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A GENETIC ALGORITHM FOR A GLOBAL OPTIMIZATION PROBLEM ARISING IN THE DETECTION OF GRAVITATIONAL WAVES

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A genetic algorithm for a global optimization problem arising in the detection of gravitational waves

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Abstract

The detection of gravitational waves is a long-awaited event in modern physics and, to achieve this challenging goal, detectors with high sensitivity are being used or are under development. In order to extract gravitational signals, emitted by coalescing binary systems of compact objects (neutron stars and/or black holes), from noisy data obtained by interferometric detectors, the matched filter technique is generally used. Its computational kernel is a box-constrained global optimization problem having many local solutions and a highly nonlinear and expensive objective function, whose derivatives are not available. To tackle this problem, we designed a real-coded genetic algorithm, which exploits characteristic features of the problem itself; special attention was devoted to the choice of the initial population and of the recombination operator. Computational experiments, carried out on a representative test set, showed that the genetic algorithm is able to compute a reasonably accurate solution of the optimization problem and that its success is strongly dependent on a problem-driven choice of the initial population. Furthermore, the genetic algorithm requires a much smaller number of function evaluations than the grid search, which is the algorithm generally used to solve the optimization problem at hand.

1 Introduction

The detection of gravitational waves (DGW) from astrophysical sources is a very challenging goal in modern physics. A direct evidence of the existence of such waves will provide a validation of Einstein's general relativity theory and will open a path toward a new view of the universe [25]. Networks of detectors have been recently deployed with

unprecedented capabilities, but gravitational waves have not yet been observed because of many difficulties arising in the detection process. Among them, the weakness of the gravitational signal and the rarity of the events that produce such waves, call for highly effective data analysis techniques to filter the detector data streams.

Coalescing binary systems of compact objects (neutron stars and/or black holes) are very promising sources of gravitational waves for ground-based laser interferometric detectors. This is because a model of the emitted waves is available and a relatively large number of events per year is expected (tens per year within a few hundred Mpc) [18]. In this case, the most widely used detection technique is the *matched filter*, which exploits the waveform of the signal and assumes that the instrumental noise is a stationary white or withened Gaussian stochastic process [2, 10, 23].

A crucial issue in this methodology is the solution of a box-constrained global optimization problem, which is hard to solve, because of the strong nonlinearity of the objective function, the unavailability of its derivatives, the presence of many local maxima, and the high computational cost of its evaluation. Furthermore, the objective function is a stochastic process because of the presence of noise, and hence, for a given gravitational signal, its maximum value changes with a specific realization of noise.

In the astrophysics community, this problem is usually solved by applying the *grid search*, which evaluates the objective function in a suitable discrete set of points belonging to the feasible domain (the grid) and approximates the maximum of the objective function with the maximum over the discrete set [2]. The strength of this method is that the grid points can be chosen to ensure that the computed maximum satisfies certain accuracy requirements, in a sense that will be clarified later (see Section 2). The drawback is that a very large number of grid points, and hence of function evaluations, is needed to obtain solutions which can be considered "accurate enough" by the astrophysics community. In order to reduce the number of function evaluations, two-step hierarchical grid search strategies have been proposed [20, 21, 24]; in the first step a coarse grid is applied to identify "promising" sub-domains where a fine grid can be used to find a good approximation of the maximum. A hierarchical approach where the first step is perfomed by the Price algorithm has also been developed [18]. However, such strategies might lead to disregard, at the first step, a sub-domain containing the solution, thus increasing the probability of missing the signal even if it is present.[1]

In this work we investigate the application of Genetic Algorithms (GAs) to the above global optimization problem, with the aim of reducing the computational cost with respect to the grid search. These algorithms have the advantage, over deterministic optimization methods, that a priori information on the problem can be incorporated in the design of genetic operators, allowing the solution of difficult problems (see, e.g., [4, 5]). Indeed, we designed a real-coded GA that takes into account characteristic features of the problem. The results obtained on a set of representative test problems show that our approach is able to compute reasonably accurate solutions with a much lower computational cost than the grid search.

[1] The matched filter technique provides a solution to the DGW problem with a certain confidence, identified by the so-called probabilities of false alarm and detection, as explained in Section 2.

The paper is organised as follows. In Section 2 we briefly describe the mathematical model of the DGW problem and the matched filter technique, focusing on the global optimization problem arising in this context; we provide also a few details on the grid search. In Section 3 we discuss the design of a genetic algorithm for our problem. We go into the choice of the initial population and on the selection of suitable genetic operators, to build an algorithm that takes into account specific features of the problem. In Section 4 we show the results of numerical experiments carried out on a representative set of test problems, with the aim of analyzing the effectiveness of our approach with respect to the grid search. We provide a few concluding remarks in Section 5.

2 Mathematical formulation of the DGW problem

The DGW problem basically consists in discovering if the output of the detector contains a gravitational signal or if it is just the noise background. In the presence of a signal, this output is generally modeled as

$$x(t) = r(t) + h(t; \boldsymbol{\theta}),$$

where t is the time, $r(t)$ is the noise, $h(t; \boldsymbol{\theta})$ is the gravitational signal and $\boldsymbol{\theta}$ is a vector of parameters. We assume that $r(t)$ is strictly white noise, i.e. a wide-sense stationary Gaussian stochastic process with mean 0 and variance 1 [22]. We focus our attention on gravitational wave signals emitted by coalescing binary systems, which can be modeled as *chirp* signals [20], i.e.

$$h(t; \boldsymbol{\theta}) = Aa(t - t_0; m_1, m_2) \cos(\varphi(t - t_0; m_1, m_2) + \varphi_0),$$

where

$$\boldsymbol{\theta} = (A, \varphi_0, t_0, m_1, m_2),$$

with A, φ_0 and t_0 denoting the amplitude, the initial phase, and the arrival time of the signal, respectively, and m_1 and m_2 the masses of the coalescing binary system. The functions $a(t - t_0; m_1, m_2)$ and $\varphi(t - t_0; m_1, m_2)$ are expressed in the so-called second-order restricted post-Newtonian approximation [20]. We note that the vector of parameters $\boldsymbol{\theta}$ is unknown.

In practice, the output of the detector is sampled with a certain time step, thus a segment of data is analyzed, which is an N-dimensional vector $\boldsymbol{x} = (x[0], \ldots, x[N-1])$; the corresponding sampled gravitational signal, if present, is an M-dimensional vector $\boldsymbol{h}(\boldsymbol{\theta}) = (h[0], \ldots, h[M-1])$, with $M < N$ (the dependence on $\boldsymbol{\theta}$ has been neglected for simplicity). In the following, we assume $\boldsymbol{\theta} = (A, \varphi_0, n_0, m_1, m_2)$, i.e. we substitute the arrival time t_0 with the index n_0 of the corresponding sample.

As noted in Section 1, a widely used technique for solving the DGW problem is the matched filter, which is an optimal linear filter for detecting signals of known shape in stationary Gaussian noise [2, 10, 23]. The application of this technique requires the following steps:

1. correlating the output of the detector with a family of templates, consisting of chirp signals $\boldsymbol{h}(\boldsymbol{\theta})$, with $\boldsymbol{\theta}$ varying in a suitable manifold;

2. finding the maximum of the correlations with respect to all the parameters;

3. comparing this maximum with a suitable threshold to decide if the output of the detector contains a gravitational signal (a detection is announced if the maximum exceeds the threshold).

This procedure is based on the observation that the highest Signal-to-Noise Ratio (SNR)[2] of the filter output is obtained when the values of the parameters identifying the template are the same as in the signal, and that this SNR is equal to the maximum, with respect to the signal parameters, of the mean value of the correlation, which is a stochastic process because of the stochastic nature of the noise [2]. The choice of the threshold is related to the probability of false alarm, i.e. of stating a detection in the absence of a signal, and to the probability of detection, i.e. of stating a detection when the output contains a gravitational signal [21].

It can be shown that the maximization in step 2 can be performed with respect to only three parameters, i.e. the masses m_1 and m_2, and the index n_0; furthermore, it can be carried out separately for (m_1, m_2) and n_0 [2]. Thus, steps 1 and 2 of the matched filter can be formulated as the following box-constrained global optimization problem:

$$\underset{(m_1,m_2)\in\Omega}{\text{maximize}}\, F(m_1, m_2), \tag{1}$$

with

$$\Omega = \{(m_1, m_2) \in \Re^2 \ : \ l \leq m_1, m_2 \leq u\} \tag{2}$$

and

$$F(m_1, m_2) = \sqrt{\max_{n_0\in\{0,\ldots,N-M\}} \Big(C_0^2(n_0, m_1, m_2) + C_{\pi/2}^2(n_0, m_1, m_2)\Big)}, \tag{3}$$

where $C_0(n_0, m_1, m_2)$ and $C_{\pi/2}(n_0, m_1, m_2)$ are the correlations between $\boldsymbol{x}$ and the so-called normalized quadrature components of the template, $\hat{\boldsymbol{h}}_0(m_1, m_2)$ and $\hat{\boldsymbol{h}}_{\pi/2}(m_1, m_2)$ [21], i.e.

$$C_0(n_0, m_1, m_2) = \sum_{k=n_0}^{n_0+M-1} x[k]\hat{h}_0[k - n_0]$$

[2]Throughout the paper we assume the following operating definition of SNR:

$$SNR = \sqrt{\frac{E^2}{\sigma^2}}, \quad E^2 = \frac{2}{f}\sum_i h[i],$$

where σ^2 is the power of the noise, assumed to be white Gaussian, f is the sampling frequency and $h[i]$ are the samples of the signal, previously defined. This definition is adopted by the LAL package [1], which has been used for the generation of test problems in our computational experiments (see Section 4).

and

$$C_{\pi/2}(n_0, m_1, m_2) = \sum_{k=n_0}^{n_0+M-1} x[k]\hat{h}_{\pi/2}[k - n_0].$$

We note that the maximum SNR is the maximum of the mean value of the objective function in (1); as shown in Section 4, this property can be used to assess the "accuracy" of the solution of problem (1) computed by using a selected algorithm.

Problem (1) is the most critical issue in the application of the matched filter. Its solution is a difficult task, because the objective function F is highly nonlinear and with many local maxima (see Figure 1), and its derivatives are not available. Furthermore, its evaluation is computationally expensive, since it requires the solution of two ordinary differential equations (ODEs) to generate the quadrature components of each template [6], and the execution of three FFTs of length N to compute the correlations of $\boldsymbol{x}$ with them [20]. Common values of N are $O(10^5)$; the time for solving the ODEs depends on the masses of the gravitational signal (the smaller the masses the larger the time), and is highly variable (from about 2% to 650% of the time for computing the correlations, in our experience).

In the astrophysics community, the most widely used method for solving problem (1) is the grid search; it discretizes the feasible domain Ω by using a suitable grid of points and evaluates the objective function F at each point to determine an approximation of the global maximum. Note that the search for the maximum can be carried out in half of the feasible domain, since F is symmetric with respect to m_1 and m_2. The main reason for choosing the grid search is that it provides information on the "accuracy" of the computed maximum. Indeed, the grid can be built by ensuring that the mean value of the maximum of F over the grid is not lower than a given percentage of the maximum SNR; this percentage is called *minimal match* [2]. In practice, a minimal match of at least 97% is required, leading to a large number of grid points and hence of objective function evaluations; for example, a grid of 27376 points is needed to get a minimal match of 97% over the domain $[1, 30] \times [1, 30]$ (see Figure 2). We also note that the grid is highly non-uniform, with more points in the regions of greater variability of the objective function.

As already observed, reducing the computational cost in the solution of problem (1), while achieving a comparable accuracy in the mean value of the maximum, is a main goal in the application of the matched filter, since it would increase the number of data segments that can be analyzed. In the next section, we present a genetic algorithm developed to achieve this goal.

3 A genetic algorithm for the DGW problem

The genetic algorithms (GAs), first proposed by Holland in [15], are a class of evolutionary algorithms which apply the principles of natural evolution to find an optimal solution to a problem. In a GA, an initial population of candidate solutions, called *individuals* or *chromosomes*, is randomly taken, so that coverage of the search space is assumed; three basic operators, *selection*, *recombination* (or *crossover*) and *mutation*,

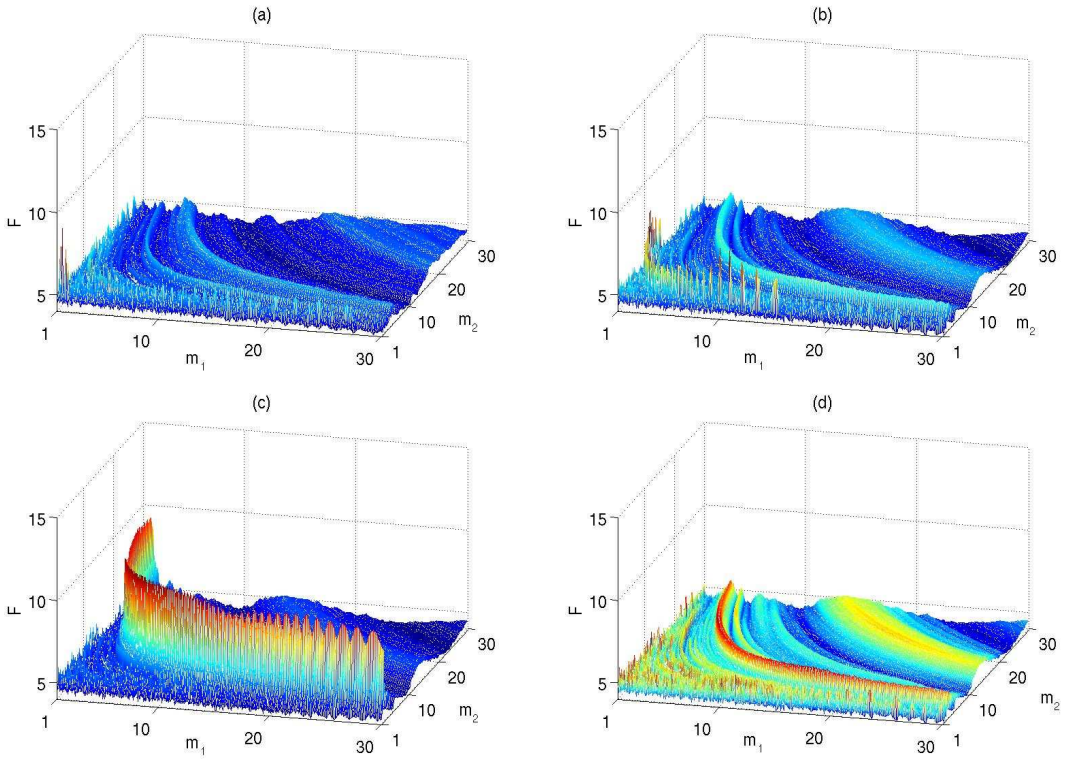

Figure 1: 3D plot of the objective function F, in case of noise plus gravitational signal from a binary system with masses $m_1 = m_2 = 1.4\ M_\odot$ (a), $m_1 = 1.4\ M_\odot$ and $m_2 = 10\ M_\odot$ (b), $m_1 = 5\ M_\odot$ and $m_2 = 10\ M_\odot$ (c), and in case of noise only (d). The SNR is equal to 10. $M_\odot$ denotes the solar mass.

which mimic the corresponding natural processes, are iteratively applied to evolve the individuals toward a better population. A *fitness* function is used to measure the "goodness" of each individual. In other words, individuals with higher fitness are selected over the time to create, by means of genetic operators, new individuals destined to enter the population by replacing some or all the old ones, with the objective of obtaining a better population. The basic structure of a GA is outlined in Figure 3. The solution of an optimization problem is provided by the fittest individual after the last evolution step.

Typically, GAs use fixed-length and binary-coded strings for representing each individual, but other representations have been devised that result more adequate for particular problems [17]. For numerical optimization problems on continuous domains,

$$\underset{x \in \Gamma \subset R^n}{\text{maximize}}\, f(x),$$

using a real-coded representation is more natural; in this case each individual is a real n-dimensional vector, having as genes the vector components [14]. In particular, in problem (1), an individual is a pair of masses $\boldsymbol{m} = (m_1, m_2)$ and its genes are the single masses m_i, $i = 1, 2$. The way GAs perform strongly depends on the design of the

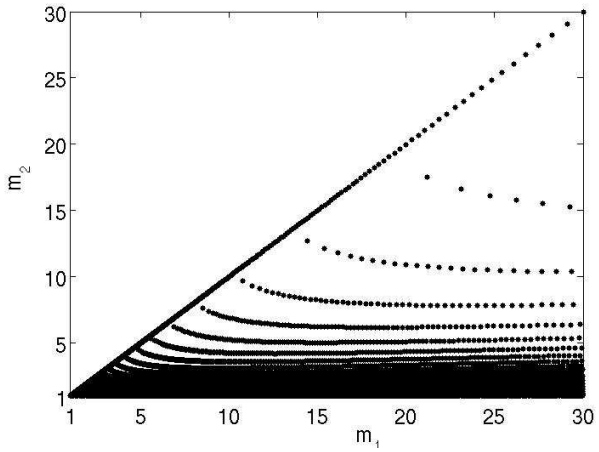

Figure 2: Grid ensuring a minimal match of 97% over the domain $[1, 30] \times [1, 30]$ (27376 points).

```
begin
    initialize the population with random potential solutions
    while (stopping criterion not satisfied)
       select parents
       recombine pairs of parents to generate offspring
       replace some parents with some offspring
       mutate the resulting population
    end while
end
```

Figure 3: Basic structure of a genetic algorithm.

genetic operators and their tuning to the specific problem under consideration. Next we describe the genetic operators as they have been designed in our algorithm. Before them, we discuss the choice of the initial population, which plays a fundamental role in the solution of our problem.

3.1 Initial population

Much has been done in the last years in developing smart basic operators aimed at preventing the stagnation around local solutions or sub-optimal points and at making GAs competitive with other more theoretically sound approaches. However, as noticed by Maaranen et al. in [16], the effect of the initial population on the performance of GAs has been widely ignored. In particular, they provide numerical evidence that the initial population may strongly affect the speed of GAs and that a "good" initial population should combine *genetic diversity*, i.e. the ability to reach the whole feasible set during

the evolution process, with *uniform coverage*, i.e. a spatial distribution in the feasible set which avoids clustering and uncovered regions. On the other hand, in the GDW problem, a suitable choice of the templates for the matched filter technique leads to a nonuniform discretization of the feasible set, with points clustered in the areas where the objective function shows greater variability. This is an a priori specific information on the problem which can be included into the selection of the initial population. The most straightforward way to do this is to randomly select the individuals from a grid G corresponding to a widely accepted value of minimal match, such as 97%.

In order to foster a uniform coverage, we combined the previous strategy with a *nonaligned systematic sampling* (NSS), in which the feasible box Ω is splitted into b^2 elementary boxes with equal side lenghts, and one individual is selected in each elementary box according to some rule [16]. In our case, the individual is randomly chosen among the points of G belonging to the box; furthermore, the algorithm has been slightly modified to handle the (possible) case that a box does not contain any grid point. Given the size N_p of the population, i.e. the number of its individuals, the NSS is applied first, to select a part of the initial population; then, the remaining individuals, are randomly taken from G. The parameter b is chosen to guarantee that no large areas of the feasible domain are left uncovered by the initial population. Note that $b = 0$ corresponds to an initial population randomly selected from G, whereas $b^2 = N_p$ corresponds to an initial population resulting only from the grid-based NSS. Because of the symmetry of the global optimization problem with respect to m_1 and m_2, only the triangle $m_1 \geq m_2$ of Ω, and the elementary boxes covering this triangle, are actually considered.

The random selection of any individual from G is performed by labelling each point of G with an integer number from 1 to N_p and by using the following formula:

$$q = 1 + int(rnd(0,1) \cdot N_p), \tag{4}$$

where $rnd(0,1)$ is a random number from a uniform distribution in $(0,1)$ and $int(x)$ is the integer part of x. The same rule is applied in each elementary box, considering only the points of G contained into the box.

3.2 Selection of parents

The purpose of the selection operator is to choose, from the current population, a mating pool of individuals that will potentially generate offspring through the recombination of their chromosomes. The selection of these individuals, called parents, is based on the principle of elitism: the individuals with higher fitness have higher probability to be picked. On the other hand, population diversity must be kept in order to avoid a premature convergence of the genetic algorithm, and therefore too much elitism in the selection might result in a serious drawback, especially when many local solutions exist.

Several selection operators have been devised, such as proportional selection [11], tournament selection [7], rank-based selection [12] and truncation selection [8]. These operators are characterized by the so-called *selective pressure*, which is related to the takeover time, i.e. the number of generations needed by the best individual in the initial

population to fill up the whole population, by the application of the selection operator alone [9]. If the takeover time is large then the selective pressure is small, and vice versa. The selective pressure provides information on the number of generations after which the mutation becomes the primary operator of exploration.

For our problem, we choose the *binary tournament without replacement*, that has a medium selective pressure with respect to the other selection operators [8], and hence appears suitable for handling the existence of a large number of local solutions. Furthermore, the binary tournament does not require for the individuals in the population to be ranked. This operator randomly picks two individuals from the population and selects the one with better fitness as potential parent to be put into the mating pool; the picked individuals are removed from the population and the process is repeated again, until no individuals are available. This procedure is repeated twice, to have a number of parents equal to the size N_p of the population. In this way the best individual is selected at least twice and the worst one is discharged. We also note that the same individual can be present in the mating pool twice, depending on its fitness. The random selection of each individual is carried out according to the rule (4), where the current number of individuals is used instead of N_p at each step of the tournament.

3.3 Recombination

Once the mating pool is defined, pairs of individuals are randomly taken from it and mated. The number of actual parents depends on a parameter $P_R \in (0, 1)$, called *probability of recombination*; for each individual in the mating pool, a random number r from a uniform distribution in (0,1) is generated and, if $r < P_R$, the individual is selected as parent. The pair of parents are formed by taking two individuals consecutively selected.

As basic recombination operator we choose the *BLX-α* one, which is a well established and studied technique for real-coded genetic algorithms [13, 14]. Each pair of parents $\boldsymbol{m}^m, \boldsymbol{m}^f$ generates three offspring, $\boldsymbol{m}^1, \boldsymbol{m}^2, \boldsymbol{m}^3$. The recombination is carried out separately on each gene by taking

$$m_i^j = rnd(I_i) \quad (i = 1, 2;\ j = 1, 2, 3), \tag{5}$$

where the action interval I_i is defined as

$$I_i = [g_i - \alpha M_i, G_i + \alpha M_i]$$

with $g_i = \min\{m_i^m, m_i^f\}$, $G_i = \max\{m_i^m, m_i^f\}$, and $M_i = G_i - g_i$.

We note that α is related to the size of the region around each parent, and thus its value controls the degree of "resemblance" to a parent. In particular, $\alpha > 0$ fosters *exploration*, i.e. the tendency to expand the search space, whereas $\alpha < 0$ fosters *exploitation*, i.e. the tendency to deepen the knowledge in areas of the search space that have been already visited. For $\alpha = 0$ the so-called flat recombination is obtained, in which m_i^j is randomly chosen between the corresponding genes of its parents. In our problem we considered $\alpha = 0.5$, a choice which allows to balance exploration and exploitation

[14], since the new gene has the same probability to lie inside or outside the interval defined by its parents.

However, taking $\alpha > 0$ might bring to an action interval which is not feasible, i.e. contains a subinterval not included in $[l, u]$, where l and u are defined in (2). In order to handle the box constraints, we devised two variants of the BLX-α strategy. In the former, a gene m_j^i generated according to (5), which does not belong to $[l, u]$, is replaced by its projection onto the interval

$$I_i^P = [g_i, G_i].$$

This strategy is called *PBLX-α*. In the latter, if I_i is not feasible, one considers as action interval the largest feasible interval $I_i^S \subset I_i$ obtained by symmetrically shrinking I, i.e.

$$I_i^S = [g_i - \bar{\alpha} M_i, G_i + \bar{\alpha} M_i],$$

where $\bar{\alpha}$ is the largest value such that I_i^S is feasible. This strategy is called *SBLX-α*. We observe that SBLX-α is more conservative with respect to PBLX-α since it works on a smaller action interval; furthermore, the closer is a parent gene to one of its bounds, the higher is the probability for the generated genes to be equal to the parent one.

3.4 Replacement of parents

In order to choose which individuals will survive, two main approaches can be adopted: the *overlapping-generation* model and the *nonoverlapping-generation* model [8]. In the former case, the parents and the offspring will compete with each other for survival; in the latter, all parents die at each generation and the offspring compete for survival.

The overlapping-model is more elitist, thus implying a loss of genetic diversity which is likely to lead to premature convergence to a local maximum. Because of the specific features of our problem, we decided to use a nonoverlapping-generation model, in which every pair of parents generates three offspring and the two offspring with better fitness survive. This simple model is combined with an elitist strategy guaranteeing that a copy of the best individual in the current population is forced to be selected into the new one. This individual is not replaced by its offspring.

3.5 Mutation

The mutation operator is aimed at randomly altering some individuals in the population, in order to introduce genetic diversity. We use the non-uniform mutation described in [17]. In this case, a gene m_i to be mutated becomes a new gene m_i^{new} according to the following formula:

$$m_i^{new} = \begin{cases} m_i + \Delta(k, u - m_i) & \text{if} \quad r \geq 0.5, \\ m_i + \Delta(k, m_i - l) & \text{if} \quad r < 0.5, \end{cases}$$

where r is a random number taken from a uniform distribution in $(0, 1)$ and $\Delta(t, y)$ is defined as

$$\Delta(k, y) = y \cdot \left(1 - r^{\left(1 - \frac{k}{N_G}\right)^2}\right),$$

where k is the number of generations obtained so far and N_G is the maximum number of generations of the GA. This operator allows to explore the feasible domain uniformly in the first generations, and locally in the later generations.

The number of genes to be mutated depends on a parameter P_M called *probability of mutation*. For each gene of each individual, a random number r is taken from a uniform distribution in (0,1) and the gene is mutated if $r < P_M$. To avoid the best individual to be lost through the generations, we use an elitist strategy as in the replacement of parents, i.e. we preserve a copy of the best individual by avoiding mutating it.

4 Computational experiments

Extensive computational experiments were carried out to evaluate the effectiveness of our GA approach in the solution of the DGW problem and its competitiveness with the grid search. Particular attention was devoted to analyzing the effects of different choices of the initial population and of recombination strategies handling the box constraints, which are the more distinctive features of the GA described in the previous section.

We generated three sets of test problems, in which the detector output consists of strictly white noise and gravitational signal from three pair of masses, corresponding to representative binary systems:

- $m_1 = m_2 = 1.4M_\odot$ (two neutron stars),
- $m_1 = 1.4M_\odot$ and $m_2 = 10M_\odot$ (one neutron star and one black hole),
- $m_1 = 5M_\odot$ and $m_2 = 10M_\odot$ (two black holes),

where $M_\odot$ denotes the solar mass. For each pair of masses we considered 30 realizations of noise, thus obtaining a set of 30 detector outputs to be analyzed. The length N of such outputs is 131072, while the length M of the signal varies with the masses (51207 for $m_1 = m_2 = 1.4$, 10823 for $m_1 = 1.4$ and $m_2 = 10$ and 3216 for $m_1 = 5$ and $m_2 = 10$). For all the problems, a SNR equal to 10 was chosen. The lower bound l and the upper bound u on the masses, defining the feasible domain, were set to 1 and 30, respectively. All the data were obtained by using the LAL package [1], which is gaining wide acceptance as a reference tool for gravitational wave data analysis.

For each set of test problems, the GA was run using 30 different seeds for initializing its pseudo-random number generator. The algorithm was stopped when the maximum number of generations, N_G, was achieved. We note that we did not stop the algorithm as soon as the detection threshold was exceeded (see step 3 of the matched filtering in Section 2), since in this case the computed maximum of the objective function may be very far from the actual one, thus providing poor information on the signal. On the other hand, the algorithm might not be able to compute a maximum exceeding the threshold. However, the threshold was used to evaluate the performance of the algorithm, as explained below. A threshold equal to 8 was chosen, which is a typical value in the DGW problem [19]. We also verified experimentally that other stopping criteria, e.g. based on the variation of the masses, may halt the algorithm prematurely.

parameter	value
N_P (number of individuals)	100
P_R (probability of recombination)	0.7
P_M (probability od mutation)	0.05
N_G (maximum number of generations)	50

Table 1: Values of the GA parameters

The GA parameters, i.e. the probabilities of recombination and mutation, the size of the initial population and the maximum number of generations were set as in Table 1. These values were selected mainly on the basis of computational experiments, which are not reported here for the sake of space; the choice of values for P_R and P_M was supported also by the literature [3, 26].

Our first experiments were aimed at studying the impact of the choice of the initial population on the GA behaviour. We compared three different strategies:

- random generation of individuals from a uniform distribution in $[1, 30] \times [1, 30]$ (RAND);
- NSS with $b^2 = N_P$ elementary boxes, with a random choice of individuals from a uniform distribution in each box (RAND-NSS);
- combination of random choice of individuals from the grid corresponding to a minimal match of 97% and of grid-based NSS with $b^2 = 4$, as explained in section 3.1 (RAND-GRID).

Initial populations generated with these three strategies are shown in Figure 4. Table 2 shows the numerical results obtained by running the GA with the three strategies, using PBLX-0.5 as recombination operator, for each set of test problems. The mean value of the computed maximum of the objective function F over 900 runs (30 realizations of noise $\times$ 30 seeds for the pseudo-random generator) and the related standard deviation are reported in the *mean* and *std* columns; the percentage of runs in which the maximum of F exceeds the selected threshold, and hence a signal detection is stated, is reported in the *success* column; finally, the absolute value of the difference between the mean value of the maximum of F computed by the grid search and that computed by the GA, divided by the first one, is reported in the *relerr* column. We recall that the reference value for the mean of the computed maximum of F is the SNR, i.e. 10.

As expected, a choice of the initial population which provides a uniform coverage of the feasible domain without taking into account the specific characteristics of the objective function (RAND-NSS) does not produce any improvement with respect to a uniform random choice of the population in the whole feasible domain (RAND). On the other hand, a very strong improvement can be observed when the problem-driven approach is adopted (RAND-GRID), which confirms that great benefit to the GA can be gained by selecting an initial population suggested by a priori knowledge of the problem. This is particularly evident for the case $m_1 = m_2 = 1.4$, in which neglecting

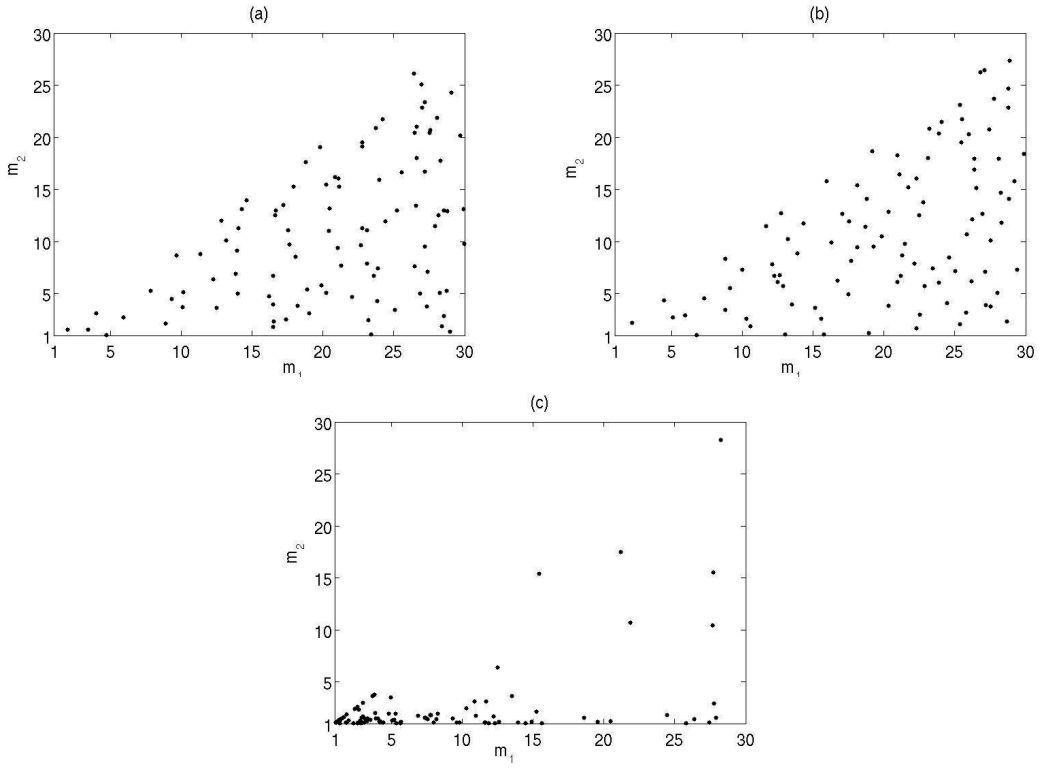

Figure 4: Initial populations of 100 individuals generated by using the RAND (a), RAND-NSS (b) and RAND-GRID (c) strategies.

m_1	m_2	*init. population*	*mean*	*std*	*success (%)*	*relerr*
1.4	1.4	RAND	6.8133	1.6225	20.9	0.321
		RAND-NSS	6.9053	1.6852	24.0	0.312
		RAND-GRID	9.7036	1.1397	92.8	0.034
1.4	10	RAND	9.6069	1.5763	87.9	0.065
		RAND-NSS	9.5351	1.5918	86.0	0.072
		RAND-GRID	10.0770	1.1449	96.4	0.020
5	10	RAND	10.2730	1.0383	99.0	0.001
		RAND-NSS	10.2770	1.0319	99.3	0.001
		RAND-GRID	10.1290	1.0717	97.8	0.015

Table 2: GA behaviour with different strategies for the selection of the initial population. PBLX-0.5 is used as recombination operator.

the information provided by the grid leads to a mean value of the maximum of F that is very far from the SNR and exceeds the threshold in at most 24% of the runs. Similar results hold if SBLX-0.5 is used as recombination operator, confirming that the problem-driven choice of the initial population is a key issue in the problem at hand.

Taking into account the previous results, we performed a deeper analysis of the GA behaviour with the RAND-GRID strategy, varying the value of the parameter b in the grid-based NSS, and applying the recombination rules PBLX-0.5 and SBLX-0.5 described in Section 3.3. The corresponding results are reported in Table 3 for PBLX-0.5 and in Table 4 for SBLX-0.5. By looking at Table 3, we see that $b = 0$ and $b = 4$ lead to very close results for all the test sets; their behaviour is satisfactory, as shown by the mean value of the computed maximum, which, in the problem at hand, can be considered very close to the mean value of the maximum over the grid (see the *relerr* column), and by the high percentage of success. We note that the lower percentage of success for $m_1 = m_2 = 1.4$ is also due to the fact that one of the 30 instances of this class of problems has a maximum value of F lower than the detection threshold (the maximum computed by the grid search algorithm is 7.02). Therefore, the success cannot exceed 96.7% in this case. The choice $b = 8$, in which more than one third of the population is generated by the grid-based NSS, degrades the GA performance for $m_1 = m_2 = 1.4$, while slightly improves it for $m_1 = 5$ and $m_2 = 10$. The previous comments apply also to the results in Table 4, concerning SBLX-0.5. However, we see that SBLX-0.5 generally leads to greater mean values of the maximum of F and greater percentages of success (about 99% for the problems with larger masses). This suggests that a more conservative strategy to handle the constraints should be preferred. It is worth noting that computational experiments carried out by using the BLX-0 operator, which never violates the box constraints, led to poor percentages of success.

We finally observe that the mean number of function evaluations required by the GA in our experiments is 5820, which is about 20% of that required by the grid search (27376). Therefore, the GA approach allows a significant saving of computational time with respect to the grid search.

PBLX-0.5						
m_1	m_2	b	*mean*	*std*	*success (%)*	*relerr*
1.4	1.4	0	9.7514	1.0979	93.6	0.029
		4	9.7036	1.1397	92.8	0.034
		8	9.5714	1.2943	88.4	0.047
1.4	10	0	10.1140	1.1139	96.8	0.017
		4	10.0770	1.1449	96.4	0.020
		8	10.0440	1.1221	96.4	0.023
5	10	0	10.1340	1.0571	98.0	0.014
		4	10.1290	1.0717	97.8	0.015
		8	10.2430	1.0449	99.4	0.004

Table 3: GA behaviour with the RAND-GRID strategy, varying the parameter b in the grid-based NSS, and with the PBLX-0.5 recombination operator.

SBLX-0.5						
m_1	m_2	b	*mean*	*std*	*success (%)*	*relerr*
1.4	1.4	0	9.8631	1.2057	93.7	0.017
		4	9.8058	1.2655	92.0	0.024
		8	9.6272	1.4666	87.2	0.041
1.4	10	0	10.2530	1.0399	99.4	0.003
		4	10.2360	1.0716	99.0	0.004
		8	10.1990	1.1088	98.3	0.008
5	10	0	10.1590	1.0421	98.8	0.012
		4	10.1990	1.0448	98.7	0.008
		8	10.2670	1.0361	98.9	0.002

Table 4: GA behaviour with the RAND-GRID strategy, varying the parameter b in the grid-based NSS, and with the SBLX-0.5 recombination operator.

5 Concluding remarks

We investigated the use of GAs for solving a global optimization problem arising in the detection of gravitational waves. Computational experiments showed that a GA tailored to the specific problem is able to compute solutions that are comparable to those obtained by the grid search, which is a well established and widely used approach to the problem. On the other hand, we found that the GA allows a strong reduction of the computational cost with respect to the grid search, thus providing a more powerful tool in the analysis of the noisy outputs of interferometric detectors. The key issue in designing our algorithm was to choose the initial population moving from the grid of templates used in the grid search approach. Such idea, coupled with rather standard genetic operators and a careful handling of the constraints, led to a quite efficient and robust algorithm for our problem.

Future work will include a comparison of our GA with other global optimization algorithms on the problem at hand. Work will be also devoted to improve the GA by introducing in the genetic operators a priori information on the problem.

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