Optimal location of family homes for dual career couples

M. Gugat, K. Klamroth, A. Abele

Abstract

The number of dual-career couples with children is growing fast. These couples face various challenging problems of organizing their lifes, in particular connected with childcare and time-management. As a typical example we study one of the difficult decision problems of a dual career couple from the point of view of operations research with a particular focus on gender equality, namely the location problem to find a family home. This leads to techniques that allow to include the value of gender equality in rational decision processes.

1 Introduction

The number of dual-career couples (DCCs) with children is growing fast. These couples face various problems of organizing their lifes, in particular connected with childcare and time-management. Some of the organizational problems that are typical for DCCs may be addressed by methods from the field of Operations Research. This includes, for example, scheduling issues and time management problems (keeping in mind the possibility to "outsource" selected housekeeping tasks), as well as logistical issues as, for example, the selection of an appropriate place to live. Having efficient solutions for organizational problems at hand may significantly reduce the stress level in DCC partnerships and improve well-being and life satisfaction. While the focus of this project is on logistical questions, similar approaches can be developed to aid DCCs also in other areas with the efficient organization of their coupled lives.

In a society where gender equality is highly valued it is important to take gender equality into account in the decision problems that occur in the life of a DCC, since an unequal distribution of the contributions of the partners to childcare may lead to dissatisfaction and high stress levels in the partnership.

The following example is used to illustrate the type of logistical problems that occur in DCCs. The underlying concepts and solution approaches are, however, more general and can be applied also in different constellations. Consider a DCC with two children (one kindergarten child and one school child) that has successfully found two workingplaces with a sufficiently small distance. Then they face the problem of finding a location for their home that satisfies their needs in an optimal way.

As a *mathematical model* for certain aspects of the decision problem of the DCC, we consider the location problem for their living place.

The aim of the couple is to find a location H for their home. In such a decision, a large number of criteria play a role. The inclination towards a location H is influenced by various factors in particular the possibilities to build up a social network, the quality of the cultural life and other factors that determine the quality of life (subjective well-being) for a given location.

The inclination of the partners in the couple towards a given location H that are determined by the above factors are modelled by utility functions $u_{loc,f}(H)$ and $u_{loc,m}(H)$.

These utility functions do not include the logistic cost connected with the tours to the working place and the tours caused by childcare. It may happen that $u_{loc,f}(H)$ and $u_{loc,m}(H)$ are almost identical depending on the preferences of the partners in the couple.

The cost of the tours to the working places and the corresponding "childcare juggling tours" is considered separately. The childcare juggling tours consist of tours that bring the children to one of the acceptable schools/kindergartens and to additional locations for activities like music school and sport clubs. So the construction of the optimal tour includes the decision for one of the schools and one of the kindergartens. Note that for each fixed location H the choice of the optimal tours is again an optimization problem. So the cost of the optimal tours that appears in the objective function is determined as the optimal value of a parametric optimization problem with parameter H.

The resulting decision problem can be considered as a multicriteria optimization problem (see Refs. 4, 5). In the case of concave objective functions and a convex admissible set for the location H, the efficient points can by found by minimizing overall utility functions that are positive linear combinations of the utility functions that model the preferences for the different criteria that are relevant for the decision. Our model leads to an optimization problem where for each possible location H of a family home the corresponding evaluation of the objective function requires the construction of an optimal "childcare juggling tour" for H including the choice of the corresponding childcare facilities.

2 The model in mathematical terms

In this section we consider the decision problem of finding a home for a DCC where the decision is based upon the maximization of a joint utility function for both partners that is obtained as a weighted sum of the utility functions that model the preferences for the different criteria that are relevant for the decision.

We call this problem **DCCLO** as an abbreviation for **D***ual* **C***areer* **C***ouple* **Lo***cation problem with joint utility function.*

For the sake of simplicity, we present here a model that takes only the schools and kindergartens into account. These facilities should be seen as examples for any kind of childcare facility that is scheduled.

Let the two workingplaces W_f and W_m be given. Let the acceptable schools

S_i, $i = 1, \dots, I$ and the acceptable kindergartens K_j, $j = 1, \dots, J$ be given. All these locations are given as points in the space $\mathbb{R}^2$.

Problem **DCCLO** is defined as follows: Find a place for the home $H \in \mathbb{R}^2$ that is optimal in the sense that it minimizes the joint utility function U_{joint} of the DCC that is defined below.

The utility function U_{joint} includes the logistical cost of H which is the cost of an optimal feasible childcare juggling tour, where different possible sequences like home-kindergarten-school-work have to be distinguished for the two partners.

For a given location H an admissible tour consists of one tour for each partner that starts at H and ends at the corresponding working-place. In addition, an acceptable school has to be included in one of the tours and also an acceptable kindergarten. Let $d(A, B)$ denote the cost of transportation from facility A to facility B. Then for a given choice of S_i and K_j we have six types of admissible tours with the corresponding costs c_1,...,c_6 defined as

$$
\begin{aligned}
c_1(H, S_i, K_j) &= d(H, K_j) + d(K_j, S_i) + d(S_i, W_m) + d(H, W_f) \\
c_2(H, S_i, K_j) &= d(H, S_i) + d(S_i, K_j) + d(K_j, W_m) + d(H, W_f) \\
c_3(H, S_i, K_j) &= d(H, S_i) + d(S_i, W_m) + d(H, K_j) + d(K_j, W_f) \\
c_4(H, S_i, K_j) &= d(H, K_j) + d(K_j, W_m) + d(H, S_i) + d(S_i, W_f) \\
c_5(H, S_i, K_j) &= d(H, K_j) + d(K_j, S_i) + d(S_i, W_f) + d(H, W_m) \\
c_6(H, S_i, K_j) &= d(H, S_i) + d(S_i, K_j) + d(K_j, W_f) + d(H, W_m).
\end{aligned}
$$

Let $c_{tour}(H)$ denote the cost of an optimal admissible tour. Then we have

$$
c_{tour}(H) = \min_{i \in I,\, j \in J} \min_{l \in \{1,2,\dots,6\}} c_l(H, S_i, K_j). \tag{1}
$$

If the quality of kindergarten K_i (or similarly of school S_j) is very good or if, conversely, the school fees are very high, this can be modelled as follows: An additional "selection utility" $u_{school}(i)$ and $u_{kindergarten}(j)$ should be introduced and subtracted from to the objective function that is minimized to compute the logistical cost $c_{tour}(H)$ of location H. These utilities contain the evaluation of the quality of the childcare facilities.

Similarly, additional terms of the type $t(K_i)$ and $t(S_j)$ can be added to the objective function modelling the expected time needed to drop off the children at the respective facilities.

Note that in the case of childcare juggling tours of Type 1, Type 2, Type 5 and Type 6 (corresponding to the cost functions c_1, c_2, c_5, c_6) one partner brings the children to kindergarten and school, whereas the other partner only drives to the workingplace. In Type 3 of the childcare juggling tours, one partner brings one child to school an then drives to the working place W_m and the other partner brings one child to kindergarten and then drives to her working place W_f. Type 4 is similar to Type 3 with the roles of the partners exchanged.

It is clear that to obtain a more realistic model, more complicated tours have to be considered: Every day, the children must also be transported back home from the corresponding childcare facility and possibly then there may be a need for transportation to a music school or sport club. To take into account the time constraints of the partners of the DCC at their workplaces, for example due to inflexible working hours, in some cases it may be convenient to change the contributions of the partners for the transportation back home. So for the transportation back home, and additional cost function of the type of $c_{tour}(H)$ could be included in the model. Moreover, to compute a meaningful logistial cost in some cases it may be necessary to consider an optimal transportation plan for a whole week.

In order to compute the logistic cost for a fixed home location H, the best tours have to be computed. Let weight $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ with $\lambda_1 + \lambda_2 \in [0, 1]$ be given. The joint utility function U_{joint} that is maximized in **DCCLO** is defined as follows:

$$U_{joint}(H) = \lambda_1 u_{loc,f}(H) + \lambda_2 u_{loc,m}(H) - (1 - \lambda_1 - \lambda_2) c_{tour}(H)$$

For a given set $\mathcal{H}$ of admissible homes we can state **DCCLO** in compact form as

$$\textbf{DCCLO}: \max_{H \in \mathcal{H}} U_{joint}(H)$$

which is equivalent to

$$\max_{H \in \mathcal{H}} \max_{i \in I, j \in J} \max_{l \in \{1,2,\ldots,6\}} \lambda_1 u_{loc,f}(H) + \lambda_2 u_{loc,m}(H) - (1 - \lambda_1 - \lambda_2) c_l(H, S_i, K_j). \tag{2}$$

The solution of **DCCLO** provides useful suggestions about the optimal place to live for the DCC. The basis of these suggestions is the joint utility function U_{joint}. Of course, these suggestions depend on the weights λ_1 and λ_2.

2.1 Bicriteria Approach

Note that in the objective function of the above model, the *overall* cost for childcare of the couple is minimized. Consequently, the division of the contribution may be unequal and thus considered unfair for the partners in an optimal solution. To account for this problem, different modifications of the model can be considered: In a general approach, the time spent by both partners can be considered as two separate and conflicting objective functions, leading to a bicriteria optimization problem where to each fixed home location H, a separate utility is assigned for both partners, namely

$$\begin{aligned} U_f(H) = & \quad u_{loc,f}(H) - c_{tour,f}(H) \\ U_m(H) = & \quad u_{loc,m}(H) - c_{tour,m}(H) \end{aligned}$$

where $c_{tour,f}$ and $c_{tour,m}$ denote the minimal cost of transportation for the female and male partners (taking into account the utility function that model the quality of the childcare facility) respectively. In order to guarantee that for the considered locations H for the evaluation of the objective function the cost of a complete childcare juggling tour is considered the introduction of an additional constraint is necessary. Otherwise it might happen that a location with a large distance to childcare facilities is chosen that is close to both workingplaces since each partner expects the other to do the childcare work.

A feasible home location H is called *efficient* if if there is no other home location that improves at least one partner's utility without deteriorating the other partner's utility. In terms of inequalities, this means that H is efficient if there is no $\hat{H} \neq H$ such that $U_f(\hat{H}) \geq U_f(H)$ and $U_m(\hat{H}) \geq U_m(H)$ with at least one strict inequality (we say that in this case, $\hat{H}$ *dominates* H).

Such efficient locations are rational candidates for the solution, and different techniques can be employed to identify such a solution. For example, in the convex case the two objective functions can be combined into an overall utility function (for example, a weighted sum) as in **DCCLO**. Another technique is the ε-constraint method that is described for example in Refs. 4, 5, 8.

The problem of optimal home location based upon the bicriteria approach is denoted as **DCCBI**, where BI stands for the bicriteria approach.

DCCBI: Find the set of efficient locations of the home H.

If problem **DCCBI** is solved, still the couple has to select among the efficient locations. Different decision making techniques can be employed to aid the couple with the final decision making process.

3 Including the value of gender equality in the decision process

Fortunately, there has been a value shift towards equality in gender roles which should also be reflected in the optimization problem that models the decision process. In the next sections we describe possibilities to include the value of gender equality in the model.

3.1 Gender Inequality Tolerance Constraints

First, constraints can be included into the formulation of the problem that only allow solutions where the contributions of the partners do not differ too much. Such inequality constraints can be seen as a model for the tolerance of the partners to an unequal distribution of the contributions to childcare. The inequality constraints can be included in the models from Section 2 and Section 2.1. For each home location H, the constraints lead to a smaller set of feasible childcare juggling rours. In the formulation of the constraint, the total contribution of the partners to the childcare during a certain time-interval which is a subset of the period of family life where such a contribution is necessary should be used to allow different models for different phases in the family life. Such a *gender-equality constraint with memory* has the form of the inequality

$$\left| \int_0^T contributions_m(t) - contributions_f(t)\, dt \right| \leq tol$$

which is equivalent to

$$-tol \leq \int_0^T contributions_m(t) - contributions_f(t)\, dt \leq tol.$$

Here $[0, T]$ denotes the considered time interval, the integrals

$$\int_0^T contributions_m(t)\, dt \text{ and } \int_0^T contributions_f(t)\, dt$$

measure the contributions of each partner during this time interval and *tol* denotes the tolerance of the couple towards inequality of the contributions. This model reflects the fact that in different life-phases different solution can make sense. Of course, here we have a problem of non-anticipativity: Only data from the past are available for a decision at a given time t_0 where the decision is taken. However, instead of unknown data from the future an estimate based upon the commitment for future contributions can be used.

Another possibility is to replace the upper bound T by t_0. Moreover, the tolerance toward unequality may change in time, so *tol* can be considered as a function of t_0. This leads to a constraint of the form

$$\left| \int_0^{t_0} contributions_m(t) - contributions_f(t)\, dt \right| \leq tol(t_0).$$

The integral should be seen as a model of the fact that the contributions sum up over time. For the measurement of the contributions different approaches can be used: For example contributions can consist of invested time and activity but also on invested money that can be used for example for outsourcing of childcare activities. We do not discuss the related question of the right trade-off between time and money here. However, we want to mention that in the context of a DCC this question is particularly interesting in the case that the income of one of the partners is substantially higher than the income of the other partner.

To include the consequences in the future of the decision for a location H in the model, we propose a constraint of the form

$$\left| \int_0^{t_0} contributions_m(t) - contributions_f(t)\, dt + w_f\,(\, c_{tour,m}(H) - c_{tour,f}(H)\,) \right| \leq tol(t_0). \tag{3}$$

Here the future contributions are measured in the term

$$w_f\,(\, c_{tour,m}(H) - c_{tour,f}(H)\,)$$

that has to be scaled appropriately with the factor w_f. For the scaling, the time that the DCC expects to live at location H is important. Let

$$C_P = \int_0^{t_0} contributions_m(t) - contributions_f(t)\, dt$$

denote the difference of the contributions in the past. Using this notation the inequality constraint (3) can be written in the form of the two inequalities

$$C_P - tol(t_0) \quad \leq \quad w_f \, (\, c_{tour,f}(H) - c_{tour,m}(H) \,), \tag{4}$$

$$-C_P - tol(t_0) \quad \leq \quad w_f \, (\, c_{tour,m}(H) - c_{tour,f}(H) \,) \tag{5}$$

where the absolute value function does not appear.

The gender inequality tolerance constraints have a serious disadvantage: If the tolerance *tol* is too small, the problem may become infeasible in the sense that is there does not exist any location that satisfies all constraints. This illustrates the fact that fixed rules of quota type may lead to difficulties in the decision process. Therefore, we prefer the second approach that we propose in the next section.

3.2 Penalization of Gender Inequality

As a second approach, a term that measures the value of gender equality can be included in the objective function that is maximized. In optimization, this method is called a penalization approach since deviations from gender equality lead to a decreasing value of the objective function and thus are penalized. This additional term models the utility of gender equality. For our paradigmatic example, we base the measurement of gender equality upon the difference between the cost of the tours of both partners. One possibility to include the value of gender equality in **DCCLO** is to modify the objective function that is maximized to

$$U_{joint}(H) = \tag{6}$$

$$\lambda_1 u_{loc,f}(H) + \lambda_2 u_{loc,m}(H) - (1 - \lambda_1 - \lambda_2) c_{tour}(H) - w_e \left(c_{tour,m}(H) - c_{tour,f}(H) \right)^2,$$

where $w_e > 0$ is a factor that models the weight that is assigned to gender equality. Definition (6) is a differentiable penalization which is convenient for numerical computations. Alternatively, a non-differentiable penalization

$$U_{joint}(H) = \tag{7}$$

$$\lambda_1 u_{loc,f}(H) + \lambda_2 u_{loc,m}(H) - (1 - \lambda_1 - \lambda_2) c_{tour}(H) - w_e \left| c_{tour,m}(H) - c_{tour,f}(H) \right|$$

can be used, where the absolute value function appears in the joint utility function, which is therefore in general not differentiable.

A different approach based upon multicriteria optimization is to consider the utility of gender equality as a third objective function in the multicriteria approach presented in Section 2.1.

It is of importance to analyze the influence of the tolerance of the partners to an unequal distribution of the contributions to childcare to the overall cost of the couple, that is modelled in the objective function. It is clear that the inclusion of gender equality has its price and will almost all the time lead to a decrease in the joint economic utility of the couple. However gender equality should be seen as an integral part of the overall utility function and therefore its maximization contributes to a successful management of the coupled lives.

4 Numerical Treatment

In this section we discuss the numerical solution of problem **DCCLO**. However, numerical results will be presented in future work.

4.1 Finite sets $\mathcal{H}$

If the set $\mathcal{H}$ of admissible homes is finite, **DCCLO** is a nonlinear combinatorial optimization problem with a finite number of choices. that can be solved using one of the algorithms of nonlinear integer programming, see for example Ref. 7.

To include the gender inequality constraints (4) and (5) in the model, it can be written in the equivalent form

$$\max_{H\in\mathcal{H}} \max_{i\in I,\, j\in J} \max_{l\in\{1,2,\ldots,6\}} \lambda_1 u_{loc,f}(H)+\lambda_2 u_{loc,m}(H)-(1-\lambda_1-\lambda_2)c_l(S_i,K_j,H) \quad (8)$$

subject to

$$\begin{aligned} C_P - tol(t_0) &\leq w_f\,(\,c_{l,f}(S_i,K_j,H) - c_{l,m}(S_i,K_j,H)\,), && (9)\\ -C_P - tol(t_0) &\leq w_f\,(\,c_{l,m}(S_i,K_j,H) - c_{l,f}(S_i,K_j,H)\,). && (10) \end{aligned}$$

Here we use the definitions

$$\begin{aligned} c_{1,f}(H,S_i,K_j) &= d(H,W_f)\\ c_{1,m}(H,S_i,K_j) &= d(H,K_j)+d(K_j,S_i)+d(S_i,W_m)\\ c_{2,f}(H,S_i,K_j) &= d(H,W_f)\\ c_{2,m}(H,S_i,K_j) &= d(H,S_i)+d(S_i,K_j)+d(K_j,W_m)\\ c_{3,f}(H,S_i,K_j) &= d(H,K_j)+d(K_j,W_f)\\ c_{3,m}(H,S_i,K_j) &= d(H,S_i)+d(S_i,W_m)\\ c_{4,f}(H,S_i,K_j) &= d(H,S_i)+d(S_i,W_f)\\ c_{4,m}(H,S_i,K_j) &= d(H,K_j)+d(K_j,W_m)\\ c_{5,f}(H,S_i,K_j) &= d(H,K_j)+d(K_j,S_i)+d(S_i,W_f)\\ c_{5,m}(H,S_i,K_j) &= d(H,W_m)\\ c_{6,f}(H,S_i,K_j) &= d(H,S_i)+d(S_i,K_j)+d(K_j,W_f)\\ c_{6,m}(H,S_i,K_j) &= d(H,W_m). \end{aligned}$$

4.2 Convex sets $\mathcal{H}$

Assume that the set $\mathcal{H}\subset\mathbb{R}^2$ of admissible homes is nonempty, compact and convex. Assume that the functions $u_{loc,f}$ and $u_{loc,m}$ are concave. For a given facility $A=(a_1,a_2)$ and $H=(h_1,h_2)$ let

$$d(H,A)=d(A,H)=c\,(|h_1-a_1|+|h_2-a_2|)$$

that is we consider the Manhattan metric with a constant $c>0$. Similar as in Ref. 6 we use the big-M method (see Ref. 3) to transform problem **DCCLO** in the form of a convex mixed integer optimization problem. Choose the number $M > 0$ sufficiently large. Then we can obtain a solution of **DCCLO** by solving the problem

$$\max \lambda_1 u_{loc,f}(H) + \lambda_2 u_{loc,m}(H) - (1 - \lambda_1 - \lambda_2)\nu$$

subject to $H \in \mathcal{H}$, $\nu \geq 0$, $\omega(i,j,l) \in \{0,1\}$ for all $i \in I$, $j \in J$, $l \in \{1,...,6\}$

$$\sum_{i \in I} \sum_{j \in J} \sum_{l \in \{1,...,6\}} \omega(i,j,l) = 1$$

$$-\gamma_f^1 \leq c(h_1 - (W_f)_1) \leq \gamma_f^1$$
$$-\gamma_f^2 \leq c(h_2 - (W_f)_2) \leq \gamma_f^2$$
$$-\gamma_m^1 \leq c(h_1 - (W_m)_1) \leq \gamma_m^1$$
$$-\gamma_m^2 \leq c(h_2 - (W_m)_2) \leq \gamma_m^2$$

and for all $j \in J$

$$-\sigma_j^1 \leq c(h_1 - (K_j)_1) \leq \sigma_j^1$$
$$-\sigma_j^2 \leq c(h_2 - (K_j)_2) \leq \sigma_j^2$$

and for all $i \in I$

$$-\delta_i^1 \leq c(h_1 - (S_i)_1) \leq \delta_i^1$$
$$-\delta_i^2 \leq c(h_2 - (S_i)_2) \leq \delta_i^2$$

and for all $i \in I$, $j \in J$, $l \in \{1,...,6\}$

$$\nu \geq \gamma_f^1 + \gamma_f^2 + \sigma_j^1 + \sigma_j^2 + d(K_j, S_i) + d(S_i, W_m) - M[1 - \omega(i,j,1)]$$
$$\nu \geq \gamma_f^1 + \gamma_f^2 + \delta_i^1 + \delta_i^2 + d(S_i, K_j) + d(K_j, W_m) - M[1 - \omega(i,j,2)]$$
$$\nu \geq \sigma_j^1 + \sigma_j^2 + \delta_i^1 + \delta_i^2 + d(S_i, W_m) + d(K_j, W_f) - M[1 - \omega(i,j,3)]$$
$$\nu \geq \delta_i^1 + \delta_i^2 + \sigma_j^1 + \sigma_j^2 + d(S_i, W_f) + d(K_j, W_m) - M[1 - \omega(i,j,4)]$$
$$\nu \geq \sigma_j^1 + \sigma_j^2 + \gamma_m^1 + \gamma_m^2 + d(K_j, S_i) + d(S_i, W_f) - M[1 - \omega(i,j,5)]$$
$$\nu \geq \delta_j^1 + \delta_j^2 + \gamma_m^1 + \gamma_m^2 + d(S_i, K_j) + d(K_j, W_f) - M[1 - \omega(i,j,6)].$$

5 Conclusions

We have presented different models for the rational decision making of a DCC that illustrate the complexity of the relevant objectives and values. We have discussed the approaches to maximize a joint economic utility or to consider efficient points in a bicriteria (or multiobjective) optimization problem with the separate utility functions of the partners.

To include the value of gender equality in the model we have discussed two approaches, namely gender equality constraints or to include the value of gender equality in the utility functions. The latter approach may even lead to a mathematical definition of gender mainstreaming. In mathematical term, it is the inclusion of the value of gender equality in the utility function that is maximized. This holds in particular for the operation of public institutions. As an alternative approach hard constraints on the tolerated gender inequality can be introduced.

In order to do this in practice, a measure for the approximation of gender equality has to be introduced. In our paradigmatic example, as a measurement of gender inequality the size of the difference between the contributions of the partners can be used.

This model is a contribution to a gender oriented branch of operations research. It may lead to important insights into the problems of dual-career couples: For example, in some cases the cost of optimal childcare juggling tours may increase if both parents work at the same place.

Moreover, it is interesting to analyze how the structure of the childcare facilities influences the childcare cost of the dual career couple. This is a strong argument for integrated childcare facilities at the workplaces, which makes the workplaces much more attractive for qualified parents.

The optimization problems that appear naturally in these models have an interesting structure that deserves mathematical analysis and requires the construction of numerical methods that are adapted to the specific problem structure. This will be the subject of future research.

References:

1. Abele, A.E., Vollmer, J. (in press). *Dual-Career Couples: Specific Challenges for Work-Life Integration.* Preprint, Social Psychology Unit, University of Erlangen-Nuremberg, Germany. To appear in Creating balance?! International Perspectives on the Work-Life Integration of Professionals.

2. Abele, A.E. (in press). *Doppelkarrierepaare - Entstehung und Relevanz der Thematik.* In J. Funk, E. Gramespacher and I. Rothusler (Hrsg.), Dual career couples in Theorie und Praxis, Opladen: Verlag B. Budrich.

3. Chang, T.-T., Chang C.-C. *A linearization method for mixed 0-1 polynomial programs.* Comput. Oper. Res. 27: 1005-1016, 2000.

4. Ehrgott, M. *Multicriteria Optimization* Springer, Heidelberg 2008.

5. Gugat, M. and Eichfelder, G. *Optimierung mit mehreren konkurrierenden Zielen,* erscheint in WISU: Das Wirtschaftsstudium.

6. Gugat, M. and Pfeiffer, B. *Weber problems with mixed distances and regional demand,* Mathematical Methods of Operations Research 66, 419-449, 2007. DOI: 10.1007/s00186-007-0165-x

7. Hemmecke, R., Köppe, M., Lee, J. and Weismantel, R. *Nonlinear Integer Programming* to appear in: M. Jünger, T. Liebling, D. Naddef, G. Nemhauser, W. Pulleyblank, G. Reinelt, G. Rinaldi, and L. Wolsey (eds.), 50 Years of Integer Programming 1958–2008: The Early Years and State-of-the-Art Surveys, Springer-Verlag, 2009, ISBN 3540682740

8. Jahn, J. *Vector Optimization: Theory, applications and extensions,* Springer, Heidelberg 2004.

9. Klamroth, K. and Miettinen, K. *Integrating Approximation and Interactive Decision Making in Multicriteria Optimization,* Operations Research 56, 224-234, 2008.

10. Pfeiffer, B. and Klamroth, K. *A Unified Model for Weber Problems with Continuous and Network Distances,* Computers and Operations Research 35, 312-326, 2008.

Authors' Adresses:

- Andrea Abele-Brehm

  Lehrstuhl Sozialpsychologie

  Universität Erlangen-Nürnberg

  Bismarckstr. 6

  91054 Erlangen, Germany

- Kathrin Klamroth

  Department of Mathematics and Natural Sciences

  Optimization and Approximation

  University of Wuppertal

  Gaußstr. 20

  42097 Wuppertal, Germany

- Martin Gugat
  Friedrich-Alexander-Universität Erlangen-Nürnberg
  Lehrstuhl II für Angewandte Mathematik
  Institut für Angewandte Mathematik
  Martensstr. 3
  91058 Erlangen, Germany