Numerical Study of Affine Supply Function Equilibrium in AC Network-Constrained Markets

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***Abstract*—An affine supply function equilibrium (SFE) approach is used to discuss voltage constraints and reactive power issues in the modeling of strategic behavior. Generation companies (GenCos) can choose their bid parameters with no restrictions for both energy and spinning reserves. The strategic behavior of generators is formulated as a multi-leader single-follower game. Each GenCo is modeled as a leader, while the central market operator running a cost minimization process is the sole follower. An ac model is considered to represent the transmission system. A three-node system is used to illustrate several cases, and study the implications of the incentives of the strategic players to exploit active and reactive power, and spinning reserves in order to maximize profits. Results for a 14-node system are also presented.**

***Index Terms*—Ac system, complementarity, market power, mathematical problem with complementarity constraints, Nash equilibrium, oligopoly, spinning reserves, supply function equilibrium.**

A. Indices and Sets

h	Indices for generation units.
i, j	Indices for nodes in the system.
ij	Composed index for a transmission line between nodes i and j.
ν	Index for generation companies (GenCos).
$\mathcal{H}_{\nu,i}$	Set of generation units owned by ν and placed at i.
$\mathcal{N}$	Set of nodes.
$\mathcal{L}$	Set of transmission lines.

B. Constants

a^f, b^f	Intersection and slope for the field limit approximation of the D-curve (MVAR, MVAR/MW).
a^u, b^u	Intersection and slope for the under-excitation limit approximation of the D-curve (MVAR, MVAR/MW).
δ_i	Slope for demand function at i ($\$/\mathrm{MW}^2\mathrm{h}$).
$\beta_{\nu,h,i}$	Linear parameter of the cost function of unit ν, h, i ($/MWh).
$\gamma_{\nu,h,i}$	Quadratic parameter of the cost function of unit ν, h, i ($\$/\mathrm{MW}^2\mathrm{h}$).
ρ_{o_i}	Price intercept for demand curve at i ($/MWh).
D_{L_i}	Fixed demand of active power at i (MW).
Q_{L_i}	Fixed demand of reactive power at i (MVAR).
R_L	Spinning reserves requirement (MW).

C. Variables

d_i	Demand of active power at i (MW).
e_i, f_i	Real and imaginary coordinates of voltage at i (p.u.).
$g_{\nu,i,h}$	Active power produced by unit ν, h, i (MW).
$q_{\nu,i,h}$	Reactive power produced by unit ν, h, i (MWAR).
$r_{\nu,i,h}$	Spinning reserve provided by unit ν, h, i (MW).
P_i, Q_i	Active and reactive power injections at i (MW, MVAR).
$\tilde{S}_{ij}$	Average apparent power flow in line ij (MVA).
V_i	Voltage magnitude at i (p.u.).
λ_i	Energy price at i ($/MWh).
μ^r	Spinning reserves price ($/MWh).
ε	Percentage of the demand to be set as spinning reserves (p.u.).
$\hat{\beta}_{\nu,h,i}$	Linear bid parameter for unit ν, h, i.
$\hat{\gamma}_{\nu,h,i}$	Quadratic bid parameter for unit ν, h, i.
$\hat{\alpha}_{\nu,h,i}$	Spinning reserve bid parameter for unit ν, h, i.

D. Symbols

$\bar{(\cdot)}$, $\underline{(\cdot)}$	Maximum and minimum values for $(\cdot)$.
$(\cdot)^\star$	Value for $(\cdot)$ at equilibrium.
$\lvert(\cdot)\rvert$	Magnitude of $(\cdot)$.
$a \perp b$	Complementarity condition between a and b.
$\boldsymbol{\varphi}_{-\nu}$	It means $\boldsymbol{\varphi}_\ell\ \forall \ell \neq \nu$.

I. Introduction

With the introduction of competition in power systems, one of the main concerns is the strategic behavior of market participants. Several simulation models have been proposed to characterize oligopolistic competition in electricity markets. The models vary depending on the rationality taken for competition, the degree of stylization of the transmission system, the market clearing scheme, and the incentives from

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other market activities, among others. From the classical theory of industrial organization, the Cournot–Nash competition is the most common game-theoretical framework to model the interaction among participants. This game is in terms of quantities, which are the generation levels in the context of power systems. In order to maximize profits, each generation company (GenCo) takes its rivals' decisions variables as fixed within its profit-maximization problem. In this case, the strategic variables of GenCos are the generation levels, and the market equilibrium is such that no GenCo (or any other market participant) can improve its profit unilaterally by altering its generation levels.

Among market structures, there is a centralized version in which a central market operator is in charge of clearing the market and managing the transmission congestion simultaneously. Market participants, both GenCos and demands, can submit bids to trade power. Then the market operator implements a cost-minimization process, typically an optimal power flow (OPF), to clear the market. Although the bid format varies from market to market, usually the GenCo's bids are a supply function in the same form of the marginal cost function [1]. Because of this bid-based format, a Cournot framework (in terms of quantities) may not represent well the interaction among participants. This concern has motivated the development of alternative models for strategic behavior, using a supply function equilibrium (SFE) approach. In this case, the strategic variables are the parameters that define the GenCo bids, and the market equilibrium is such that no GenCo can improve its profit unilaterally by altering its bids.

The SFE approach was originally developed by Klemperer and Meyer [2]. The authors provide a general functional form for demands, costs, and supply functions, and they characterize the SFE with differential equations. Because of the complexity to solve for a general SFE, several studies have relied on the analysis using an affine SFE, where the marginal cost, demand, and the supply function are considered to be affine, and players are able to manipulate the coefficients of its affine supply functions [1], [3]–[5]. The affine SFE approach assumes that the bid-based market rules require the bid of a supply function that is similar in form to the affine marginal cost function [6]. In this paper, an affine SFE approach is used to study the interactions of suppliers in a joint market for energy and reserves. In [3], the first SFE model for electricity markets is proposed. The authors simulate a multi-period market where participants submit the same bid for all periods under uncertainty in demand. However, the transmission system is not considered. In [4], a model that accounts for the transmission system impact, using a linear (dc) approximation, is proposed. Each strategic player problem is formulated as a mathematical problem with complementarity constraints (MPCC) and solved by a penalized interior-point algorithm. A heuristic scheme is then used to set the starting points and to build the bids in their algorithm. In [5], an ac model is used to account for the transmission system impact. The sensitivities of an ac OPF are used to determine the changes in the players' bids. An iterative procedure is then used to find the optimal bids for each player. Although in that paper the authors use an ac model, the study focuses only on the active power, as it is usually done with the dc approach. In this context, the gain from using the ac model was to account accurately for the transmission losses.

The use of dc approximations for modeling strategic behavior is ubiquitous in the technical literature. Dc linear approximations are suitable to calculate active power flows and voltage angles, but they do not provide any insight regarding voltage magnitudes and reactive power flows. The aim in using an ac approach is to capture the reactive power and voltage issues that a dc model cannot [7]. The ac model introduces significant technical complications though, and this explains its limited use. The main assumption for the usual dc approximation is that sufficient reactive power compensation is available at all nodes to keep voltage levels constant at one per unit values. However, power systems operate within a voltage band. This provides some degree of flexibility in the operation of the system to accommodate insufficient local provision of reactive power. With the introduction of competition, the transmission system is used more intensively, resulting in more frequent congestion from voltage limits. Therefore, deviations from the ideal case of one per unit voltage profile assumed for the dc model may not be negligible.

The study of reactive power in a market-oriented environment is a complex matter; even its pricing is not a resolved issue [8]–[10]. Nonetheless, it is of paramount importance in the operation and security of the system. In the study of strategic behavior in electricity markets, a common practice to account for the impact of the transmission system, i.e., for congestion, is to use fixed thermal limits in MW. However, voltage constraints are also a source of congestion. Therefore, reactive power and voltage constraints may have an impact on the strategic behavior of market participants as well. Due to the complexities in implementing an ac-based model, this impact has been overlooked in the study of market power. In [11], a detailed ac model is used to represent the transmission system for a multi-leader single-follower Cournot game. It is shown how GenCos can manipulate not only active but also reactive generation to maximize their profits. In this paper, the work in [11] is extended to the study of strategic behavior with an affine SFE model.

Besides markets for energy, there are markets for spinning reserves. Joint markets allow one to price and procure energy and spinning reserves (SR) simultaneously and are becoming more common [12], [13]. With this approach, the coupling nature of resources, in addition to the system constraints, can be explicitly considered. Hence, a greater consistency between prices and the physical dispatch can be achieved. This fact makes the market prices represent more accurately the actual value of energy and SR. Moreover, generators can supply different commodities, namely, 1) active power, 2) reactive power, and 3) spinning reserves. These commodities are interrelated and limited by the generators' capability curve, known as D-curve. However, the provision of one commodity can affect the provision of the others, as the generator's capacity is consumed. This gives rise to opportunity costs due to the trade-offs among commodities. As GenCos may participate and profit from both energy and SR markets, they have to determine their optimal bids for both markets. As in the energy-only market, issues of strategic behavior can arise in joint energy and SR markets. The strategic behavior of GenCos in joint markets has been studied using Cournot [14] and conjectured functions [15].

The contribution of this paper is threefold. First, a detailed model for the formulation of an affine SFE in electricity markets is used. This model includes voltage constraints, MVA line limits, and a linear approximation of the generators' capability curve. The SFE formulation allows one to model the flexibility of GenCos to manipulate the coefficients of the energy and SR bids. Second, the results from experimenting with different computational strategies for solving the SFE model are briefly discussed. Third, a numerical study of the impact of voltage constraints, reactive power, and SR on the incentives for strategic behavior of generators is presented.

This paper is organized as follows. In Section II, the affine SFE model using an ac system is presented. Several study cases are discussed in Section III. Conclusions are given in Section IV.

II. SFE Formulation

Let us consider a pool-like market in which suppliers and consumers submit their respective bids for energy and spinning reserves. A market operator computes a market equilibrium using those bids and taking into account the system constraints. The power market has $|\mathcal{V}|$ generation companies, indexed over $\nu = 1, \ldots, |\mathcal{V}|$. Each GenCo is composed of a set $\mathcal{H}_{\nu,i}$ of generation units placed at node i. The true cost function of each generation unit is represented as

$$c_{\nu,h,i}(g_{\nu,h,i}) = \beta_{\nu,h,i} g_{\nu,h,i} + \gamma_{\nu,h,i} g_{\nu,h,i}^2, \quad \forall \nu, h, i \qquad (1)$$

where $\beta_{\nu,h,i} \in \mathbb{R}$ and $\gamma_{\nu,h,i} \in \mathbb{R}_+$ are parameters associated with the linear and quadratic terms, and $g_{\nu,h,i}$ is its generation output. Although the value of $\beta_{\nu,h,i}$ is unrestricted, the value of $\gamma_{\nu,h,i}$ is assumed to be nonnegative to have a convex cost function. The corresponding marginal cost function characterizes the true cost bid, which is the inverse of the supply function, i.e.,

$$c'_{\nu,h,i}(g_{\nu,h,i}) = \beta_{\nu,h,i} + 2\gamma_{\nu,h,i} g_{\nu,h,i}, \quad \forall \nu, h, i. \qquad (2)$$

However, in a pool-like market, GenCos submit bids that do not necessary represent their true costs. A GenCo will submit a bid defined by a linear ($\hat{\beta}_{\nu,h,i} \in \mathbb{R}$) and a quadratic ($\hat{\gamma}_{\nu,h,i} \in \mathbb{R}_+$) parameter. Thus, the parameterized bid for each generation unit is

$$c'_{\nu,h,i}(g_{\nu,h,i}) = \hat{\beta}_{\nu,h,i} + 2\hat{\gamma}_{\nu,h,i} g_{\nu,h,i}, \; \forall \nu, h, i. \qquad (3)$$

In addition, the cost for providing SR is represented by

$$c^R_{\nu,h,i}(r_{\nu,h,i}) = \alpha_{\nu,h,i} r_{\nu,h,i}, \quad \forall \nu, h, i \qquad (4)$$

where $\alpha_{\nu,h,i} \in \mathbb{R}_+$ is a true cost parameter, and $r_{\nu,h,i}$ is the spinning reserves provided by the generation unit. A generator providing reserves foregoes profits from the energy market. Therefore, the cost of providing reserves is the opportunity cost of not producing energy, which can be estimated as the difference between the energy price and the energy cost and, in some instances, may be zero [16]. The cost for providing SR can also account for other components such as capital and operating costs [17]. Unlike energy bids, the bids for SR cost are usually constant. Hence, a GenCo will submit a bid defined by a constant parameter, $\hat{\alpha}_{\nu,h,i}$. This formulation allows GenCos to choose the value of each bid parameter for energy and SR separately.

A. Formulation of the Game

For an ac power network, the finite set of nodes is denoted by $\mathcal{N}$, and $\mathcal{N}_i$ stands for the set of all nodes directly connected to node i. The set $\mathcal{L} := \{(i,j) | i \in \mathcal{N} \text{ and } j \in \mathcal{N}_i, \text{and } j > i\}$ stands for sending node i and receiving node j of every transmission line in the system. Therefore, the ISO problem can be stated as

$$\min \sum_{\nu,h,i} \left\{ \hat{\beta}^\star_{\nu,h,i} g_{\nu,h,i} + \hat{\gamma}^\star_{\nu,h,i} g^2_{\nu,h,i} \right\} + \sum_{\nu,h,i} \left\{ \hat{\alpha}^\star_{\nu,h,i} r_{\nu,h,i} \right\} - \sum_i \left\{ \rho_{oi} d_i - \delta_i d_i^2 \right\} \qquad (5)$$

s.t.

$$\sum_{\nu,h} g_{\nu,h,i} - d_i - D_{Li} - P_i(\boldsymbol{e},\boldsymbol{f}) = 0, \quad : \lambda_i, \forall i \qquad (6)$$

$$\sum_{\nu,h} q_{\nu,h,i} - Q_{Li} - Q_i(\boldsymbol{e},\boldsymbol{f}) = 0, \qquad : \lambda_i^q, \forall i \qquad (7)$$

$$\sum_{\nu,h,i} g_{\nu,h,i} \geq R_L, \qquad : \mu^r \qquad (8)$$

$$\left| \tilde{S}_{ij}(\boldsymbol{e},\boldsymbol{f}) \right|^2 \leq \overline{S}^2_{ij}, \qquad : \overline{\mu}_{ij}, \qquad \forall (i,j) \in \mathcal{L} \qquad (9)$$

$$\underline{V}_i^2 \leq V_i^2(\boldsymbol{e},\boldsymbol{f}) \leq \overline{V}^2_i, \quad : \underline{\mu}_i^v, \overline{\mu}_i^v, \quad \forall i \qquad (10)$$

$$g_{\nu,h,i} + r_{\nu,h,i} \leq \overline{g}_{\nu,h,i}, \quad : \overline{\mu}^g_{\nu,h,i}, \quad \forall \nu, h, i \qquad (11)$$

$$\underline{g}_{\nu,h,i} \leq g_{\nu,h,i} \quad : \underline{\mu}^g_{\nu,h,i}, \quad \forall \nu, h, i \qquad (12)$$

$$q_{\nu,h,i} \leq b^f_{\nu,h,i} g_{\nu,h,i} + a^f_{\nu,h,i}, \; : \overline{\mu}^q_{\nu,h,i}, \forall \nu, h, i \qquad (13)$$

$$q_{\nu,h,i} \geq b^u_{\nu,h,i} g_{\nu,h,i} + a^u_{\nu,h,i}, \; : \underline{\mu}^q_{\nu,h,i}, \forall \nu, h, i \qquad (14)$$

$$r_{\nu,h,i}, d_i \geq 0, \qquad \forall \nu, h, i. \qquad (15)$$

The problem (5)–(15) is a classical OPF in rectangular coordinates. The objective function (5) stands for the net cost to be minimized, given the bids of all market participants. Its first term stands for the generation cost defined by the bid functions of all GenCos. The second term stands for the cost to get the requirements of SR. The third term represents the demands' benefit. The benefit of the demand at node i is given by a non-decreasing concave function, where ρ_{o_i} and δ_i are parameters to denote the price intersection and demand slope, respectively, and d_i stands for the demand level. The demands are assumed to be competitive.

If a GenCo is modeled as competitive, then $\hat{\beta}^\star_{\nu,h,i} \equiv \beta_{\nu,h,i}$, $\hat{\gamma}^\star_{\nu,h,i} \equiv \gamma_{\nu,h,i}$, and $\hat{\alpha}^\star_{\nu,h,i} \equiv \alpha_{\nu,h,i}$, since it will bid its true costs. On the other hand, if a GenCo is assumed to behave strategically, then it will bid parameters $\hat{\beta}^\star_{\nu,h,i}$, $\gamma^\star_{\nu,h,i}$, and $\hat{\alpha}^\star_{\nu,h,i}$.

Because the bid parameters are exogenous within the OPF problem (5)–(15), they are denoted with the symbol $\star$, but they are decision variables within each leader problem. Hence, given all the players' bids, the market operator dispatches the generation units; i.e., the market operator (control) decision variables are the active and reactive generation as well as the demand levels. Also, the state variables are the voltage angles and magnitudes.

Expressions (6) and (7) are the power flow balances of the system (supply meets demand) for both active and reactive power, respectively. Vectors $\boldsymbol{e} \in \mathbb{R}^{|\mathcal{N}|}$ and $\boldsymbol{f} \in \mathbb{R}^{|\mathcal{N}|}$ are the voltage rectangular components. Any fixed demand for active or reactive power is denoted by D_{L_i} or Q_{L_i}, respectively. Expression (8) is the reserve balance that ensures that the SR requirement is met. The SR requirement can be modeled as a fixed amount or as a percentage (ε) of the total demand of the system [15]; i.e., $R_L = \varepsilon \sum_i (d_i + D_{L_i})$. However, if the SR requirement is defined in this way, there will be an impact of the spinning reserve market on the energy market because the SR is also defined in terms of the demand level of active power.

Expression (9) accounts for the transmission line limits in either direction in terms of the magnitude of the average flow of apparent power (in MVA). Expression (10) stands for the lower and upper nodal voltage limits. Expressions (11) and (12) denote the maximum and minimum generation levels of active power, respectively. Expressions (13) and (14) are linear approximations of the D-curve used to define the maximum amount of reactive power that a generator can produce or adsorb, respectively. These two expressions allow us to model the trade-off between active and reactive power generation.

To the right-hand side of each constraint, its corresponding dual variables are introduced. In particular, the dual variables λ_i, associated with the power flow balance of active power, stand for the locational marginal prices (LMPs).

In compact notation, the problem (5)–(15) can be stated as

$$\min \quad b(\boldsymbol{w}_0, \boldsymbol{w}_1; \boldsymbol{\varphi}) \tag{16}$$
$$\text{s.t.} \quad \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{w}_0, \boldsymbol{w}_1) = \boldsymbol{0}, \quad : \boldsymbol{\lambda} \tag{17}$$
$$\boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_0, \boldsymbol{w}_1) \geq \boldsymbol{0}, \quad : \boldsymbol{\mu} \tag{18}$$
$$\boldsymbol{w}_0 \geq \boldsymbol{0}. \tag{19}$$

All of the bid parameters for each leader are comprised in a decision-variable vector $\boldsymbol{\varphi}_\nu = (\hat{\boldsymbol{\beta}}^*, \hat{\boldsymbol{\gamma}}^\star, \hat{\boldsymbol{\alpha}}^\star)$, and all the leaders decision vectors are comprised in a vector $\boldsymbol{\varphi} = (\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_{|\mathcal{V}|})$, and the OPF can be seen as parameterized over the bids $\boldsymbol{\varphi}$ of all leaders. The vector $\boldsymbol{w}_0$ has the demand and spinning reserve variables for all periods, while $\boldsymbol{w}_1$ contains the generation (active and reactive) and state system (voltage) variables. All equality and inequality constraints are gathered into the vector constraints $\boldsymbol{c}_{\mathcal{E}}$ and $\boldsymbol{c}_{\mathcal{I}}$, respectively. Similarly, the dual variables associated with the equality and inequality constraints for all periods are comprised into the vectors $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$, respectively. This formulation yields the following compact form of the Karush–Kuhn–Tucker (KKT) conditions of the cost minimization problem:

$$\boldsymbol{0} = \nabla_{\boldsymbol{w}_1} b(\boldsymbol{w}_0, \boldsymbol{w}_1; \boldsymbol{\varphi}) - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{w}_0, \boldsymbol{w}_1)\boldsymbol{\lambda} - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_0, \boldsymbol{w}_1)\boldsymbol{\mu} \tag{20}$$
$$\boldsymbol{0} = \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{w}_0, \boldsymbol{w}_1) \tag{21}$$
$$\boldsymbol{0} \leq \nabla_{\boldsymbol{w}_0} b(\boldsymbol{w}_0, \boldsymbol{w}_1; \boldsymbol{\varphi}) - \nabla^T_{\boldsymbol{w}_0} \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{w}_0, \boldsymbol{w}_1)\boldsymbol{\lambda} \perp \boldsymbol{w}_0 \geq \boldsymbol{0} \tag{22}$$
$$\boldsymbol{0} \leq \boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_0, \boldsymbol{w}_1) \perp \boldsymbol{\mu} \geq \boldsymbol{0}. \tag{23}$$

The symbol $\perp$ is henceforth used to compactly denote a complementarity condition. Because the OPF problem is nonconvex, its KKT conditions are only stationarity conditions, rather than equilibrium conditions. By introducing slack variables ($\boldsymbol{s}$) into the complementarity conditions, the OPF problem is characterized by the following stationary conditions:

$$\boldsymbol{0} = \nabla_{\boldsymbol{w}_1} b(\boldsymbol{w}_0, \boldsymbol{w}_1; \boldsymbol{\varphi}) - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{w}_0, \boldsymbol{w}_1)\boldsymbol{\lambda} - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_0, \boldsymbol{w}_1)\boldsymbol{\mu} \tag{24}$$
$$\boldsymbol{0} = \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{w}_0, \boldsymbol{w}_1) \tag{25}$$
$$\boldsymbol{0} = \nabla_{\boldsymbol{w}_0} b(\boldsymbol{w}_0, \boldsymbol{w}_1; \boldsymbol{\varphi}) - \nabla^T_{\boldsymbol{w}_0} \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{w}_0, \boldsymbol{w}_1)\boldsymbol{\lambda} - \boldsymbol{s}^{\boldsymbol{w}_0} \tag{26}$$
$$\boldsymbol{0} = \boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_0, \boldsymbol{w}_1) - \boldsymbol{s}^{\boldsymbol{\mu}} \tag{27}$$
$$\boldsymbol{0} \leq \boldsymbol{w}_0 \perp \boldsymbol{s}^{\boldsymbol{w}_0} \geq \boldsymbol{0} \tag{28}$$
$$\boldsymbol{0} \leq \boldsymbol{\mu} \perp \boldsymbol{s}^{\boldsymbol{\mu}} \geq \boldsymbol{0}. \tag{29}$$

From participating in both the energy and SR markets, GenCo ν has a net profit given by its revenue minus its costs of each activity

$$\Pi_\nu(\boldsymbol{w}_0, \boldsymbol{w}_1, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_{h,i} \left\{ \lambda_i g_{\nu,i,h} - \beta_{\nu,h,i} g_{\nu,h,i} - \gamma_{\nu,h,i} g^2_{\nu,h,i} \right\} + \sum_{h,i} \left\{ (\mu^r - \alpha_{\nu,h,i}) r_{\nu,i,h} \right\}. \tag{30}$$

A price-maker GenCo will alter its bids in order to influence the market outcome and maximize its profit. Therefore, the bid parameters $\hat{\beta}_{\nu,h,i}$, $\hat{\gamma}_{\nu,h,i}$, and $\hat{\alpha}_{\nu,h,i}$ are the strategic variables of GenCos. The values of $\hat{\beta}_{\nu,h,i}$ and $\hat{\alpha}_{\nu,h,i}$ are unrestricted, but the value of $\hat{\gamma}_{\nu,h,i}$ is assumed to be nonnegative to preserve the positiveness of the bid slope. This nonnegativity requirement is incorporated in the standard box constraints for the parameters in the model below.

Each GenCo has to take into account its rivals' decisions and the market mechanism. A rationality for competition commonly used is to consider that each GenCo chooses its best strategy taking the decisions of its rivals as given. Thus, each GenCo problem is parameterized over its rivals' decisions. Given GenCo ν, its rivals' decisions $\{\boldsymbol{\varphi}_\ell | \ell \neq \nu\}$ are denoted as $\boldsymbol{\varphi}_{-\nu}$; this vector is considered to be fixed within each GenCo problem.

Since the market is characterized by its stationarity conditions (24)–(29), the fact that each GenCo is able to identify how its decisions impact the market is modeled by appending (24)–(29) as constraints in its profit-maximization problem. This gives rise to the following mathematical problem with complementarity constraints (MPCC) to denote each leader problem:

$$\min -\Pi_\nu(\boldsymbol{w}_0, \boldsymbol{w}_1, \boldsymbol{\lambda}, \boldsymbol{\mu}) \tag{31}$$
$$\text{s.t.} \tag{32}$$
$$\underline{\boldsymbol{\varphi}}_\nu \leq \boldsymbol{\varphi}_\nu \leq \overline{\boldsymbol{\varphi}}_\nu \tag{33}$$
$$\nabla_{\boldsymbol{w}_1} b(\boldsymbol{\varphi}_\nu, \boldsymbol{w}_0, \boldsymbol{w}_1; \boldsymbol{\varphi}_{-\nu}) - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{w}_0, \boldsymbol{w}_1)\boldsymbol{\lambda} - \nabla^T_{\boldsymbol{w}_1} \boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_0, \boldsymbol{w}_1)\boldsymbol{\mu} = \boldsymbol{0} \tag{34}$$
$$\boldsymbol{c}_{\mathcal{E}}(\boldsymbol{w}_0, \boldsymbol{w}_1) = \boldsymbol{0} \tag{35}$$
$$\nabla_{\boldsymbol{w}_0} b(\boldsymbol{\varphi}_\nu, \boldsymbol{w}_0, \boldsymbol{w}_1; \boldsymbol{\varphi}_{-\nu}) - \nabla^T_{\boldsymbol{w}_0} \boldsymbol{c}_{\mathcal{E}}(\boldsymbol{w}_0, \boldsymbol{w}_1)\boldsymbol{\lambda} - \boldsymbol{s}^{\boldsymbol{w}_0} = \boldsymbol{0} \tag{36}$$
$$\boldsymbol{c}_{\mathcal{I}}(\boldsymbol{w}_0, \boldsymbol{w}_1) - \boldsymbol{s}^{\boldsymbol{\mu}} = \boldsymbol{0} \tag{37}$$
$$\boldsymbol{0} \leq \boldsymbol{w}_0 \perp \boldsymbol{s}^{\boldsymbol{w}_0} \geq \boldsymbol{0} \tag{38}$$
$$\boldsymbol{0} \leq \boldsymbol{\mu} \perp \boldsymbol{s}^{\boldsymbol{\mu}} \geq \boldsymbol{0}. \tag{39}$$

The set of MPCC resembles a multi-leader single-follower game and conforms an equilibrium problem with complementarity constraints (EPCC).

B. Implementation

In this section, various schemes to solve the problem (31)–(39) are discussed.

1) Mixed Complementarity Formulation: An alternative to solve the EPCC is to form a mixed nonlinear complementarity problem (MNCP). To do this, the complementarity constraints (28)–(29) are expressed in terms of smooth nonlinear programming (NLP) constraints. For instance, expression (28) is defined as $\boldsymbol{w}_0 \circ \boldsymbol{s}^{w_0} = \mathbf{0}$, $\boldsymbol{w}_0 \geq \mathbf{0}$, and $\boldsymbol{s}^{w_0} \geq \mathbf{0}$, where $\circ$ denotes the Hadamard product of $\boldsymbol{w}_0$ and $\boldsymbol{s}$. With this reformulation, each leader problem becomes an NLP problem. Then the KKT conditions of each leader are derived and concatenated to compose a single square MNCP. This formulation was implemented with PATH [18]. Unfortunately, no solution for the game problem could be attained.

2) Nonlinear Programming Formulation: A second alternative is to follow the derivation in [19] to set an NLP formulation of the game. The complementarity constraints are reformulated as NLP inequality constraints; e.g., expression (28) is defined as $\boldsymbol{w}_0 \circ \boldsymbol{s}^{w_0} \leq \mathbf{0}$, $\boldsymbol{w}_0 \geq \mathbf{0}$, and $\boldsymbol{s}^{w_0} \geq \mathbf{0}$. Then the corresponding MNCP is defined as above, and the terms of the kind $\boldsymbol{w}_0 \circ \boldsymbol{s}^{w_0} \leq \mathbf{0}$ are aggregated in an objective function. This manipulation allows one to overcome the violation of the constraint qualifications from the complementarity conditions [19]. Although this formulation was successfully implemented for a Cournot game in [11], for the SFE game, a poor performance is observed. Only for the monopoly case does the formulation reach optimal solutions for the game problem.

3) Diagonalization Approach: As an EPCC is a set of MPCC with shared constraints, the most common solution method is to rely on diagonalization procedures, such as Gauss or Jacobi methods. In this way, the effort is focused on solving one MPCC at a time. MPCCs are still challenging to solve because they have salient features inherited from the complementarity expressions [19], [20]. Computational methods to solve MPCCs are still evolving and are an active area of current research (see, e.g., [21]–[23]). In this paper, each leader problem is solved with NLPEC [24]. This solver tackles MPCCs by reformulating the complementarity constraints in various ways, such as: 1) smooth equality and inequality product constraints; 2) penalty functions appended into the objective function; and 3) smoothing functions such as the well-known Fischer–Burmeister function. In some formulations, a parameter can be introduced to drive the complementarity gap to zero, as well as slack variables.

After testing all these variants, the inequality product formulation was found to give the best results in terms of computational performance. For instance, for the 14-node system, the NLP formulation has 862 variables and 945 constraints, and it can take 200–600 iterations (depending on the starting point) using the solver CONOPT to find a solution for one leader. Depending on the starting point, the convergence of the diagonalization scheme can be as fast as three cycles.

C. Opportunity Cost in a Joint Market

Due to capacity limits given by the D-curve, a GenCo has to optimize the simultaneous dispatch of active and reactive power and SR, and hence, it can face opportunity costs among commodities. This opportunity cost may lead generators to shift capacity from one market to another depending on profitability. From the KKT conditions with respect to active power, reactive power, and spinning reserves, the corresponding expressions can be obtained as follows:

$$\begin{aligned}
&\hat{\beta}^{\star}_{\nu,h,i} + 2\hat{\gamma}^{\star}_{\nu,h,i} g_{\nu,h,i} + \lambda_i + \overline{\mu}^{g}_{\nu,h,i} \\
&\quad - \overline{\mu}^{q}_{\nu,h,i} b^{f}_{\nu,h,i} - \underline{\mu}^{g}_{\nu,h,i} + \underline{\mu}^{q}_{\nu,h,i} b^{u}_{\nu,h,i} = 0 \qquad (40)\\
&\lambda^{q}_{i} + \overline{\mu}^{q}_{\nu,h,i} - \underline{\mu}^{q}_{\nu,h,i} = 0 \qquad (41)\\
&\hat{\alpha}^{\star}_{\nu,h,i} + \overline{\mu}^{g}_{\nu,h,i} - \mu^{r} = 0, \quad \forall r_{\nu,h,i} > 0. \qquad (42)
\end{aligned}$$

Without loss of generality, the opportunity cost between spinning reserves and active and reactive is discussed for maximum capacity constraints. Thus, if maximum capacity limits may be binding, minimum capacity limits cannot be active at the same time, and $\underline{\mu}^{g}_{\nu,h,i} \equiv \underline{\mu}^{q}_{\nu,h,i} \equiv 0$. Using a similar derivation as in [15], the substitution of expressions (40) and (41) into (42) yields the following relationship for the SR price:

$$\mu^{r} = \left\{\lambda_i - \left(\hat{\beta}^{\star}_{\nu,h,i} + 2\hat{\gamma}^{\star}_{\nu,h,i} g_{\nu,h,i}\right)\right\} + \lambda^{q}_{i} b^{f}_{\nu,h,i} + \hat{\alpha}^{\star}_{\nu,h,i}. \qquad (43)$$

The first term in brackets is the opportunity cost for providing spinning reserve instead of active power. This is given by the difference between the active power price and the marginal bid. The second term is the trade-off between SR and reactive power. This is given by the slope of the field limit (from the D-curve) and the dual variable of the reactive power balances. The last term is the optimal bid for spinning reserve of unit ν, h, i.

The opportunity cost among commodities occurs even in a competitive setting. The SR price for competitive conditions is obtained when using the actual cost parameters, i.e.,

$$\mu^{r} = \{\lambda_i - (\beta_{\nu,h,i} + 2\gamma_{\nu,h,i} g_{\nu,h,i})\} + \lambda^{q}_{i} b^{f}_{\nu,h,i} + \alpha_{\nu,h,i}. \qquad (44)$$

Similar derivations can be obtained for the trade-off for active power with reactive power and SR.

III. Computational Results

Let us consider the three-node system shown in Fig. 1. The transmission network consists of three identical transmission lines. Each line's resistance is 0.005 p.u., the inductive reactance is 0.01 p.u., and the shunt susceptance is 0.4 p.u. For the sake of illustration, only line 1–3 has a limit of 600 MVA. This power system setting is taken from an example in [8] and [9].

There are two GenCos that are placed at nodes 1 and 2 and can provide active and reactive power to a sole demand at node 3 and also SR to the system. Their cost data are given in Fig. 1. The load at node 3 is represented by a quadratic benefit function $B(d) = 200d - 0.05d^2$ for active demand (d) as well as a fixed demand of 200 MVAR. Also, upper voltage limits of 1.03 p.u. and lower voltage limits of 0.97 p.u. are enforced at each node.

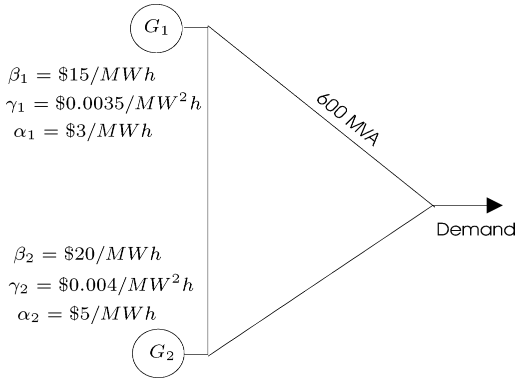

Fig. 1. Three-node power system.

TABLE I
MARKET OUTCOME UNDER DIFFERENT STRATEGIES

	Competitive	Monopoly
D_{L3} (MW)	1,477.5	1,477.5
Q_{L3} (MVAR)	200.0	200.0
g_1 (MW)	288.12	267.41
g_2 (MW)	1,255.69	1,276.42
q_1 (MVAR)	-0.02	99.89
q_2 (MVAR)	211.91	111.31
$\hat{\beta}_1$ ($/MWh)	15.0	-16.48
$\hat{\beta}_2$ ($/MWh)	20.0	20.0
$\hat{\gamma}_1$ ($\$/MW^2h$)	0.0035	0.08845
$\hat{\gamma}_2$ ($\$/MW^2h$)	0.0040	0.00400
Π_1 ($/hr)	290.54	3,979.33
Π_2 ($/hr)	6,307.07	6,516.98
V_1 (pu)	1.008	1.014
V_2 (pu)	1.030	1.030
V_3 (pu)	0.970	0.973
λ_1 ($/MWh)	17.01	30.81
λ_2 ($/MWh)	30.04	30.21
λ_3 ($/MWh)	52.25	33.72

A. Gaming With Reactive Power

First, as a benchmark, an energy-only market is simulated. A competitive case is considered using a fixed demand of 1477.5 MW together with the 200 MVAR. This demand level is obtained when using the affine function for demand described above in a competitive setting.

1) Competitive Market: In this setting, GenCo 1 has the strongest impact on the transmission line 1–3. For that reason, GenCo 2 provides most of the active power, even though it is the most expensive unit. To supply the reactive demand, GenCo 2 provides 211.91 MVAR, and in contrast, GenCo 1 adsorbs 0.024 MVAR. The average active and reactive power flows in line 1–3 are 79.55 MVAR and 594.70 MW. Both the voltage upper limit at node 2 and the line 1–3 limit are binding. Active losses amount to 66.31 MW. The results are summarized in Table I.

2) Monopoly Market: Because in a multi-leader setting multiple equilibria may arise, the impact of reactive demand and voltage constraints can be difficult to characterize. Therefore, to highlight reactive power issues in a basic fashion, the following one-leader one-follower example is chosen as a second case. For this setting, GenCo 1 is assumed to behave strategically by altering its bid parameters to maximize its profit (monopolist), while GenCo 2 remains competitive. The market outcome is also given in Table I.

The monopolist makes its bid steeper by increasing its quadratic parameter while decreasing its linear bid parameter. These manipulations increase the price at node 1 from $17.01/MWh to $30.81/MWh, resulting in a monopolist's profit 13.69 times higher.

Interestingly, in this monopoly outcome, the dual variables associated with the binding limits of line 1–3 and the upper voltage at node 2 are nonzero, which implies that the system is congested and the monopolist is not able to capture the congestion rents. As discussed by Joskow [25], the fact that leaders can anticipate how they impact the system and how the ISO acts is not enough to capture the congestion rents arising from binding constraints, as the system configuration plays a role in this kind of degeneracy. In this numerical example, the leader is unable to avoid congestion because the competitive generator limits the leader's manipulation of the constraints. For instance, in the line 1–3, the power flows of both players are substitutes, and therefore, the leader cannot fully control the line's flow.

In general, existence and uniqueness of the solution cannot be guaranteed when the model for competition relies on a leader-follower setting [26]. For the monopolist results reported in this paper, since there is only one leader with two decision variables, the global optimality and uniqueness of the solutions have been established with reasonable confidence by sampling the ranges of the leader's decision variables. For the description of this test, refer to [11].

The price at node 1 is controlled by the monopolist with its bid function. Hence, at the solution point and with no capacity limits binding, the price at node 1 is simply the parameterized bid of GenCo 1; i.e., expression (3)

$$\lambda_1^\star = \hat{\beta}_1 + 2\hat{\gamma}_1 g_1^\star. \tag{45}$$

Using the values of the solution point, the following relationship is obtained:

$$\hat{\beta}_1 = -534.824468\hat{\gamma}_1 + 30.816826, \quad \forall \hat{\gamma}_1 \geq 0. \tag{46}$$

Expression (46) defines pairs of parameters $(\hat{\beta}_1, \hat{\gamma}_1)$ that the monopolist can set as its bid to obtain the same level of profit. Two salient pairs are: 1) the linear-only bid (28.9449, 0.0035), where the monopolist alters only the linear parameter but bids its actual quadratic parameter; and 2) the quadratic-only bid (15, 0.02957), where GenCo 1 alters only the quadratic parameter but bids its actual linear parameter. The negative slope implies that a bid with a larger quadratic parameter requires larger negative values of the linear parameter, as hypothesized by Baldick [1].

3) Impact of Reactive Power Demand: Considering the monopolist setting, let us now vary the reactive demand level from 0 up to 300 MVAR in order to see how the monopolist impacts the market outcome. As in the previous example, a linear function is obtained to determine the bids that the monopolist can set to maximize its profit for each reactive demand level and for any nonnegative value of the quadratic bid parameter. The functions are summarized in Table II.

For higher values of reactive demand, the monopolist requires less steep functions (see the coefficients of the quadratic bid parameter in Table II) that relate the bid parameters used to maximize its profits. This also implies that the cost bid functions of

TABLE II
FUNCTIONS GIVING THE OPTIMAL BIDS OF GENCO 1

Q_3 (MVAR)	Optimal bids satisfy	$\|\Pi_1\|$ (pu)	$\|\Pi_2\|$ (pu)
0	$\hat{\beta}_1 = -597.883877\hat{\gamma}_1 + 30.896551$	13.48	1.01
50	$\hat{\beta}_1 = -586.163955\hat{\gamma}_1 + 30.963261$	13.07	1.02
100	$\hat{\beta}_1 = -567.700380\hat{\gamma}_1 + 31.067239$	12.56	1.04
150	$\hat{\beta}_1 = -542.257396\hat{\gamma}_1 + 31.210029$	12.48	1.05
200	$\hat{\beta}_1 = -534.824468\hat{\gamma}_1 + 30.816826$	13.69	1.03
250	$\hat{\beta}_1 = -502.735289\hat{\gamma}_1 + 31.016508$	17.20	1.00
300	$\hat{\beta}_1 = -384.483550\hat{\gamma}_1 + 32.142509$	24.47	1.00

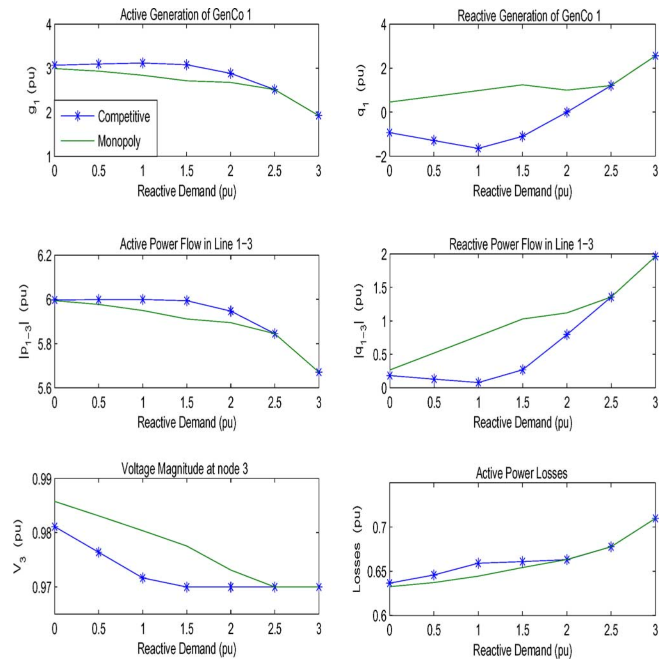

Fig. 2. Market outcomes comparison for different requirements of reactive demand. Fixed active demand.

GenCos will be steeper for higher values of reactive demand.[1] By changing the requirement of reactive demand, not only will the monopoly outcome vary but also the competitive outcome. Hence, in order to account for the impact of reactive demand on the monopoly outcome, the relative variation with respect to the competitive outcome needs to be considered as well. The profits of both GenCos from the monopoly outcome are normalized with respect to the profits of the competitive outcome. The profit of GenCo 2 (which is competitive) in both settings varies accordingly in both the competitive and the monopoly setting, and its normalized profit is around 1 p.u. for any level of reactive demand. However, the profit of the monopolist varies nonlinearly and markedly in both the competitive and monopoly setting. The normalized profits are shown in Table II. For both the competitive and monopoly outcomes, the trends of different variables are shown in Fig. 2.

Higher values of reactive demand require more production of reactive power from both GenCos and, hence, the use of more

[1]Given the same value of the linear parameter $\hat{\beta}_1$ for all the functions in Table II, the corresponding value for the quadratic parameter $\hat{\gamma}_1$ will be larger for higher values of reactive demand.

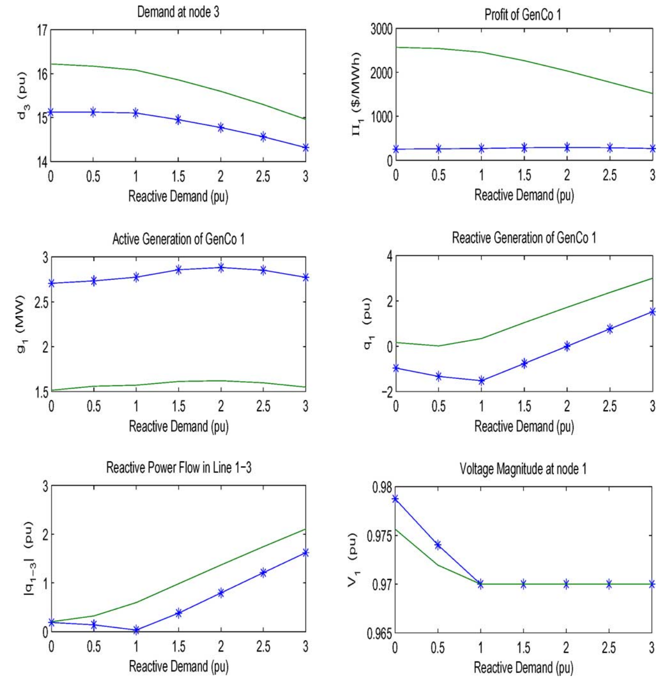

Fig. 3. Market outcomes comparison for different values of reactive demand. Price-responsive active demand.

capacity of transmission lines 1–3 and 2–3. Because line 1–3 is limited to 600 MVA and more reactive power flows through it, its active power flow has to decrease, and the active power flow in line 2–3 increases correspondingly (as it has no limit). To accommodate the change of power flows in line 1–3, the monopolist reduces its generation of active power as it has the strongest impact on the line, and on the other hand, GenCo 2 increases its generation as it is more attractive to provide not only active but also reactive power. As can be seen, the trends of power flows in line 1–3 are inherited from the trends of monopolist's generations.

For higher values of reactive demand, the profit of the monopolist decreases because its generation becomes less attractive as less transmission capacity in line 1–3 is available for active power, and the monopolist congests this transmission line the most. Hence, it is more economical to use generation from GenCo 2, which congests less the line 1–3. This will make GenCo 2 increase its profit, even if it is competitive. The higher the reactive demand is, the greater the voltage drop at the demand node. The monopolist sets its optimal strategy by including the impact of its reactive power generation. As long as the lower voltage limit at node 3 is not reached, the monopolist can fully internalize its reactive generation in its optimal bid. However, when the reactive demand is high enough (around 250 MVAR) to make the lower voltage limit binding at the demand node, the monopolist has no more room to account strategically for its reactive generation, and thus, it sets its bid so that its generation will be at the same level as the competitive setting.

For the sake of comparison, the same simulation is implemented using the price-responsive demand introduced at the beginning of this section. For different values of reactive demand, the market outcomes are shown in Fig. 3. The main difference

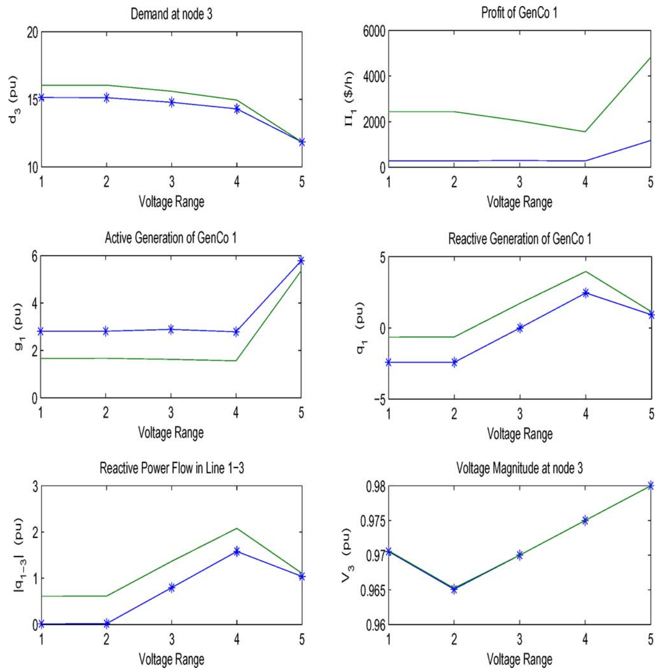

Fig. 4. Trends of the market outcome for different voltage limits.

is that the active demand can now decrease when reactive demand increases.

Because of the responsiveness of the demand, the trends of the monopolist outcomes are more similar to that of the competitive setting. The inflexion point in most curves (see Fig. 3) between 0.5 and 1.0 MVARpu is due to two facts: 1) the monopolist reaches a point of no generation/adsorbsion of reactive power; and 2) the reactive power flow reverses direction in line 1–3.

4) Impact of Voltage Constraints: Let us now consider how changes in the voltage constraints impact the strategies of the monopolist. For the results presented in Fig. 4, the price-responsive demand for active power and a fixed value of 200 MVAR for reactive demand are used. The voltage range of operation is then tightened from 0.96–1.04 to 0.98–1.02 p.u. by variations of 0.005 p.u. The first voltage range is labeled as 1 and the last voltage range is labeled as 5.

By tightening the voltage range of operation, reactive power generation from the competitive generator needs to be shifted to the monopolist so that the voltage limits at node 2 (where the competitive generator is placed) can be enforced. This will also imply an increase of the voltage at the monopolist node and an increase of the reactive power flow in line 1–3. In order to accommodate the extra reactive power flow in line 1–3, both GenCos have to reduce their active power generation, resulting in lower demand and profit levels. In the last portion of the graphs, the trends change because with the voltage limits of 0.98–1.02 p.u., the market outcome (in both the competitive and monopolist scenario) is such that the limit of line 1–3 is not binding anymore. Hence, congestion is only due to voltage constraints. In order to enforce such tight limits (0.98–1.02) and supply the fixed reactive demand, the demand for active power has to be reduced markedly. In fact, if the same voltage limits are put in place with the inelastic demand setting, no solution

TABLE III
VARIATION OF PROFITS WITH DIFFERENT VOLTAGE LIMITS FOR TWO DIFFERENT REACTIVE DEMAND LEVELS

Voltage Range	Q_{L3}=200 MVAR $\lvert\Pi_1\rvert$ (pu)	Q_{L3}=200 MVAR $\lvert\Pi_2\rvert$ (pu)	Q_{L3}=0 MVAR $\lvert\Pi_1\rvert$ (pu)	Q_{L3}=0 MVAR $\lvert\Pi_2\rvert$ (pu)
1	8.85	1.36	10.03	1.40
2	8.83	1.36	10.02	1.40
3	6.98	1.38	10.01	1.40
4	5.72	1.35	9.33	1.38
5	4.11	1.13	13.22	1.19

can be obtained because the problem is infeasible: the demand of 200 MVAR leads to a voltage drop beyond the lower limit of 0.98 p.u.

As can be seen, the market outcomes from increasing the reactive demand and from tightening the voltage constraints are similar for this study case. This is due to the strong coupling between the voltage magnitude and reactive power. Also, for this study case, the pattern of the functions that relate the linear and quadratic bid parameters is similar for all the voltage ranges above.

In another simulation, a zero reactive demand is also used. Reactive power is still required from both generators in order to provide the requirements of the transmission system. In Table III, the comparison of normalized profits (ratio monopolist/competitive) are presented for both cases of reactive demand.

For both cases, the profit of the GenCos increases. However, the monopolist's profits are markedly higher in the case of no reactive demand. With no reactive demand to be supplied, there is more transmission available and more active power can flow through the transmission line 1–3, which implies a higher active demand can be supplied. In this case, the monopolist has more room to behave strategically. The pattern observed for the case of no reactive demand is in general similar to that of a reactive demand of 200 MVAR.

B. Gaming With Spinning Reserves

For the following study cases, consider the monopoly setting with the price-responsive demand for active power and with the 200 MVAR for reactive demand. Also, assume a fixed SR requirement of 147.75 MW (10% of the system demand at the competitive level). Five cases are to be described; they are summarized in Table IV.

1) Competitive Case: The competitive case is used as a benchmark. As GenCo 1 is the cheapest unit for SR, and there are no limits on the amount of SR it can provide, all the SR requirement is provided by this GenCo and its marginal cost sets the price for the SR (\$3/MWh). In this case, there is no profit from participating in the SR market.

2) Monopolist Case: For this case, no limits on the generators' capacity are considered, and again GenCo 1 is modeled as a monopolist. In order to maximize profits, the monopolist adjusts its bids for both energy and SR. As there is no constraint in the capacity of both GenCos, the SR market is independent from the energy market, even though they are cleared simultaneously. Hence, the strategic setting of the SR bid is exclusively from the incentive of the SR market. The SR bid of the

TABLE IV
COMPARISON OF CASES FOR GAMING ENERGY AND SPINNING RESERVES.†

	Case				
	1	2	3	4	5
$\Pi_1(g_1)$	290.54	2,029.63	2,020.98	3,339.24	4,672.60
$\Pi_2(g_2)$	6,307.06	8,725.17	8,921.71	8,839.83	21,460.51
$\Pi_1(r_1)$	0.0	295.5	295.5	287.44	0.0
$\Pi_2(r_2)$	0.0	0.0	0.0	0.0	1,325.08
$\hat{\beta}_1$	15.00	5.98	9.87	19.41	17.66
$\hat{\beta}_2$	20.00	20.00	20.00	20.00	20.00
$\hat{\gamma}_1$	0.0035	0.0682	0.0621	0.0532	0.0707
$\hat{\gamma}_2$	0.0040	0.0040	0.0040	0.0040	0.004
$\hat{\alpha}_1$	3.00	5.00	5.00	5.00	13.96
$\hat{\alpha}_2$	5.00	5.00	5.00	5.00	5.00
λ_1	17.01	28.09	28.80	36.91	42.37
λ_2	30.04	31.81	31.94	31.89	40.58
λ_3	52.25	43.03	43.45	43.69	45.06
μ^r	3.00	5.00	5.00	5.00	13.96
d_3	1,477.50	1,558.90	1,565.44	1,563.06	1,549.37
g_1	288.11	161.94	152.25	156.28	174.58
g_2	1,255.69	1,476.91	1,493.46	1,486.59	1,452.25
q_1	-2.44	171.68	184.47	179.16	165.59
q_2	211.91	65.50	54.88	59.29	68.11
r_1	147.75	147.75	147.75	143.71	0.0
r_2	0.0	0.0	0.0	4.03	147.75
V_1	1.008	1.013	1.013	1.013	1.013
V_2	1.030	1.030	1.030	1.030	1.030
V_3	0.970	0.970	0.970	0.970	0.970

† Profits in \$/h; linear and quadratic bid parameters in \$/MWh and $\$/\mathrm{MW}^2\mathrm{h}$; prices in \$/MWh; demands, SR, and generations in MW and MVAR; and voltage in p.u.

monopolist is, however, bounded by the bid of the competitive player. Thus, the optimal bid of the monopolist to maximize its SR profit is \$5/MWh to match the bid of the competitive player. However, if the monopolist bids exactly \$5/MWh, there is an infinite combination of SR schedules that lead to the same market outcome. Hence, the SR could be allocated in any proportion to both GenCos, resulting in different levels of profit for the monopolist. The provision of SR of the monopolist can be $r_1 \in [0, 147.75]$ MW, and the competitive player would provide the remaining SR; i.e., $r_2 = 147.5 - r_1$ MW, with a resulting monopolist's profit of $\Pi_1 \in [2029.63, 2325.13]$ \$/h. However, the monopoly equilibrium is attained only when the monopolist provides all the SR. Recognizing this problem, the monopolist's bid would be just an epsilon under \$5/MWh in order to outbid the competitive player.

3) Monopolist With Limited Active-Power Capacity: In the previous case, the monopolist provides 161.94 MW for energy and 147.75 MW for SR, which amount to 309.69 MW of capacity. If we consider that only the monopolist is now constrained up by, say, 300 MW, it has to scale down its schedule to remain feasible. As energy and SR are interrelated in the capacity of generators, the monopolist best bid for energy is such that its provision of energy is reduced in order to keep the same level of SR. This also implies that the monopolist sacrifices some profit from the energy market to maintain the same level of profit from the SR market. This is simply because of the incentives of the SR that is now joined to the energy market [15].

4) Monopolist With Limited Active- and Reactive-Power Capacities: Consider that the field limit of the D-curve for the monopolist is defined by a maximum reactive generation of $a^f = 200$ MVAR with a slope of $b^f = -0.133$ MVAR/MW. In this case, the monopolist is limited up to provide reactive power, and therefore, it is unable to supply the 184.47 MVAR of Case III-B-3, unless it decreases its generation of active power. Active power is now tied not only with the SR but also with the reactive power. The D-curve limit makes the monopolist decrease its reactive generation to 179.16 MVAR. In order to compensate for this, the competitive generator has to increase its reactive generation. With this, less reactive power flows through line 1–3 (but more in line 2–3), allowing more active power to flow in the constrained line, resulting in a shift of active generation from the competitive player to the monopolist and in a higher energy bid of the monopolist. This is what produces the apparently counter-intuitive result of higher profits for the monopolist when it is more limited.

5) Competitive Player With Limited Capacity: If now we consider that only the competitive player is constrained up by, say, 1600 MW, apparently, this limit would not impact the market outcome because the competitive player is providing only 193.46 MW (see Case 3). Even the limited capacity of the competitive player can be used by the monopolist to exercise market power. Because of the opportunity costs between energy and SR, the monopolist can manipulate its energy bid to make the competitive player absorb all the SR. Hence, the competitive player will have to reduce its provision of energy (with respect to Case 3), making the monopolist more attractive in the energy market. In this case, the monopolist is foregoing its profit from the SR in order to increase its energy profit. The SR price is \$13.9684/MWh, which is defined mainly by the opportunity cost between energy and SR. Hence, the monopolist can bid any value higher than \$13.9684/MWh and the same market outcome would be obtained as the monopolist would be still too expensive to be scheduled. In contrast, for any value lower than \$13.9684/MWh, the monopolist would become attractive to provide some SR as its bid is lower than the combined cost (SR and opportunity costs) of SR from the competitive player, which is constrained up. In this case, the monopolist is exploiting the opportunity cost for SR from the constraint on the competitive player.

C. Larger System

The SFE model is also implemented for the IEEE 14-node system [27]. This system has 20 transmission lines, and their limits have been scaled down accordingly to induce congestion in the system. The voltage limits used here are the standard values of 0.94–1.06 p.u. provided with the data. Five GenCos and eight demands are considered for the simulations; two GenCos are assumed to be strategic, while the other GenCos and all the demands are assumed to be competitive. D-curves are considered as well for all GenCos, and there is a SR requirement of 140 MW.

First, the competitive market outcome is presented for comparison purposes. Second, GenCos enumerated as 1 and 4 are assumed to behave strategically and considered as leaders in both the energy and the SR markets. The comparison of the market outcomes is presented in both Fig. 5 and Table V.

The optimal strategy for the leaders is to make their bids more extreme by decreasing the linear parameter while increasing the

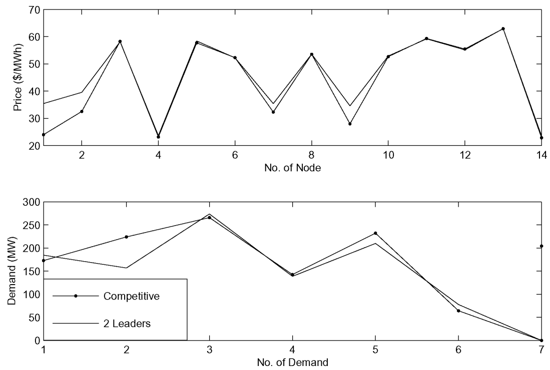

Fig. 5. Prices and demands for the 14-node system.

TABLE V
COMPARISON OF MARKET OUTCOMES FOR THE 14-NODE SYSTEM

	Competitive	Strategic
$\hat{\beta}_1$ $(\$/MWh)$	21.00	-5.229
$\hat{\beta}_4$ $(\$/MWh)$	22.00	-27.036
$\hat{\gamma}_1$ $(\$/MW^2h)$	0.0050	0.1000
$\hat{\gamma}_4$ $(\$/MW^2h)$	0.0060	0.1200
$\hat{\alpha}_1$ $(\$/MW)$	3.0	4.0
$\hat{\alpha}_4$ $(\$/MW)$	3.5	4.0
μ^r $(\$/MW)$	3.0	4.0
g_1 (MW)	299.418	203.385
g_2 (MW)	193.961	299.832
g_3 (MW)	400.000	400.000
g_4 (MW)	403.959	253.351
g_5 (MW)	85.704	157.503
q_1 (MVAR)	37.280	37.378
q_2 (MVAR)	107.069	100.011
q_3 (MVAR)	90.000	90.000
g_4 (MVAR)	93.841	99.865
g_5 (MVAR)	10.531	-10.306
r_1 (MW)	140.00	0.0–140.0
r_4 (MW)	0.0	140.0–0.0
$\Pi_1(g_1)$ ($/hr)	448.25	2,731.51
$\Pi_2(g_2)$ ($/hr)	459.65	747.50
$\Pi_3(g_3)$ ($/hr)	9,735.74	9,704.27
$\Pi_4(g_4)$ ($/hr)	1,444.87	2,810.38
$\Pi_5(g_5)$ ($/hr)	36.72	124.03
$\Pi_1(r_1)$ ($/hr)	0.0	0.0-140
$\Pi_4(r_4)$ ($/hr)	0.0	70-0.0

quadratic parameter. Due to the competitive fringe composed by GenCos 2, 3, and 5, the influence of the leaders on prices is limited mostly to the nodes where they are placed and to their adjacent nodes. Because the leaders submit higher energy bids, their corresponding prices increase, and consequently, they provide less active power. This decrease in active generation is compensated by the increased active generation from the fringe.

The two leaders (GenCos 1 and 4) have the cheapest SR cost, which are $3/MWh and $3.5/MWh, respectively, followed by GenCo 2, whose cost is $4/MWh. For the competitive setting, the SR requirement is totally fulfilled by GenCo 1 ($3/MWh), with a zero profit as it is marginal. However, for the strategic case, the equilibrium is more complex. The optimal strategy for each leader is to bid $4/MWh in order to match the bid of the cheapest competitive GenCo that has a cost of $4/MWh. The leaders' bids are bounded by the cost of this competitive player. Although there is an equilibrium in terms of bids, there is a continuum of optimal dispatches in terms of the schedule for SR.[2] With the strategic bids for SR, there are three GenCos with the same bid value. So the market operator can fulfill the SR requirements with any linear combination of reserves from those three GenCos. Obviously, a rational player would recognize this condition and might submit a bid just under $4/MWh to outbid the others. With the diagonalization approach, each leader considers that it can provide all the SR and, therefore, capture all the SR profit. For instance, from the point of view of leader 1, it will provide 140 MW of SR, resulting in a profit of $140/h. Similarly, leader 2 believes it will provide all the SR and, hence, would have a profit of $70/h. This fact has no impact on the energy outcome but impacts on the SR profit component and hence on the net profit of the leaders. Under this rationality, the bid of $4/MWh is strictly an equilibrium because no leader can unilaterally improve its profit. If one leader makes a higher bid, then it would be too expensive. If that leader makes a lower bid, then it will have a profit lower than that possibly obtained with a bid of $4/MWh. This would happen even if the SR cost of the leaders were zero. Although 100 different starting points were tested, only the reported equilibrium was attained.

IV. CONCLUSION

A parametric SFE model with a detailed ac transmission system is proposed to study market power. The aim of using an ac model is to analyze the potential impact of voltage constraints, reactive power, and SR in the strategic behavior of GenCos. Because the main concern of market power is with active (real) power, reactive power and SR have been usually omitted in the modeling of strategic behavior. However, as reactive power and SR are also provided by generators, they may bias their incentives to maximize their profits. Through numerical examples, the main aspects of strategic behavior in a full ac model are discussed, and it is found that generators do change their bids when facing different requirements of voltage, reactive demand, and SR.

One of the main reasons for the deviations is because the reactive power congests the system to the same extent as active power. Therefore, when voltage or reactive demand requirements change, the available transmission for active power will change as well, and the strategic players will take account of it in the computation of their bids. In the modeling of competition, the reactive power is taken as a support, and hence, there is no profit component for reactive power within each leader problem. However, generators have to internalize the impact of their reactive generation into the computation of their optimal bids. Although reactive power can be cheap to supply with static sources, when its supply is insufficient, the resulting market outcomes may be inefficient. This highlights the importance of developing sound schemes for ancillary services, such as for voltage support.

The strategic setting of bids for SR has also been illustrated. It is shown that incentives from the SR market happen in an ac model as well. The joint clearing of energy and SR can lead to higher SR prices, even if the competitive generators are

[2]However, as the rationality of competition is á la SFE, the leaders cannot decide the level of SR that they could provide but rather only the bid for SR.

those constrained up by capacity limits. In some instances, the SR market will drive generators to deviate from the optimal schedule and forego profits from the energy market in order to maximize the overall profit.

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