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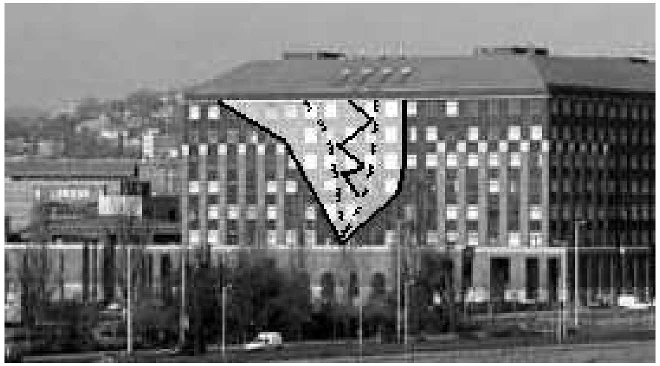

Strengthening weak sandwich theorems in the presence of inconnectivity

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Abstract

In the paper we consider degree, spectral, and semidefinite bounds on the stability and chromatic numbers of a graph: so-called weak sandwich theorems. We examine the additivity properties of the bounds (the sum of two graphs is their disjoint union), and as an application we tighten the bounds in the weak sandwich theorems, if the graph or its complementer is not connected.

Mathematics Subject Classifications (2000). 90C22, 90C27

1 Introduction

In the paper we will consider additivity properties of (lower resp. upper) bounds on the stability and chromatic numbers of a simple graph. As an application we can tighten the bounds when the graph or its complementer is not connected.

We start this paper with stating the main additivity result. First we fix some notation. Let $n \in \mathcal{N}$, and let $G = (V(G), E(G))$ be an undirected graph, with vertex set $V(G) = \{1, \ldots, n\}$, and with edge set $E(G) \subseteq \{\{i, j\} : i \neq j\}$. Let $A(G)$ be the 0-1 adjacency matrix of the graph G, that is let

$$A(G) := (a_{ij}) \in \{0, 1\}^{n \times n}, \text{ where } a_{ij} := \begin{cases} 0, & \text{if } \{i, j\} \notin E(G), \\ 1, & \text{if } \{i, j\} \in E(G). \end{cases}$$

The complementer graph $\overline{G}$ is the graph with adjacency matrix $A(\overline{G}) := J - I - A(G)$, where I is the identity matrix, and J denotes the matrix with all elements equal to one. The disjoint union of the graphs G_1 and G_2 is the graph $G_1 + G_2$ with adjacency matrix

$$A(G_1 + G_2) := \begin{pmatrix} A(G_1) & 0 \\ 0 & A(G_2) \end{pmatrix}.$$

Let $(d_1, \ldots, d_n)$ be the sum of the row vectors of the adjacency matrix. The elements of this vector are the degrees of the vertices of the graph G. Let $d_{\max}$ be the maximum of the degrees in the graph. We define similarly the values $\overline{d}_1, \ldots, \overline{d}_n$ and $\overline{d}_{\max}$ in the complementer graph $\overline{G}$ instead of G.

By Rayleigh's Theorem (see [6]) for a symmetric matrix $M = M^T \in \mathcal{R}^{n \times n}$ the maximum eigenvalue, $\lambda_{\max}(M)$, can be expressed as

$$\lambda_{\max}(M) = \max_{||u||=1} u^T M u.$$

The maximum eigenvalue of the adjacency matrix $A(G)$ (resp. $A(\overline{G})$) is denoted by $\alpha_{\max}$ (resp. $\overline{\alpha}_{\max}$).

A symmetric matrix $M = M^T \in \mathcal{R}^{n \times n}$ is called positive semidefinite if $u^T M u \geq 0$ for all $u \in \mathcal{R}^n$. The set of the n by n real symmetric positive semidefinite matrices is denoted by PSD.

The stability number, $\alpha(G)$, is the maximum cardinality of the (so-called stable) sets $S \subseteq V(G)$ such that $\{i, j\} \subseteq S$ implies $\{i, j\} \notin E(G)$. The chromatic number, $\chi(G)$, is the minimum number of stable sets covering the vertex set $V(G)$.

In the seminal paper [3] L. Lovász proved the following result, now popularly called *sandwich theorem*:

$$\alpha(G) \leq \vartheta(G) \leq \chi(\overline{G}),$$

where $\vartheta(G)$ is the Lovász number of the graph G. The Lovász number has several equivalent descriptions, for example it is the optimal value of the Slater-regular primal-dual semidefinite programs

$$\min \lambda, \begin{cases} x_{ii} = \lambda - 1 \ (i \in V(G)), \\ x_{ij} = -1 \ (\{i, j\} \in E(\overline{G})), \\ (x_{ij}) \in PSD \end{cases}$$

and

$$\max \sum_{i,j} m_{ij}, \begin{cases} m_{11} + \ldots + m_{nn} = 1, \\ m_{ij} = 0 \ (\{i, j\} \in E(G)), \\ (m_{ij}) \in PSD. \end{cases}$$

Following [7], we will call *weak sandwich theorem* a result of the type

$$L(G) \leq \alpha(G) \leq \chi(\overline{G}) \leq U(\overline{G}),$$

where $L(G) \geq 1$, $U(G) \leq n$ are some kind of (degree, spectral, ...etc.) bounds.

In Section 2 we define four pairs of bounds, $L_k(G), U_k(G)$ $(k = 1,2,3,4)$, and prove the corresponding four weak sandwich theorems, immediate consequences of results of Brooks ([4]), Alon-Spencer ([1]), Wilf ([8], [9]), and Lovász ([3]). Our main result relates the value of the bounds L_k, U_k resp. their complements $\overline{L}_k, \overline{U}_k$ on $G_1 + G_2$ to their values on G_1 and G_2 $(k = 1,2,3,4)$. (Here we use the notation $\overline{L}_k(G) := L_k(\overline{G})$ and $\overline{U}_k(G) := U_k(\overline{G})$.)

THEOREM 1.1. *Given two graphs, G_1, G_2, the following relations hold for $k = 1,2,3,4$:*

a) $U_k(G_1 + G_2) \geq \max\{U_k(G_1), U_k(G_2)\}$;

b) $\overline{U}_k(G_1 + G_2) \geq \overline{U}_k(G_1) + \overline{U}_k(G_2)$;

c) $L_k(G_1 + G_2) \leq L_k(G_1) + L_k(G_2)$;

d) $\overline{L}_k(G_1 + G_2) \leq \max\{\overline{L}_k(G_1), \overline{L}_k(G_2)\}$.

Equality holds in part a) for $k = 1,2$, and in part c) for $k = 1$.

In Section 3 we will prove Theorem 1.1. Then we apply these inequalities for tightening the bounds in the weak sandwich theorems in the case of the inconnectivity of the graph or its complementer.

2 Weak sandwich theorems

In this section we will describe the weak sandwich theorems and examine the relations between the bounds in them.

The weak sandwich theorems are immediate consequences of results of Brooks, Alon-Spencer, Wilf, and Lovász.

PROPOSITION 2.1. *Let us denote*

$$L_1(G) := \sum_{i \in V(G)} \frac{1}{d_i + 1}, \; U_1(G) := d_{\max} + 1.$$

Then, $L_1(G) \leq \alpha(G) \leq \chi(\overline{G}) \leq U_1(\overline{G})$.

Proof. The first inequality is proved by Alon-Spencer in [1] (for another proof see [7]). The last inequality is part of the Brooks' Theorem (see e.g. [4]). □

PROPOSITION 2.2. *Let us denote*

$$L_2(G) := \frac{n}{\alpha_{\max} + 1}, \; U_2(G) := \alpha_{\max} + 1.$$

Then, $L_2(G) \leq \alpha(G) \leq \chi(\overline{G}) \leq U_2(\overline{G})$.

Proof. The last inequality is proved by Wilf in [8]. The first inequality is an easy consequence of the last one, as $\alpha(G) \cdot \chi(G) \geq n$, obviously. □

PROPOSITION 2.3. *Let us denote*

$$L_3(G) := \frac{n}{n + 1 - \vartheta(G)}, \; U_3(G) := n + 1 - \vartheta(G).$$

Then, $L_3(G) \leq \alpha(G) \leq \chi(\overline{G}) \leq U_3(\overline{G})$.

Proof. The last inequality follows from Lovász's sandwich theorem, and from the fact that $\chi(G) + \chi(\overline{G}) \leq n + 1$ (see Exercise 9.5 in [4]). The remaining $L_3(G) \leq \alpha(G)$ inequality can be proved similarly as the $L_2(G) \leq \alpha(G)$ inequality in Proposition 2.2. □

PROPOSITION 2.4. *Let us denote*

$$L_4(G) := \frac{n}{n - \overline{\alpha}_{\max}}, \; U_4(G) := n + 1 - \frac{n}{n - \overline{\alpha}_{\max}}.$$

Then, $L_4(G) \leq \alpha(G) \leq \chi(\overline{G}) \leq U_4(\overline{G})$.

Proof. The first inequality is proved by Wilf in [9] as a consequence of a theorem of Motzkin-Straus. The last inequality follows from the first one, and from the fact that $\chi(G) + \chi(\overline{G}) \leq n + 1$ (see Exercise 9.5 in [4]). □

Now, we compare the bounds in the weak sandwich theorems.

PROPOSITION 2.5. *Between the bounds defined above the following relations hold: a) $L_2(G) \leq L_1(G)$; b) $U_2(G) \leq U_1(G)$; c) $L_3(G) \leq L_4(G)$; d) $U_3(G) \leq U_4(G)$; e) $L_2(G) \leq L_4(G)$.*

Proof. a) is proved in [7].

b) is well-known, see e.g. Exercise 11.14 in [4].

c) follows from the fact that $\vartheta(G) \leq U_2(\overline{G})$ (a consequence of Lovász's sandwich theorem and Proposition 2.2).

d) follows from Lovász's sandwich theorem and Proposition 2.4 similarly as part c).

e) The inequality $L_2(G) \leq L_4(G)$ can be written as

$$n + 1 \leq \alpha_{\max} + 1 + \overline{\alpha}_{\max} + 1,$$

and this latter inequality is true, as by Rayleigh's Theorem

$$\begin{aligned} \alpha_{\max} + \overline{\alpha}_{\max} &= \max_{||u||=1} u^T A(G) u + \max_{||v||=1} v^T A(\overline{G}) v \\ &\geq \max_{||u||=1} u^T (A(G) + A(\overline{G})) u \\ &= \max_{||u||=1} u^T (J - I) u = n - 1. \end{aligned}$$

Hence, the proof is complete. □

We remark that for example for regular graphs and complete bipartite graphs the inequality $U_2(G) \leq U_4(G)$ holds too, in other words $L_4(G) \cdot L_4(\overline{G}) \leq n$. It is an open problem, whether this inequality holds or not, generally.

3 Additivity properties

In this section we will prove Theorem 1.1, and then we present the motivating application.

Proof of Theorem 1.1:

Case 1: $k = 1$. The statements a), b), and c) are obvious from the definitions.

d) We will use the identity

$$\frac{1}{a + x} = \frac{1}{a} - \frac{x}{a(a + x)}. \tag{1}$$

Let us suppose that

$$\sum_{G_1} \frac{1}{\overline{d}_i + 1} \geq \sum_{G_2} \frac{1}{\overline{d}_i + 1}. \tag{2}$$

Then, we have to show that

$$\sum_{G_1} \frac{1}{\overline{d}_i + n_2 + 1} + \sum_{G_2} \frac{1}{\overline{d}_i + n_1 + 1} \leq \sum_{G_1} \frac{1}{\overline{d}_i + 1},$$

which can be written as

$$\sum_{G_2} \frac{1}{\overline{d}_i + n_1 + 1} \leq \sum_{G_1} \frac{n_2}{(\overline{d}_i + 1) \cdot (\overline{d}_i + n_2 + 1)}. \tag{3}$$

In (3), by (1), the left hand side is

$$\sum_{G_2} \frac{1}{\overline{d}_i + n_1 + 1} = \sum_{G_2} \frac{1}{\overline{d}_i + 1} - \sum_{G_2} \frac{n_1}{(\overline{d}_i + 1) \cdot (\overline{d}_i + n_1 + 1)},$$

while, by (2), the right hand side is

$$\sum_{G_1} \frac{n_2}{(\overline{d}_i + 1) \cdot (\overline{d}_i + n_2 + 1)} \geq \frac{n_2}{n} \sum_{G_2} \frac{1}{\overline{d}_i + 1}.$$

Hence, in order to prove (3), it is enough to show that

$$\frac{n_1}{n} \sum_{G_2} \frac{1}{\overline{d}_i + 1} \leq \sum_{G_2} \frac{n_1}{(\overline{d}_i + 1) \cdot (\overline{d}_i + n_1 + 1)},$$

and this inequality is true as $\overline{d}_i + n_1 + 1 \leq n$.

Case 2: $k = 2$. a) is obvious from the definitions.

b) We have to prove that

$$\lambda_{\max} \begin{pmatrix} J - A(G_1) & J \\ J & J - A(G_2) \end{pmatrix} \geq \lambda_{\max}(J - A(G_1)) + \lambda_{\max}(J - A(G_2)).$$

Let us choose nonnegative unit vectors v_1, v_2 such that

$$\lambda_{\max}(J - A(G_i)) = v_i^T (J - A(G_i)) v_i, \; i = 1, 2$$

holds, by the Perron-Frobenius Theorem ([5]) this can be done.

By Rayleigh's Theorem it is enough to verify that there exist constants $\varepsilon_1, \varepsilon_2 > 0$ such that the vector $v := (\varepsilon_1 v_1^T, \varepsilon_2 v_2^T)^T$ satisfies both $v^T v = 1$ and

$$v^T \begin{pmatrix} J - A(G_1) & J \\ J & J - A(G_2) \end{pmatrix} v \geq v_1^T (J - A(G_1)) v_1 + v_2^T (J - A(G_2)) v_2.$$

Let us introduce the following notation:

$$p_{12} := v_1^T J v_2, \; p_{ii} := v_i^T (J - A(G_i)) v_i \; (i = 1, 2).$$

We have to find constants $\varepsilon_1, \varepsilon_2 > 0$ satisfying

$$\varepsilon_1^2 + \varepsilon_2^2 = 1, \; 2\varepsilon_1\varepsilon_2 \cdot p_{12} \geq \varepsilon_2^2 \cdot p_{11} + \varepsilon_1^2 \cdot p_{22}.$$

Note that by the inequality $p_{ii} \leq v_i^T J v_i$ $(i = 1, 2)$, we have $p_{12}^2 \geq p_{11} \cdot p_{22}$. (We used here the nonnegativity of the vectors v_1, v_2.) Let us choose the constants $\varepsilon_1, \varepsilon_2 > 0$ so that they satisfy both $\varepsilon_1^2 + \varepsilon_2^2 = 1$ and

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{p_{12} + \sqrt{p_{12}^2 - p_{11}p_{22}}}{p_{22}}.$$

It can be easily verified that these constants $\varepsilon_1, \varepsilon_2$ meet the requirements.

c) and d) follows from part a) and b), respectively.

Case 3: $k = 3$. a) is a consequence of the equation

$$\vartheta(G_1 + G_2) = \vartheta(G_1) + \vartheta(G_2) \tag{4}$$

(see [2], Section 18.), and the fact that $\vartheta(G) \leq n$.

b) is a consequence of the equation

$$\vartheta(\overline{G_1 + G_2}) = \max\{\vartheta(\overline{G_1}), \vartheta(\overline{G_2})\} \tag{5}$$

(see [2], Section 19.), and the fact that $\vartheta(G) \geq 1$.

c) follows from formula (4) as with the notation $z_1 := U_1(G_1)$, $z_2 := U_1(G_2)$ we have $z_1, z_2 \geq 1$, and thus

$$\frac{n}{z_1 + z_2 - 1} \leq \min\left\{\frac{n}{z_1}, \frac{n}{z_2}\right\} = \frac{n_1 + n_2}{\max\{z_1, z_2\}} \leq \frac{n_1}{z_1} + \frac{n_2}{z_2}.$$

d) follows from formula (5) and from the fact that $1 \leq m \leq n' \leq n$ implies the inequality $n/(n + 1 - m) \leq n'/(n' + 1 - m)$.

Case 4: $k = 4$. a) is an easy consequence of part c).

b) is obvious from the definitions.

c) Let us denote

$$\overline{\alpha} := \lambda_{\max}(A(\overline{G_1 + G_2})), \; \overline{\alpha}_1 := \lambda_{\max}(A(\overline{G_1})), \; \overline{\alpha}_2 := \lambda_{\max}(A(\overline{G_2})).$$

Notice that

$$1 - \frac{1}{L_4(G_1 + G_2)} = \frac{\overline{\alpha}}{n},$$

and that

$$1 - \frac{1}{L_4(G_1) + L_4(G_2)} = \frac{n_1 n_2 - \overline{\alpha}_1 \overline{\alpha}_2}{2n_1 n_2 - n_1 \overline{\alpha}_2 - n_2 \overline{\alpha}_1}.$$

Hence, introducing the notation $x := \overline{\alpha}_1/n_1$, $y := \overline{\alpha}_2/n_2$, we have to prove the inequality

$$\frac{\overline{\alpha}}{n} \leq \frac{1 - xy}{2 - x - y}. \tag{6}$$

Let $u = (u_1, u_2) \in \mathcal{R}^n$ be a unit vector such that

$$\overline{\alpha} = u^T \begin{pmatrix} J - I - A(G_1) & J \\ J & J - I - A(G_2) \end{pmatrix} u.$$

Then by Rayleigh's Theorem, and by the obvious inequality

$$u_1^T J u_2 \leq \sqrt{n_1 n_2} \cdot ||u_1|| \cdot ||u_2||,$$

we have

$$\overline{\alpha} \leq ||u_1||^2 \cdot \overline{\alpha}_1 + ||u_2||^2 \cdot \overline{\alpha}_2 + 2\sqrt{n_1 n_2} \cdot ||u_1|| \cdot ||u_2||.$$

Hence, in order to prove (6), it is enough to show that

$$||u_1||^2 \cdot n_1 x + ||u_2||^2 \cdot n_2 y + 2\sqrt{n_1 n_2} \cdot ||u_1|| \cdot ||u_2|| \leq n \cdot \frac{1 - xy}{2 - x - y}. \tag{7}$$

As $||u_1||^2 + ||u_2||^2 = 1$ holds, by Rayleigh's Theorem, the left hand side of (7) satisfies

$$\begin{pmatrix} ||u_1|| \\ ||u_2|| \end{pmatrix}^T \begin{pmatrix} n_1 x & \sqrt{n_1 n_2} \\ \sqrt{n_1 n_2} & n_2 y \end{pmatrix} \begin{pmatrix} ||u_1|| \\ ||u_2|| \end{pmatrix} \leq \frac{n_1 x + n_2 y + \sqrt{(n_1 x - n_2 y)^2 + 4n_1 n_2}}{2}.$$

Hence, in order to prove (7), it suffices to show that

$$n_1 x + n_2 y + \sqrt{(n_1 x - n_2 y)^2 + 4n_1 n_2} \leq 2n \cdot \frac{1 - xy}{2 - x - y},$$

or in other words, with the notation $n_1' := n_1/n$, $n_2' := n_2/n$, that

$$\sqrt{(n_1' x - n_2' y)^2 + 4n_1' n_2'} \leq \frac{n_1'(x-1)^2 + n_2'(y-1)^2 + 1 - xy}{2 - x - y} \tag{8}$$

holds.

Let us denote

$$a := \frac{(x-1)^2}{2 - x - y}, \; b := \frac{(y-1)^2}{2 - x - y}, \; c := \frac{1 - xy}{2 - x - y}.$$

Then squaring the inequality (8), taking into account that $n_1' + n_2' = 1$, we get the inequality

$$\begin{pmatrix} n_1' \\ n_2' \end{pmatrix}^T \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} n_1' \\ n_2' \end{pmatrix} \leq 0, \tag{9}$$

which is equivalent to (8), and where

$$m_{11} = x^2 - (a+c)^2,\ m_{22} = y^2 - (b+c)^2$$
$$m_{12} = m_{21} = 2 - xy - (a+c)(b+c).$$

It can be easily verified that

$$m_{11} \leq 0,\ m_{22} \leq 0,\ m_{11}m_{22} - m_{12}m_{21} \geq 0.$$

(Really, for example the inequality $m_{11}m_{22} \geq m_{12}m_{21}$ can be written as

$$(a+c+x)(b+c+y) \geq \left(\frac{x(b+c)+y(a+c)}{2} + 1\right)^2,$$

and this inequality follows from the definitions.) Hence, (9) is satisfied for all n_1', n_2', part c) is proved.

d) Let us introduce the notation

$$\alpha := \lambda_{\max}(A(G_1+G_2)),\ \alpha_1 := \lambda_{\max}(A(G_1)),\ \alpha_2 := \lambda_{\max}(A(G_2)).$$

As obviously $\alpha_1 \geq \alpha_2$ implies

$$\frac{n}{n-\alpha} = \frac{n}{n-\alpha_1} \leq \frac{n_1}{n_1-\alpha_1},$$

and $\alpha_2 \geq \alpha_1$ implies

$$\frac{n}{n-\alpha} = \frac{n}{n-\alpha_2} \leq \frac{n_2}{n_2-\alpha_2},$$

so the statement follows. Hence, the theorem is proved. □

We mention an application of Theorem 1.1. Suppose that G is not connected, $G = G_1 + G_2$ for some subgraphs G_1, G_2. Then, as

$$\alpha(G) = \alpha(G_1) + \alpha(G_2),$$

we have besides the bound $\alpha(G) \geq L_k(G)$ the bound

$$\alpha(G) \geq L_k(G_1) + L_k(G_2)$$

also ($k = 1,2,3,4$). By Theorem 1.1 the latter bound is tighter than the former one. If some of the graphs G_1, G_2 is not connected then we can iterate the method and obtain even tighter bounds.

Similarly, if the complementer graph $\overline{G}$ is not connected, $\overline{G} = G_1' + G_2'$, then by

$$\alpha(G) = \overline{\alpha}(\overline{G}) = \overline{\alpha}(G_1' + G_2') = \max\{\overline{\alpha}(G_1'), \overline{\alpha}(G_2')\},$$

besides $\alpha(G) \geq \overline{L}_k(\overline{G})$, we have the tighter bound

$$\alpha(G) \geq \max\{\overline{L}_k(G_1'), \overline{L}_k(G_2')\}$$

also ($k = 1,2,3,4$).

The method carries over to bounds on $\overline{\chi}(G)$ too, as the equalities

$$\overline{\chi}(G_1+G_2) = \overline{\chi}(G_1) + \overline{\chi}(G_2),\ \ \chi(G_1'+G_2') = \max\{\chi(G_1'), \chi(G_2')\}$$

hold.

Hence, in the case of inconnectivity of G or $\overline{G}$, the weak sandwich theorems

$$L_k(G) \leq \alpha(G) \leq \overline{\chi}(G) \leq \overline{U}_k(G)\ (k = 1,2,3,4)$$

can be strengthened.

We conclude with a simple example: Let G_0 be the cherry graph

$$G_0 := (\{1,2,3\}, \{\{1,2\},\{1,3\}\}).$$

Then,

$$L_1(G_0) = \frac{4}{3},\ L_2(G_0) = \frac{3}{\sqrt{2}+1},\ L_3(G_0) = L_4(G_0) = \frac{3}{2}.$$

Now, $\overline{G}_0 = G_1' + G_2'$, where

$$G_1' = (\{1\}, \emptyset),\ \ G_2' = (\{2,3\}, \{\{2,3\}\}),$$

so by the above considerations

$$\alpha(G_0) \geq \max\{\overline{L}_k(G_1'), \overline{L}_k(G_2')\}\ (k = 1,2,3,4),$$

which gives the best possible bound $\alpha(G_0) \geq 2$.

Finally, we mention an open problem: Find spectral or degree bounds with multiplicativity properties. (Multiplicativity properties of the Lovász's $\vartheta(G)$ number were studied in [3].)

Conclusion. In this paper we studied additivity properties of various bounds on the stability and chromatic numbers of a graph. These well-known degree, spectral, and semidefinite bounds are due to Brooks, Alon-Spencer, Wilf, and Lovász. As an application of our main result we obtained stronger bounds in weak sandwich theorems in the case of inconnectivity of the graph or its complementer.

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