g_{\nu,h,i})$ (see Section III-B). Also, the generation limits for leaders, given by (10)–(12), are comprised into $\boldsymbol{h}_\nu(\boldsymbol{x}_\nu)$. Thus, the game problem can be cast as follows:

$$\begin{pmatrix} \min & -\Pi_\nu = f_\nu(\boldsymbol{x}_\nu, \boldsymbol{y}_1) \\ \text{s.t.} & \boldsymbol{h}_\nu(\boldsymbol{x}_\nu) \geq \boldsymbol{0}, \\ & \boldsymbol{h}_{\mathcal{E}}(\boldsymbol{x}_\nu, \boldsymbol{y}_0, \boldsymbol{y}_1; \boldsymbol{x}_{-\nu}) = \boldsymbol{0}, \\ & \boldsymbol{h}_{\mathcal{I}}(\boldsymbol{y}_0, \boldsymbol{y}_1) - \boldsymbol{s} = \boldsymbol{0}, \\ & \boldsymbol{0} \leq \boldsymbol{y}_0 \quad \perp \quad \boldsymbol{s} \geq \boldsymbol{0}, \end{pmatrix}, \quad \forall \nu. \tag{51}$$

As the rationality taken for competition among leaders is *à la* Cournot, each leader takes its rivals decisions as given. This is applied in both active and reactive power balances. The inclusion of the ISO complementarity constraints into each leader problem implies that each leader anticipates how the ISO will react to its decisions. Because of this inclusion, each leader problem is an MPEC. Because of the nonconvexity of the ISO problem, the KKT conditions are only stationarity conditions, and the leader problem is, indeed, an MPCC.

In a multi-leader setting, the problem becomes harder because all leaders share the complementarity constraints given by the KKT conditions of the ISO. These constraints link the leader problems with each other. Therefore, the leaders-follower problem becomes an EPEC; in fact, an equilibrium problem with complementarity constraints (EPCC).

V. Solution Methods

A. Mixed Nonlinear Complementarity Model

A straightforward approach is to form a mixed nonlinear complementarity problem (MNCP). To do this, the complementarity constraint of (51) is expressed in terms of nonlinear constraints. For instance, it can be defined as $\boldsymbol{y}_0 \circ \boldsymbol{s} = \boldsymbol{0}$, $\boldsymbol{y}_0 \geq \boldsymbol{0}$, and $\boldsymbol{s} \geq \boldsymbol{0}$, where $\circ$ denotes the Hadamard product of $\boldsymbol{y}_0$ and $\boldsymbol{s}$. With this formulation, the problem (51) becomes an NLP. Concatenating the KKT conditions of all leaders, the game is defined as a nonsquare MNCP. However, there is only one set of variables $\boldsymbol{y}_0, \boldsymbol{y}_1$, and $\boldsymbol{s}$ for all leaders. Furthermore, expressions associated with $\bar{\boldsymbol{\vartheta}}_\nu, \underline{\boldsymbol{\vartheta}}_\nu$, and $\boldsymbol{\xi}_\nu$ are the same for all leaders; thus, only one set of such equations is kept. This results in the following square MNCP:

$$\boldsymbol{0} = \nabla_{\boldsymbol{x}_\nu} f - \nabla^T_{\boldsymbol{x}_\nu} \boldsymbol{h}_\nu \boldsymbol{\phi}_\nu - \nabla^T_{\boldsymbol{x}_\nu} \boldsymbol{h}_{\mathcal{E}_\nu} \bar{\boldsymbol{\vartheta}}_\nu, \quad \forall \nu \tag{52}$$
$$\boldsymbol{0} = \nabla_{\boldsymbol{y}_0} f - \nabla^T_{\boldsymbol{y}_0} \boldsymbol{h}_{\mathcal{E}_\nu} \bar{\boldsymbol{\vartheta}}_\nu - \nabla^T_{\boldsymbol{y}_0} \boldsymbol{h}_{\mathcal{I}_\nu} \underline{\boldsymbol{\vartheta}}_\nu - \boldsymbol{s} \circ \boldsymbol{\xi}_\nu - \boldsymbol{\psi}_\nu, \quad \forall \nu \tag{53}$$
$$\boldsymbol{0} = \nabla_{\boldsymbol{y}_1} f - \nabla^T_{\boldsymbol{y}_1} \boldsymbol{h}_{\mathcal{E}_\nu} \bar{\boldsymbol{\vartheta}}_\nu - \nabla^T_{\boldsymbol{y}_1} \boldsymbol{h}_{\mathcal{I}_\nu} \underline{\boldsymbol{\vartheta}}_\nu, \quad \forall \nu \tag{54}$$
$$\boldsymbol{0} = \underline{\boldsymbol{\vartheta}}_\nu - \boldsymbol{y}_0 \circ \boldsymbol{\xi}_\nu - \boldsymbol{\sigma}_\nu, \quad \forall \nu \tag{55}$$
$$\boldsymbol{0} \leq \boldsymbol{h}_\nu \quad \perp \quad \boldsymbol{\phi}_\nu \geq \boldsymbol{0}, \quad \forall \nu \tag{56}$$
$$\boldsymbol{0} \leq \boldsymbol{y}_0 \quad \perp \quad \boldsymbol{\psi}_\nu \geq \boldsymbol{0}, \quad \forall \nu \tag{57}$$
$$\boldsymbol{0} \leq \boldsymbol{s} \quad \perp \quad \boldsymbol{\sigma}_\nu \geq \boldsymbol{0}, \quad \forall \nu \tag{58}$$
$$\boldsymbol{0} = \boldsymbol{y}_0 \circ \boldsymbol{s} \tag{59}$$
$$\boldsymbol{0} = \boldsymbol{h}_{\mathcal{E}} \tag{60}$$
$$\boldsymbol{0} = \boldsymbol{h}_{\mathcal{I}} - \boldsymbol{s}. \tag{61}$$

[1]Because the concern of market power is emphasized in the generation sector, it is assumed that no demand facilities are controlled by GenCos. Nonetheless, it would be straightforward to include into the leaders' objective function a demand-side component.

The squareness of the above MNCP is only possible by stating the complementarity constraint of (50) as an equality constraint, $\boldsymbol{y}_0 \circ \boldsymbol{s} = \mathbf{0}$. The index ν over the dual variables $\boldsymbol{\xi}_\nu, \boldsymbol{\psi}_\nu$, and $\boldsymbol{\sigma}_\nu$ implies that each leader individually prices the shared set of constraints arising from the ISO, and in general, their prices will differ. Problem (52)–(61) is nonconvex because of both the constraints of the ISO problem and the complementarity constraints. Since the ISO problem (13)–(21) is nonconvex, its KKT conditions characterize stationarity conditions rather than equilibrium conditions. Hence, a solution of (52)–(61) can be not only a minimum but also a stationary point or even a maximum point. This also holds for the NLP formulation of Section V-B. Attempts were made to solve the above MNCP using PATH [28] under NEOS [29]. Although PATH has been explicitly designed to tackle complementarity constraints, its performance on this specific problem has been quite poor; even with a simpler formulation, a poor performance has been reported elsewhere [19].

B. NLP Formulation

With the vast work done in NLP, an attractive proposition is to state problem (51) as an NLP and then use widely available optimization software. For the NLP formulation of the game, we have followed the derivation proposed in [19]. If the complementarity constraint in (51) is now formulated as $\boldsymbol{y}_0 \circ \boldsymbol{s} \leq \mathbf{0}$ with $\boldsymbol{y}_0, \boldsymbol{s} \geq \mathbf{0}$, problem (51) becomes

$$\left(\begin{array}{lll} \min & -\Pi_\nu = f_\nu(\boldsymbol{x}_\nu, \boldsymbol{y}_1) & \\ \text{s.t.} & \boldsymbol{h}_\nu(\boldsymbol{x}_\nu) \geq \mathbf{0}, & : \boldsymbol{\phi}_\nu \\ & \boldsymbol{h}_\mathcal{E}(\boldsymbol{x}_\nu, \boldsymbol{y}_0, \boldsymbol{y}_1; \boldsymbol{x}_{-\nu}) = \mathbf{0}, & : \bar{\boldsymbol{\vartheta}}_\nu \\ & \boldsymbol{h}_\mathcal{I}(\boldsymbol{y}_0, \boldsymbol{y}_1) - \boldsymbol{s} = \mathbf{0}, & : \underline{\boldsymbol{\vartheta}}_\nu \\ & -\boldsymbol{y}_0 \circ \boldsymbol{s} \geq \mathbf{0}, & : \boldsymbol{\xi}_\nu \\ & \boldsymbol{y}_0 \geq \mathbf{0}, & : \boldsymbol{\psi}_\nu \\ & \boldsymbol{s} \geq \mathbf{0}, & : \boldsymbol{\sigma}_\nu \end{array}\right), \quad \forall \nu. \tag{62}$$

Because each leader problem (an MPEC) does not meet any of the classical constraint qualifications, by defining the complementarity constraints as NLP constraints, and using the notion of strong stationarity [20], the following stationarity conditions can be obtained:

$$\mathbf{0} = \nabla_{\boldsymbol{x}_\nu} f - \nabla^T_{\boldsymbol{x}_\nu} \boldsymbol{h}_\nu \boldsymbol{\phi}_\nu - \nabla^T_{\boldsymbol{x}_\nu} \boldsymbol{h}_{\mathcal{E}_\nu} \bar{\boldsymbol{\vartheta}}_\nu, \quad \forall \nu \tag{63}$$
$$\mathbf{0} = \nabla_{\boldsymbol{y}_0} f - \nabla^T_{\boldsymbol{y}_0} \boldsymbol{h}_{\mathcal{E}_\nu} \bar{\boldsymbol{\vartheta}}_\nu - \nabla^T_{\boldsymbol{y}_0} \boldsymbol{h}_{\mathcal{I}_\nu} \underline{\boldsymbol{\vartheta}}_\nu + \boldsymbol{s} \circ \boldsymbol{\xi}_\nu - \boldsymbol{\psi}_\nu, \quad \forall \nu \tag{64}$$
$$\mathbf{0} = \nabla_{\boldsymbol{y}_1} f - \nabla^T_{\boldsymbol{y}_1} \boldsymbol{h}_{\mathcal{E}_\nu} \bar{\boldsymbol{\vartheta}}_\nu - \nabla^T_{\boldsymbol{y}_1} \boldsymbol{h}_{\mathcal{I}_\nu} \underline{\boldsymbol{\vartheta}}_\nu, \quad \forall \nu \tag{65}$$
$$\mathbf{0} = \underline{\boldsymbol{\vartheta}}_\nu + \boldsymbol{y}_0 \circ \boldsymbol{\xi}_\nu - \sigma_\nu, \quad \forall \nu \tag{66}$$
$$\mathbf{0} \leq \boldsymbol{h}_\nu \quad \perp \quad \boldsymbol{\phi}_\nu \geq \mathbf{0}, \quad \forall \nu \tag{67}$$
$$\mathbf{0} \leq \boldsymbol{y}_0 \quad \perp \quad \boldsymbol{\psi}_\nu \geq \mathbf{0}, \quad \forall \nu \tag{68}$$
$$\mathbf{0} \leq \boldsymbol{s} \quad \perp \quad \sigma_\nu \geq \mathbf{0}, \quad \forall \nu \tag{69}$$
$$\mathbf{0} \leq -\boldsymbol{y}_0 \circ \boldsymbol{s} \quad \perp \quad \boldsymbol{\xi}_\nu \geq \mathbf{0}, \quad \forall \nu \tag{70}$$
$$\mathbf{0} = \boldsymbol{h}_\mathcal{E} \tag{71}$$
$$\mathbf{0} = \boldsymbol{h}_\mathcal{I} - \boldsymbol{s}. \tag{72}$$

This is a nonsquare MNCP because (64), (66), and (72) cannot be solely matched to either variables $\boldsymbol{y}_0$, $\boldsymbol{s}$, or $\underline{\boldsymbol{\vartheta}}_\nu$. In order to recover the initial complementarity condition of (51), expressions (68)–(70) can be alternatively defined as [19]

$$\begin{aligned} &\mathbf{0} \leq \boldsymbol{y}_0 \quad \perp \quad \boldsymbol{s} \geq \mathbf{0}, \quad \forall \nu \\ &\mathbf{0} \leq \boldsymbol{y}_0 \quad \perp \quad \boldsymbol{\psi}_\nu \geq \mathbf{0}, \quad \forall \nu \\ &\mathbf{0} \leq \boldsymbol{s} \quad \perp \quad \boldsymbol{\sigma}_\nu \geq \mathbf{0}, \quad \forall \nu \\ &\boldsymbol{\xi}_\nu \geq \mathbf{0}, \quad \forall \nu. \end{aligned} \tag{73}$$

In addition, a slack variable $\boldsymbol{\tau}_\nu$ can be introduced in constraint (67) in order to have only equality constraints

$$\boldsymbol{h}_\nu - \boldsymbol{\tau}_\nu = 0, \quad \forall \nu \tag{74}$$
$$0 \leq \boldsymbol{\tau}_\nu \quad \perp \quad \boldsymbol{\phi}_\nu \geq 0, \quad \forall \nu. \tag{75}$$

To overcome the violation of the constraint qualifications, all the complementarity conditions are now formulated as NLP constraints with the form $\boldsymbol{a}^T \boldsymbol{b} \leq 0$ with $\boldsymbol{a}, \boldsymbol{b} \geq 0$, and all the terms of the form $\boldsymbol{a}^T \boldsymbol{b}$ are aggregated in an objective function to be minimized; this results in the following NLP problem:

$$\min \sum_\nu \left\{ \boldsymbol{\phi}^T_\nu \boldsymbol{\tau}_\nu + \boldsymbol{\sigma}^T_\nu \boldsymbol{s} + \boldsymbol{\psi}^T_\nu \boldsymbol{y}_0 \right\} + \boldsymbol{y}^T_0 \boldsymbol{s} \tag{76}$$
$$\text{s.t.}\ \mathbf{0} = \nabla_{\boldsymbol{x}_\nu} f - \nabla^T_{\boldsymbol{x}_\nu} \boldsymbol{h}_\nu \boldsymbol{\phi}_\nu - \nabla^T_{\boldsymbol{x}_\nu} \boldsymbol{h}_{\mathcal{E}_\nu} \bar{\boldsymbol{\vartheta}}_\nu, \quad \forall \nu \tag{77}$$
$$\mathbf{0} = \nabla_{\boldsymbol{y}_0} f - \nabla^T_{\boldsymbol{y}_0} \boldsymbol{h}_{\mathcal{E}_\nu} \bar{\boldsymbol{\vartheta}}_\nu - \nabla^T_{\boldsymbol{y}_0} \boldsymbol{h}_{\mathcal{I}_\nu} \underline{\boldsymbol{\vartheta}}_\nu + \boldsymbol{s} \circ \boldsymbol{\xi}_\nu - \boldsymbol{\psi}_\nu, \quad \forall \nu \tag{78}$$
$$\mathbf{0} = \nabla_{\boldsymbol{y}_1} f - \nabla^T_{\boldsymbol{y}_1} \boldsymbol{h}_{\mathcal{E}_\nu} \bar{\boldsymbol{\vartheta}}_\nu - \nabla^T_{\boldsymbol{y}_1} \boldsymbol{h}_{\mathcal{I}_\nu} \underline{\boldsymbol{\vartheta}}_\nu, \quad \forall \nu \tag{79}$$
$$\mathbf{0} = \underline{\boldsymbol{\vartheta}}_\nu + \boldsymbol{y}_0 \circ \boldsymbol{\xi}_\nu - \boldsymbol{\sigma}_\nu, \quad \forall \nu \tag{80}$$
$$\mathbf{0} = \boldsymbol{h}_\nu - \boldsymbol{\tau}_\nu, \quad \forall \nu \tag{81}$$
$$\mathbf{0} = \boldsymbol{h}_\mathcal{E} \tag{82}$$
$$\mathbf{0} = \boldsymbol{h}_\mathcal{I} - \boldsymbol{s} \tag{83}$$
$$\mathbf{0} \leq \boldsymbol{\tau}_\nu, \boldsymbol{\phi}_\nu, \boldsymbol{\psi}_\nu, \boldsymbol{\sigma}_\nu, \boldsymbol{\xi}_\nu, \quad \forall \nu \tag{84}$$
$$\mathbf{0} \leq \boldsymbol{y}_0, \boldsymbol{s}. \tag{85}$$

With no standing objective function to account for, there is no need for a penalty factor for the aggregated terms. With a slight variation of constraints (73), Leyffer and Munson [19] enforce a stronger penalization (two times the number of leaders) on the complementarity constraints associated with the follower terms. The larger the number of leaders in the market, the stronger the penalization on the ISO complementarity conditions. The NLP formulation (76)–(85) aims to provide a feasible solution to the EPCC by leading all complementarity conditions to zero while enforcing all the original system constraints, which are now well defined. An objective value of zero means that a feasible solution has been found for the EPCC. Such a solution may or may not be a Nash Equilibrium for the leader-follower game. In our implementations, an objective function value of the order of 10^{-4} sufficed to yield a reasonable solution to the game problem.

VI. Computational Results

Let us consider the three-node system shown in Fig. 1. The transmission network consists of three identical transmission

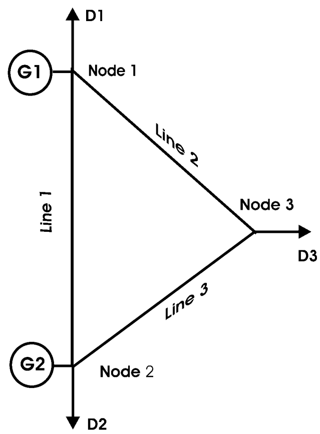

Fig. 1. Three-node power system.

TABLE I
GENERATION DATA FOR TWO GENCOS

GenCo ν	β_v ($\$/MWh$)	γ_v ($\$/MW^2h$)	MW point	MVAR min	MVAR max
1	15	0.004	0	-600	600
			3000	-400	400
2	20	0.004	0	-600	600
			3000	-400	400

TABLE II
NODE CHARACTERISTICS

Node i	$\underline{V}_i$ (pu)	$\overline{V}_i$ (pu)	$\rho_{o\,i}$ ($\$/MWh$)	δ_i ($\$/MW^2h$)	Q_{Li} (MVAR)
1	0.97	1.03	120	0.05	250
2	0.97	1.03	120	0.05	250
3	0.97	1.03	125	0.05	250

lines; they are accordingly labeled in Fig. 1. The lines parameters are $R_{ij} = 0.005, X_{ij} = 0.01$ and $b_{ij}^{\text{sh}} = 0.4$. They are in per unit (p.u.) with a base of 100 MVA; this network setting is taken from [30]. For the sake of illustration, only line 2 has a limit of 350 MVA.

There are two GenCos, which are placed at nodes 1 and 2 and can provide active and reactive power. Their generation characteristics are given in Table I. Demands for active power are considered to be price responsive, while reactive demands are considered as inelastic. Voltage and demand requirements are listed in Table II.

A. *Case A: Competitive Setting*

In a first case, both GenCos are assumed to behave competitively. For this setting, both GenCos are scheduled to supply active and reactive power. The highest price is at node 3, where there is no generation. Congestion arises because both the upper voltage limit at node 2 and the power flow limit of line 2 are binding. In addition, GenCo 2 is providing reactive power on its D-curve limit. Results are summarized in Tables III and IV. Net losses in the transmission system are 20.74 MW.

B. *Case B: Cournot Setting*

In this study case, both GenCos are assumed to behave *à la* Cournot. Thus, both are leaders competing against each other,

TABLE III
MARKET OUTCOME UNDER DIFFERENT STRATEGIES

Node i	V_i (pu)	λ_i $(\$/MWh)$	θ_i $(Degrees)$	d_i (MW)	Q_{Li} $(MVAR)$
			Competitive		
1	1.012	24.312	0	956.875	250
2	1.030	32.206	0.475	877.928	250
3	0.989	42.644	-1.775	823.588	250
			Cournot		
1	1.018	58.375	0	616.254	250
2	1.030	58.585	-0.635	614.150	250
3	0.996	60.055	-1.844	649.454	250

TABLE IV
GENCO'S PROFIT UNDER DIFFERENT STRATEGIES

GenCo ν	g_ν (MW)	q_ν $(MVAR)$	Π_ν $(\$/hr)$
		Competitive	
1	1,164.080	169.918	5,420.391
2	1,515.022	498.998	9,312.411
		Cournot	
1	1,012.411	111.788	39,812.95
2	880.805	541.279	30,882.53

whereas the demands and the ISO behave competitively (they are the follower). Results are summarized in Tables III and IV. Due to strategic behavior, demands drop around one third of consumption; all nodal prices goes up to a range of \$60/MWh. As a result, profits increase from \$5420.4/hr to \$39 812.9/MWh for GenCo 1 and from \$9312.4/hr to \$30 882.5/hr for GenCo 2. However, the voltage profile only changes slightly. Because less demand is consumed, losses are reduced to 13.35 MW.

Observing the generation levels, both GenCos withhold active generation to exercise market power, as one may expect; for reactive power, in contrast, GenCo 2 increases generation. This seemingly counter-intuitive result comes from the fact that reactive power is used just as a means to enhance market power, as there is no profit component for reactive power in the objective function and also because reactive demand is fixed. Hogan [4] has shown that even within an active-power-only market, GenCos can exercise market power by increasing generation in order to congest the system.

It is worth mentioning the fact that, at the solution point, Cournot suppliers set generation outputs at a level where the system becomes weakly congested. Congestion can arise from constraints of either nodal voltage levels or transmission lines flows. It turns out that in this outcome, voltage at node 2 is on the upper limit of 1.03 p.u., and the apparent flow in transmission line 2 is also on its limit of 350 MVA. However, their corresponding dual variables, i.e., the associated congestion multipliers, are both zero. From the optimization point of view, this is nonstrict complementarity [31]. This fact may also imply a kind of degeneracy at the solution point. Hence, there is no congestion or the system is weakly congested. However, this is not a surprising fact at all if we recall the rationality behind the Cournot setting. These weakly active constraints simply reflect the fact that Cournot suppliers are able to identify their impact on all the system constraints and prices, and therefore, they choose outputs at a level where they can profit the most. If they choose their outputs differently, congestion may arise and then prices change with a corresponding drop of profits. This effect was first highlighted by Oren [5] and later by Hobbs and Helman [17] using three- and two-node systems with a (dc) linear approximation. With the example shown in this section, it is clear that dominant suppliers will exploit not only constraints related to active power but also to reactive power, such as MVA and voltage limits; this is the by-product of the Cournot rationality. The outcome from this sophisticated behavior also means that Cournot suppliers are able to capture the ISO congestion rents. Moreover, GenCo 2 is also on the limit of its capability curve. If its generation is 880.8 MW, then the reactive limit given by its D-curve is $600 - (2/30) \times 880.8 = 541.27$ MVAR. Similarly, the associated dual variable for its D-curve limit is also zero.

C. Characterization of Solutions

With the vast work done for NLP problems, the formulation of the game as an NLP problem allows one to have available different tools to tackle the problem. Nonetheless, there are three inherent facts to be considered with an NLP formulation.

1) Number of Variables: Because the optimality conditions for each leader are derived and then set as constraints, there are more (dual) variables to be used within this formulation. For instance, for the three-node system above described, the number of variables is in the range of ten for the classical cost minimization problem; in the range of 80 variables for a diagonalization approach; and in the range of 200 for this NLP formulation. Accordingly, the number of constraints in the formulation increases because the ISO constraints are compounded. All this has a final impact on the performance of the solver. For instance, using the solver CONOPT on NEOS, [29], the classical OPF takes in the range of 30 iterations, while for the game problem, it can take between 60 and 300 iterations, depending on the starting point.

2) Nature of the Solution: In this NLP formulation, a computed solution fits the classical definition of stationary points in NLP theory. We still need to verify the nature of the computed solution to determine whether or not it is an actual maximum point for the game problem. Because the leader problems are highly nonconvex, only local optimality can be achieved by using standard NLP solvers, which are based on local optimization techniques. Current research is being carried out on verifying the second-order optimality conditions. A practical approach implemented so far has been to carry out a heuristic check for local optimality of the solution points computed from the NLP formulation of the game (76)–(85). First, a classical OPF [similar to (13)–(21)] is used to emulate the outcome of the game. In this OPF, the leaders are modeled as competitive players, together with the competitive fringe. In this way, we have a competitive setting of the market under study, and the strategic behavior of leaders is emulated by fixing the leaders' variables to the levels given by the outcome of the game. Therefore, a solution point for leader ν (i.e., the generation levels of active and reactive power, x_ν^*) computed from the game is emulated with the OPF. The generation levels of all leaders except ν are fixed to the values obtained from the game. Then for leader ν, different generation levels are checked in a neighborhood of the solution to the game. We do this as follows: 1) the generation levels are chosen by varying the entries of x_ν^* by up to $\pm 1\%$, in such a way that any perturbed generation level remains feasible; and 2) for each of these points, the profit of leader ν is calculated using an OPF. This allows us to have a reasonable sense of whether or not the profit in its neighborhood can be improved. If any perturbed value gives a higher profit, the solution point of the game under study is disregarded. This procedure is applied in turn to each leader. It is cheap to implement because it can be done by analyzing as few as two points (lower and higher values with respect to the solution point) per decision variable. This analysis is simply a heuristic check to determine whether or not a computed solution is a Nash point, i.e., that no leader can unilaterally improve its profits. Of course, more than two points can be checked in a neighborhood to get a more conclusive insight into the optimality of the computed solutions. For instance, for the three-node system described above, 30 points for each generation point were checked. The profit curves for the neighborhood of the solution point for each GenCo are presented in Figs. 2 and 3. Graphically, it can be seen that for both GenCos, the market outcome is a Nash equilibrium, as there is no more profitable point in their corresponding neighborhood. Hence, no GenCo can improve its profit unilaterally. With this post-optimality analysis, Nash equilibria are verified but also spurious points (from the game point of view) are disregarded. Among feasible solutions of the EPCC problem, three kinds of points have been identified: 1) Nash equilibria, where no leader can improve its profits unilaterally; 2) saddle points, where leaders cannot increase profits through one decision variable, say, active power, but they are able to increase profit with the other variable, say, reactive power; and 3) minimum points, where the solution point attains a maximum value for some leaders but attains a minimum value for other leaders.

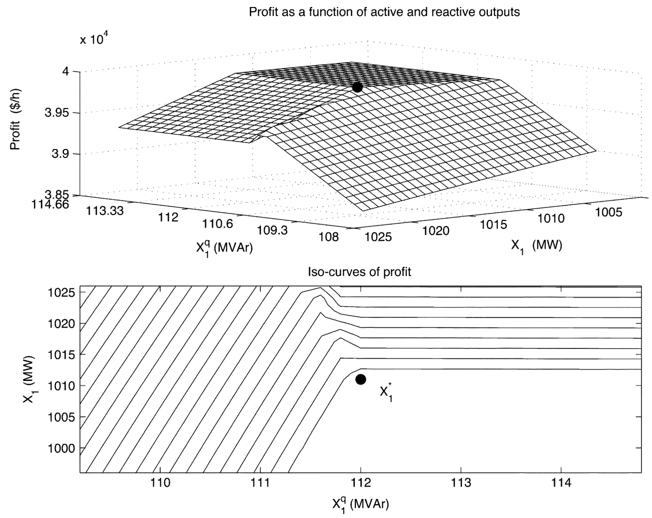

Fig. 2. Profit functions for GenCo 1 around the solution point.

3) Starting Point: Because of the nonconvexity of the NLP formulation, sometimes a global minimizer (i.e., a point where the sum of all the slack complementarities is nonzero) may not be attained, which also implies that a feasible solution for the game is not attained. This problem can typically be overcome by using different starting points. A flat starting point is commonly used for a classical OPF, where voltage magnitude are set to 1 p.u., nodal angles set to zero, and generation levels set to average values; also, dual variables are usually set to zero.

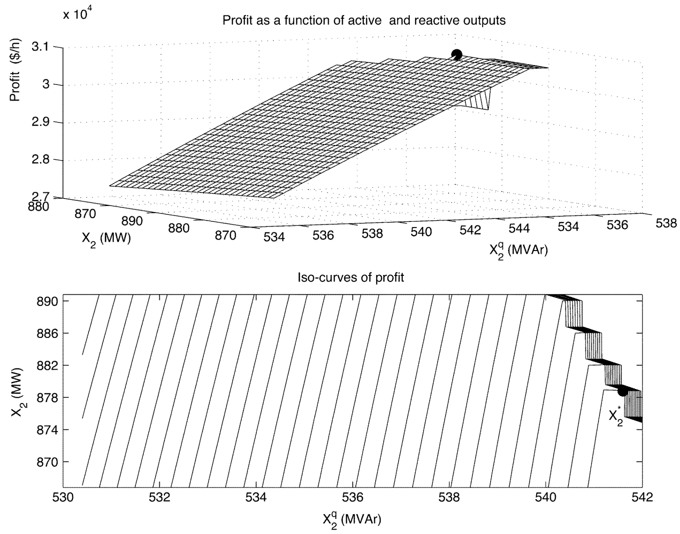

Fig. 3. Profit functions for GenCo 2 around the solution point.

TABLE V
MARKET OUTCOME UNDER COURNOT BEHAVIOR FOR EQUILIBRIUM II

Node i	V_i (pu)	λ_i $(\$/MWh)$	θ_i $(Degrees)$	d_i (MW)	Q_{Li} $(MVAR)$
1	1.030	60.695	0	593.052	250
2	1.017	60.494	0.622	595.050	250
3	0.997	62.186	-1.158	628.140	250

TABLE VI
GENCO'S PROFIT UNDER COURNOT BEHAVIOR FOR EQUILIBRIUM II

GenCo ν	g_ν (MW)	q_ν $(MVAR)$	Π_ν $(\$/hr)$
1	854.794	543.013	36,136.896
2	974.128	108.677	35,650.688

For the proposed ac model, the starting point has been set randomly (around a flat start) in order to search a broader region. An important feature to mention is that for 250 random starting points, less than 25 times a solution with a nonzero objective function was attained (a total complementary slackness of 10^{-5} or less has shown to be accurate enough). For the rest of solutions, more than 100 times the same optimal solution point was attained, and more than 50 of those points attained a complementarity slackness of (numerical) zero.

4) Multiple Equilibria: For the illustrative example shown in this section, multiple Nash equilibria were found. A second equilibrium point is next presented. The market outcome associated with this equilibrium is summarized in Tables V and VI. It turns out that this second equilibrium point shows a kind of symmetry, in terms of reactive power, with respect to the equilibrium described in Section VI-B. In this second equilibrium, the reactive power levels of GenCos are interchanged. Now GenCo 1 is providing most of it, and it is also on the limit of its D-curve. Because of this, now the voltage limit is binding at node 1. In addition, there are higher nodal prices with the resulting lower demands. Still, the feature of weakly active constraints arises at this equilibrium point for the constraints of power flow in line 2, voltage at node 1, and D-curve of GenCo 1. Losses decrease slightly to 12.67 MW.

TABLE VII
COMPARISON OF COMPUTATIONAL ISSUES FOR DIFFERENT MARKET SETTINGS

No. Leaders	Variables	Constraints	Iterations	Time (sec)
0	53	87	12	0.13
2	1543	925	266	53.20
3	2171	1295	366	252.86
4	2799	1665	415	245.16

TABLE VIII
GENCO'S PROFIT IN $/HR WITH DIFFERENT NUMBER OF LEADERS

	Leaders			
GenCo ν	–	1, 2	1, 2, 3	1, 2, 3, 4
1	448.25	2,934.03	4,083.95	5,746.03
2	1,444.86	3,071.06	4,523.34	6,731.63
3	459.65	961.54	3,359.40	4,895.60
4	36.72	133.81	125.90	2,458.15
5	9,735.74	9,706.53	9,922.92	10,163.62

TABLE IX
NODAL PRICES IN $/MWH WITH DIFFERENT NUMBER OF LEADERS

	Leaders			
Node i	–	1, 2	1, 2, 3	1, 2, 3, 4
1	23.99	38.58	44.39	46.21
2	32.54	42.22	48.91	49.64
3	58.27	58.06	58.79	58.11
4	23.14	24.39	45.99	44.52
5	57.75	58.54	57.46	57.27
6	52.33	52.26	52.80	53.40
7	32.32	36.63	44.91	46.73
8	53.57	53.63	54.08	53.50
9	28.00	35.24	40.04	46.25
10	52.67	52.98	53.43	53.60
11	59.35	59.18	59.37	59.06
12	55.51	54.84	56.73	56.08
13	62.95	63.01	63.09	63.03
14	22.85	23.63	23.58	44.39

Interestingly, other Nash equilibria have been identified. A key feature of these equilibria is that they always occur in the field limit of the D-curve of either generator. This may suggest a kind of continuum of equilibria along the D-curve. To some extent, this may come from the trade-off between active and reactive generation that the D-curve introduces. However, this is just the reality of the generation limits. If the D-curve is simplified to box constraints (constant maximum and minimum levels for reactive power), all these equilibria would not be captured.

D. Fourteen-Node System

In a second simulation, the standard IEEE transmission system of 14 nodes and 20 transmission lines has been used to simulate a larger market [32]. Transmission limits were set so that some limits would be binding in the solutions. Furthermore, five GenCos are considered; for each GenCo, quadratic generation costs and D-curves were included. There are eight nodes with active and reactive demands. In Table VII, a summary of the computational requirements for different market settings is shown. For these simulations, the solver SNOPT has been used under GAMS, via NEOS, on a Pentium IV, 1.6-GHz PC [29]. Although there are over 1000 variables and constraints for two or more leaders, the computation is inexpensive. GenCos' profits and nodal prices for such settings are presented in Tables VIII and IX, respectively. Each column has the market outcome of the setting for the GenCos listed in the head acting as leaders. Notice how nodal prices and profits increase as the number of leaders increases.

VII. Conclusion

The dc approximation may be a poor approach in the study of market power since it neglects the impact of reactive power. Although reactive power can be produced cheaply, it may impact the active power generation and may bias the market outcome because 1) active and reactive power are coupled through the capability curve; and 2) reactive power congests the transmission system to the same extent as active power. Transmission lines capacity is defined in MVA; hence, if a transmission line becomes congested, active and reactive power will be competing for transmission capacity.

The proposed formulation extends the game theoretical approach of market power studies to the reactive power side, since it formally accommodates the potential strategic behavior with reactive power. Notice that in this formulation, no market for reactive power is assumed. Thus, reactive power is not considered as an explicit good. Instead, it is modeled as a support to provide active power. Therefore, reactive power is taken as a means to enhance market power in the active-power market. This setting is used because it better represents some real-world markets, where usually there is an incipient market for reactive power, and payments for the provision of reactive power are set apart.

In addition, this formulation comprises a detailed ac model where issues of reactive power and voltage constraints can be explicitly analyzed. Given the formulation, it is straightforward to study market power for the provision of reactive power. Also, a market for reactive power with either fixed or price-responsive demand of reactive power can be included, and stability constraints can be further analyzed. The formulation based on polar coordinates also has been implemented; however, a disappointing performance is observed. We suspect this poor performance is due to the highly nonconvex nature of the model obtained in terms of trigonometric functions. As a result, the solvers simply fail to find local optima.

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