Multi-Objective Stochastic Linear Programming with General form of Distributions

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Abstract

Probabilistic or Stochastic programming is a framework for modeling optimization problems that involve uncertainty. The basic idea used in solving stochastic optimization problems has so far been to convert a stochastic model into an equivalent deterministic model and is possible when the right hand side resource vector follows some specific distributions such as normal, lognormal and exponential distributions. In this paper, a multi-objective stochastic programming problem has been considered with right hand side resource vector following general form of distributions $F(b_i) = 1 - B_i e^{-A_i h(b_i)}$, which include many distributions such as Power Function distribution, Pareto distribution, Beta distribution of first kind, Weibull distribution, and Burr type XII distribution. In this approach, the multi-objective stochastic programming problem is converted into an equivalent deterministic model, which is then solved by a multi-objective programming genetic algorithm using Matlab. Few numerical examples are presented to illustrate the proposed approach.

Keywords: Multi-objective stochastic programming; Deterministic equivalence; General form of distributions; Joint constraints.

1. Introduction

One of the common problems in the practical application of mathematical programming is the difficulty for determining the proper values of model parameters. The values of these parameters are often influenced by random events that are impossible to predict i.e., some or all of the model parameters may be random variables. What is needed is a way to formulating the problem so that the optimization will directly consider the uncertainty. One such approach for mathematical programming under uncertainty is

Stochastic Programming (SP). The SP is an optimization technique in which the constraints and/or the objective function of an optimization problem contains certain random variables. In the stochastic linear programming literature (Infanger 1993, Kall and Wallace 1994), several researchers suggested various models. A bibliography has been presented by Stancu-Minasian and Wets (1976). Model coefficients of most of these models are assumed to follow independent normal distribution because deriving the deterministic equivalent of the objective function and/or constraints of the model is well known (Kall and Wallace, 1994) in this case.

SP models were first formulated by Dantzig (1955) who suggested a two stage programming technique that involves conversion of SP models into their equivalent deterministic programming models. However, this technique suffers from the limitation that it does not allow any constraint to be violated even at specific probability level. This gave rise to the concept of Chance Constrained Programming (CCP), where constraints containing random variables are guaranteed to be satisfied with a certain probability. Charnes and Cooper (1959, 1963) developed the concept of CCP. Jagannathan (1974) discussed a single objective chance constraint programming with joint constraints. Biswal et al. (1998) obtained deterministic equivalent of the objective function and left hand side constraint coefficients following exponential distributions. Charles and Dutta (2005) also derived deterministic equivalent of the objective function and constraint coefficients with normal random variables. Joint probabilistic constraints for independent random variables were used initially by Miller and Wagner (1965).The properties of stochastic programming problems and methods for obtaining optimal solution have been described in Rao (1989), Kall and Wallace (1994), Birge and Louveaux (1997) and Pre'kopa (1995).

In the recent past, SP has been applied to the problems having multiple, conflicting and non-commensurable objectives where generally there does not exist a single solution which can maximize (minimize) all the objectives. However, in a multiple criteria decision-making system, the decision-maker generally follows on satisfaction of criteria rather than maximization (minimization) of objectives. Several methods for solving Multi-Objective Stochastic Linear Programming (MOSLP) problems have been developed by Leclercq (1982), Goicoechea et al. (1982). Baba and Morimoto (1993) proposed a stochastic approximation method for solving the MOSLP problem and Caballero et al. (2001) provided efficient solution concepts in stochastic multi-objective programming problem. Suwarna et al. (1997) converted MOSLP problem into a deterministic problem and then applied fuzzy programming approach to find the compromise solution. Solution procedure for solving multi-objective stochastic fractional programming problems is found in Charles and Dutta (2003, 2006). Abdelaziz et al. (2007) presented multi-objective programming techniques to choose the portfolio best satisfying the decision makers' aspirations and preferences. Most of the probabilistic models assume normal distribution for model coefficients. Sahoo and Biswal (2005) presented some deterministic equivalents for the probabilistic problem involving normal and log-normal random variables for joint constraints. However, most of the papers fail to address more than two distributions, moreover we cannot only relay on normal, lognormal and exponential distributions when we deal with real time empirical modeling. Hence, this paper addresses the general form of distributions, wherein one can model relevant application when the elements of resource vector follow some special form of distributions. In this paper, we have converted the multi-objective stochastic linear programming model into deterministic model, where right hand side follows the general form of distributions $F(b_i) = 1 - B_i e^{-A_i h(b_i)}$. We have then deduced from it, the expression for some special distributions like Power Function distribution, Pareto distribution, Beta distribution of first kind, Weibull distribution, Burr type XII distribution.

This paper is organized as follows. Following the introduction section, in Section 2 the mathematical model of a MOSLP problem has been defined and its deterministic equivalent form is derived for marginal constraints. Section 3 defines a mathematical model of MOSLP problem with joint constraints and its deterministic equivalence is derived. Numerical examples and conclusion are given in Sections 4 and 5 respectively.

2. MOSLP with marginal constraint for general form of distributions

The mathematical model of a multi-objective probabilistic linear programming problem can be expressed as:

$$max\, z_k = \sum_{j=1}^{n} c_j^k x_j, \quad k = 1, 2, ..., K \tag{2.1}$$

Subject to

$$P\left(\sum_{j=1}^{n} a_{ij} x_j \le b_i \right) \ge p_i, \; i = 1,2,...,m \tag{2.2}$$

$$x_j \ge 0, \quad j = 1,2,...,n \tag{2.3}$$

where $0 < p_i < 1$, and usually close to 1. We are assuming that the parameters a_{ij} and c_j are deterministic constants and only b_i are random variables having a general form of distributions $F(b_i) = 1 - B_i e^{-A_i h(b_i)}$. It is also given that the i^{th} random variable b_i has two known parameters namely $A_i (\ne 0)$, and $B_i (> 0)$, where A_i and B_i are such that $F(\alpha_i) = 0, F(\beta_i) = 1$ and $h(b_i)$ is a monotonic and differentiable function of b_i in the interval (α_i, β_i). In this model, the decision variables $x_j, j = 1,2,...n,$ are treated as deterministic decision variables. The probability density function of the random variable b_i be given by

$$f(b_i) = A_i B_i e^{-A_i h(b_i)} h'(b_i) \tag{2.4}$$

Equation (2.2) can be expressed as

$$P(b_i \ge y_i) \ge p_i, \; i = 1,2,...,m \tag{2.5}$$

where $\quad y_i = \sum_{j=1}^{n} a_{ij} x_j$

Equation (2.5) can be restated as

$$\int_{y_i}^{\beta_i} A_i B_i e^{-A_i h(b_i)} h'(b_i)\, db_i \ge p_i, \quad i = 1,2,...,m$$

After integration, we have

$$B_i e^{-A_i h(y_i)} \ge p_i, \text{ as } B_i e^{-A_i h(\beta_i)} = 0 \tag{2.6}$$

2.1 Few special class of distributions

(1) *When* b_i*'s follow Weibull distribution*

The distribution function of Weibull distribution is given by

$$F(b_i) = 1 - e^{-\theta_i b_i^{a_i}}, 0 \le b_i < \infty,\ a_i > 0,\ \theta_i > 0 \tag{2.7}$$

Here $A_i = \theta_i,\ B_i = 1,\ h(b_i) = b_i^{a_i}$

Now from equation (2.6), we have

$$e^{-\theta_i y_i^{a_i}} \ge p_i$$

This can be simplified as

$$y_i \le \left[-\frac{1}{\theta_i} \log(p_i) \right]^{1/a_i} \tag{2.8}$$

So, the deterministic multi-objective mathematical model can be expressed as

$$\left.\begin{aligned} &max\, z_k = \sum_{j=1}^{n} c_j^k x_j, \quad k = 1, 2, ..., K \\ &subject\ to \\ &\qquad \sum_{j=1}^{n} a_{ij} x_j \le \left[-\frac{1}{\theta_i} \log(p_i) \right]^{1/a_i}, i = 1, 2, ..., m \\ &and \qquad\qquad x_j \ge 0,\ j = 1, 2, ..., n \end{aligned}\right\} \tag{2.9}$$

Note: we can also take $A_i = 1,\ B_i = 1,\ h(b_i) = \theta_i b_i^{a_i}$ for the Weibull distribution.

(2) *When* b_i*'s follow Burr type XII distribution*

The distribution function of Burr type XII distribution is given by

$$F(b_i) = 1 - \left(1 + \theta_i b_i^{a_i}\right)^{-\lambda_i}, 0 \le b_i < \infty,\ a_i > 0,\ \theta_i > 0, \lambda_i > 0 \tag{2.10}$$

Here $A_i = \lambda_i,\ B_i = 1,\ h(b_i) = \log\left(1 + \theta_i b_i^{a_i}\right)$

Now from equation (2.6), we have

$$e^{-\lambda_i \log\left(1 + \theta_i y_i^{a_i}\right)} \ge p_i$$

This can be simplified as

$$y_i \le \left[\frac{p_i^{-1/\lambda_i} - 1}{\theta_i} \right]^{1/a_i} \tag{2.11}$$

So, the deterministic multi-objective mathematical model can be expressed as

$$\left.\begin{aligned} & max\, z_k = \sum_{j=1}^{n} c_j^k x_j, \quad k = 1, 2, ..., K \\ & subject\ to \\ & \qquad \sum_{j=1}^{n} a_{ij} x_j \le \left[\frac{p_i^{-1/\lambda_i} - 1}{\theta_i} \right]^{1/a_i}, i = 1, 2, ..., m \\ & and \qquad\qquad x_j \ge 0, \ j = 1, 2, ..., n \end{aligned}\right\} \tag{2.12}$$

Note: we can also take $A_i = 1, \ B_i = 1, \ h(b_i) = \lambda_i \log\left(1 + \theta_i b_i^{a_i}\right)$ for the Burr Type XII distribution.

(3) *When b_i's follow Beta distribution of first kind*

The distribution function of Beta distribution of first kind is given by

$$F(b_i) = 1 - \left(\frac{\lambda_i - b_i}{\lambda_i - \delta_i} \right)^{a_i}, 0 < \delta_i \le b_i \le \lambda_i, a_i > 0 \tag{2.13}$$

Here $A_i = -a_i$, $B_i = 1, \ h(b_i) = \log\left(\frac{\lambda_i - b_i}{\lambda_i - \delta_i} \right)$

Now from equation (2.6), we have

$$e^{a_i \log\left(\frac{\lambda_i - y_i}{\lambda_i - \delta_i}\right)} \ge p_i$$

This can be simplified as

$$y_i \le \lambda_i \left(1 - p_i^{1/a_i}\right) + p_i^{1/a_i} \delta_i \tag{2.14}$$

So, the deterministic multi-objective mathematical model can be expressed as

$$\left.\begin{aligned}
&max\, z_k = \sum_{j=1}^{n} c_j^k x_j, \quad k = 1, 2, \dots, K \\
&subject\ to \\
&\qquad \sum_{j=1}^{n} a_{ij} x_j \le \lambda_i \left(1 - p_i^{1/a_i}\right) + p_i^{1/a_i} \delta_i\, , i = 1, 2, \dots, m \\
&and \qquad\qquad x_j \ge 0,\ j = 1, 2, \dots, n
\end{aligned}\right\} \tag{2.15}$$

Note: we can also take $A_i = -1,\ B_i = 1,\ h(b_i) = a_i \log\left(\dfrac{\lambda_i - b_i}{\lambda_i - \delta_i}\right)$ for the Beta distribution of first kind.

(4) *When* b_i*'s follow Pareto distribution*

The distribution function of Pareto distribution is given by

$$F(b_i) = 1 - \lambda_i^{a_i} b_i^{-a_i}, 0 < \lambda_i \le b_i < \infty,\ a_i > 0 \tag{2.16}$$

Here $A_i = a_i,\ B_i = \lambda_i^{a_i},\ h(b_i) = \log(b_i)$

Now from equation (2.6), we have

$$\lambda_i^{a_i} e^{-a_i \log(y_i)} \ge p_i$$

This can be simplified as

$$y_i \le \frac{\lambda_i}{p_i^{1/a_i}} \tag{2.17}$$

So, the deterministic multi-objective mathematical model can be expressed as

$$\left.\begin{aligned}
&max\, z_k = \sum_{j=1}^{n} c_j^k x_j, \quad k = 1, 2, \dots, K \\
&subject\ to \\
&\qquad \sum_{j=1}^{n} a_{ij} x_j \le \frac{\lambda_i}{p_i^{1/a_i}}, i = 1, 2, \dots, m \\
&and \qquad\qquad x_j \ge 0,\ j = 1, 2, \dots, n
\end{aligned}\right\} \tag{2.18}$$

Note: We can also take $A_i = 1,\ B_i = \lambda_i^{a_i},\ h(b_i) = a_i \log(b_i)$ for the Pareto distribution.

(5) *When* b_i*'s follow Power Function distribution*

The distribution function of Power Function distribution is given by

$$F\left(b_i\right)=\lambda_i^{-a_i} b_i^{a_i}, 0 \le b_i \le \lambda_i,\ a_i>0,\ \lambda_i>0 \qquad (2.19)$$

Here $A_i=-1,\ B_i=1,\ h\left(b_i\right)=\log\left(1-\lambda_i^{-a_i} b_i^{a_i}\right)$

Now from equation (2.6), we have

$$e^{\log\left(1-\lambda_i^{-a_i} y_i^{a_i}\right)} \le p_i$$

This can be simplified as

$$y_i \le \lambda_i\left(1-p_i\right)^{1/a_i} \qquad (2.20)$$

So, the deterministic multi-objective mathematical model can be expressed as

$$\left.\begin{aligned}
& max\, z_k=\sum_{j=1}^{n} c_j^k x_j, \quad k=1,2,\ldots,K \\
& subject\ to \\
& \qquad \sum_{j=1}^{n} a_{ij} x_j \le \lambda_i\left(1-p_i\right)^{1/a_i}\ , i=1,2,\ldots,m \\
& and \qquad\qquad x_j \ge 0,\ j=1,2,\ldots,n
\end{aligned}\right\} \qquad (2.21)$$

3. MOSLP with joint constraint for general form of distributions

The mathematical model of a multi-objective probabilistic linear programming problem with a joint constraint can be expressed as

$$max\, z_k=\sum_{j=1}^{n} c_j^k x_j, \quad k=1,2,\ldots,K \qquad (3.1)$$

subject to

$$P\left(\sum_{j=1}^{n} a_{1j} x_j \le b_1, \sum_{j=1}^{n} a_{2j} x_j \le b_2, \ldots, \sum_{j=1}^{n} a_{mj} x_j \le b_m\right) \ge p \qquad (3.2)$$

$$x_j \ge 0, \quad j=1,2,\ldots,n \qquad (3.3)$$

where $0<p<1$, usually close to 1. We are assuming that the parameters a_{ij} and c_j are deterministic constants and only b_i are random variables having a general form of distributions $F(b_i) = 1 - B_i e^{-A_i h(b_i)}$. It is also given that the i^{th} random variable b_i has two known parameters $A_i (\neq 0)$, and $B_i (> 0)$, where A_i and B_i are such that $F(\alpha_i) = 0, F(\beta_i) = 1$ and $h(b_i)$ is a monotonic and differentiable function of b_i in the interval (α_i, β_i). In this model, the decision variables $x_j, j = 1,2,...n,$ are treated as deterministic variables.

Now the joint probabilistic constraint (3.2) can be written as

$$\prod_{i=1}^{m} P(b_i \geq y_i) \geq p \tag{3.4}$$

where $y_i = \sum_{j=1}^{n} a_{ij} x_j$, b_i's are independent random variables and

$$P(b_i \geq y_i) = \int_{y_i}^{\beta_i} A_i B_i e^{-A_i h(b_i)} h'(b_i) db_i$$

The above expression can be simplified as

$$P(b_i \geq y_i) = B_i e^{-A_i h(y_i)} \tag{3.5}$$

So the joint probabilistic constraint can be transformed into a deterministic constraint

$$\prod_{i=1}^{m} \left[B_i e^{-A_i h(y_i)} \right] \geq p, \text{ as } B_i e^{-A_i h(\beta_i)} = 0 \tag{3.6}$$

Solve (3.6) for y_i, we get the deterministic form of the given multi-objective probabilistic linear programming problem which can be solved by using fuzzy or goal programming method or any of the existing hybrid algorithm techniques.

3.1 Few special class of distributions

(1) *When b_i's follow Weibull distribution*

The distribution function of Weibull distribution is given by

$$F(b_i) = 1 - e^{-\theta_i b_i^{a_i}}, 0 \leq b_i < \infty, \; a_i > 0, \; \theta_i > 0 \tag{3.7}$$

Here $A_i = \theta_i, \; B_i = 1, \; h(b_i) = b_i^{a_i}$

Now from equation (3.6), we have

$$\prod_{i=1}^{m}\left(e^{-\theta_i y_i^{a_i}}\right) \ge p \tag{3.8}$$

So, the deterministic multi-objective mathematical model for joint constraint can be expressed as

$$\left.\begin{aligned}
&max\, z_k = \sum_{j=1}^{n} c_j^k x_j, \quad k = 1, 2, ..., K \\
&subject\ to \\
&\qquad \sum_{j=1}^{n} a_{ij} x_j = y_i\,, i = 1, 2, ..., m \\
&\qquad \prod_{i=1}^{m}\left(e^{-\theta_i y_i^{a_i}}\right) \ge p \\
&and \qquad x_j \ge 0,\ j = 1, 2, ..., n
\end{aligned}\right\} \tag{3.9}$$

Note: we can also take $A_i = 1,\ B_i = 1,\ h(b_i) = \theta_i b_i^{a_i}$ for the Weibull distribution.

(2) *When b_i's follow Burr type XII distribution*

The distribution function of Burr type XII distribution is given by

$$F(b_i) = 1 - \left(1 + \theta_i b_i^{a_i}\right)^{-\lambda_i}, 0 \le b_i < \infty,\ a_i > 0,\ \theta_i > 0, \lambda_i > 0 \tag{3.10}$$

Here $A_i = \lambda_i,\ B_i = 1,\ h(b_i) = \log\left(1 + \theta_i b_i^{a_i}\right)$

Now from equation (3.6), we have

$$\prod_{i=1}^{m}\left(1 + \theta_i y_i^{a_i}\right)^{-\lambda_i} \ge p \tag{3.11}$$

So, the deterministic multi-objective mathematical model for joint constraint can be expressed as

$$\left.\begin{aligned}
&max\, z_k = \sum_{j=1}^{n} c_j^k x_j, \quad k = 1,2,...,K \\
&Subject\ to \\
&\qquad \sum_{j=1}^{n} a_{ij} x_j = y_i \,, i = 1,2,...,m \\
&\qquad \prod_{i=1}^{m} \left(1+\theta_i y_i^{a_i}\right)^{-\lambda_i} \ge p \\
&and \qquad x_j \ge 0,\ j = 1,2,...,n
\end{aligned}\right\} \tag{3.12}$$

Note: we can also take $A_i = 1,\ B_i = 1,\ h(b_i) = \lambda_i \log\left(1+\theta_i b_i^{a_i}\right)$ for the Burr Type XII distribution.

(3) *When b_i's follow Beta distribution of first kind*

The distribution function of Beta distribution of first kind is given by

$$F(b_i) = 1 - \left(\frac{\lambda_i - b_i}{\lambda_i - \delta_i}\right)^{a_i}, 0 < \delta_i \le b_i \le \lambda_i, a_i > 0 \tag{3.13}$$

Here $A_i = -a_i,\ B_i = 1,\ h(b_i) = \log\left(\frac{\lambda_i - b_i}{\lambda_i - \delta_i}\right)$

Now from equation (3.6), we have

$$\prod_{i=1}^{m} \left(\frac{\lambda_i - y_i}{\lambda_i - \delta_i}\right)^{a_i} \ge p \tag{3.14}$$

So, the deterministic multi-objective mathematical model for joint constraint can be expressed as

$$\left.\begin{aligned}
&max\, z_k = \sum_{j=1}^{n} c_j^k x_j, \quad k = 1,2,...,K \\
&subject\ to \\
&\qquad \sum_{j=1}^{n} a_{ij} x_j = y_i \,, i = 1,2,...,m \\
&\qquad \prod_{i=1}^{m} \left(\frac{\lambda_i - y_i}{\lambda_i - \delta_i}\right)^{a_i} \ge p \\
&and \qquad x_j \ge 0,\ j = 1,2,...,n
\end{aligned}\right\} \tag{3.15}$$

Note: we can also take $A_i = -1,\ B_i = 1,\ h(b_i) = a_i \log\left(\frac{\lambda_i - b_i}{\lambda_i - \delta_i}\right)$ for the Beta distribution of first kind.

(4) *When* b_i*'s follow Pareto distribution*

The distribution function of Pareto distribution is given by

$$F(b_i) = 1 - \lambda_i^{a_i} b_i^{-a_i},\ 0 < \lambda_i \le b_i < \infty,\ a_i > 0 \tag{3.16}$$

Here $A_i = a_i,\ B_i = \lambda_i^{a_i},\ h(b_i) = \log(b_i)$

Now from equation (3.6), we have

$$\prod_{i=1}^{m}\left(\lambda_i^{a_i} y_i^{-a_i}\right) \ge p \tag{3.17}$$

So, the deterministic multi-objective mathematical model for joint constraint can be expressed as

$$\left.\begin{aligned} & max\, z_k = \sum_{j=1}^{n} c_j^k x_j, \quad k = 1, 2, ..., K \\ & subject\ to \\ & \qquad \sum_{j=1}^{n} a_{ij} x_j = y_i,\ i = 1, 2, ..., m \\ & \qquad \prod_{i=1}^{m}\left(\lambda_i^{a_i} y_i^{-a_i}\right) \ge p \\ & and \qquad x_j \ge 0,\ j = 1, 2, ..., n \end{aligned}\right\} \tag{3.18}$$

Note: we can also take $A_i = 1,\ B_i = \lambda_i^{a_i},\ h(b_i) = \log(b_i)$ for the Burr Type III distribution.

(5) *When* b_i*'s follow Power Function distribution*

The distribution function of Power Function distribution is given by

$$F(b_i) = \lambda_i^{-a_i} b_i^{a_i},\ 0 \le b_i \le \lambda_i,\ a_i > 0,\ \lambda_i > 0 \tag{3.19}$$

Here $A_i = -1,\ B_i = 1,\ h(b_i) = \log\left(1 - \lambda_i^{-a_i} y_i^{a_i}\right)$

Now from equation (3.6), we have

$$\prod_{i=1}^{m}\left(1-\lambda_i^{-a_i} y_i^{a_i}\right) \geq p \tag{3.20}$$

So, the deterministic multi-objective mathematical model for joint constraint can be expressed as

$$\left.\begin{aligned} & max\, z_k = \sum_{j=1}^{n} c_j^k x_j , \quad k = 1, 2, ..., K \\ & subject\ to \\ & \qquad \sum_{j=1}^{n} a_{ij} x_j = y_i , i = 1, 2, ..., m \\ & \qquad \prod_{i=1}^{m}\left(1-\lambda_i^{-a_i} y_i^{a_i}\right) \geq p \\ & and \qquad x_j \geq 0,\ j = 1, 2, ..., n \end{aligned}\right\} \tag{3.21}$$

4. Numerical examples

Example 1:

Let the following MOSLP problem involving random variables follow different deduction of general form of distributions:

$$\begin{aligned} & \max z_1 = 5x_1 + 6x_2 + 3x_3 \\ & \max z_2 = 6x_1 + 3x_2 + 5x_3 \\ & \max z_3 = 2x_1 + 5x_2 + 8x_3 \end{aligned} \tag{4.1}$$

subject to

$$\begin{aligned} & P\left(3x_1 + 2x_2 + 2x_3 \leq b_1\right) \geq .90 \\ & P\left(2x_1 + 8x_2 + 5x_3 \leq b_2\right) \geq .98 \\ & P\left(5x_1 + 3x_2 + 2x_3 \leq b_3\right) \geq .95 \\ & P\left(0.5x_1 + 0.5x_2 + 0.25x_3 \leq b_4\right) \geq .90 \\ & P\left(8x_1 + 3x_2 + 4x_3 \leq b_5\right) \geq .99 \\ & x_{1,} x_2, x_3 \geq 0 \end{aligned}$$

where b_1 follow Power Function distribution with parameter $\lambda = 10, a = 5$; b_2 follow Pareto distribution with parameter $\lambda = 8, a = 2$; b_3 follow Beta of first kind distribution with parameter $\lambda = 15, a = 10, \beta = 3$; b_4 follow Weibull distribution with parameter

$\theta = 1/5, a = 10$; b_5 follow Burr type XII distribution with parameter $\lambda = 1/10, \theta = 1/15, a = 1/5$.

Using the well known concept of weighting characterization of the objective functions and from equations (2.9), (2.12), (2.15), (2.18) and (2.21) the deterministic model of the above MOSLP problem is

$$\max z = \lambda_1(5x_1 + 6x_2 + 3x_3) + \lambda_2(6x_1 + 3x_2 + 5x_3) + \lambda_3(2x_1 + 5x_2 + 8x_3) \tag{4.2}$$

subject to

$$\begin{aligned}
&3x_1 + 2x_2 + 2x_3 \le 6.3096\\
&2x_1 + 8x_2 + 5x_3 \le 8.0812\\
&5x_1 + 3x_2 + 2x_3 \le 4.7115\\
&0.5x_1 + 0.5x_2 + 0.25x_3 \le 0.9379\\
&8x_1 + 3x_2 + 4x_3 \le 10.0321\\
&\lambda_1 + \lambda_2 + \lambda_3 = 1\\
&x_{1,} x_2, x_3, \lambda_{1,} \lambda_2, \lambda_3 \ge 0
\end{aligned}$$

This problem is solved using the genetic algorithm inbuilt functions from Matlab and the optimal solution is obtained as follows, z = 8.5089, at x_1=0.3727, x_2=0.2319, x_3=1.0761, λ_1=0.3882, λ_2=0.2001, and λ_3=0.4117 with the individual objective values z_1 =6.4834, z_2=8.3125, and z_3=10.5140.

Example 2:

Consider the following MOLSP problem involving random variables follow different deduction of general form of distributions:

$$\begin{aligned}
&\max z_1 = 3x_1 + 8x_2 + 5x_3\\
&\max z_2 = 7x_1 + 4x_2 + 3x_3\\
&\max z_3 = 6x_1 + 7x_2 + 10.5x_3
\end{aligned} \tag{4.3}$$

subject to

$$P\begin{pmatrix} 5x_1 + 4x_2 + 2x_3 \le b_1, 7x_1 + 3x_2 + x_3 \le b_2, 2x_1 + 7x_2 + 3x_3 \le b_3, \\ 2x_1 + 3x_2 + 2.5x_3 \le b_4, 5x_1 + 2x_2 + 1.5x_3 \le b_5 \end{pmatrix} \ge 0.95$$

$$x_1, x_2, x_3 \ge 0$$

where b_1 follow Power Function distribution with parameter $\lambda = 3, a = 2$; b_2 follow Pareto distribution with parameter $\lambda = 10, a = 2$; b_3 follow Beta of first kind distribution with parameter $\lambda = 15, a = 1, \beta = 5$; b_4 follow Weibull distribution with parameter $\theta = 2, a = 1$; b_5 follow Burr type XII distribution with parameter $\lambda = 1, \theta = 3, a = 2$.

Using the well known concept of weighting characterization of the objective functions and from equations (3.9), (3.12), (3.15), (3.18) and (3.21) the deterministic model of the above stochastic linear programming problem is

$$\max z = \lambda_1(3x_1 + 8x_2 + 5x_3) + \lambda_2(7x_1 + 4x_2 + 3x_3) + \lambda_3(6x_1 + 7x_2 + 10.5x_3) \tag{4.4}$$

subject to

$$\left[\frac{y_1^2}{9}\right]\left[\frac{y_2^2 - 100}{y_2^2}\right]\left[\frac{y_3 - 5}{10}\right]\left[\frac{e^{2y_4} - 1}{e^{2y_4}}\right]\left[\frac{3y_5^2}{1 + 3y_5^2}\right] \geq 0.95$$

$$5x_1 + 4x_2 + 2x_3 = y_1$$
$$7x_1 + 3x_2 + x_3 = y_2$$
$$2x_1 + 7x_2 + 3x_3 = y_3$$
$$2x_1 + 3x_2 + 2.5x_3 = y_4$$
$$5x_1 + 2x_2 + 1.5x_3 = y_5$$
$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$
$$x_1, x_2, y_1, y_2, y_3, y_4, y_5, \lambda_1, \lambda_2, \lambda_3 \geq 0$$

This problem is solved using the genetic algorithm inbuilt functions from Matlab and the optimal solution is obtained as follows, z = 3.2081, at x_1=0.1939, x_2=0.2810, x_3=0.1968, y_1 =2.4872, y_2 =2.3971, y_3 =2.9454, y_4 =1.7229, y_5 =1.8267, λ_1=0.1764, λ_2=0.8211, and λ_3=0.0026 with the individual objective values z_1 =3.8139, z_2=3.0717, and z_3=5.1968.

5. Conclusion

In this paper, we have established the deterministic constraints (2.6) for the given probabilistic constraints where b_i 's follow general form of distribution for independent constraints. We have also established the deterministic constraints (3.6) for joint constraints. With the help of (2.6) and (3.6), one can easily obtain the deterministic multi-objective mathematical programming problem corresponding to the given MOSLP problem with independent or joint constraints. The main contribution of this paper is the derivation of deterministic equivalence for independent and joint constraints where the right hand side resource vector following different distributions like Power Function Distribution, Pareto Distribution etc. by taking the suitable value of A_i, B_i and $h(b_i)$. After the conversion of MOSLP problem into a deterministic equivalent, the resultant multi-objective mathematical programming problem is still an NP- Hard problem. Therefore, one can try some soft computing techniques to solve MOSLP problem directly without applying any conversion technique.

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