A MATHEMATICAL PROGRAMMING MODEL FOR THE EVALUATION AND DESIGN OF AIRPORT CONFIGURATIONS.

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ABSTRACT

A mathematical programming model for assessing the design of an optimal airport topology is presented herein. It takes into account the efficient and safe taxiing of aircraft on the ground. We balance a set of conflicting factors that depend directly on aircraft trajectories on the ground, such as the number of arriving and departing flights within a planning period, delays and the avoidance of risk situations. The results, which are shown as a set of configurations for a set of test airports, illustrate design decisions relating to the allocation of waiting points and their capacities, and the number of terminals.

Keywords: Airport design; Taxi Planning; Airport management; Binary capacitated multicommodity flow network.

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1 Introduction

This paper describes an airport design model in which taxi planning plays a central role. This introduction describes the integration of the taxiing process and its planning in the context of managing airport operations after landing and before take-off as experienced by the authors. Then a literature review of the main contributions related to the taxi planning models is presented and finally, the contribution of this paper is highlighted.

1.1 Current role of the taxi planning in the airport operations

The taxiing time taken by aircraft operations on the ground after landing and previous to take-off is a major factor in aircraft delays. Airport congestion is caused by the ground infrastructure being unable to meet the need for flight movement. Major hubs suffer aircraft delay on the ground, and this is sometimes aggravated by poor visibility and other contingencies. Airport delays are especially significant in the case of domestic flights because of their impact on the total trip time. Aircraft taxiing is critical for security reasons, particularly in departure and arrival management operations.

Taxi planning optimization is being developed as part of the European Commission's Leonardo project (2004). Taxi planning (TP) optimization is an improvement on current airport planning in which pre-calculated tables are usually used to fix taxiing times, because these tables do not take into account the location and congestion of airport taxiways and facilities. Taxi planning optimization is used by the surface manager (SMAN) module as a mathematical model that allows the movement of aircraft on the ground to be optimized. One of the functionalities of this module is that it can assign the optimum taxiing route to each aircraft to maximize the use of airport capacity. It does so by determining the most efficient operational configuration on the basis of the current and predicted traffic situation. In addition, it calculates the taxiing time for each aircraft. It has been integrated as an experimental tool in the implementation of the Leonardo project at Madrid Barajas airport.

The SMAN module is divided into the following tasks: gate management (GMAN), which assigns stands to arriving aircraft; departure management (DMAN), which gives pushback orders to departing aircraft at the stands and establishes the desired (or required) take-off time windows; and arrival management (AMAN), which plans the arrival sequence. In this context, TP is not intended as a standalone tool. Rather, it must be coordinated with DMAN tools (for departing traffic), with AMAN and GMAN tools (for arriving traffic) and obviously with the airport's actual taxi planner, known as the aircraft ground controller (AGC). At present, each activity is modelled separately and then coordinated with all of the other activities.

Updated information, such as a delay in the boarding process, arises frequently during an airport's daily operations, especially in the case of outbound traffic. Thus, TP must be flexible enough to accommodate changing inputs while being consistent regarding routes and schedules that have already been delivered in past executions. In this dynamic context, the AGC's requirement is that the TP execution time must not exceed a few minutes.

1.2 Literature review

Authors who have researched taxiing operations include Anderson et al. (2002) and Anagnostakis (2001). In Anderson (2002) queuing models are developed for the evaluation of taxiing operation times and in Anagnostakis (2001) the planning of runway operations is formulated. Traditionally, the evaluation of routing decisions for aircraft on the ground, that is, taxi planning (TP), is carried out using numerical simulation. The limited literature that exists on optimization models does not explicitly refer to the methods used to solve the taxi planning approaches considered. Idris et al. (1998) investigated the main factors that create problems in departure operations at major airports. They conclude that the runway system is the main bottleneck and source of delay. They also proposed strategies for improving departure operations and for determining control points in the airport at which the departure sequence on the runway can be improved.

Pujet et al. (1999) developed a queuing model for departure operations at congested airports. The model was validated and calibrated using traffic data for available runway configurations. The taxiing model of Gotteland et al. (2000) is based on the characterization of conflicts using pattern recognition and is solved using genetic algorithms. Idris et al. (2002) estimated the taxiing time in terms of factors such as runway/terminal configurations, downstream restrictions and take-off queues. Stoica (2004) proposed an adaptive approach to the management of available aircraft routes at airports.

Under a different modelling framework, Smeltink et al. (2005) describe an explicit optimization model based on linear integer programming for taxiing optimization at Amsterdam Schiphol airport. A similar approach is followed by Balakrisnan and Jung (2007) for the Dallas-Fort Worth airport with the aim of optimizing taxiing operations using rerouting and controlled pushback. They use current data and a projected data set with twice the traffic density. To optimize routing and scheduling, Viser and Roling (2003) develop a model for coordinating surface traffic movements. They try to obtain robust solutions in the presence of uncertain data, but they do not describe the formulation used in their model. Rathinam and Jung (2007) model the aircraft's surface movements, minimizing total taxiing time or maximizing throughput while satisfying safety and operational constraints, although the formulation of the model is not included.

Another approach is the optimization model developed in Marín and Salmerón (2003) and Marín (2006a). This model is focused on the routing and scheduling problem of aircraft on the ground for a fixed airport topology and is the backbone of the network design model presented here.

It must be noted that these taxi planning models rely on the assumption that the optimization of routing on the ground is a task that will in the future be performed using centralized guidance and control systems to obtain major improvements in delay savings and safety.

1.3 Contribution

Because of the relevance of taxiing operations on the ground, for the purposes of this paper taxi planning has been considered to be the most important factor in the design of airport configurations. Safety and performance are also taken into account.

The main contribution of this paper is that it presents a consistent formulation based on linear integer programming for the design of airport configurations. The formulation takes the taxi planning model of Marín and Salmerón (2003) and Marín (2006a) as its keystone and is an extension of the model presented in Marín and Codina (2008). Although aspects regarding taxi planning have been taken into account in the design of airports, the evaluation of factors relating to the routing and scheduling of aircraft has been carried out using simulation tools (ATAC Corporation (2008)). It must be noted that there are currently no equivalent models based on mathematical programming techniques that can guarantee the optimality of the design solutions.

1.4 Outline

This paper is organized as follows. Section 2 introduces the airport network model. Section 3 gives an overview of the taxi planning model's structure. In Section 4 a set of airport performance indexes are formulated. The airport design model is defined in Section 5. Section 6 discusses the computational experience and several examples illustrating the effects of the critical parameters of the model and its response. Section 7 contains the conclusions. Although notation is inrtroduced in the paper when it is required, for the sake of legibility all variables and sets are given in the Appendix.

2 Airport design

The model presented in this paper may be considered a tool for assessing the design decisions that must be taken concerning the routing and scheduling of aircraft. The airport design model is an extension of the one by Marín and Codina (2006). Design decisions are taken once a basic topological structure of the airport has been defined by major aeronautical aspects that take into account the airport surface area and its aerospace constraints. The core of the model reproduces the trajectories of aircraft on the ground under realistic assumptions and in accordance with the technical limitations inherent to the mandatory separation time between aircraft in take-off and landing operations in mixed and separate runways. Emphasis is placed on limiting the risks that may be created by given routing solutions for aircraft on the ground.

The evaluation of configurations and the resulting decisions given by the model are based on a set of indexes or objectives that are relevant to the performance of the airport. The resulting configuration is chosen by optimizing an objective function that is the weighted sum of these indexes. The indexes comprise the following: *a*) the characteristics of the on-the-ground aircraft times (routing, delays, worst routing times), *b*) the input/output airport's throughput, *c*) the avoidance of risk situations at a set of predetermined locations and *d*) the economic or location costs associated with the alternatives being modelled.

The approach for evaluating airport configurations presented herein considers the design decisions at an upper level and aircraft routing and scheduling at a lower level. At the upper level a set of alternatives for the airport's configuration are evaluated and compared. These alternatives are considered by means of a set of design decision variables (binary or integer) in relation to the capacity of the topology of the airport considered. At the lower level, the core of the model reproduces the on-the-ground evolution of aircraft by means of a multicommodity network flow problem with a set of additional constraints that ensures that the routes followed by aircraft are realistic and obey the capacity limitations imposed by blocking times at runways, a limited surface area and the safety of taxiing operations. It must be noted that although direct collision is always avoided by the constraints of the model, additional conditions that enforce safety are included.

Although the aforementioned assumptions describe the TP model, others may be considered, such as those in Marín and Salmerón (2003) that take into account time windows. Other assumptions have been considered, such as in the case in which a predefined departure time is assumed and the optimal time at which a flight can leave its gate is optimized. These possibilities are studied in view of the relation established between TP and the other modules in the SMAN.

3 Taxi Planning Model

This section describes the multicommodity network flow structure on which the TP model relies. It will be shown that its structure is that of a time-space network based on a network model of the airport configuration and on a planning period.

3.1 Space Time Network Representation

The taxi planning formulation is based on considering decisions during certain periods of time, t, within the planning time set $T = \{1, 2, \ldots, |T|\}$. All time periods are assumed to be of equal duration. It is assumed that the airport's model is expressed as a directed graph $G = (N,E)$.

For the sake of convenience, the set of nodes N can be divided into the following subsets:

N^W - A subset of nodes at which one or more aircraft can remain waiting. The model assumes that the maximum number of aircraft that can remain at a given node at a given instant is related to the dimensions of the aircraft and the area of the node.

N^O - A subset of ordinary nodes at which aircraft cannot remain waiting.

N^{ER} - A subset of nodes at which aircraft enter the airport after leaving the landing runways.

N^{AR} - A subset of nodes with access to runways that enables take-off operations to be initiated immediately.

N^F - A subset of artificial nodes that are used to model queues in corridors. At them, overtaking is prohibited and only one aircraft can remain waiting.

Each feasible edge has an associated time $t_{i,j}$ corresponding to the delay time or the average time taken by aircraft to move from "origin" node i to "destination" node j . This time is assumed to be the same for all aircraft and constant along the PP; it is given by airport regulations. In the model, $t_{i,j}$ is taken to be an integer number of time subperiods.

The TP model has a multicommodity flow structure with a set of additional constraints; a commodity is every one of the flights $w \in W$. Every flight $w \in W$ in the set of flights W covered during the PP is defined by an origin node $o(w)$, a destination node $d(w)$, and a starting time $t(w)$. In the expanded space-time network the origin is a single node, but the destination consists of a set of nodes associated with the destination $d(w) = i$ for the different time subperiods $t \in T$ if the aircraft arrives at its destination during the PP or a sink node at the end of the PP. This is equivalent to saying that, for landing flights, the time instant at which entering aircraft leave the landing runway is known, whereas for departing flights it is the time instant for pushback that is known.

3.2 Main flow variables

The binary variables that define the network structure of the model are associated with each of the flights $w \in W$. They are defined as follows:

$E^w_{i,t} = 1$, if aircraft w waits at node i at period t and 0 otherwise.

$X^w_{i,j,t} = 1$, if the aircraft w leaves node i at time t in order to go towards node j following link $(i, j) \in E$ and 0 otherwise.

$S^w_{d(w),t} = 1$, if aircraft w reaches its destination $d(w)$ at time t within the PP and 0 otherwise.

3.3 Flow conservation constraints

If $N_S = \{\ i \in N |\ i = o(w),\ w \in W\}$ is the set of origin nodes, then the flow conservation constraints at non-origin nodes are

$$(1)\quad E^w_{i,t} + \sum_{j\in R(i)} X^w_{j,i,t+1-t_{j,i}} = E^w_{i,t+1} + \sum_{j\in F(i)} X^w_{i,j,t+1},\ \forall t < |T|,\ \forall w \in W,\ \forall i \in N - N_S$$

$F(i)$ being the set of emerging nodes from node i and $R(i)$ the set of incoming nodes at node i. These are usually known as the forward and reverse of node i, respectively. The following figure illustrates the aforementioned relationship (1).

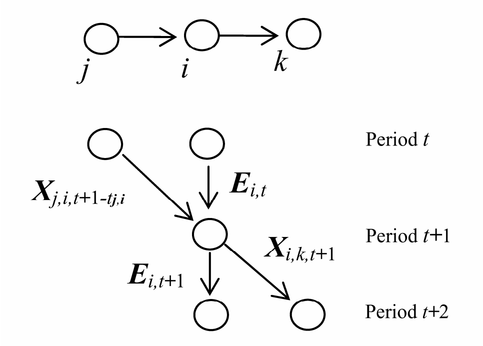

Figure 1: Node-arc representation of relationship (1) with $t_{j,i} = 1$.

Aircraft $w \in W$ starts its time-space trajectory at node $o(w)$ and time $t(w)$, so it may wait or move at $(o(w),t(w))$. This is reflected in relationship (2) by means of the corresponding variables $E^w_{o(w),t(w)}$ and $X^w_{o(w),j,t(w)}$. (If aircraft $w \in W$ waits at node $o(w)$ at time $t(w)$, $E^w_{o(w),t(w)}$ should be 1 and 0 if it does not; if the aircraft moves from node $o(w)$ towards node j at time $t(w)$, $X^w_{o(w),j,t(w)}$ should be 1 and 0 otherwise.)

$$(2)\qquad E^w_{o(w),t(w)} + \sum_{j\in F(o(w))} X^w_{o(w),j,t(w)} = 1,\ \forall w \in W$$

If an aircraft cannot reach its destination during the PP, then it must be located at a given node at the end of the PP. If its destination is reached during the PP, the aircraft disappears from the network. A sink node captures both possibilities.

$$(3)\qquad \sum_{i\in N} E^w_{i,|T|} + \sum_{t<|T|} S^w_{d(w),t} = 1,\ \forall w \in W$$

3.4 Capacity constraints

The model also takes into account a set of constraints that reflect physical limitations related to space, conflicts and the prohibition to stop at given places. These constraints are formulated below.

Capacity constraints at wait nodes $i \in N^W$:

$$(4)\qquad \sum_{w\in W} e_w\, E^w_{i,t} \le q_i,\ \forall t < |T|,\ \forall i \in N^W$$

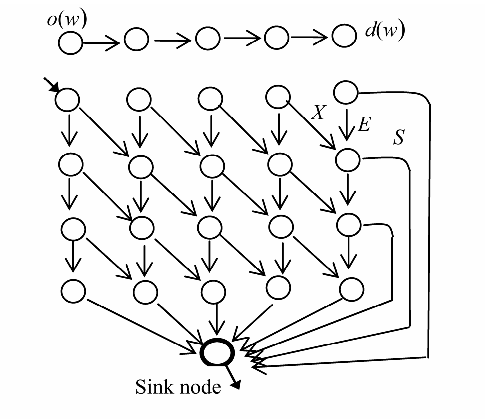

Figure 2: A simple time-expanded network showing inflows and outflows for aircraft $w \in W$.

where q_i is the capacity (in surface units) of node i, and e_w is the space required by aircraft $w \in W$.

An aircraft cannot remain waiting at ordinary nodes $i \in N^O$ and exit runway nodes $i \in N^{ER}$

$$E^w_{i,t} = 0, \; \forall t < |T|, \; \forall i \in N^O \cup N^{ER} \tag{5}$$

At the access to runway nodes $i \in N^{AR}$ and artificial nodes $i \in N^F$ only one aircraft is allowed at a time:

$$\sum_{w \in W} E^w_{i,t} \leq 1, \; \forall t < |T|, \; \forall i \in N^{AR} \cup N^F \tag{6}$$

Constraint (7) below avoids the direct overlap of several aircraft at certain nodes. It states that the total number of aircraft arriving at these nodes during each time subperiod is limited to 1 if they are unoccupied and 0 if they are occupied.

$$\sum_{w \in W} E^w_{i,t} + \sum_{w \in W} \sum_{j \in R(i)} X^w_{j,i,t+1-t_{j,i}} - 1 \leq 0, \quad \forall t < |T|, \; \forall i \in N^O \cup N^{AR} \cup N^F \tag{7}$$

Other constraints are defined to characterize other incompatibilities, such as the simultaneous use of taxiways for more than one aircraft and the use of bidirectional links as stated in Marín (2006a).

3.5 Flow constraints on runways

A realistic formulation of the problem must also take into account limitations due to operation times on the runways, which are set for safety purposes. The formulation presented here deals with runways used for *a*) take-off only, *b*) landing only and *c*) mixed runways. For the sake of simplicity, it is assumed that no corridors overlap at access/exit points to runways and also that all of the aircraft are of the same type.

Now let P be the set of runways, W^A the set of aircraft entering the airport with a given destination on the ground and W^D the set of departing aircraft. When an aircraft $w \in W^D$ gains access to a runway $p \in P$ at time $t \in T$ for take-off purposes, that runway must be reserved for that operation during a given blocking time $\delta_D(w)$ that is dependent on the aircraft's characteristics and no other aircraft must start to take off on that runway during the period $[t,\ t+\delta_D(w)]$. If $N^{AR}(p)$ is the set of access nodes to runway p, let OAN_j^t be defined as the occupation of access node $j \in N^{AR}(p)$ for each runway p:

$$(8) \qquad OAN_j^t = \sum_{w \in W^D} \sum_{i \in R(j)} \left\{ \sum_{\substack{t' = t \\ (t_{ij} < t' \leq |T|)}}^{t+\delta_D(w)} X^w_{i,j,t'-t_{ij}} \right\},\ \ j \in N^{AR}(p),\ t < |T|$$

When an aircraft enters at the airport's network at time $t \in T$, leaving runway $p \in P$, there are two periods of time in which that runway is blocked: a) a period of time *after* time t of length $\delta_L^1(w)$ and b) a period of time *before* time t of length $\delta_L^2(w)$. The first blocking time prevents other aircraft from encountering potential turbulence left by aircraft $w \in W$ that could affect their take-off operations. The second blocking time is the time required by aircraft w to land on the runway.

Let now $N^{ER}(p)$ be the set of nodes that exit from runway $p \in P$ and, as before, let $N^{AR}(p)$ be the set of nodes with access to runway p. The number of concurrent aircraft to a node $i \in N^{ER}(p)$ is given by the variable occupation of exit runway OER_i^t, defined as for each runway p as

$$(9) \qquad OER_i^t = \sum_{w \in W^A} \sum_{j \in R(i)} \left\{ \sum_{\substack{t' = t - \delta_L^1(w) \\ (t_{ij} < t' \leq |T|)}}^{t+\delta_L^2(w)} X^w_{i,j,t'} \right\},\ \ i \in N^{ER}(p),\ t < |T|$$

Then, the limitation of concurrence of aircraft on take-off/landing runways can be expressed as

$$(10) \qquad \sum_{i \in N^{ER}(p)} OER_i^t + \sum_{j \in N^{AR}(p)} OAR_j^t \leq 1,\ \ t < |T|,\ p \in P$$

Note that, for take-off only runways $N^{ER}(p) = \emptyset$ and the first summation in (10) vanishes; the same applies to the second summation at landing-only runways. It must also be noted that previous relationship (9) could be adapted to take into account the dependence of blocking times δ_L^1 and δ_L^2 on the aircraft type.

3.6 Modelling of risk situations

Design decisions regarding aspects of airport topology must be taken so that the risk of operations can be minimized. In this model a risk situation is considered to arise when either a) two or more aircraft approach a point and at least one of them is going to cross it and the crossing trajectories either coincide or are separated by a short period of time, b) an aircraft enters a zone that contained another aircraft a few moments ago or that still contains it or c) an aircraft stops at a certain point where stopping is not strictly forbidden, but because of that stop a risk situation may arise under certain circumstances (for instance, in the presence of reduced visibility because of bad weather). At this point it must be remarked that this approach for the modelling of risk situations must be understood in the context of a centralized guidance system for taxiing. Under these conditions, the centralized guidance system must update routes if any disruption of the planned ones occurs. Within this context, permitting aircraft to halt at some points may be a risk if planned routes are not followed. For these purposes an index will be defined which provides the number of times that risk situations may happen within the PP, as defined earlier.

Let K be the set of predetermined areas in the airport where risk situations need to be explicitly minimized. From now on, these areas will be referred to as conflict areas. Let $K_A \in \mathrm{K}_A$ be the set of incoming arcs to a conflict area and let $K_N \in \mathrm{K}_N$ be the set of nodes inside the conflict area. In order to detect risk situations in an area $K \in \mathrm{K}$, a control time-space box of time length ν in the time-space network is defined for each time $t \in \{2, ..., |T|\}$, so that previous situations of type a), b) and c) can be detected. A variable $C_{K,t}$ can be defined by relationship (11) for each control time-space box at time t for a conflict area K.

$$C_{K,t} = \sum_{\ell=\hat{\ell}(t)}^{t} \left(\sum_{\substack{(i,j)\in K_A \\ \ell - t_{i,j} \geq 1}} x_{i,j,\ell-t_{i,j}} + \sum_{i \in K_N} e_{i,\ell-1} \right) - 1\,,\; K \in \mathrm{K},\; 2 \leq t \leq |T| \tag{11}$$

where $\hat{\ell}(t) = max\{t-\nu, 2\}$, $x_{i,j,t} = \sum_{w \in W} X^w_{i,j,t}$ is the total outgoing traffic flow for link $(i,j) \in A$ at time t and $e_{i,t} = \sum_{w \in W} E^w_{i,t}$ is the total number of aircraft waiting at $i \in N$ at time t. Variable $C_{K,t}$ can be used for any of the purposes a) b) or c) described above. Figure 3 below illustrates a simple example of a control time-space box, as well as the links and nodes intervening in relationship (11).

An interpretation of variable $C_{K,t}$ might be the following:

$$C_{K,t} = \sum_{\ell=\hat{\ell}(t)}^{t} \left(\sum_{\substack{(i,j)\in K_A \\ \ell - t_{i,j} \geq 1}} x_{i,j,\ell-t_{i,j}} \right) + \sum_{i \in K_N} e_{i,\hat{\ell}(t)-1} - 1 + \left(\sum_{\ell=\hat{\ell}(t)+1}^{t} e_{i,\ell-1} \right) = \tag{12}$$

$$= \left\{ \begin{array}{c} \texttt{no. of aircraft} \\ \texttt{whose trajectories} \\ \texttt{will conflict at } K \\ \texttt{or will conflict with} \\ \texttt{aircraft inside } K \end{array} \right\} + \left\{ \begin{array}{c} \texttt{If there are no} \\ \texttt{entering aircraft at } K \\ \texttt{then, total stopping time} \\ \texttt{for aircraft} \\ \texttt{inside } K \end{array} \right\}$$

Proposition 3.1 *Assume that a) there are two or more aircraft whose trajectories will enter a conflict area $K \in \mathrm{K}$ and cross each other by a time lag that is less than or equal to ν time intervals and/or b) one or more aircraft enter a conflict area that less than or equal to ν time subintervals ago contained other aircraft and/or c) the total stopping time of aircraft at $K \in \mathrm{K}$ is greater than 1 time subinterval.*

Then, there are at least some $\tilde{t} \in \{2, ..., |T|\}$ so that $C_{K,\tilde{t}} > 0$.

Proof: Situation a) occurs when for at least two aircraft $w, w' \in W$, $X^w_{j,i,t-t_{j,i}+1} = 1$ and $X^{w'}_{n,i,t'-t_{n,i}+1} = 1$, with $(j,i),(n,i) \in K_A$ and $|t'-t| \leq \nu$. In this case, $C_{K,\tau} > 0$ for any $\tau \in [max\{t,t'\}, min\{t,t'\}+\nu+1] \cap \{2, ..., |T|\}$. Also, in situation b), if for $t \geq t'$ and $t - t' \leq \nu$ and for two aircraft $w, w' \in W$, $X^w_{j,i,t-t_{j,i}+1} = 1$ and $E^{w'}_{i',t'} = 1$ at an entering link $(j,i) \in K_A$ and a point $i' \in K_N$, then $C_{K,t'+\nu+1} > 0$. Finally, because of the definition of variables $C_{K,t}$ in (11) and their decomposition in (12), in c) also $C_{K,t} > 0$ for some $\{2, ..., |T|\}$. $\square$

Let $\gamma_{K,t}$ be a binary variable that takes a value of 1 when $C_{K,t} > 0$. In other words, $\gamma_{K,t}$ is an indicator variable if a risk situation occurs at a control time-space box at time t for a conflict area $K \in \mathrm{K}$.

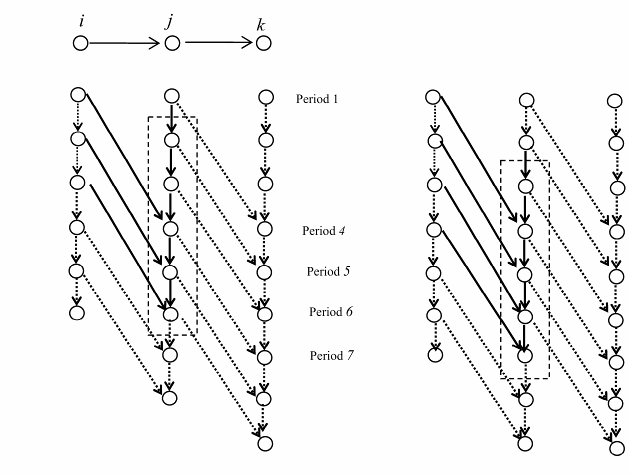

Figure 3: Control time-space boxes at $t = 6$ and $t = 7$ for a time lag of $\nu = 4$ for a conflict area defined by $K_N = \{j\}$. Link $(i, j) \in K_A$ and $t_{i,j} = 3$.

As no more than one aircraft can cross a link or stay at a node at a time, it must be noticed that, because of (11), $C_{K,t}$ must be less than or equal to $\nu(\,|\,K_A\,|+|\,K_N\,|\,)$. Using this fact the following constraint is equivalent to the implication that $C_{K,t} > 0 \;\Rightarrow\; \gamma_{K,t} = 1$:

(13) $$C_{K,t} \leq \nu(\,|\,K_A(K)|+|\,K_N(K)|\,)\,\gamma_{K,t},\; \gamma_{K,t} \in \{0,1\},\;\; K \in \mathrm{K},\;\; 2 \leq t \leq\; |T|$$

The total number I_C of control time-space boxes affected by a risk situation can then be defined as

(14) $$I_C = \sum_{K \in \mathrm{K}} \sum_{t=2}^{|T|} \gamma_{K,t}$$

and can be considered to be an index measuring the degree of risk of the trajectories. Then, an objective will be to keep this index I_C as low as possible during the PP.

Another interpretation of a risk situation in a control time-space box is that it would require surveillance or even preventive action by on-the-ground controllers. If n_c is the number of controllers available for control tasks in the planning period, then at each $t \in T$ no more than n_c risk situations might occur and the following constraint would be required in the model:

(15) $$\sum_{K \in \mathrm{K}} \gamma_{K,t} \leq n_c,\;\; 2 \leq t \leq |T|$$

4 Modelling airport performance

The TP model is defined by the previous constraints and the following (conflicting) objectives are to be optimized under low to medium congestion situations during the PP:

1. Index of risk situations

2. Delay of outgoing traffic

3. Delay of incoming traffic
4. Routing time until take-off or arrival at a final space.
5. Worst travel time
6. Number of arrivals within the PP
7. Number of departures within the PP

In situations of high congestion an objective that takes priority is the total number of arrivals at the final destination on the ground (parking or terminal) plus the total number of take-offs. We will now present the formulation of the objectives mentioned above.

4.1 Routing times

Let the travel time z^w for aircraft $w \in W$ be defined in terms of the flow variables X^w, E^w as:

$$z^w(X^w, E^w) = \sum_{t=1}^{|T|-1} \left(\sum_{(i,j)\in E} t_{i,j} \; X^w_{i,j,t} + \sum_{i\in N^W} E^w_{i,t} \right) + \sum_{i\in N^W} r_i^{d(w)} \; E^w_{i,n}, \tag{16}$$

where $r_i^{d(w)}$ is the estimated travel time from node $i \in N^W$ to destination $d(w)$ in the next PP. The estimation method depends on the purpose for which the taxi planning model is used. Using a shortest path algorithm, $r_i^{d(w)}$ is evaluated and updated for each planning period. In the context of this paper we suggest that $r_i^{d(w)}$ be estimated using historical data.

If λ^w is a coefficient for the priority of aircraft $w \in W$ then the weighted total travel time z can be expressed as

$$z = \sum_{w\in W} \lambda^w z^w. \tag{17}$$

4.2 Delays and worst travel times

In addition to the weighted total travel times z given by (17) it is advisable to also take into account the worst routing time $\hat{z}$ for all the flights considered in the PP. The worst routing time is given by

$$\hat{z} = Max_{w\in W} \{z^w\} \tag{18}$$

The worst travel time can be considered into the model by means of the following constraints:

$$z^w \leq \hat{z}, \quad w \in W \tag{19}$$

with z^w as defined in (16).

Delays D_{IN} and D_{OUT} of incoming and outgoing traffic are given by

$$\begin{aligned} D_{IN} &= \sum_{w\in W^A} \sum_{t<|T|} \sum_{i\in N^W} E^w_{i,t}, \\ D_{OUT} &= \sum_{w\in W^D} \sum_{t<|T|} \sum_{i\in N^W} E^w_{i,t} \end{aligned} \tag{20}$$

Travel times for aircraft, either z or the delays D_{IN} and D_{OUT}, may be in conflict with I_C, the total number of concurrences in the case of congestion, because reducing the number of times that space-time trajectories of two or more aircraft are close to each other must generally result in delays for waiting or adopting routes with longer travel times.

4.3 Number of arrivals and take-offs

The model also takes into account the total throughput $\mathcal{T} = \mathcal{T}^- + \mathcal{T}^+$ comprised by the total number of take-offs $\mathcal{T}^-$ and arrivals at destinations on the ground $\mathcal{T}^+$, within the PP. The number of arrivals on ground $\mathcal{T}^+$ and the number of take-offs $\mathcal{T}^-$ for each runway p are given by

$$\begin{aligned} \mathcal{T}^+ &= \sum_{t<|T|} \sum_{w \in W^A} \sum_{i \in R(j)} \sum_{j \in N^P} X^w_{i,j,t} \ , \\ \mathcal{T}^- &= \sum_{t<|T|} \sum_{w \in W^D} \sum_{j \in F(i)} \sum_{j \in N^{AR}} X^w_{i,j,t} \end{aligned} \tag{21}$$

where $N^P = \{ \ q \in N \mid \exists w \in W : \ d(w) = q \ \}$ is the set of nodes which are the end destination of aircraft on the ground.

As for the aircraft travel times and delays discussed in previous subsections, the number of arrivals and take-offs may be in conflict with I_C, as the effect of minimizing I_C may lead to a reduction in the airport's capacity.

5 Design decision constraints

The design possibilities considered in this paper comprise decisions of two classes: a) the possibility of waiting at a node (with capacity for one or more aircraft) and b) the possibility to open/close simple facilities such as nodes or links. All these decisions are associated with binary design variables that appear in the constraints discussed in this section. Clearly, combined decisions regarding the inclusion of a subset of these facilities can also be considered.

The possibility that a corridor can be used for queuing is also considered. In this case, the corridor will be modelled with a sequence of nodes at which an aircraft can wait. To this end, let $N^F_d \subseteq N^F$ be a subset of the fictitious nodes used for modelling the corridor as one of the design alternatives. The previous set of nodes includes the fixed sets and the design sets. A subindex "d" will be added to the subsets of design nodes. Constraints (22) must be added to the model.

$$\sum_{w \in W} E^w_{it} \leq y_c \ , \ \ i \in N^F_d, \ t < |T| \tag{22}$$

Clearly, if $y_c = 1$, waiting at nodes $i \in N^F_d$ is enabled and disabled if $y_c = 0$.

Design decisions would also need to be made with regard to the possibility of considering one or several nodes as the exit from runways or access to runways. To this end, let N^{ER}_d and N^{AR}_d be subsets of design nodes of the sets N^{ER} and N^{AR}, respectively. Then, the following constraints:

$$\sum_{w \in W} \sum_{j \in F(i)} X^w_{i,j,t} \leq y_i \ , \ \ i \in N^{ER}_d \cup N^{AR}_d, \ t < |T| \tag{23}$$

state that a node $i \in N_d^{ER} \cup N_d^{AR}$ can be the exit from/access to the runway, or must otherwise be eliminated.

The possibility of concurrence of at most b_i aircraft at a node $i \in N_d^O \cup N_d^{AR}$ can be also avoided by adding the following constraints:

$$\sum_{w \in W} E_{it}^w + \sum_{w \in W} \sum_{j \in R(i)} X_{j,i,t-t_{j,i}+1}^w \leq y_i \ b_i \ \ ,t < |T|, \ i \in N_d^O \cup N_d^{AR} \tag{24}$$

It must be noted that the decision to discard a node $i \in N_d^O \cup N_d^{AR}$ also involves discarding all its incoming arcs.

For a given subset N_d^W of wait nodes in the network topology, the possibility of allocating sufficient capacity for several aircraft to remain waiting at them and simultaneously be crossed by aircraft may be considered. Nodes $i \in N_d^W$ can be either wait nodes on corridors or wait nodes located in terminals and shall be referred to as wait nodes with capacity q_i.

$$\sum_{w \in W} e_w \, E_{i,t}^w \leq q_i \, y_i, \ i \in N_d^W, \ t < |T| \tag{25}$$

Optionally, for nodes $i \in N_d^W$ the following constraints can be defined in order to eliminate them from the network if they are not going to be capacitated waiting nodes

$$\sum_{w \in W} \sum_{j \in F(i)} X_{i,j,t}^w \leq y_i, \ \ i \in N_d^W, \ t < |T| \tag{26}$$

Several links in the network topology can be considered for inclusion (or not) as capacitated links in the design by means of the following constraints:

$$\sum_{w \in W} X_{i,j,t}^w \leq q_{ij} \ \ y_{ij}, \ \ \forall (i,j) \in E_d, \ \ t < |T| \tag{27}$$

where q_{ij} is the link capacity and $E_d \subseteq E$ is the subset of such links.

Other design constraints may simultaneously involve several design variables such as the limitation in the available extension Q_{ext} in a given zone of the airport for capacitated nodes if they are close to each other. If the subset of possible wait nodes in one of these zones is denoted by $N_{ext} \subseteq N_d^W$, then

$$\sum_{i \in N_{ext}} q_i \, y_i \leq Q_{ext} \tag{28}$$

5.1 Formulation of the network design model

We will formulate the taxi planning network design (TPND) model in the form of a network design problem, in which the objective function will be a weighted sum of conflicting objectives:

$$\begin{aligned} \phi = \alpha_z \, z + \alpha_L \ g(y) + \alpha_{IC} \, I_c + \alpha_{OUT} \, D_{OUT} + \\ + \alpha_{IN} \, D_{IN} + \alpha_{\hat{z}} \, \hat{z} - \alpha_{\mathcal{T}^+} \, \mathcal{T}^+ - \alpha_{\mathcal{T}^-} \, \mathcal{T}^- \end{aligned} \tag{29}$$

where the weights α are non-negative and add up to one and $g(y)$ is a linear function of location costs associated with design decision variables y.

The model can be stated as a linear integer programming problem consisting of the minimization of objective function (29) and subject to the following constraints: a) (1) to (3) in order to impose that trajectories must be in the airport's time-expanded network, *b*) (4) to (7) in order to impose limitations on capacity and to avoid aircraft collision, *c*) (8) to (10) in order to take into account blocking times on runways, *d)* (11) and (13) in order to take into account risk situations during the PP and, optionally (15) to limit the total amount of them and finally *e*) accordingly to the characteristics of the design problem under consideration, a set of relationships (22) to (28) (or a subset of them), which are relationships amongst design decision variables and flow variables.

6 Computational experience

The TPND model was implemented using the GAMS modelling language and solved using the branch-and-bound algorithm of the CPLEX system. Computational experiments were run in order to solve the model for simplified airport networks, taken from actual data supplied by Aeropuertos Españoles y Navegación Aérea, the Spanish airport management corporation, in order to better illustrate the key aspects of the model.

TPND is a large mathematical model that includes a TP model as a submodel for each design alternative. The TP model for Madrid Barajas airport, for example, for a scenario with 30 flights and a planning period of 30 minutes (divided into sixty 30-second periods), the TP network submodel described in the previous section leads to a model with over 600,000 binary decision variables and over a million constraints, including nearly 200,000 balance constraints.

In theory, the TP model can be solved as a mixed-integer optimization problem using the branch-and-bound algorithm. However, due to the model's size, this approach is impractical for operational use. As alternatives, we have explored the fix-and-relax (FR) method and Lagrangian decomposition (LD). Marín (2006a) describes the specifics of using the FR method for the TP problem, while Marín (2006b) describes the specifics of using LD for the TP problem. One advantage of the LD approach is that the optimization subproblems derived from the decomposition have special structures for which specialized algorithms exist, so they can be solved efficiently without relying on commercial optimization software.

This section shows examples to which the design model has been applied. The first example is used to illustrate the effect of the parameters of the landing and take-off constraints and of the time lag of the control time-space boxes on conflict areas. The second example shows the influence of the headway nodes on the waiting point location. The effect of considering larger conflict areas is considered in Examples 1 and 2.

The third example is a comparison between two configurations that differ only in the structure of the runways. In one configuration a mixed runway for take-off/landing is used, whereas in the second there are two separate runways for each of the operations. The fourth example shows the effect of prioritizing the location costs versus the routing by altering the weighting factors of the objective function.

The fifth example is based on two alternative terminal configurations for the airport with a mixed take-off/landing runway used in Example 1. It illustrates the airport's performance under these two alternatives.

6.1 Example 1

The first example is for the airport depicted in figure 4 below using a mixed runway for take-offs and landings. This example is used to illustrate the effect of the parameters ν and δ_D for the time lag of the control time-space boxes on conflicting areas and the blocking of take-off runways on the aircraft routing.

In all the runs in Examples 1 and 2 a location decision is made on a set of points for waiting purposes. Three points in the configurations shown are considered to be waiting points for aircraft. A constraint is added to the model so that it is only possible to locate one of these waiting points. A large value for capacity is assumed at each of these nodes so that there is no need to queue at other locations. Additionally, in the second of the examples the different possibilities of opening or closing access points to take-off runways are considered.

The results for Example 1 are shown in Table 1. All the runs correspond to a demand of 25 aircraft for a PP of 30 minutes. For all executions, the weight factors of objective function ϕ were set to the following values: α_z= 0.1, α_L= 0.5, α_{IC}= 0.2, $\alpha_{\texttt{OUT}}$= 0.05, $\alpha_{\texttt{IN}}$= 0.05, $\alpha_{\bar{z}}$= 0 and α_{T+}=α_{T-}= 0.05. In these runs, the advisability of permitting or banning waiting points for aircraft at nodes NW1, NW2 and NW3 is examined and the decision variables $y = (y_{NW1}, y_{NW2}, y_{NW3})$ show which nodes are more convenient for waiting. NW1 and NW2 are waiting areas to access the take-off runway at heading points NAR1 and NAR2, respectively. NW3 is a waiting area for both arriving and departing traffic.

The conflict area is made up of nodes O6, NW2 and NW3 in runs 1 to 8.

Run 1 is for aircraft that require a blocking time for take-off $\delta_D = 1$ or $\delta_D = 2$ (1, 2 in the Table) according to the individual characteristics of each aircraft. The blocking time after landing and the blocking time previous to landing were set to $\delta_L^1 = \delta_L^2 = 1$.

Table 1. Runs for the airport in Figure 4 (mixed runway)

#Run	δ_D	ν	$\texttt{T}_{\texttt{CPU}}$	y	ϕ	I_C	z	$D_{\texttt{IN}}$	$D_{\texttt{OUT}}$	$\mathcal{T}^-$	$\mathcal{T}^+$
1	1, 2	3	57.03	0,1,0	30.2	7	260	1	65	13	7
2	1	3	38.1	1,0,0	25.95	9	233	1	30	17	7
3	2	3	51.05	1,0,0	37.05	8	307	1	100	9	7
4	3	3	130.22	1,0,0	41.3	8	336	1	125	7	7
5	1, 2	1	1.13	0,1,0	28	0	256	1	57	13	7
6	1, 2	2	1.17	0,1,0	28.55	1	259	2	57	13	7
7	1, 2	5	30.6	0,1,0	31.2	16	260	1	59	13	7
8	1, 2	3	216.09	0,1,0	31.25	15	257	1	60	13	7
9	1, 2	3	6.05	0,1,0	31.95	19	257	1	58	13	7
10	1, 2	3	8.2	0,1,0	31.95	19	257	1	58	13	7

Run 1 can be taken as a reference for runs 2 to 7. In runs 2, 3 and 4, parameter δ_D is increased from 1 to 3, respectively, and made equal for all the aircraft. As expected, the higher the blocking time, the higher the congestion. The total routing time z grows from 233 to 336, and there is also a significant increase in delay D_{OUT} for departing flights. It must be also noted that the total number of flights $T^+ + T^-$ that arrive at their destination within the PP (parking for arriving flights or take-off for departures) decreases as the blocking time increases. There is a slight decrease in index I_C. This is because the increase in the blocking time between departures makes the crossing trajectories for a fixed lag time ν in conflict area less necessary. It must be noted that for runs 2 to 4 node NW1 is chosen for waiting whereas in run 1 node NW2 is chosen for waiting.

Runs 5 to 7 show the influence of the ν lag time parameter, whereby $\nu = 1, 2, 5$ for runs 5, 6 and 7, respectively. The requirement for larger lag times has the effect of increasing the values for I_C, the delays for departing flights and the total routing time z. However, the total number of flights $T^+ + T^-$ that reach their destination within the PP is not altered by this factor.

Runs 8 to 9 show the influence of a more extended conflict area. This area for run 8 is comprised by nodes O6, NW2 and NW3. The conflict area for run 9 adds nodes O7 and O8 to the ones in run 8. The conflict area for run 10 adds nodes O9 and O2 to the ones in run 8. The effect of the extending the areas of conflict does not contribute to increasing congestion and only increases index I_C.

As a conclusion, the previous runs show that for the airport in Figure 4 the blocking time δ_D between departing flights contributes to congestion and increases delays much more than trying to obtain trajectories that create fewer risk situations does. Moreover, it shows that the routing mechanism for aircraft on the ground performs differently and leads to different decisions with regard to the required waiting nodes. It must be also noted that, whereas δ_D is a parameter that depends on the airport's demand of departing flights during the PP, lag time ν is a parameter that depends on the safety requirements for the aircraft routes on the ground.

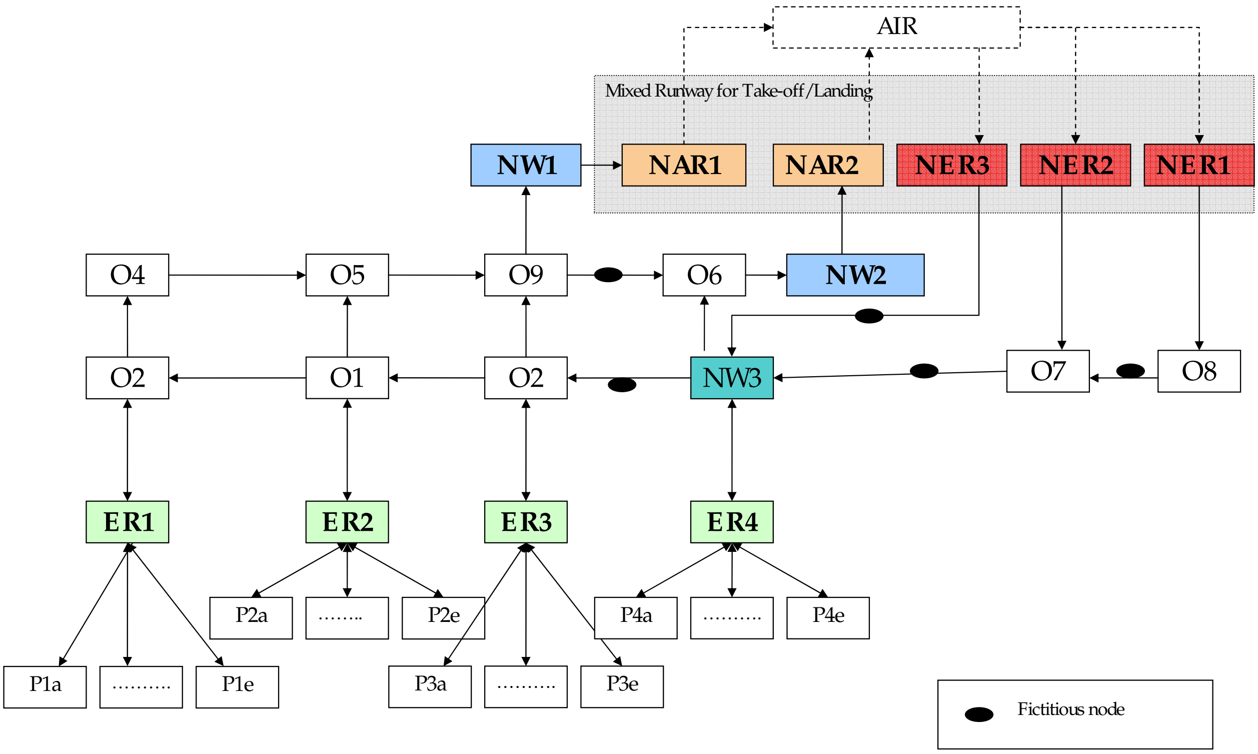

Figure 4: Test airport with a mixed runway.

6.2 Example 2

In this example, several runs are made for the airport shown in Figure 5, with separate runways for take-off and landing. In this example the influence of the ν parameter is studied.

The results of this example appear in table 2. All the runs are for the same set of 25 aircraft used in example 1 and with the same weight factors of the objective function ϕ . In this example the headway nodes NAR1, NAR2 can be closed or open as shown in the corresponding column of table 2 below. All the runs in this example have been made with a blocking time δ_D =1 or δ_D =2 depending on the aircraft characteristics.

The results of this example appear in Table 2. All the runs are for the same set of 25 aircraft used in Example 1 and with the same weight factors of objective function ϕ. In this example, the headway nodes NAR1 and NAR2 can be closed or open as shown in the corresponding column in Table 2. All the runs in this example were made with a blocking time δ_D =1 or δ_D =2 depending on the aircraft's characteristics.

Table 2. Runs for the airport in Figure 5 (separate runways)

#Run	$\mathtt{T_{CPU}}$	NAR1	NAR2	y	ϕ	I_C	z	$D_{\mathtt{IN}}$	$D_{\mathtt{OUT}}$	$\mathcal{T}^-$	$\mathcal{T}^+$
1	0.95	1	1	0,1,0	30.35	11	257	2	57	13	7
2	56.06	1	1	0,1,0	31.9	19	256	1	59	13	7
3	1.66	1	1	0,1,0	28	0	256	1	57	13	7
4	169.52	1	1	0,1,0	29.7	4	264	3	57	13	7
5	21.5	1	1	0,1,0	31.9	18	258	1	59	13	7
6	61.34	1	1	0,1,0	31.25	15	257	1	60	13	7
7	4.28	1	0	1,0,0	27.4	4	238	4	61	14	5
8	55.25	0	1	0,1,0	31.1	19	247	2	58	13	5

Runs 1 and 2 show the effect of different conflict areas. For run 1 the conflict area is made up of nodes O7, O8 and NW3, which are associated with arrivals, whereas for run 2 the conflict area is made up of nodes O2, O6, O9, NW1, NW2 and NW3, which are associated with departures. Runs 3 to 5 show the effect of increasing the ν parameter, which slightly increases the congestion results. Index I_C increases as the conflict area becomes larger, but this has a negligible effect on delays D_{IN}, D_{OUT} and and total throughput $T^+ + T^-$. These effects are similar to that which occurs in Example 1.

Runs 6 to 8 are for different combinations of access nodes NAR1 and NAR2 to the take-off runway. As can be observed, run 6 with both NAR1 and NAR2 provides more throughput (a total of 20 aircraft) than the one obtained with runs 7 and 8. It is also shown that it is better to locate NAR1 rather than NAR2 because this increases the total throughput $T^+ + T^-$ (19 aircraft). This option also has a shorter routing time z and incurs significantly fewer risk situations.

The performance of the airports used in Examples 1 and 2 is strongly influenced by the fact that in Example 1 there is a mixed runway used for two purposes (take-off and landing) whereas in Example 2 the airport has two runways, one for landing and the other for take-off. Run 1 in Example 1 and run 1 in Example 2 are made under the same conditions with equal values for parameters $\nu, \delta_D, \delta^1_{\mathtt{L}}, \delta^2_{\mathtt{L}}$ and demands for the same PP. Performance is clearly better for the configuration with separate runways.

6.3 Example 3

In order to better illustrate this fact, Table 3 shows a set of runs in two airports, which take the values $\delta^1_{\mathtt{L}} = 1$, $\delta^2_{\mathtt{L}} = 2$ and $\delta_D = 1, 2$. In this case, with greater lag times blocking the runways, the advantage of separate runways is clear in terms of the total travel time z, delays and total throughput $\mathcal{T}^- + \mathcal{T}^+$. The number of risk situations is determined by the conflict area being used and when the access node NAR1 is located the number of available routes increases and the number of risk situations decreases. It must also be observed that this factor is better in the case of separate runways than it is in the case of mixed runways.

Table 3. Comparative tests: mixed versus separate runways

#Run	T_{CPU}	NAR1	NAR2	y	ϕ	I_C	z	D_{IN}	D_{OUT}	$\mathcal{T}^-$	$\mathcal{T}^+$
	Mixed runways										
1	40.36	1	1	0,1,0	37.9	12	305	1	106	10	7
2	4.42	1	0	1,0,0	37.2	4	309	1	116	10	7
3	20.73	0	1	0,1,0	37.95	15	308	1	89	10	7
	Separated runways										
4	50	1	1	0,1,0	30.2	7	260	1	65	13	7
5	0.92	1	0	1,0,0	28.45	4	250	1	63	14	7
6	89.05	0	1	0,1,0	31.25	11	267	1	56	13	7

Since the number of combinations for our design variable y is small, the headway nodes NAR1 and NAR2 were set to close or open manually (i.e. as input data, rather than as optimization variables), allowing us to compare the optimal headway choice with a sub-optimal design. Thus, in these examples, y will only determine whether or not waiting nodes NW1, NW2 and NW3 are necessary, and their capacity (if applicable). In this example, we make the following assumptions:

- Only one waiting node of unlimited capacity can be chosen.
- The capacity of terminal nodes ER1, ER2, ER3, and ER4 is also unlimited.

For the mixed-runway runs (1 and 2), the total throughput is 17 (10 arrivals and 7 take-offs), regardless of the available headway node. It appears that the use of NAR1 as the only headway point significantly reduces conflicts. This may be due to the fact that node O6 and waiting nodes NW2 and NW3 form a "control zone", where deconfliction is strictly enforced. The use of NAR2 may reduce delays for take-off traffic. Similarly, for separate runways (runs 3 and 4) NAR1 also produces minimum conflicts and even an improvement of one arriving aircraft and a better routing time. Overall, the advantage of separate runways is clear in terms of routing time, delays and total throughput.

6.4 Example 4

The decision variables in previous examples determined which of the design waiting nodes NW1, NW2 and NW3 should or should not be located, assuming there was sufficient capacity at each node to allocate several aircraft. Because of the demand conditions, these capacities were never surpassed and as only one of the waiting nodes had to be located, delays did not increase too much.

In this example, however, decision variables are the capacities themselves and location costs are proportional to these capacities. There is no limitation on the number of nodes to be allocated for waiting. Runs have been made in the airport shown in Figure 4 with mixed runways and the same parameters $\nu, \delta_D, \delta_{\mathtt{L}}^1, \delta_{\mathtt{L}}^2$, demand and PP as those in Example 3. In the runs both access runway nodes are considered to have been located.

In Table 4, runs 1 to 3 show the optimal capacity location at the design waiting nodes for different location weights, although the capacity of the terminal nodes ER1, ER2, ER3 and ER4 is limited to one aircraft. In the runs in the previous examples these capacities were set to infinity. The recommendation is clear: the location of NW2 must be decided with a capacity of up to six aircraft. When the location factor has less weight in the objective function, less capacity is required at the design wait nodes and a progressive degradation of the routing factors is detected. The decrease in index I_C can be explained by the increase in the delays experienced by aircraft waiting at other points with capacity for only one aircraft far from the conflict area. This previous recommendation is kept if the capacity of the terminal nodes is increased to 2 or 5 (runs 4 and 5).

Table 4. Prioritization of location costs

#Run	$\alpha_{\mathtt{L}}$	$\mathrm{T}_{\mathtt{CPU}}$	q	y	ϕ	I_C	z	$D_{\mathtt{IN}}$	$D_{\mathtt{OUT}}$	$\mathcal{T}^-$	$\mathcal{T}^+$
1	0.1	18.54	1,1,1	1,6,0	67.57	15	302	1	95	10	7
2	0.5	23.64	1,1,1	1,4,0	40.2	15	303	1	104	10	7
3	0.9	22.76	1,1,1	1,3,0	10.89	14	324	8	108	9	7
4	0.5	73.52	2,2,2	0,3,0	38.9	12	305	1	106	10	7
5	0.5	351.49	3,3,3	0,2,0	37.85	12	302	1	101	10	7

6.5 Example 5

The following example compares two alternative configurations for the terminal areas for an airport with mixed runways. The first option is shown in Figure 6. It has three terminal areas, each with capacity for 21 aircraft, whereas the second option shown in Figure 4 has four terminal areas with capacity for 20 aircraft. In both configurations the advisability of setting nodes NW1, NW2 and NW3 as wait nodes is considered. The conflict area is made up of nodes O6, NW2 and NW3, as before.

Runs of the TPND model have been done with 30 aircraft for both configurations distributing evenly the locations on the parking areas. Table 5 below shows the distribution of arrivals and departures for the terminal areas on both configurations. The results of these runs (decision variables and objective function components) are shown in table 6. The objective function weights assumed have been:α_L= 0.5, α_z= 0.1, α_{IC}= 0.2, $\alpha_{\mathtt{OUT}}$= 0.05, $\alpha_{\mathtt{IN}}$= 0.05, α_{T+}=α_{T-}= 0.05.

Runs of the TPND model were carried out with 30 aircraft for both configurations. The locations in the parking areas were evenly distributed. Table 5 below shows the distribution of arrivals and departures for the terminal areas in both configurations. The results of these runs (decision variables and objective function components) are shown in Table 6. The objective function weights assumed were α_L= 0.5, α_z= 0.1, α_{IC}= 0.2, $\alpha_{\mathtt{OUT}}$= 0.05, $\alpha_{\mathtt{IN}}$= 0.05 and α_{T+}=α_{T-}= 0.05.

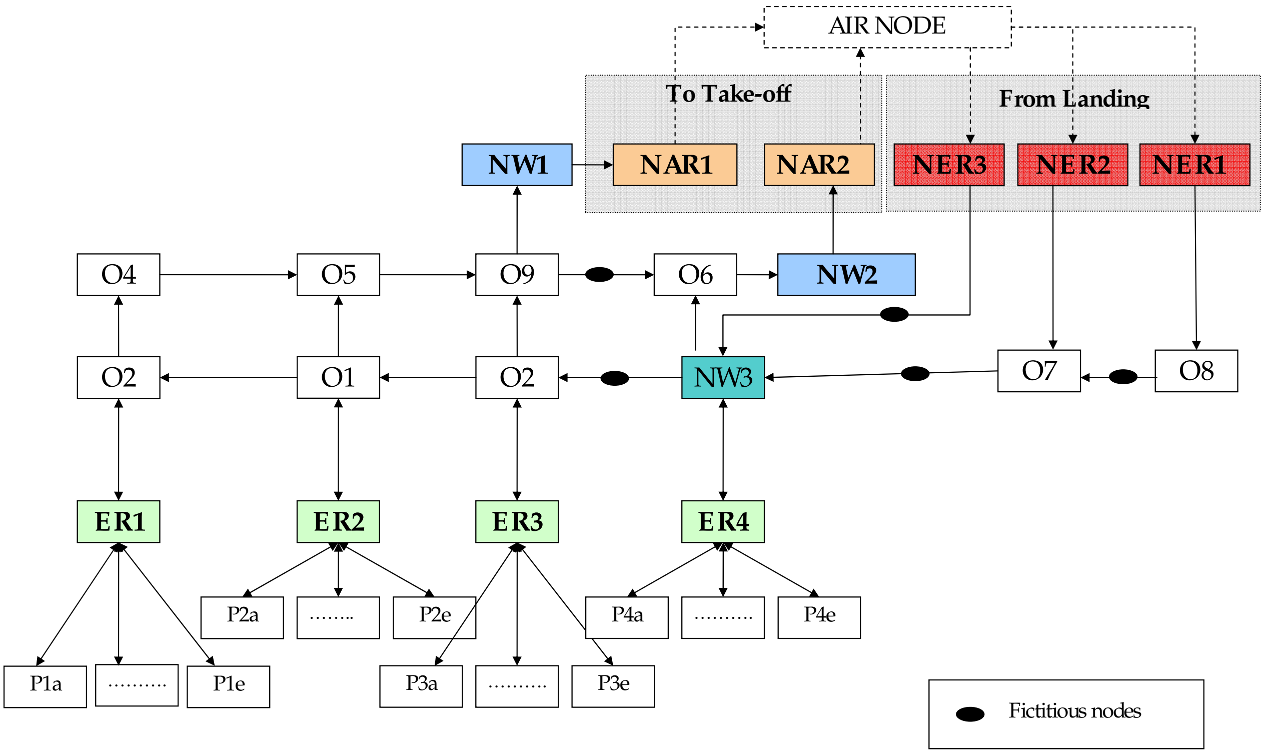

Figure 5: Test airport with separate runways.

Table 5. Distribution of arrivals/departures in Example 5				
Alternatives	Terminals	Departing	Arrivals	Airplanes
I: 3 terminals	1	4	6	10
	2	5	5	10
	3	4	6	10
II: 4 terminals	1	4	5	9
	2	3	4	7
	3	3	4	7
	4	3	4	7

Table 6. Runs of the TPND model in Example 5									
	ϕ	$g(y)$	y	z	I_C	D_{IN}	D_{OUT}	$\mathcal{T}^-$	$\mathcal{T}^+$
3 terminals	78.4	2	1,1,0	306	17	11	90	10	7
4 terminals	63.7	1	0,1,0	257	17	1	64	13	7

Table 6 shows that the configuration with four terminal areas (i.e. more distributed parking lots) is preferable to the one with three terminal areas. In both cases, the index for risk situations is 17 in the conflict area. It must be remarked that in both cases the planning period cannot allocate all of the flights. In the configuration with three

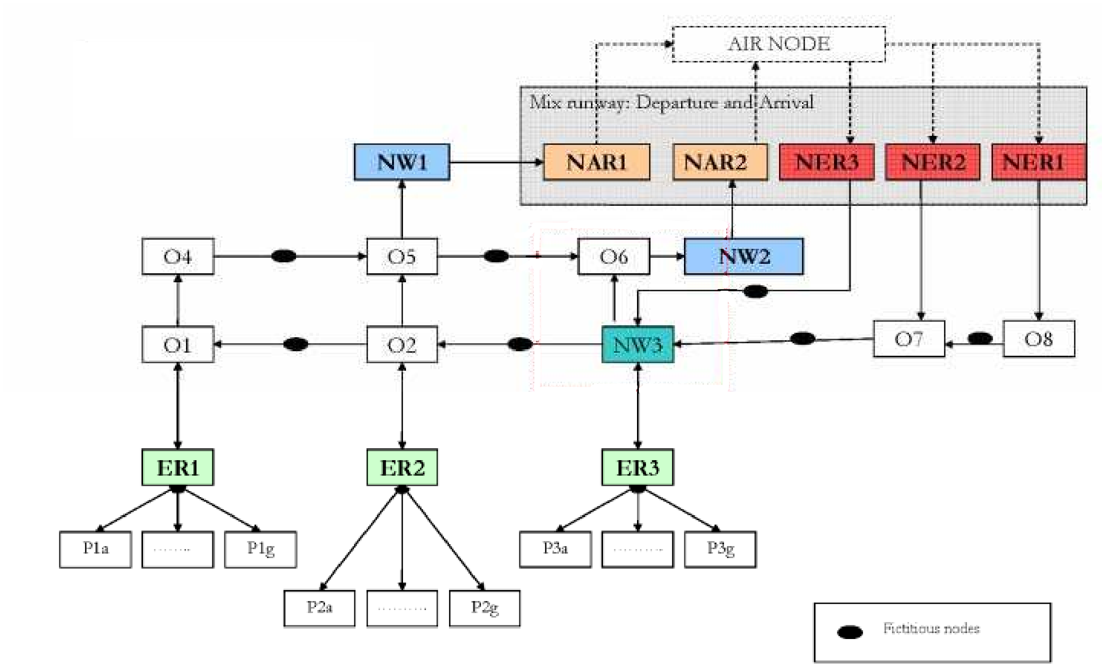

Figure 6: Test airport with a mixed runway. Configuration 1 for Example 3.

terminals, only 17 out of 30 aircraft are able to reach their destination (a terminal or a runway), whereas in the configuration with four terminals the number of aircraft that complete their trip on the ground is 20 out of 30. This fact is reflected mainly in the total routing time spent by aircraft during the planning period z , which is greater in the three-terminal configuration. In addition to all these facts, in the four-terminal configuration only node NW1 is necessary as a wait node (which implies a lower fixed cost component g), whereas in the configuration with three terminals nodes NW1 and NW2 are required.

7 Conclusions

Up to now, the evaluation and design of airport configurations have been studied using simulation tools. These tools lack the ability to preemptively optimize aircraft routes and schedules. Moreover, although the optimization of taxiing operations has been tackled using mathematical programming models, the problem of designing airport configurations using these tools and taking into account the taxiing and routing on the ground has not yet been tackled and is the main contribution of this paper. Our model adopts a multi-objective approach by balancing the factors that are generally taken into account in real practice using simulation.

It must be remarked that the taxi planning model used as the core model also incorporates realistic aspects such as the possibility of analyzing configurations with mixed runways for both take-off and landing and separate runways for each operation with blocking times depending on the aircraft operations.

The examples shown in this paper were carried out with simplified airport networks, taken from actual data supplied by Aeropuertos Españoles y Navegación Aérea. The computational times are satisfactory for this range of network sizes, although a specialized algorithmic approach is required for more complex and larger airport configurations and longer planning periods.

Computational tests were run in the examples using a predetermined set of weights of the objective function in order to show the most relevant aspects of the implicit multicriteria decision process on which the model relies, and to show the effects of two important parameters of the model that determine the airport's performance, namely the blocking time of runways and the minimum lag time between crossing trajectories at nodes in the airport's network. The examples show that it is the former that has the most significant impact on delays.

Our main conclusion is that the airport design model presented herein has been shown to be appropriate for comparing alternative airport configurations while considering aircraft congestion, and also for locating distinct airport facilities such as waiting points, parking locations and the direction of corridors.

8 Acknowledgments

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9 Appendix. Notation

T - The planning time set $T = \{1, 2, \ldots, |T|\}$.

G - $G = (N, E)$ is the graph modelling the airport layout. N is the set of nodes and E is the set of directional links.

W - is the set of aircraft whose movements are considered within the planning period.

N^W - Subset of nodes at which one or more aircraft can remain waiting. The model assumes that the maximum number of aircraft that can remain at them at a given instant is related to the dimensions of the aircraft and the area of the node.

N^O - Subset of ordinary nodes at which aircraft cannot remain waiting.

N^{ER} - Subset of nodes at which an aircraft enters the airport after leaving the landing runways.

N^{AR} - Subset of nodes with access to runways that enables take-off operations to be initiated immediately.

N^F - Subset of artificial nodes that are used to model queues in the corridors. At these nodes, overtaking is prohibited and only one aircraft can remain waiting.

N_S - $= \{\ i \in N |\ \exists w \in W : o(w) = i\ \}$ is the set of origin nodes at which a trajectory starts at the beginning of the planning period.

N^P - $= \{\ q \in N \mid \exists w \in W :\ d(w) = q\ \}$ is the set of nodes that are end destinations for aircraft on the ground.

$o(w)$ - Origin node in N at which aircraft $w \in W$starts its trajectory.

$d(w)$ - Destination node in N at which aircraft $w \in W$ finishes its trajectory.

$t(w)$ - Time period in T during which aircraft $w \in W$ starts its trajectory.

$t_{i,j}$ - Number of time intervals required to cross link $(i, j) \in E$.

$F(i)$ - The set of emerging nodes from node i.

$R(i)$ - The set of incoming nodes at node i.

$E^w_{i,t}$ - =1 , if aircraft w waits at node i during period t, and 0 otherwise.

$X^w_{i,j,t}$ - =1 , if aircraft w leaves node i at time t to go towards node j following link $(i, j) \in E$, and 0 otherwise.

$S^w_{d(w),t}$ - =1 , if aircraft w reaches its destination $d(w)$ at time t within the PP and 0 otherwise.

$E^w_{o(w),t(w)}$ - should be 1 if aircraft w waits at node $o(w)$ at time $t(w)$, or 0 if it does not.

$X^w_{o(w),j,t(w)}$ - should be 1 if the aircraft moves from node $o(w)$ towards node j at time $t(w)$, and 0 otherwise.

q_i - is the capacity (in surface units) of node i.

e_w - is the space required by aircraft $w \in W$.

P - is the set of runways.

W^A - is the set of aircraft entering the airport with a given destination on the ground.

W^D - is the set of departing aircraft.

$\delta_D(w)$ - is the blocking time that is dependent on the aircraft's characteristics.

$N^{AR}(p)$ - is the set of access nodes to runway p.

$N^{ER}(p)$ - is the set of nodes that exit from runway $p \in P$.

OAN^t_j - is defined as the occupation of access node $j \in N^{AR}(p)$ for each runway $p \in P$.

$\delta^1_L(w)$ - is a blocking time *after* time t.

$\delta^2_L(w)$ - is a blocking time *before* time t of length.

OER^t_i - is the number of concurrent aircraft to node $i \in N^{ER}(p)$.

OAN_j^t - is defined as the occupation of access node $j \in N^{AR}(p)$ for each runway p.

K - is the set of predetermined areas in the airport where risk situations need to be explicitly minimized.

ν - Number of time subintervals in a control time-space box.

$C_{K,t}$ - For a control box defined at time t in conflict area K, this variable counts either the number of aircraft entering at the control box or the number of time intervals of aircraft waiting inside the control box.

$x_{i,j,t}$ - is the total outgoing traffic flow for link $(i, j) \in A$ at time t.

$e_{i,t}$ - is the total number of aircraft waiting at $i \in N$ at time t.

$\gamma_{K,t}$ - is a binary variable that takes a value of 1 when $C_{K,t} > 0$.

I_C - is the total number of control time-space boxes affected by a risk situation.

n_c - is the number of controllers available for control tasks in the planning period.

z^w - is the travel time for aircraft $w \in W$.

z - is the weighted total travel time.

$r_i^{d(w)}$ - is the estimated travel time from node $i \in N^W$ to destination $d(w)$ in the next PP.

λ^w - is a coefficient for the priority of aircraft $w \in W$.

$\hat{z}$ - is the worst routing time.

D_{IN} - is the total delay for incoming traffic.

D_{OUT} - is the total delay for outgoing traffic.

$\mathcal{T}^-$ - The total number of take-offs within the PP.

$\mathcal{T}^+$ - The total number of arrivals at destinations on the ground within the PP.

N_d^F, N_d^{ER}, N_d^{AR} and N_d^O, N_d^W - are subsets of nodes in N^F, N^{ER}, N^{AR}, N^O and N^W, respectively which are considered for inclusion in or exclusion from the design model.

E_d - is a subset of links which are considered for inclusion in or exclusion from the design model.

y_c, y_i and y_{ij} - are design decision binary variables for the design model. y_c is a binary variable associated with a corridor, y_i is a binary variable associated with a node in any of the subsets N_d^F, N_d^{ER}, N_d^{AR}, N_d^O and N_d^W and y_{ij} is a binary variable associated with a link in E_d.

ϕ - Objective function of the linear integer programming design model.

$g(y)$ - is a linear function of location costs associated with design decision variables y.

α_z, α_L, α_{IC}, α_{OUT}, α_{IN}, $\alpha_{\hat{z}}$, $\alpha_{\tau T^+}$ and $\alpha_{\tau T^-}$ - are weighting parameters in the objective ϕ for the distinct factors considered in the design, i.e. z, location costs, I_c, D_{OUT}, D_{IN}, $z_{\hat{z}}$, $\mathcal{T}^+$ and $\mathcal{T}^-$.