An novel ADM for finding Cournot equilibria of bargaining problem with alternating offers

Zheng Peng

Department of Mathematics, NanJing University, P R. China, 210093

Abstract: Bargaining is a basic game in economic practice. Cournot duopoly game is an important model in bargaining theory and is well studied in the literatures. Recently, asymmetry information [20] and incomplete information [19], limited individual rationality [2] and slightly altruistic equilibrium [10] are introduced into bargaining theory. And computational game theory also comes into being a new hot-research field. In this paper, we propose a novel method to compute Cournot equilibria of bargaining problem with alternating offers. The method is *Inexact Proximal Alternating Directions Method*. In the proposed method, the idea of alternating directions method corresponds to alternating offers, and the inexact term corresponds to asymmetry information and limited individual rationality in practice. Under some suitable conditions, we prove convergence of the proposed method (i.e., the strategic sequence generated by the proposed method converges to the Cournot equilibria of this game). Numerical tests show rationality, efficiency and applicability of the proposed method.

Keywords: Bargaining problem; Cournot duopoly game; Cournot equilibria; inexact proximal alternating directions method.

1 Introduction

In a duopoly, where there are two firms, each firm has to take into account its rival's behavior, when it decides how much output to produce. In particular, we focus on the case in which each firm has to forecast the other firm's output choice. Given its forecast, each firm chooses a profit-maximizing output for itself. A Cournot equilibrium is a situation where each firm finds its beliefs about the other firm to be confirmed.

In this paper, we consider a bargaining problem in the following circumstances: The total demand of a certain product in a market is Q. Two firms, Firm 1 and Firm 2, will partake this market. Set Firm 1s' output is x, and Firm 2s' is y. The price of this product is as follows:

$$p(x, y) = p_0 + p(Q - x - y).$$

Then Firm 1 and Firm 2 bargain on their outputs, x and y, to maximize profit for itself respectively. This is a typical bargaining problem in Cournot duopoly circumstances, and it is a basic and common economic behavior in practice.

Most of the previous investigations are connecting with existence and uniqueness of the solution of bargaining problem with various forms and applications. Rubinstein [16] gave some intelligible results on bargaining problem with two players, by using the following example: Two players have to reach an agreement on the partition of a pie of size 1. Each has to make in turn, a proposal as to how it should be divided. After one player has made an offer, the other must decide either to accept it, or to reject it and continue the bargaining. Under some reasonable conditions, Rubinstein proved existence and uniqueness of the solution of this bargaining problem, and gave an explicit expression of this solution. For the other literatures, one can see [3, 12, 21] ect., and the references therein.

However, finding a solution of a bargaining problem is also an important task in practice. Recently, computational game theory which associated this task is in attention. See, for examples, [11, 17, 18, 22, 24].

Our goal in this paper is to propose an intuitionistic explanation of bargaining process with alternating offers between two players, in Cournot duopoly game. And to give a method for finding its solution, i.e., the Cournot equilibrium. Existence and uniqueness of the solution of this problem are presumed as to be positive.

The Cournot duopoly game is well studied in the literatures. Qiu [13] compares Bertrand and Cournot equilibria in a differentiated duopoly with R&D (research and development) competition, and shows that Cournot competition induces more R&D effort than Bertrand competition. In [14], a Cournot model with an arbitrary nonlinear demand function and where firms do not observe their rival's actions directly is shown to allow mistaken beliefs to persist. These alter the original equilibrium state and, in a range of beliefs, destroy its stability and create cycles. The dynamics of the Cournot model are therefore fundamentally affected. Which is more interesting for us, Hakan [9] presents a characterization of internal Cournot equilibrium based on first-order conditions corresponding to profit maximization over prices. This characterization may yield significant computational advantage as demand functions need not be inverted. And simple first-order conditions are obtained in Hakan's paper.

This paper is organized as follows: In section 2, we give an intuitionistic explanation of bargaining problem with alternating offers between two players. In section 3, we proposed an inexact proximal alternating directions methods for solving this bargaining problem, and prove convergence of the proposed method. In section 4, we give some simulative results on using the proposed method to solve Cournot's duopoly game models. These results show rationality, efficiency and applicability of the proposed method. Finally, some conclusions are provided in section 5.

2 An explanation of a bargaining process with alternating offers

Let us consider a bargaining process with alternating offers between two players in Cournot duopoly circumstances. Let A and B denote this two players. Let X be the strategies set of player A and Y be the strategies set of player B, where $X,\ Y$ are subsets of R^n. Let $u(x,y) : X \times Y \to U \subseteq R$ be the utility function of player A, and player Bs' utility function is $v(x,y) : X \times Y \to V \subseteq R$.

Under the economic considerations, we assume:

Assumption 1. For all agreement pairs $(x,y) \in X \times Y$, we have $u(x,y) \geq d_A$ and $v(x,y) \geq d_B$, where (d_A, d_B) is refereed as to disagreement point.

Assumption 2. The utility function $u(x,y)$ is quasi-concave and differentiable with respect to $x \in X$, and utility function $v(x,y)$ is quasi-concave and differentiable with respect to $y \in Y$, respectively.

Assumption 3. The strategies sets $X, Y \subseteq R^n$ are closed convex and compact. Furthermore, we assume X and Y are simple convex subsets of R^n. The simple convex subsets are refereed as to, for examples, $R^n,\ R^n_+$ or a box defined by $B = \{x \in R^n |\ \ |x_i - c_i| \leq b_i,\ b_i > 0.\ i = 1, 2, ..., n\}$, ect.

An explanation of bargaining process with alternating offers in Cournot duopoly circumstances can be described as the following:

Bargaining process of Cournot duopoly game:

Step 0. Initiation: Player A gives an offer, say x_0. Player B gets y_0 via solving the following maximizing problem:

$$y_0 = \operatorname{Arg\,max}\{v(x_0, y) - d_B | y \in Y\}. \tag{2.1}$$

Step 1. Repeat: For given (x_k, y_k), player A gets his offer via solving the following maximizing problem:

$$x_{k+1} = \operatorname{Arg\,max}\{u(x, y_k) - d_A | x \in X\} \tag{2.2}$$

and player B gets his offer via solving the following maximizing problem:

$$y_{k+1} = \operatorname{Arg\,max}\{v(x_{k+1}, y) - d_B | y \in Y\} \tag{2.3}$$

Step 2. Until: some terminating criterion is met, an agreement is reached.

Let

$$\begin{cases} g(x, y_k) = \nabla_x u(x, y_k) \\ h(x_{k+1}, y) = \nabla_y v(x_{k+1}, y). \end{cases} \tag{2.4}$$

By Assumption 2, g and h are monotone operators with respect to X and Y respectively. Then the maximal problems (2.2) and (2.3) are equivalent to the following monotone variational inequalities:

$$x \in X, \quad (x' - x)^T g(x,\ y_k) \geq 0, \quad \forall x' \in X, \tag{2.5}$$

and

$$y \in Y, \quad (y' - y)^T h(x_{k+1},\ y) \geq 0, \quad \forall y' \in Y. \tag{2.6}$$

Exactly, the variational inequality (2.5) is equivalent to the following implicit projection equation

$$\widehat{x}_k = P_X\left\{\widehat{x}_k - g(\widehat{x}_k, y_k)\right\}, \tag{2.7}$$

and the variational inequality (2.6) is equivalent to the following implicit projection equation

$$\widehat{y}_k = P_Y\left\{\widehat{y}_k - h(\widehat{x}_k, \widehat{y}_k)\right\}. \tag{2.8}$$

Where $P_\Omega\{\cdot\}$ denotes projection operator on closed convex set Ω, which means:

$$P_\Omega\{x\} = \operatorname{Arg\,min}\{\|x - y\| | y \in \Omega\}.$$

Generally speaking, it is not easy to solve implicit projection equations (2.7) and (2.8) unless X and Y are R^n.

However, when X and Y are the other simple convex sets, we have some approaches to solve (2.4) inexactly. In the next section, we will propose an inexact proximal alternating directions method for solving this problem, and prove convergence of this method.

3 A inexact proximal alternating directions method for Cournot equilibria

Alternating directions methods (ADM) is a class of effective methods for solving monotone variational inequalities with separable operators. Alternating direction methods were first introduced by Peaceman and Rachford [1, 15]. The original procedure was applied to the numerical solution of the heat equation and to the iterative solution of the linear algebraic equations associated with the usual difference approximation to the Laplace equation. In recent twenty years, many authors proposed various version of alternating directions methods to solve convex programming and variational inequality with separable structure. For examples, see [4, 5, 6, 7, 8, 23].

In the interest of overcoming the drawback of implicit projection, such as (2.7) and (2.8), we introduce inexact terms into (2.5) and (2.6), and proposed an inexact proximal alternating directions method for solving variational inequalities (2.5–2.6). Under suitable conditions, we prove convergence of the proposed method.

Algorithm: (inPADMtoCBS)

Step 0. Initiation: Let $\varepsilon > 0$, $\nu \in (0,1)$ and $\gamma \in (0,2)$. Given $x_0 \in X$, we get y_0 via estimating the solution of the following variational inequality:

$$(y' - y)^T h(x_0, y) \geq 0. \tag{3.1}$$

And let $k = 0$.

Step 1. For given $w_k = (x_k, y_k)$, we first get $\widehat{x}_k$ via solving the following variational inequality:

$$\text{Find } \widehat{x}_k \in X, \text{ such that } (x - \widehat{x}_k)^T [g(\widehat{x}_k, y_k) + r_k(\widehat{x}_k - x_k) + \xi_x^k] \geq 0, \quad \forall x \in X. \tag{3.2}$$

where r_k and ξ_x^k satisfy the following relationship:

$$\|\xi_x^k\| \leq \nu r_k \|x_k - \widehat{x}_k\|, \quad \xi_x^k = g(x_k, y_k) - g(\widehat{x}_k, y_k). \tag{3.3}$$

Then we get $\widehat{y}_k$ via solving the following variational inequality:

$$\text{Find } \widehat{y}_k \in Y, \text{ such that } (y - \widehat{y}_k)^T [h(\widehat{x}_k, \widehat{y}_k) + s_k(\widehat{y}_k - y_k) + \xi_y^k] \geq 0, \quad \forall y \in Y. \tag{3.4}$$

where s_k and ξ_y^k satisfy the following relationship:

$$\|\xi_y^k\| \leq \nu s_k \|y_k - \widehat{y}_k\|, \quad \xi_y^k = h(\widehat{x}_k, y_k) - h(\widehat{x}_k, \widehat{y}_k). \tag{3.5}$$

Step 2. Produce w_{k+1} from w_k and $\widehat{w}_k = (\widehat{x}_k, \widehat{y}_k)$ by using the following formula:

$$w_{k+1} = w_k - \alpha_k d(w_k, \widehat{w}_k). \tag{3.6}$$

Where $d(w_k, \widehat{w}_k)$ is the search direction, and α_k is the step-length, which are defined in the following:

$$d(w_k, \widehat{w}_k) = \begin{pmatrix} (x_k - \widehat{x}_k) - \dfrac{1}{r_k}\xi_x^k \\ (y_k - \widehat{y}_k) - \dfrac{1}{s_k}\xi_y^k \end{pmatrix}$$

and $\alpha_k = \gamma \alpha_k^*$,

$$\alpha_k^* = \frac{\varphi(w_k, \widehat{w}_k)}{\|w_k - \widehat{w}_k\|_G^2}. \tag{3.7}$$

$$\varphi(w_k, \widehat{w}_k) = (w_k - \widehat{w}_k)^T G d(w_k, \widehat{w}_k), \quad G = \begin{pmatrix} r_k & \\ & s_k \end{pmatrix}.$$

Step 3. Check the terminating criterion: let $e_k = \max(\|x_k - \widehat{x}_k\|_\infty, \|y_k - \widehat{y}_k\|_\infty)$. If $e_k \geq \varepsilon$, let $k := k+1$, goto step 1, else goto next.

Step 4. Let $\widehat{w}^* = w_{k+1}$ be the approximating solution, and stop this process.

Remark 3.1. Correspondingly, the variational inequality (3.2) can be solved by the following explicit projection:

$$\widehat{x}_k = P_X\left\{x_k - \frac{1}{r_k} g(x_k, y_k)\right\}, \tag{3.8}$$

and the variational inequality (3.4) can be solved by the following explicit projection:

$$\widehat{y}_k = P_Y \left\{ y_k - \frac{1}{s_k} h(\widehat{x}_k, y_k) \right\}. \tag{3.9}$$

Let $W = X \times Y$ and

$$D(w_k, \widehat{w}_k) = \begin{bmatrix} g(\widehat{x}_k, y_k) \\ h(\widehat{x}_k, \widehat{y}_k) \end{bmatrix},$$

variational inequalities (3.2) and (3.4) can be rewritten into a compact form:

$$\text{Find } \widehat{w}_k \in W \text{ such that } (w - \widehat{w}_k)^T [D(w_k, \widehat{w}_k) - Gd(w_k, \widehat{w}_k)] \geq 0, \ \forall w \in W. \tag{3.10}$$

We are now to prove convergence of the proposed algorithm.

Lemma 3.1. For given $w_k = (x_k, y_k)$, let $\widehat{w}_k = (\widehat{x}_k, \widehat{y}_k)$ be generated by the proposed method inPADMtoCBS, and $w^c = (x^c, y^c)$ be the Cournot equilibria of this BP, i.e., $w^c = (x^c, y^c)$ satisfied the conditions (2.5–2.6). Then we have

$$(\widehat{w}_k - w^c)^T D(w_k, \widehat{w}_k) \geq 0, \quad \forall \widehat{w}_k \in X \times Y. \tag{3.11}$$

Proof: Since $g(x, y)$ and $h(x, y)$ are monotone with respect to $x \in X$ and $y \in Y$ respectively, we have

$$(\widehat{w}_k - w^c)^T [D(w_k, \widehat{w}_k) - D(w_k, w^c)] \geq 0 \tag{3.12}$$

Combining (3.12) and (2.5–2.6), we get (3.11) directly.

Lemma 3.2. Under the same conditions of Lemma 3.1, we have

$$(w_k - w^c)^T Gd(w_k, \widehat{w}_k) \geq \varphi(w_k, \widehat{w}_k). \tag{3.13}$$

Proof: Using (3.10) by letting $w = w^c$, we get

$$(\widehat{w}_k - w^c)^T Gd(w_k, \widehat{w}_k) \geq (\widehat{w}_k - w^c)^T D(w_k, \widehat{w}_k) \tag{3.14}$$

By using (3.11),

$$(\widehat{w}_k - w^c)^T Gd(w_k, \widehat{w}_k) \geq 0,$$

which implies

$$(w_k - w^c)^T Gd(w_k, \widehat{w}_k) \geq (w_k - \widehat{w}_k)^T Gd(w_k, \widehat{w}_k) = \varphi(w_k, \widehat{w}_k). \tag{3.15}$$

Theorem 3.1. For given $w_k = (x_k, y_k)$, let $\widehat{w}_k = (\widehat{x}_k, \widehat{y}_k)$ be generated by the proposed method inPADMtoCBS. Then we have

$$3\varphi(w_k, \widehat{w}_k) \geq \|d(w_k, \widehat{w}_k)\|_G^2 + \phi(w_k, \widehat{w}_k), \tag{3.16}$$

where

$$\phi(w_k, \widehat{w}_k) = (1 - \nu)(2 + \nu)\|w_k - \widehat{w}_k\|_G^2 > 0, \quad (\nu \in (0, 1)). \tag{3.17}$$

Proof: By computing directly, we have

$$\begin{aligned}
3\varphi(w_k, \widehat{w}_k) - [d(w_k, \widehat{w}_k)]^T Gd(w_k, \widehat{w}_k) &= 3(w_k - \widehat{w}_k)^T Gd(w_k, \widehat{w}_k) - d(w_k, \widehat{w}_k) Gd(w_k, \widehat{w}_k) \\
&= [3(w_k - \widehat{w}_k) - d(w_k, \widehat{w}_k)]\, Gd(w_k, \widehat{w}_k) \\
&= \begin{pmatrix} 2(x_k - \widehat{x}_k) + \frac{1}{r_k}\xi_x^k \\ 2(y_k - \widehat{y}_k) + \frac{1}{s_k}\xi_y^k \end{pmatrix}^T G \begin{pmatrix} (x_k - \widehat{x}_k) - \frac{1}{r_k}\xi_x^k \\ (y_k - \widehat{y}_k) - \frac{1}{s_k}\xi_y^k \end{pmatrix} \\
&= 2r_k\|x_k - \widehat{x}_k\|^2 + 2s_k\|y_k - \widehat{y}_k\|^2 \\
&\quad -\frac{1}{r_k}\|\xi_x^k\|^2 - \frac{1}{s_k}\|\xi_y^k\|^2 && (3.18) \\
&\quad -(x_k - \widehat{x}_k)^T \xi_x^k - (y_k - \widehat{y}_k)^T \xi_y^k && (3.19)
\end{aligned}$$

By using the conditions (3.3) and (3.5), we get

$$-\frac{1}{r_k}\|\xi_x^k\|^2-\frac{1}{s_k}\|\xi_y^k\|^2 \geq -\nu^2 r_k\|x_k-\widehat{x}_k\|^2-\nu^2 s_k\|y_k-\widehat{y}_k\|^2 \tag{3.20}$$

Using Cauchy-Schwarz inequality and using the conditions (3.3) and (3.5) again, we get

$$\begin{aligned} -(x_k-\widehat{x}_k)^T\xi_x^k-(y_k-\widehat{y}_k)^T\xi_y^k &\geq -\|x_k-\widehat{x}_k\|\|\xi_x^k\|-\|y_k-\widehat{y}_k\|\|\xi_y^k\| \\ &\geq -\nu r_k\|x_k-\widehat{x}_k\|^2-\nu s_k\|y_k-\widehat{y}_k\|^2 \end{aligned} \tag{3.21}$$

Substituting (3.20) and (3.21) into (3.18) and (3.19) respectively, we have

$$3\varphi(w_k,\widehat{w}_k)-[d(w_k,\widehat{w}_k)]^T Gd(w_k,\widehat{w}_k) \geq (2-\nu-\nu^2)r_k\|x_k-\widehat{x}_k\|^2+(2-\nu-\nu^2)s_k\|y_k-\widehat{y}_k\|^2 \tag{3.22}$$

By rearrangement of (3.22), we get (3.16-3.17) and complete this proof.

Theorem 3.1 implies that

$$\alpha_k^* \geq \frac{1}{3}, \quad \forall k=1,2,... \tag{3.23}$$

Theorem 3.2. Let $\{w^k\}$ be the sequence generated by the proposed method (inPADMtoCBS) for variational inequalities (2.5–2.6), and let w^c be a solution of variational inequalities (2.5–2.6). Then we have

$$\|w^{k+1}-w^c\|_G^2 \leq \|w^k-w^c\|_G-\frac{1}{9}\gamma(2-\gamma)\|w^k-\widehat{w}^k\|_G^2. \tag{3.24}$$

Proof: By using the iterative formula (3.6) and the notation α_k^* in (3.7), we have

$$\begin{aligned} \|w^{k+1}-w^c\|_G^2 &= \|w_k-w^c-\alpha_k d(w_k,\widehat{w}_k)\|_G^2 \\ &= \|w_k-w^c\|_G^2-2\alpha_k(w_k-w^c)^T Gd(w_k,\widehat{w}_k)+\alpha_k^2\|d(w_k,\widehat{w}_k)\|_G^2 \\ &= \|w_k-w^c\|_G^2-2\gamma\alpha_k^*(w_k-w^c)^T Gd(w_k,\widehat{w}_k)+\gamma^2(\alpha_k^*)^2\|d(w_k,\widehat{w}_k)\|_G^2 \\ &= \|w_k-w^c\|_G^2-\frac{2\gamma\varphi(w_k,\widehat{w}_k)(w_k-w^c)^T Gd(w_k,\widehat{w}_k)}{\|d(w_k,\widehat{w}_k)\|_G^2}+\left[\frac{\gamma\varphi(w_k,\widehat{w}_k)\|d(w_k,\widehat{w}_k)\|_G}{\|d(w_k,\widehat{w}_k)\|_G^2}\right]^2 \\ &\leq \|w_k-w^c\|_G^2-2\gamma\frac{\varphi^2(w_k,\widehat{w}_k)}{\|d(w_k,\widehat{w}_k)\|_G^2}+\gamma^2\frac{\varphi^2(w_k,\widehat{w}_k)}{\|d(w_k,\widehat{w}_k)\|_G^2} \\ &= \|w_k-w^c\|_G^2-2\gamma\frac{\varphi(w_k,\widehat{w}_k)}{\|d(w_k,\widehat{w}_k)\|_G^2}\varphi(w_k,\widehat{w}_k)+\gamma^2\frac{\varphi(w_k,\widehat{w}_k)}{\|d(w_k,\widehat{w}_k)\|_G^2}\varphi(w_k,\widehat{w}_k) \\ &= \|w_k-w^c\|_G^2-2\gamma\alpha_k^*\varphi(w_k,\widehat{w}_k)+\gamma^2\alpha_k^*\varphi(w_k,\widehat{w}_k) \\ &= \|w_k-w^c\|_G^2-\gamma(2-\gamma)\alpha_k^*\varphi(w_k,\widehat{w}_k) \\ &\leq \|w_k-w^c\|_G^2-\frac{1}{9}\gamma(2-\gamma)\|w_k-\widehat{w}_k\|_G^2 \end{aligned}$$

in the first inequality we use Lemma 3.2 (3.13), and in the last inequality we use Theorem 3.1 (3.16) and (3.23).

This is the key theorem for convergence of the proposed method. By using Theorem 3.2 and the sequence $\{w_k\}$ is bounded (for a proof, see Theorem 3 in [8]), it is easy to get convergence of the proposed method, we omit this proof here.

Obviously, the limit point of the sequence $\{w_k\}$, i.e., $w^c=(x^c,y^c)$, is Cournot equilibria of the bargaining process in this Cournot duopoly game.

4 Simulative results and remarks

Cournot's duopoly game models a situation in which each firm chooses its output independently, and the market determines the price at which it is sold. Specifically, if Firm 1 produces the output x and Firm 2 produces the output y then the price at which each unit of output is sold is $P(x+y)$, where P is the inverse demand function. For convenience, we let $P(x+y) = p_0 + p(Q - x - y)$ in our model, where $Q \geq x + y$ is the total demand, and $p_0, p > 0$. We suppose that Firm 1s' cost of each output is a constant c_1 and Firm 2s' is c_2.

Then Firm 1s' total revenue when the pair of outputs is chosen (by the firms) as to (x, y), is $(p_0 + p(Q - x - y))x$. Thus its profit is

$$u(x, y) = (p_0 + p(Q - x - y))x - c_1 x \tag{4.1}$$

Firm 2s' revenue is $(p_0 + p(Q - x - y))y$, and hence its profit is

$$v(x, y) = (p_0 + p(Q - x - y))y - c_2 y \tag{4.2}$$

Firm 1 and Firm 2 bargain for their outputs $x,\ y$ in order to maximize their profits $u(x, y),\ v(x, y)$ severally. By the first order condition, the analytical solution $(x^c,\ y^c)$ of this problem is

$$x^c = \frac{1}{3p}(p_0 + pQ + c_2 - 2c_1), \quad y^c = \frac{1}{3p}(p_0 + pQ + c_1 - 2c_2). \tag{4.3}$$

Using inPADMtoCBS method to solve this problem, we get the result which is stated in Figure 1. The parameters in this simulative problem are given in the following: $p_0 = 3.5, p = 0.5 \times 10^{-8}, c_1 = 3.00, c_2 = 3.00, Q = 1.0 \times 10^9$.

The left part of Figure 1 shows the outputs of Firm 1 vs that one of Firm 2 in each offer, and the right part shows profit of Firm 1 vs that one of Firm 2. From this simulation, the computational result approximates to the analytical result after 5–6 iterations with an accredited error. This implies that inPADMtoCBS method is applicable.

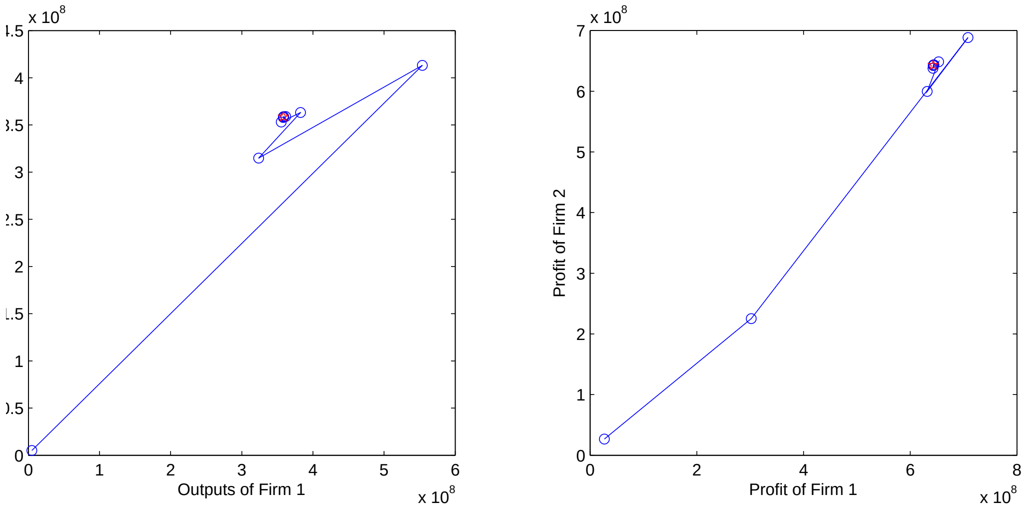

Figure 1: Non-altruistic bargaining process of Cournot's duopoly model

Slightly altruistic equilibrium is an attractive research field recently, see [10]. Let altruistic parameter of Firm 1 be ϵ_1 and Firm 2s' be ϵ_2, where $\epsilon_1,\ \epsilon_2 \in [0, 1)$. Then their profit functions are

$$U(x, y) = u(x, y) + \epsilon_1 v(x, y), \tag{4.4}$$

and

$$V(x,y) = v(x,y) + \epsilon_2 u(x,y) \tag{4.5}$$

respectively. In the same way, by maximizing their (Firm 1s' and Firm 2s') profit, i.e., maximizing

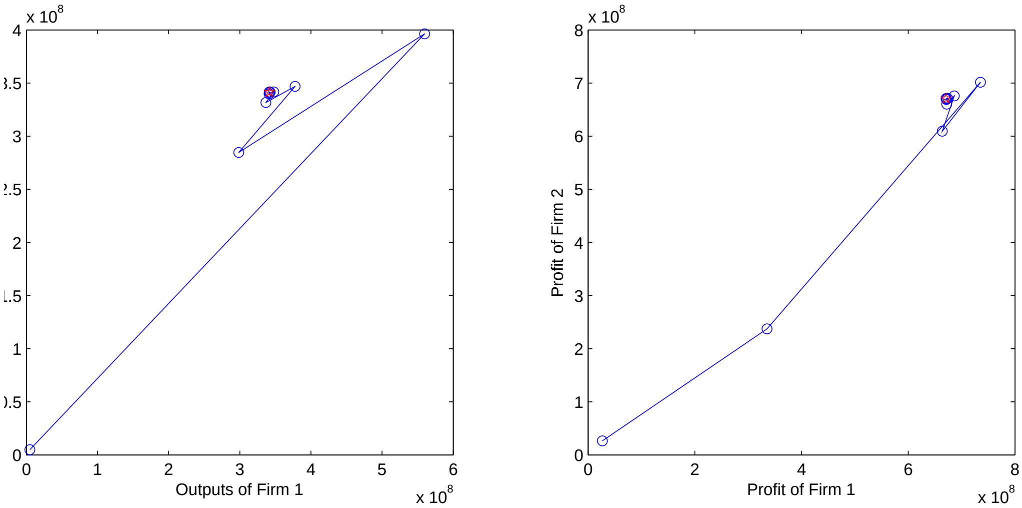

Figure 2: Slightly altruistic bargaining process of Cournot's duopoly model

$U(x,y)$ and $V(x,y)$, we get the analytical solution $(x^c,\ y^c)$ as follows:

$$\begin{aligned} x^c &= \frac{(1-\epsilon_1)(p_0+pQ)-(2c_1-(1+\epsilon_1)c_2)}{4p-p(1+\epsilon_1)(1+\epsilon_2)}, \\ y^c &= \frac{(1-\epsilon_2)(p_0+pQ)-(2c_2-(1+\epsilon_2)c_1)}{4p-p(1+\epsilon_1)(1+\epsilon_2)}. \end{aligned} \tag{4.6}$$

Let $\epsilon_1 = \epsilon_2 = 0.15$ and the other parameters be as same as the non-altruistic case, employing inPADMtoCBS method to solve this problem, we get the computational result which is stated as Figure 2.

Notice the difference of non-altruistic case and slightly altruistic case is that, the Cournot equilibria of non-altruistic case is $(x^c, y^c) = (3.667, 3.667) \times 10^8$ and their optimal profit pair in this case is $(6.722, 6.722) \times 10^8$, and the Cournot equilibria of slightly altruistic case is $(x^c, y^c) = (3.492, 3.492) \times 10^8$ and their optimal profit pair in this case is $(7.012, 7.012) \times 10^8$, respectively.

The computational results go all the way the analytical ones. In the first, the computational results (equilibrium) are matching to the analytical ones. In the second, we can see in the experimentation, before they attain Cournot equilibria, if they (Firm 1 and Firm 2) stop the bargaining process and come to an agreement at any iteration except original iteration, the firm which offers in first can get better profit than the later one. This assertion is also matching to asymmetry of bargaining of alternating offers with two players which is affirmed in literature.

Let Firm 1 and Firm 2 have different cost of each output, i.e., $c_1 \neq c_2$; or let they have different altruistic parameters, i.e., $\epsilon_1 \neq \epsilon_2$, we simulate this bargaining process repeatedly, we can get consistent results on numerical and analytical ones. For example, let $c_1 = 3.12,\ c_2 = 3.02$ and $\epsilon_1 = 0.15, \epsilon_2 = 0.20$, the analytical solution is $(x^c, y^c) = (3.403, 3.438) \times 10^8$, and their corresponding optimal profit pair in this case is $(6.668, 7.081) \times 10^8$. The bargaining process is showed in figure 3.

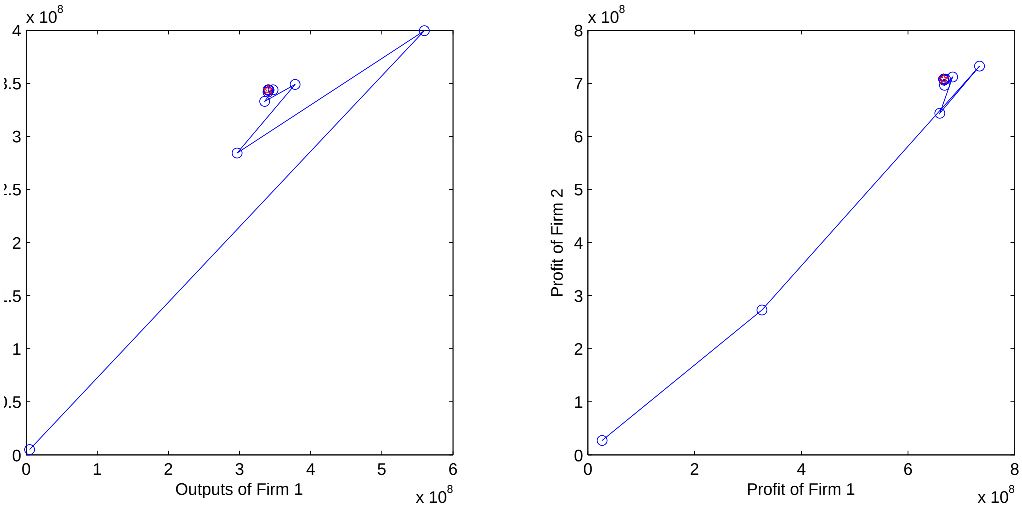

Figure 3: Slightly altruistic bargaining process of Cournot's duopoly model ($c_1 \neq c_2,\ \epsilon_1 \neq \epsilon_2$)

5 Conclusions

We develop a method for finding Cournot equilibria of bargaining problem with alternating offers. This method is refereed as to inexact proximal alternating directions method. In the proposed method, the idea of alternating directions method corresponds to alternating offers in bargaining process, and the inexact proximal point term corresponds to asymmetry information and limited individual rationality in this process. We prove convergence of the proposed method under suitable conditions. Indeed, these conditions (see (3.3) and (3.5)) restrict the players that they can not overstep a bound, when they make any error in the bargaining process. Numerical results show rationality, efficiency and applicability of the proposed method.

The proposed method can be used to find Cournot equilibria of bargaining problem with alternating offers in a very broad range. For examples, each of strategies of a firm may consist of n components in the bargaining problem, or one can restrict the total outputs of two firms in the bargaining process must be not less than the demand of the market, or there exists a discounting factor in each turn of the bargaining process, ect. However, we can not extend this method in a straight way to solve bargaining problem of alternating offers with three players, despite this case is also common in practice. Since we can not give convergence of the alternating directions method in solving three separable operators up to now.

Acknowledgement: We are grateful to Professor He Bingsheng and Professor Yu Jian, for their enlightening of the idea of this paper, and much useful advices to improve its quality.

References

[1] J. Douglas. On the numericalintegration of $U_{xx} + U_{yy} = U_t$ by implicit methods. Journal of the Society for Industrial and Applied Mathematics, 3(1955), 42–65 .

[2] Abraham Diskin, Dan S. Felsenthal. Individual rationality and bargaining, Public Choice, 133(2007), 25-29.

[3] Ezra Einy, Ori Haimanko, Diego Moreno, Benyamin Shitovitz. On the existence of Bayesian Cournot equilibrium, Games and Economic Behavior, inpress(2009).

[4] J. Eckstein, Dimitri P. Bertsekas. On the Douglas-Rachford splitting method and the proximal point algorithm for maximal monotone operators, Mathematical Programming 55 (1992), 293-318.

[5] J. Eckstein, B. F. Svaiter. A family of projective splitting methods for the sum of two maximal monotone operators, Math. Program. Ser. B 111(2008), 173-199, DOI 10.1007/s10107-006-0070-8

[6] M. Fukushima. Application of the alternating direction method of multipliers to separable convex programming problems, Computational Optimization and Applications, 1 (1992), 93-111.

[7] B.S. He, H. Yang and S.L. Wang. Alternating directions method with self-adaptive panalty parameters for monotone variational inequalities, Journal of optimization theory and applications, 106(2) (2000), 337-356

[8] B.S. He, L.Z. Liao, D.R. Han, H.Yang. A new inexact alternating directions method for monotone variational inequalities, Mathematical Programming (Ser. A), 92 (2002), 103–118.

[9] H. Orbay. Computing Cournot equilibrium through maximization over prices, Economics Letters 105 (2009) 71-73.

[10] G. De Marco, J. Morgan. Slightly Altruistic Equilibria, Journal of Optimization Theory Applications , 137(2008), 347-362

[11] Noah Stein. Polynomial Games: Characterization and Computation of Equilibria, technical report in the 20th International Symposium on Mathematical Programming, Chicago, U.S.A

[12] N. Van Long, A. Soubeyran. Existence and uniqueness of Cournot equilibrium: a contraction mapping approach, Economics Letters 67 (2000) 345-348.

[13] Larry D. Qiu. On the Dynamic Efficiency of Bertrand and Cournot Equilibria, journal of economic theory 75(1997), 213-229.

[14] Daniel Leonard and Kazuo Nishimura. Nonlinear dynamics in the Cournot model without full information, Annals of Operations Research 89(1999), 165-173.

[15] D. W. Peaceman and H. H. Racheord jr.. The numerical solution of parabolic and elliptic differential equations, Journal of the Society for Industrial and Applied Mathematics, 3(1955), 28–41 .

[16] Ariel Rubinstein. Perfect Equilibrium in a Bargaining Model, Econometrica, 50(1) (1982), 97-109

[17] Ruchira Datta. Polynomial Graphs with Applications to Game Theory, technical report in the 20th International Symposium on Mathematical Programming, Chicago, U.S.A

[18] Rahul Savani. Enumeration of Nash Equilibria for Two-player Games, technical report in the 20th International Symposium on Mathematical Programming, Chicago, U.S.A

[19] J*ó*zsef S*á*kovics. Games of incomplete information without common knowledge proiors, Theory and Decision 50(2001), 347-366.

[20] I. Ray, J. Williams. Locational asymmetry and the potential for cooperation on a canal, Journal of Development Economics, 67(2002), 129-55

[21] S. Svizzero. Cournot equilibrium with convex demand, Economics Letters 54 (1997) 155-158.

[22] Yusuke Samejima. A note on implementation of bargaining solutions, Theory and Decision, 59(2005), 175-191.

[23] Paul Tseng. Alternating Projection -Proximal methods for convex programming and variational inequalities, SIAM Journal on Optimization, 7(4) (1997), 951–965.

[24] J. Zhang et al. Some projection-like methods for the generalized Nash equilibria, Comput. Optim. Appl. DOI 10.1007/s10589-008-9173-x.