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Consideration of Gas Supply Contracts with Take-or-pay Clauses in the Brazilian Long-term Energy Planning

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Abstract In Brazil's long-term energy planning, the dispatch of thermal plants usually varies along a year. Such variation is essentially due to the predominance of the hydraulic mix in the system electric energy supply. For this reason, without preventive measures, a highly irregular cash flow occurs for natural gas (NG) providers, who supply gas for electric energy generators.

In order to achieve more regularity for NG providers' cash flows, supply contracts of this resource for electric energy generation usually contain special clauses, called take-or-pay – ToP. Such clauses force electric generators to pay each month a minimal financial amount, even if the effective use of NG in this period is smaller than the paid amount of resource. Without representing explicitly ToP clauses, the Brazilian model is currently forced to dispatch NG-fueled thermal plants in a compulsory minimal amount, corresponding to the financial lower bound required by the contract.

The explicit consideration of ToP clauses in hydrothermal dispatch models yields a better application of NG and a smaller expected operation cost for the whole power system, because it introduces some flexibility in the decision of NG purchase and its use. As shown by our numerical experience, this flexibility may result in reduced water spillages in periods with favorable hydrology.

The methodology presented in this work takes into account the characteristics of the ToP contracts in the Brazilian long-term energy planning and differs from other models found in the literature by the fact of aiming at a smaller expected operation cost of the whole system – National Interconnected System, because it treats the contracts from the operator point of view. This work presents the process of inclusion of the

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NG contracts in Brazil's long-term energy planning model NEWAVE, developed by the Electric Energy Research Centre (CEPEL), with its mathematical formulation, impacts, and results obtained from real case studies with the NIS.

Keywords Long-term Operation Planning · Thermoelectric Plants · Gas Supply · Take-or-pay Clauses

1 Introduction

In 1996 the Brazilian Electric Sector Restructuring Program (RE-SEB) was started. This project was part of an institutional reform in the electric sector, aiming at its future development. The goals of the project were to ensure a reliable and safe supply of electric energy through optimal operational processes and a higher economical efficiency in all its segments, in order to stimulate investments and reduce investors' risks.

To allow for an effective competition in the regulated environment, an economical regulation of the sector was necessary, leading to a diversification of the national energy matrix. In such a setting, generation of electric energy with thermoelectric plants was expected to occur in a competitive way, stimulating future investments.

For this reason, the Thermoelectric Priority Program (PPT) was created, aiming at the construction of natural gas fueled thermoelectric plants. This program was first created considering a future scenario in which the participation of thermoelectric plants grew from 5 to 20% of the generation market, in the year of 2010. In order to ensure competitity between thermoelectric and hydroelectric generation, thermoelectric plants costs needed to be reduced. This feature could only be possible by making Brazil's natural gas market grow considerably. In order to stimulate investments in this market, significant benefits were given to investors: the PPT program ensured natural gas supply during twenty years at certain maximal price. Moreover, the program ensured eventual access to financial support programs, by the Brazilian National Bank for Economical and Social Development (BNDES).

In 2002 the PROINFA program was created, to promote the diversification of the national energy matrix. Part of the resources applied in this program is provided by the Energetic Development Account (CDE), a tax paid by all commercial agents involved with the final consumption sector. Part of the CDE financial resources should be used to build a transportation network for the natural gas, projected to be built in the states in which, until the end of the year of 2002, there was no transportation network.

Nowadays, the natural gas fueled thermoelectric plants account for, approximately, 45% of the installed capacity of thermoelectric plants, which in turn corresponds to about 11% of the total installed capacity in Brazil. However, this doesn't mean that 11% of the total generated energy of the country is generated by these thermoelectric plants. For illustration, in the year of 2006 the participation of the natural gas fueled plants reached only 4% of the total energy generation. In Figure 1 the evolution of the Brazilian consumption of natural gas is presented, from the year of 1970 until 2004, showing that Brazil follows the world tendency of an increasing use of this energetic source.

The importance of a more detailed consideration of thermoelectric plants in the long-term energy planning problem is related to the existence of contracts for the construction of new plants, totalizing 6,266 MW. This is a significant increase to the Brazilian energy market and a more detailed consideration of these plants became nec-

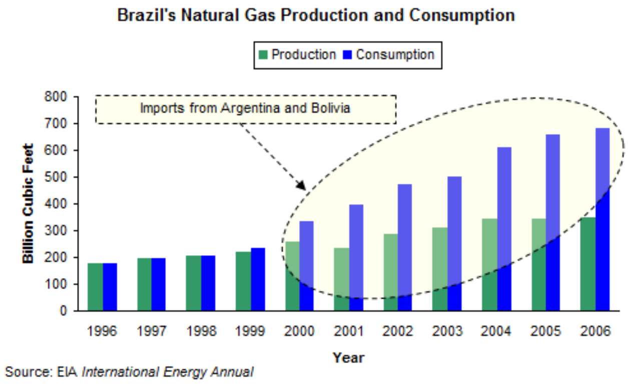

Fig. 1 Brazilian consumption of natural gas

essary.

Since Brazil's energy system is hydro-dominated, the dispatch of thermal plants usually varies along a year. The corresponding NG supply to thermal plants is highly irregular, and to achieve more regularity for NG providers' cash flows, supply contracts of this resource for electric energy generation usually contain special clauses, called take-or-pay – ToP. Such clauses force electric generators to pay each month a minimal financial amount, even if the effective use of NG in this period is smaller than the paid amount of resource. Without representing explicitly ToP clauses, the Brazilian model is currently forced to dispatch NG-fueled thermal plants in a compulsory minimal amount, corresponding to the financial lower bound required by the contract.

The explicit consideration of ToP clauses in hydrothermal dispatch models yields a better application of NG and a smaller expected operation cost for the whole power system, because it introduces some flexibility in the decision of NG purchase and its use. As shown by our numerical experience, this flexibility may result in reduced water spillages in periods with favorable hydrology.

The methodology presented in this work takes into account the characteristics of the ToP contracts in the Brazilian long-term energy planning and differs from other models found in the literature by the fact of aiming at a smaller expected operation cost of the whole system – National Interconnected System (NIS), because it treats the contracts from the independent system operator (ISO) point of view. This work presents the process of inclusion of the NG contracts in Brazil's long-term energy planning model NEWAVE, developed by the Electric Energy Research Centre (CEPEL).

This paper is organized as follows. In Section 2, the long-term energy planning model currently used in Brazil, NEWAVE, developed by the Electric Energy Research Centre (CEPEL) is presented. Section 3 describes the main features of NG contracts. The mathematical formulation of the proposed model is given in Sections 3.1 and 4. Numerical results, showing the positive impact of the proposal, and obtained from real case studies with the NIS are given in Section 5.

2 Description of NEWAVE model

This work concerns itself with a more detailed representation of natural gas supply contracts for thermoelectric plants in Braazil's energy planning process. Thus, this section presents some characteristics of NEWAVE model, which is Brazil's official computational model for this process.

In Brazil, the dispatch of thermoelectric and hydroelectric plants in the NIS (98% of the capacity of the entire country) is centralized by the ISO (ONS – which is an acronym in portuguese for national operator of the system). The system has a large number of plants, in a vast network of transmission, and the uncertainty of future rainfall needs to be considered in high detail, due to the hydro-power dominance. For this reason, the energy planning problem is divided into stages and resolved with the use of a chain of models, [11].

Model NEWAVE belongs to this chain of models and is used in the long-term energy planning since 1998, in several types of studies, [10]. Its main characteristics, [1], are:

- aggregation of plants in multiple interconnected subsystems;
- consideration of dynamic characteristics of the systems;
- representation of rain uncertainty through consideration of a hight number of energy inflow scenarios.

As there is uncertainty regarding future rainfall, model NEWAVE has as its objective to minimize the expected value of the operation cost over an horizon of 5 to 10 years. To this aim, PAR statistical models (periodic autoregressive models) are adjusted according to the inflows history, and Q scenarios are generated based on these PAR models, [8].

In a simplified way, the stochastic program solved by the NEWAVE model for a T periods horizon can be written as

$$\begin{array}{ll} \min \mathbb{E}_p \left(\sum_{t=1}^{T} (c^t)^\top x^t \right) & \\ \text{s.t.} \begin{cases} A_t x^t \leq b^t, \ \forall t \\ \quad x^t \geq 0, \ \forall t, \end{cases} \end{array} \tag{1}$$

where symbol $\mathbb{E}_p$ represents the expected value function, given the distribution of probabilities p, and $c^t \in \mathbb{R}^{n_t}$ represents the cost vector for a given period t, including penalties. The matrix $A_t \in \mathbb{R}^{m_t \times n_t}$ and the vector $b^t \in \mathbb{R}^{m_t}$ define the constraints for the problem in an abstract.

Decision variables $x^t \in \mathbb{R}^{n_t}$ represent generation of each subsystem, exchanges of energy between the subsystems, and slack variables for some constraints, at each time period t.

Constraints are the typical ones in an energy planning problem, such as satisfying the demand for power and controlling the water reservoirs' levels. These constraints entail a coupling among the time periods.

The problem is solved by combining a stagewise decomposition with a sampling method, as briefly explained next. Consider the t^{th} subproblem

$$\begin{array}{ll} \min\limits_{x^1, x^2(q)} & c_1(x^1) + \mathbb{E}_p \left(c_2(x^2(q), q) \right) \\ \text{s.t.} & \begin{cases} A_1 x^1 & \leq \quad b^1, \\ e(x^1) + A_2 x^2(q) & \leq b^2(q), \ q = 1, \cdots, Q, \end{cases} \end{array} \tag{2}$$

where the function $c_1(\cdot) : \mathbb{R}^{n_1} \to \mathbb{R}$ represents the cost of the first stage, i.e. the cost of operation for the stage t, $c_2(\cdot, q)$ represents the cost of the second stage, i.e. the stage $t+1$, for a given scenario q, and the convex function $e(x^1) : \mathbb{R}^{n_1} \to \mathbb{R}^{m_2}$ represents the coupling between the first and second stages.

Typically, the function $e(\cdot)$ is assumed to be linear with respect to x^1. But in the case of the NEWAVE model, each one of the m_2 functions $e_i : \mathbb{R}^{n_1} \to \mathbb{R}$ can be quadratic or linear, [1]. The theorem presented at the end of this section deals with the convexity of the value function, even for the case in which the function $e_i(x^1)$ are no longer linear, but convex.

As only a finite number of scenarios Q is generated to represent the inflow uncertainty, the expected value $\mathbb{E}_p\left(c_2(x^2(q), q)\right)$ is given by

$$\mathbb{E}_p\left(c_2(x^2(q), q)\right) = \sum_{q=1}^{Q} p_q \left(c_2(x^2(q), q)\right), \tag{3}$$

where p_q represents the probability of occurrence of the q^{th} scenario's energy inflow.

One can easily deduce that the cost of the problem in the second stage depends on the decisions taken in the first stage, due to the coupling between the periods given by the function $e(x^1)$. A function of the second stage cost, or value function, can be written as defined in (4), which depends on variables of the first stage, x^1, and the scenario q

$$v_2(x^1, q) := \begin{cases} \min\limits_{x^2(q) \geq 0} \; c_2(x^2(q), q) \\ A_2 x^2(q) \leq b^2(q) - e(x^1). \end{cases} \tag{4}$$

Thus, the problem (2) may be replaced by the two-level formulation

$$\begin{array}{l} \min\limits_{x^1 \geq 0} c_1(x^1) + \mathbb{E}_p v_2(x^1, q) \\ A_1 x^1 \leq b^1. \end{array} \tag{5}$$

It is not a simple task to write an analytical expression for $v_2(x^1, q)$, so the approach usually taken is to approximate it, in some sense. There are several ways to do this and model NEWAVE uses suport hyperplanes, also called Benders cuts, [3], built by using the method of cutting planes - Benders decomposition. This method is mathematically consistent, and usually efficient, if the approximated function is convex, [5]. The cutting planes technique, when used in the context of stochastic programming, is known as L-Shaped method, described in [15]. Several methods based on this one can be found in the literature, as in [4,14], see also [13].

The technique used for solving the long-term energy planning in Brazil is the Stochastic Dynamic Dual Programming, originally described in [12], for which a convergence analysis can be found in [13]. The algorithm solves the problem by a iterative approximation of the real value function, done by the inserction of Benders' cuts at each iteration. An iteration of the process contains a backward and a forward passes and Bellman's optimality criteria, [2], is used to prove the equivalence between the solution found originally and by the decomposition algorithm.

Thus, it is important to show that the value function $v_2(x^1, q)$ is convex, in order to validate the use of Benders cuts. As all the possible realizations of the second stage are represented by a finite number of scenarios, the expected value of the second stage cost is defined as the sum presented in (3). Thus, as the sum of convex functions is convex, since all probabilities p_q are nonnegative, it is sufficient to prove that the function $v_2(x^1, q)$ is convex with respect to x^1, for a given scenario q.

Theorem 1 (Convexity of the Value Function) *Let $c_2 : \mathbb{R}^{n_2} \to \mathbb{R}$ be a convex and coercive cost function, $x^1 \in \mathbb{R}^{n_1}$ a given parameter, $A_2 \in \mathbb{R}^{m_2 \times n_2}$ an array of constraints, $b^2 \in \mathbb{R}^{m_2}$ the resources vector, $e : \mathbb{R}^{n_1} \to \mathbb{R}^{m_2}$ a function composed by m_2 convex functions $e_j : \mathbb{R}^{n_1} \to \mathbb{R}$. Consider the function defined by*

$$v_2(x^1) := \begin{cases} \min\limits_{x^2 \geq 0} \; c_2(x^2) \\ s.t. \;\; A_2 x^2 \leq b^2 - e(x^1), \end{cases} \tag{6}$$

where A_2 has full rank. The following holds.

- *The feasible set of (6) is non-empty, for all x^1, and*
- *if $\lambda^*(x^1) \in \mathbb{R}^{m_2}$ denotes a optimal Lagrange multiplier associated with the constraint $A_2 x^2 \leq b^2 - e(x^1)$ at the optimal point x^{2^*}, then $v_2(\cdot)$ is convex with respect to x^1 and $Je(x^1)^\top \lambda^*(x^1)$ is a subgradient of the function $v_2(\cdot)$ at the point x^1, where $Je(x^1)$ represents the Jacobian matrix of the vector function e at the point x^1.*

Proof As A_2 has full rank, the linear system $A_2 x^2 = b^2 - e(x^1)$ has a solution for all $x^1 \in \mathbb{R}^{n_1}$, and the feasible set of (6), $\mathcal{V}_2(x^1)$, is non-empty for all $x^1 \geq 0$. Moreover, as the function c_2 is coercive, by the Weierstrass Theorem (e.g. [7]), it has a global minimum for all x^1. Let this minimum and its corresponding dual variable be given by $(x^{2^*}(x^1), \lambda^*(x^1)) \in \mathbb{R}^{n_2} \times \mathbb{R}^{m_2}$.

The Lagrangean function, $\mathcal{L}$, of the problem (6) is

$$\mathcal{L}(x^2, \lambda) = c_2(x^2) + \lambda^\top (A_2 x^2 - b^2 + e(x^1)).$$

By the weak duality property, [7], for all x^2 in the primal feasible set $\mathcal{V}_2(x^1) = \left\{x^2 \in \mathbb{R}^{m_2} : x^2 \geq 0 \text{ and } A_2 x^2 \leq b^2 - e(x^1)\right\}$ and all $\lambda \in \mathbb{R}^{m_2}$, the dual function defined as $\theta(\lambda) = \inf_{x^2 \leq 0} \mathcal{L}(x^2, \lambda)$ satisfies the relation $\theta(\lambda) \leq c_2(x^2)$. In particular, when $x^2 = x^{2^*}(x^1)$, we obtain the inequality $v_2(x^1) = c_2(x^{2^*}(x^1)) \geq \theta(\lambda)$, i.e.

$$\begin{aligned} v_2(x^1) &\geq \inf_{x^2} \left\{ c_2(x^2) + \lambda^\top (A_2 x^2 - b^2 + e(x^1)) \right\} \\ &= \inf_{x^2} \left\{ c_2(x^2) + \lambda^\top (A_2 x^2 - b^2 + e(x^1)) \right\} + \lambda^\top (e(\overline{x^1}) - e(\overline{x^1})), \end{aligned} \tag{7}$$

for all $\overline{x^1} \in \mathbb{R}^{n_1}$.

As the infimal value does not depend on x^1, one may write

$$v_2(x^1) \geq \inf_{x^2} \left\{ c_2(x^2) + \lambda^\top (A_2 x^2 - b^2 + e(\overline{x^1})) \right\} + \lambda^\top (e(x^1) - e(\overline{x^1})), \tag{8}$$

for all $\lambda \in \mathbb{R}^{m_2}$. Taking $\lambda = \lambda^*(\overline{x^1})$, as the optimal Lagrangean multiplier for the problem $v_2(\overline{x^1})$, equation (8) may be written as

$$\begin{aligned} v_2(x^1) &\geq \inf_{x^2} \left\{ c_2(x^2) + \lambda^*(\overline{x^1})^\top (A_2 x^2 - b^2 + e(\overline{x^1})) \right\} + \lambda^*(\overline{x^1})^\top (e(x^1) - e(\overline{x^1})) \\ &= v_2(\overline{x^1}) + \lambda^*(\overline{x^1})^\top (e(x^1) - e(\overline{x^1})), \\ &= v_2(\overline{x^1}) + \textstyle\sum_{j=1}^{m_2} \lambda_j^*(\overline{x^1})(e_j(x^1) - e_j(\overline{x^1})). \end{aligned}$$

By convexity of the function $e_j(\cdot)$,

$$\begin{aligned} e_j(x^1) &\geq e_j(\overline{x^1}) + Je_j(\overline{x^1})(x^1 - \overline{x^1}), \\ e_j(x^1) - e_j(\overline{x^1}) &\geq Je_j(\overline{x^1})(x^1 - \overline{x^1}). \end{aligned} \tag{9}$$

As the coordinates of $\lambda^*(\overline{x^1})$ are non-negative,

$$\begin{aligned} v_2(x^1) &\geq v_2(\overline{x^1}) + \textstyle\sum_{j=1}^{m_2} \lambda_j^*(\overline{x^1}) Je_j(\overline{x^1})(x^1 - \overline{x^1}), \\ &= v_2(\overline{x^1}) + \lambda^*(\overline{x^1})^\top Je(\overline{x^1})(x^1 - \overline{x^1}), \end{aligned}$$

which proves that $Je(\overline{x^1})^\top \lambda^*(\overline{x^1})$ is a subgradient of the function v_2 at the point $\overline{x^1}$.

To prove the convexity of the function v_2, using once more weak duality, one may write

$$\begin{aligned} v_2(x_1^1) &\geq \inf_{x^2} \left\{ c_2(x^2) + \lambda^\top (A_2 x^2 - b^2 + e(x_1^1)) \right\}, \\ v_2(x_2^1) &\geq \inf_{x^2} \left\{ c_2(x^2) + \lambda^\top (A_2 x^2 - b^2 + e(x_2^1)) \right\}. \end{aligned} \tag{10}$$

Let $\alpha \in [0, 1]$ be the scalar for which $\tilde{x} = \alpha x_1^1 + (1 - \alpha) x_2^1$. Multiplying the inequalities (10) by α and $(1 - \alpha)$ respectively and removing the terms that do not depend on x^2,

$$\begin{aligned} \alpha v_2(x_1^1) &\geq \alpha \inf_{x^2} \left\{ c_2(x^2) + \lambda^\top A_2 x^2 \right\} + \alpha \lambda^\top (-b^2 + e(x_1^1)), \\ (1 - \alpha) v_2(x_2^1) &\geq (1 - \alpha) \inf_{x^2} \left\{ c_2(x^2) + \lambda^\top A_2 x^2 \right\} + (1 - \alpha) \lambda^\top (-b^2 + e(x_2^1)). \end{aligned}$$

As the definition sets of the infimum are the same,

$$\begin{aligned} \alpha v_2(x_1^1) + (1 - \alpha) v_2(x_2^1) &\geq \inf_{x^2} \left\{ c_2(x^2) + \lambda^\top A_2 x^2 \right\} - \\ &\quad \lambda^\top b^2 + \lambda^\top (\alpha e(x_1^1) + (1 - \alpha) e(x_2^1)) \\ &= \inf_{x^2} \left\{ c_2(x^2) + \lambda^\top A_2 x^2 \right\} - \\ &\quad \lambda^\top b^2 + \textstyle\sum_{j=1}^{m_2} \lambda_j (\alpha e_j(x_1^1) + (1 - \alpha) e_j(x_2^1)). \end{aligned} \tag{11}$$

Equation (11) is valid for all λ, including $\lambda^*(\tilde{x})$, an optimal Lagrange multiplier for problem (6) written for $x^1 = \tilde{x}$:

$$\begin{aligned} \alpha v_2(x_1^1) + (1 - \alpha) v_2(x_2^1) &\geq \inf_{x^2} \left\{ c_2(x^2) + \lambda^*(\tilde{x})^\top A_2 x^2 \right\} - \\ &\quad \lambda^*(\tilde{x})^\top b^2 + \\ &\quad \textstyle\sum_{j=1}^{m_2} \lambda_j^*(\tilde{x}) (\alpha e_j(x_1^1) + (1 - \alpha) e_j(x_2^1)). \end{aligned} \tag{12}$$

As before, since the function $e_j(\cdot)$ is convex,

$$e_j(\alpha x_1^1 + (1 - \alpha) x_2^1) \leq \alpha e(x_1^1) + (1 - \alpha) e(x_2^1).$$

Using again the non-negativity of $\lambda^*(\tilde{x})$, equation (12) becomes

$$\begin{aligned} \alpha v_2(x_1^1) + (1 - \alpha) v_2(x_2^1) &\geq \inf_{x^2} \left\{ c_2(x^2) + \lambda^*(\tilde{x})^\top A_2 x^2 \right\} - \\ &\quad \lambda^*(\tilde{x})^\top b^2 + \\ &\quad \textstyle\sum_{j=1}^{m_2} \lambda_j^*(\tilde{x}) (\alpha e_j(x_1^1) + (1 - \alpha) e_j(x_2^1)) \\ &\geq \inf_{x^2} \left\{ c_2(x^2) + \lambda^*(\tilde{x})^\top A_2 x^2 \right\} - \\ &\quad \lambda^*(\tilde{x})^\top b^2 + \lambda^*(\tilde{x})^\top e(\alpha x_1^1 + (1 - \alpha) x_2^1) \\ &= \inf_{x^2} \left\{ c_2(x^2) + \lambda^*(\tilde{x})^\top (A_2 x^2 - b^2 + e(\tilde{x})) \right\} \\ &= v_2(\tilde{x}), \end{aligned}$$

which proves the desired convexity property. □

The assumption that A_2 has full rank is related to the condition of complete recourse in multi-stage stochastic programming. In energy planning problems, this property always holds.

By Theorem 1, the expected value of the second stage cost (value function) is convex and can be approximated by support hyperplanes, as used by the theory of Benders cuts. An important corollary of this theorem is the case where $e(\cdot)$ is linear, providing a subgradient formula for the function v_2 in this case.

Corollary 1 *Let the hyphotesis of Theorem 1 be valid, but with $e(x^1) = Ex^1$, where $E \in \mathbb{R}^{m_2 \times n_1}$. Then, a subgradient for the function v_2 at the point x^1 is $E^\top \lambda^*(x^1)$.*

Proof The result is direct because $Je(x^1) = E$, for all x^1, in this case. In addition, every linear function is concave and convex at the same time. □

Given the results of the theorem, the expected value of the second stage cost may be replaced by a maximum of linear functions, representing the Benders cuts generated in previous iterations, as follows:

$$\mathbb{E}_p v_2(x^1, q) \approx \max_{k \leq \kappa} \left\{ \sum_{q=1}^{Q} p_q \left(v_2(x_k^1, q) + {s_q^k}^\top (x^1 - x_k^1) \right) \right\}, \tag{13}$$

where κ is the number of Benders cuts, and s_q^k is a subgradiente of $v_2(\cdot, q)$ at the point x_k^1. By using this approximation, problem (5) can be rewritten as

$$\begin{aligned} &\min_{x^1} \; c_1(x^1) + \alpha \\ &\text{s.t.} \; \begin{cases} A_1 x^1 = b^1 \\ \alpha \geq \max_{k \leq \kappa} \left\{ \sum_{q=1}^{Q} p_q \left(v_2(x_k^1, q) + {s_q^k}^\top (x^1 - x_k^1) \right) \right\}. \end{cases} \end{aligned} \tag{14}$$

We finish by mentioning that the analysis above, applied to a 2-stage setting, can be extended to the multi-stage case. The latter problems are more involved (both the analysis and the notation), because computing the cuts involve nested approximations of the cost-to-functions.

3 Natural Gas Contracts Modelling

Due to its big extension and geographical localization, Brazil has several hydrological basins with distinct behaviors related to their hydrological inflows. Moreover, the structure of the NIS allows the utilization of resources of different regions to supply the demand for energy, converting all the system in an “unique big generator park”, dominated by hydroelectric plants, with a regularization capacity of several years. These characteristics imply in a seasonal dispatch of the thermoelectric plants along the year, being highly influenced by the inflows to the water reservoirs. Nowadays, there is a low diversification on the natural gas market in Brazil, what leads to an inconstant cash flow for natural gas suppliers. These were the motivations for the creation of supply contracts with take-or-pay clauses.

The contracts with take-or-pay clauses ensure a minimal monthly cash flow for the natural gas suppliers. The consumers of the natural gas, in this case the electric energy generators, are obligated to buy a minimal amount of gas, even if just a smaller quantity is necessary at this moment. This clause is known as the monthly take-or-pay clause. In a similar way, at the end of a year, a minimal amount, related to the total energy contracted for this year, must be paid by the electric energy generators to the natural gas suppliers; this is the annual take-or-pay clause. Typical values for the monthly and annual take-or-pay clauses are 56 and 70% of the monthly and annual contracted quantity of natural gas, respectively. If the generator does not use all the natural gas paid at a certain month, the exceeding quantity is available for generation for, usually, 7 years after the payment.

Some clauses similar to take-or-pay clauses also may exist in this kind of contracts, related to the transportation of natural gas, known as ship-or-pay clauses (SoP). The cost resulted by the incorporation of this clause may be easily incorporated to the model as a fixed cost. Due to this fact, these costs are not explicitly considered in this paper.

3.1 Problem Modeling

In this paper, the decision variables of the hydrothermal dispatch problem at a certain month are represented by lower case letters, and the known values at this month (e.g. state variables and constants) are represented by upper case letters.

3.1.1 Current Model

The computational model currently used by the national operator (ONS) for the long-term energy operation optimization is the NEWAVE model, developed by CEPEL. The model, in its current version, doesn't differentiate thermoelectric plants by their fuel, [9]. In this manner, there are no specific characteristics modeled for the natural gas thermoelectric plants, what means that supply contracts with take-or-pay clauses can't be represented exactly.

For all thermoelectric plants, monthly inflexibility values are declared, representing lower bounds for energy generation of these plants in some months of the planning horizon, represented by constraints as

$$gt_w^t \geq \underline{gt_w^t}, \tag{15}$$

where gt_w^t represents the energy generation of thermoelectric plant w at month t, and $\underline{gt_w^t}$ is the inflexibility declared by the user for this plant at this month.

Due to this representation, inflexibilities in thermoelectric plants generation are allocated to the load curve before generation of hydroelectric plants (thermoelectric inflexibility is always generated, because it is represented as a lower bound for the corresponding variables). This modeling yields spillages in some months at which the abundance of water could be used in demand supplying, instead of this thermoelectric inflexibility. In the case of consideration of the flexibility of take-or-pay clauses, the thermal resources may be used in a better way, e.g. at months with low inflows to or low levels at these reservoirs.

3.1.2 Auxiliary Models

Some models that aim at a better use of the flexibility offered by take-or-pay clauses in natural gas supply contracts can be found in the literature. As an example, in [6] an auxiliary model that optimizes the operation of a plant from the generator point-of-view is presented.

To realize this optimization, a long-term planning model that generates operation marginal costs – CMO, scenarios (spot prices of energy selling) is necessary. At this stage, the referred plant isn't considered. Once the generation of these scenarios is completed, an auxiliary model is used to find the optimal declaration of inflexibility that satisfies the take-or-pay clauses of its supply contract and maintenance constraints, and leads to the maximal expected profit. This procedure considers the referred plant isn't never (or in a very small number of scenarios) the marginal resource[1].

One of the disadvantages of an auxiliary model is the necessity for a "main" model that generates the spot price scenarios (e.g. NEWAVE model). Moreover, its goal is to maximize the expected profit of a thermoelectric plant, instead of minimizing the total operation cost for the whole system, goal of the NEWAVE model. Another disadvantage of the use of this procedure is that the plant isn't considered during the process of definition of the optimal operation of the system and the spot prices scenarios used in the profit maximization process are produced without considering this plant; the capacity of this plant available for energy generation may change in a significant way the resultant optimal operation and, as a consequence, the spot prices for energy selling.

3.1.3 Proposed Model

The model proposed in this work aims at the consideration of natural gas supply contracts take-or-pay clauses directly on the long-term energy planning model NEWAVE, in such a way to use the flexibility on the decision of buying and using the gas to achieve the smaller operation cost for the whole system, instead of achieving the maximum profit of an individual operator of the system.

The constraint represented in Eq.(15) is no longer used for the natural gas-fueled thermoelectric plants, being substituted by constraints that model the contract existent between the natural gas supplier and the electric energy generator. The decision variables of the NEWAVE model are, in this approach, the natural gas bought and used for generation, at each month.

4 Mathematical Modeling

The control of the new decision variables of the model are done through the use of thermal "reservoirs", following the modeling used for hydroelectric plants, currently used in the NEWAVE model. However, two reservoirs to model take or pay clauses contracts are used, instead of just one used in the hydroelectric modeling. The objective of the use of these reservoirs is to represent, at each month, the quantity of natural gas available for energy generation and the quantity bought initially but not used yet. In this way, the annual and monthly minimal buying of natural gas, representing the take or pay clauses can be guaranteed. The following reservoirs are the ones used in the modeling:

[1] the plant whose cost equals the marginal operation cost

– reservoir ETD: natural gas bought until the beginning of a certain month, but not used yet for energy generation (energy available for plant); and
– reservoir ETC: natural gas available at the beginning of the contract or the current year, but not bought until the beginning of a certain month.

The evolution of these reservoirs levels on time (months) is done through the consideration of the natural gas quantities (in energy) bought and effectively used on energy generation. Thus, these two variables must be considered for each natural gas fueled thermoelectric plant with take or pay clauses.

The relationship between one of these plants and its reservoirs is represented in Figure 2.

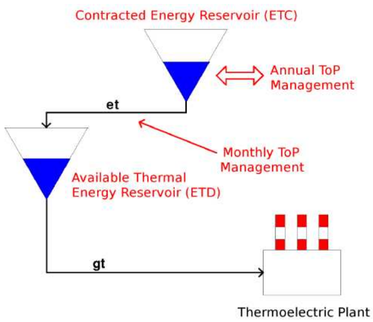

Fig. 2 Thermoelectric plant and its reservoirs

One may write the control equations of final level of the thermal reservoir after a simple analysis of the schematic representation of Figure 2. If gt_w^t represents the quantity of energy generated by thermoelectric plant w at month t, and et_w^t represents the quantity of energy bought by this plant at the same month, and if etd_w^{t-1} is the initial level of ETD reservoir, then its final level, etd_w^t is

$$etd_w^t = etd_w^{t-1} + et_w^t - gt_w^t. \tag{16}$$

Similarly, if etc_w^{t-1} is the initial level of ETC reservoir, then, if a quantity et_w^t of natural gas energy is bought at month t, the final level of this reservoir is

$$etc_w^t = etc_w^{t-1} - et_w^t. \tag{17}$$

The monthly take or pay clause is easily represented by a lower bound for the variable representing the bought energy – et_w^t. This means there is a minimal monthly transference of energy from the ETC reservoir to ETD reservoir, represented by the constraint

$$et_w^t \geq \frac{\gamma_m}{100} EC_t, \tag{18}$$

where γ_m represents the minimal monthly percentage of energy to be bought and EC_t the total energy available at this month (defined in the supply contract). In Brazil, a typical value for γ_m is $\gamma_m = 56$.

The representation of annual take or pay clauses, however, can't be found in such a direct way. To a better comprehension of its modeling, it is necessary to explain the management of ETC reservoir previously. Due to the monthly control on the contracted energy, in this work, the total energy specified in the contract is divided belong the years in which the contract is valid. The results of this division are the annual contracted energy values. If the first (resp. last) year of the contract doesn't begin (resp. finish) in the first (resp. last) month of the year, the number of months in which this year has a valid contract is taken into account when calculating the energy division.

After the contracted energy for a certain year is calculated, the model sets the initial level of ETC reservoir to this value, EC_y, where y represents the year, at the first month of contract at this year, t_0. If the contract ends at month τ at this year, then, the annual take or pay clause may be represented by the constraint

$$\sum_{t=t_0}^{\tau} et_w^t \geq \frac{\gamma_y}{100} EC_{y(t)}, \tag{19}$$

where $y(t)$ represents the year period t belongs to, and γ_y represents the minimal annual percentage of the contracted energy that must be bought. This constraint isn't easily incorporated to NEWAVE model due to its time dependence of (possibly) several months. By considering this dependence in a stochastic dynamic dual programming algorithm, as the one used in NEWAVE model, an unnecessary complexity would be aggregated to the computational program.

An alternative form for representing the annual take or pay clause is to set a upper bound for the final level of ETC reservoir at month τ. By setting this bound, a minimal bought along the current year is guaranteed, without the dependence of several months variables

$$etc_w^{\tau} \leq \left(1 - \frac{\gamma_y}{100}\right) EC_{y(t)}. \tag{20}$$

However, due to the stochastic dynamic dual programming algorithm used in NEWAVE model, which does not use feasibility cuts, it is important to ensure the operation of a period does not result in infeasibilities for the others periods (empty feasibility sets). For a intermediary month t, $t_0 \leq t \leq \tau$, one must ensure that in the $(\tau - t)$ final months it is feasible to buy at least the necessary quantity of natural gas in order to satisfy the annual take or pay clause. So, it is necessary to consider the upper bound of et_w^t, which represents the limits in natural gas monthly supplying. If this upper bound, given by the supplier, is represented by et^{max}, then the constraint to be considered at month t for the final level of ETC reservoir at this month is

$$etc_w^t \leq \left(1 - \frac{\gamma_y}{100}\right) EC_{y(t)} + (\tau - t)\, et^{max}. \tag{21}$$

Similarly, the monthly take or pay clause must be guaranteed. The quantity of natural gas bought at a period t can't be bigger than an upper bound. This value is the necessary natural gas for satisfying the monthly take or pay clauses in the $(\tau - t)$ final months. This constraint exists to simulate the real necessities of natural gas suppliers.

In a scenario in which this constraint is violated, the minimal annual gas constraint is satisfied, but a minimal natural gas buying is still necessary, due to operational aspects, usually associated to production of liquids in the natural gas extraction process (e.g. in the Bolivian gas extraction process).

Then, at a month t, $t_0 \leq t \leq \tau$, to ensure the monthly take or pay clauses for the final months, the minimal final level of the ETC reservoir at this month is

$$etc_w^t \geq (\tau - t)\,\frac{\gamma m}{100} EC_t. \tag{22}$$

The constraints of Eqs.(21) and (22) represent the expected values for ETC reservoir level at the last month τ of the year

$$0 \leq etc_w^\tau \leq \left(1 - \frac{\gamma y}{100}\right) EC_{y(t)}. \tag{23}$$

5 Results

5.1 Deterministic Cases

Initially, the model NEWAVE with the proposed modeling for take or pay clauses was tested with 75 deterministic cases. Due to the fact that the "old" modeling is a particular case of the proposed modeling, it is expected that the total operation cost of current NEWAVE model be always greater or equal to the one obtained with the proposed model. In Figure 3 are presented gains obtained using the proposed modeling for take or pay clauses in NEWAVE model. The case used is the one used by ONS for the monthly planning process in March, 2007.

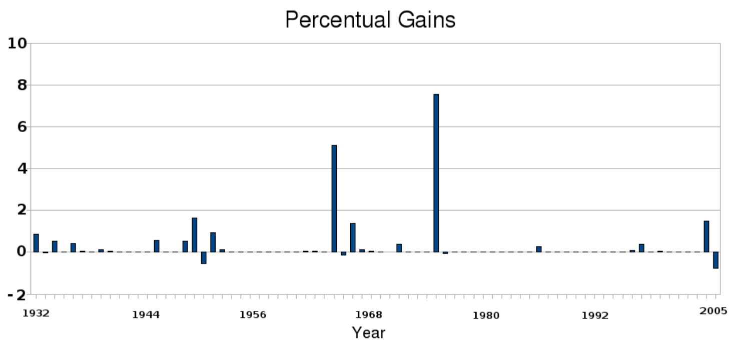

Fig. 3 Gains obtained using the proposed modeling

These results are presented in percentage of the original total operation cost. There were 6 negative gains (losses) in 75 cases; these 6 cases had an operation cost bigger than the original one. This result isn't expected and occurred only when the maximum number of iterations wasn't sufficient for the convergence of the case.

The optimal operation of the case in which was obtained the biggest gain (7.5%) when using the proposed modeling is compared to the original optimal operation for the same case. This case is chosen because significant differences of the operation cost must represent significant differences of operation, exploring the flexibility offered by the take or pay clauses.

For these deterministic cases, the supply contract simulated has the following characteristics: it is valid from July/2007 until November/2008 (17 months), with total contracted energy equal to 6800MWmês (400MWmês at each month), $\gamma_t = 56$ and $\gamma_y = 70$, default values used in brazilian contracts. By using the current modeling, the contract is represented by a lower bound equal to 280MWmês for the energy generation of the respective thermoelectric plant.

The generation of the thermoelectric plant with take or pay clauses when using or not the proposed modeling is presented in Figure 4. It is evident that the consideration of the take or pay clauses, in this case, may result in a better operation of the system, and, as a consequence, a smaller total operation cost.

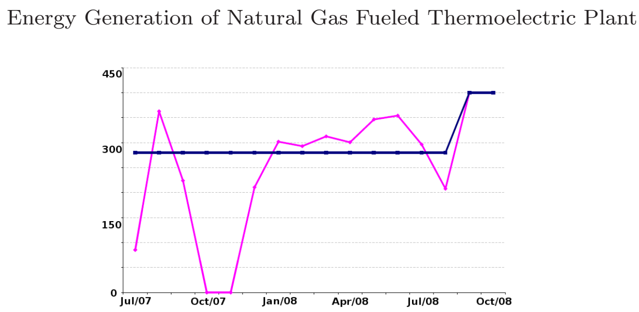

Fig. 4 Values of energy generation when using or not the proposed modeling

The evolutions of the ETD and ETC reservoirs levels are presented in Figures 5 and 6. In the first months (from July/2008 until December/2007), natural gas is bought but not completely used for generation, increasing the level of ETD reservoir, reserving this energy for the future operation of the plant. After December/2007 the plant uses more gas than the quantity bought at the respective month and the level of ETD reservoir decreases. The final level of this reservoir should be zero in the optimal case. This value isn't zero due to numerical tolerances on the convergence.

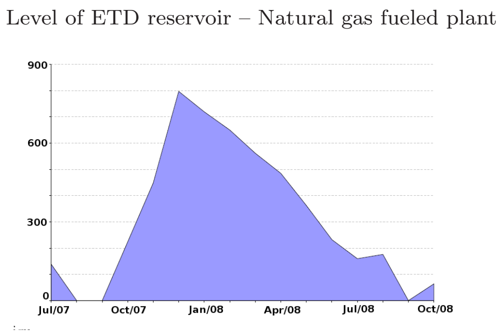

Fig. 5 Evolution of the final level of ETD reservoir

Figure 6 contains the evolution of the final level of the ETC reservoir and the acceptable range for this variable. This acceptable range is given by Eqs.(21) and (22).

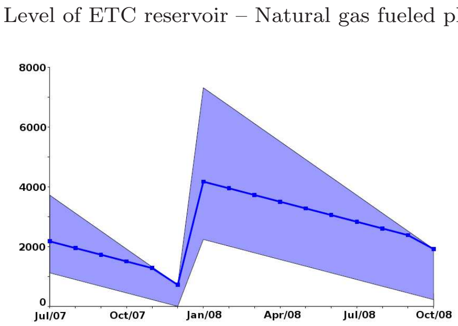

Fig. 6 Evolution of the final level of ETC reservoir

5.2 Stochastic Cases

The consideration of take or pay clauses directly in the energy long-term planning may avoid spillages in several moments, at which the inflexibility in thermoelectric generation does not allow a better use of water inflows. Due to this fact, it was done a sensibility analysis about the influence of water inflows in results obtained when using the proposed modeling for these clauses.

Initially, a case based on the real case used by ONS for the planning of the brazilian system in November/2007 had some of its natural gas fueled thermoelectric plants modeled considering explicitly the take or pay clauses. For this case, it was verified a reduction of the spilled energy, as illustrated in Figure 7. The pink line represents the expected spilled energy when using the proposed modeling, while the blue line (with squares) represents the expected spilled energy without the utilization of this modeling.

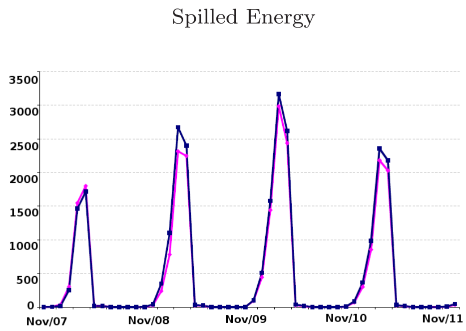

Fig. 7 Comparation of energy spillage

The influence of the water inflow is illustrated in Figures 8 and 9. These figures represent "permanence curves". A point (x,c) on the permanence curve of a certain variable should be read as: at a fixed month, the considered variable has a value greater or equal to c in x% of the series of the stochastic simulation. The blue lines with squares represent the curves for a case with low initial water inflow (30% of the long-term mean); the pink lines represent the curves for a case with high initial water inflow (300 % of the long-term mean).

The stochastic dynamic dual programming algorithm usually uses Monte Carlo simulation for the uncertainties involved on the problem. Thus, the difference of water inflows occurs, in this case, as a tendency for generation of the series for simulation. The impacts of the hydrological tendency occur, mainly, at the beginning of the series, due to the use of PAR(p) model to generate the series.

The permanence curve for generated energy of a natural gas fueled plant (UTE Norte Fluminense) is shown in Figure 8. In the case of a high water inflow as hydrological tendency (pink curve), the plant doesn't generate energy in none of the series. This behavior is also observed in Figure 9 – in all the series, the ETD reservoir level represents all the energy bought in previous months.

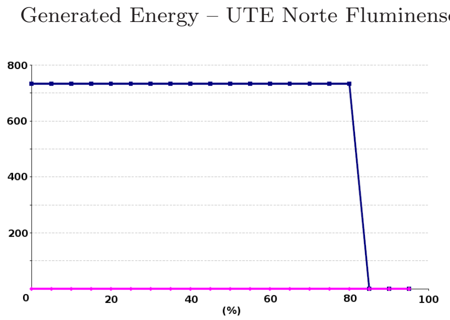

Fig. 8 Permanence curve for *gt*, Month: Apr/08 – UTE Norte Fluminense

In the case of a low water inflow as hydrological tendency, the behavior is different, as expected. In 80% of the series, the plant generates its maximal potential in April/2008, and in 60% of the series the plant uses all the gas bought until this month in energy generation.

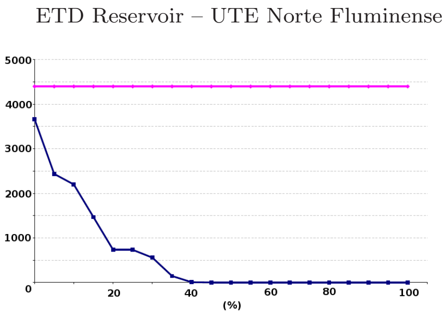

Fig. 9 Permanence curve for *etd*, Month: Apr/08 – UTE Norte Fluminense

Due to the fact of an increase in the number of state and decision variables when using the proposed modeling, the processing time also increases in about 35%, as shown in Table 1. The results of sensibility related to different hydrological tendencies are also shown in this table. The processing time for an extra case (1 Plant) represents a sensibility related to the numbers of thermoelectric plants for which the take or pay clauses are modeled.

	Processing Time (minutes)		
Case	Without	With	Increase (%)
Base	238	323	35,71
LowInflow	243	327	34,57
HighInflow	215	296	37,67
1 Plant		307	-

Table 1 Processing time – 3 iterations

6 Concluding Remarks

Nowadays, specially due to the discover of natural gas reservoirs in Brazil, potential political problems with Bolivia, and a bigger intercontinental integration of the countries that form Mercosul block, the correct modeling of the supply contracts of this resource may avoid financial losses in both parts of the contracts – supplier and consumer. Moreover, a better utilization of the natural gas allows other segments of the market to have a bigger availability of the resource, without influences on the energy planning.

The consideration of the flexibility offered by the take or pay clauses allows a more flexible definition of monthly goal for the thermoelectric plants generation, avoiding water losses. The current situation, in which generation goals are defined by the generator agents themselves, aims at, always, obtaining higher profits. When considering the take or pay clauses directly in the planning model, a better operation of the whole NIS is (may be) achieved.

By using a planning model considering more characteristics of the system (e.g. the characteristics of distinct plants), some studies aiming at the definition of values for new natural gas supply contracts can be done in such a way to optimize the operation of the whole system. Moreover, the consideration of take or pay clauses flexibility results, in a long- or long-term horizon, yields a more trustable situation of the system, due to a better use of the hydraulic potential, avoiding part of the spillages.

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