Energy Security: a robust optimization approach to design a robust European energy supply via TIAM

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Abstract

Energy supply routes to a given TIAM region (say E.U.) are subject to randomness, resulting in partial or total closure of a route (corridor). For instance: a pipeline may be subject to technical problems that reduce its capacity. Or, oil supply by tanker may be reduced for political reasons or because of equipment mishaps at the point of origin, or again by a conscious decision by the supplier in order to obtain economic benefits. This paper uses the approach of Robust Optimization to model uncertainty on the energy supply constraints for Europe in the economy-energy model TIAM. The resulting formulation provides several interesting features regarding the security of EU energy supply and has also the advantage to be numerically tractable.

Keywords. *Energy supply, Robust Optimization, Ambiguous Chance Constraint Programming, TIAM.*

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1 Introduction

Energy supply routes to a given TIAM region are subject to randomness, resulting in partial or total closure of a route (corridor). For instance: a pipeline may be subject to technical problems that reduce its capacity. Or, oil supply by tanker may be reduced for political reasons or because of equipment mishaps at the point of origin, or again by a conscious decision by the supplier in order to obtain economic benefits.

The purpose of this article is to formulate the above issue with the TIAM Integrated Model, using the approach of Robust Optimization [3], which, in our case, can be interpreted as a revival of Chance Constrained Programming [7] under the name of Distributionally Robust, or Ambiguous, Chance Constrained Programming [6, 17], and to apply the approach to improving the security of supply to the European Energy system.

Energy security is now considered a priority in any energy policy and future energy strategy. In the United States, the Energy Independence and Security Act of 2007 and the

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proposed Clean Energy and Security Act of 2009 consider energy security at a pillar of the US energy policy. In Europe, the Green Paper on energy security [9], the Green Paper on the European strategy for sustainable, competitive and secure energy [10], several directives (for example: the 2006 directive on oil stocks forcing each Member State to maintain a minimum petroleum reserve, the 2004 and 2005 directives on measures to safeguard security of, respectively, natural gas supply and electricity supply), the current proposals for new regulations on investment projects in energy infrastructure (2009) and on gas supply security (2009) as well as the 20-20-20 Energy Policy [11], aim at strengthening the European Community security of energy supply.

Several concerns or fears are behind the importance given to energy security:

- The rapidly increasing World demands of energy, mainly driven by the emerging countries like China and India [16] may have important consequences on the availability and price of energy resources at the World level. Debates on the possibility of an imminent peak oil add to these fears.
- The import dependence of Europe is expected to grow in a business-as-usual context, and energy imports might reach up to 65% of the EU consumption by 2030 [11]. Moreover, this import dependence of Europe as well as many other importing countries or regions tends to concern a relatively small number of supplying countries [16].
- The Russian-Ukrainian gas crisis of January 2009, resulting in a disruption of gas supplies to EU via Ukraine, illustrates the increased transit risks.
- Uncertain geopolitical stability or strategy of supplying regions, like Middle East, Nigeria or Venezuela is of course of concern.
- Threats to energy security can result from system failures either of the supplier, or of the import country, for different reasons, such as natural events, terrorism, poor quality or conditions of the installations (operational failures from inadequate protection, generation capacity limitations due to under-investment) [14]. The 2003 blackout in Italy and the 2006 blackout that affected several European countries are examples.
- Environmental risk must be considered, related for example to the potential damage from accidents or any future policy implemented in the supplying or consuming countries and affecting the production and consumption of energy.

Based on this description, the risks related to energy supply can be geological (possible exhaustion of the resources), economic (fluctuations in the prices), technical (system failures – for different reasons), environmental (accidents or policies), or geopolitical [2]. Moreover, the management of risks will differ if the risks are external or internal to Europe. Internal EU risks generally mean coping with low-probability events, as well as appropriate investments in supply, transmission and distribution of energy [2]. Finally, the time scale of different risks varies from short term (supply shortage due to technical failures for example) to long term (depletion of the resources, pricing, etc.) and appropriate actions will differ according to the time scale and nature of risks [15].

The proposed measures or strategies to increase the energy security can be classified in four categories [2, 11, 13, 15]:

- Diversification of the fuel mix, the geographic sources, the transportation routes and the diversification of suppliers; diversification can indeed be considered as a general “insurance” against heavy dependence, and against massive disruption.
- Definition of commercial agreement between suppliers and consumers, for example between the EU, Russia and Ukraine to secure gas supply from Russia via Ukraine to the EU.

- Appropriate investment in supply, storage, transport and distribution technologies to guarantee the quality of the energy system, to increase the available capacity of the production system and of the import and local network, and to promote an efficient management of, and recovery from, energy system disruptions.
- Decrease of the total energy demand (increase the efficiency of the energy system) and priority to energy sources considered less risky (reduction of foreign sources, reduction of sources with higher risk of accidents, etc.).

The impact of climate policies on energy security is considered as positive as regards the import dependence dimension of energy security, when considering the decrease of fossil fuel consumption and the growth of domestic renewable energy sources. However, the increase of nuclear generation raises issues of import dependence and availability of the resource, while the growth of renewable sources might affect negatively the reliability of the energy system energy security given their higher dependence on weather and intermittency [14];. Indeed, trade-offs between energy security, climate change policies but also the climate resilience of the energy systems are to be found, where not only technologies but integrated policies promoting both greenhouse gas reduction and energy security [5, 15].

In this research, we take into account most but not all of the above relevant aspects of energy security. In particular, we take a view that encompasses most of the security components mentioned above, and we quantify a measure of reliability of supply that is defined as the probability that the total available import capacity does not fall short of the planned imports of energy by EU.

2 Methodology

The supply of energy to the European Union is modeled via TIAM, a dynamic, integrated, global partial equilibrium energy model, in which the EU energy system and its energy import channels are explicitly represented. We first describe TIAM, followed by a description of Ambiguous Chance Constrained Programming (ACCP) applied to the capacity constraints of the import channels (also called corridors).

2.1 Description of TIAM

The TIMES Integrated Assessment Model (TIAM) is a global technology-rich bottom-up model that represents the entire energy system of the World divided in 16 regions: Africa, Australia-New Zealand, Canada, United States, Mexico, Central and South America, China, India, Japan, South Korea, Other Developing Asia, Middle-East, EU30, Other East Europe, Russia, Central Asia & Caucasia. It covers the procurement, transformation, trade, and end-uses of all energy forms in all sectors of the economy (see Figure 1).

TIMES'economic paradigm is the computation of a dynamic inter-temporal partial equilibrium on energy/emission markets where demands for energy services are exogenously specified (only in the reference case), and are sensitive to price changes in alternate scenarios via a set of own-price elasticities at each period [21]. Although TIMES does not encompass all macroeconomic variables beyond the energy sector, accounting for price elasticity of demands captures a major element of feedback effects between the energy system and the economy. Thus, the equilibrium is driven by the maximization (via linear programming) of the discounted present value of total surplus, representing the sum of surplus of producers and consumers, which acts as a proxy for welfare in each region of the model.

The time horizon of TIAM extends to 2100, but has been reduced to 2055 for the present application. The model contains explicit descriptions of more than one thousand technologies and one hundred commodities in each region, logically interrelated in a Reference Energy System [22]. Residential, commercial, industry, transport, power plants,

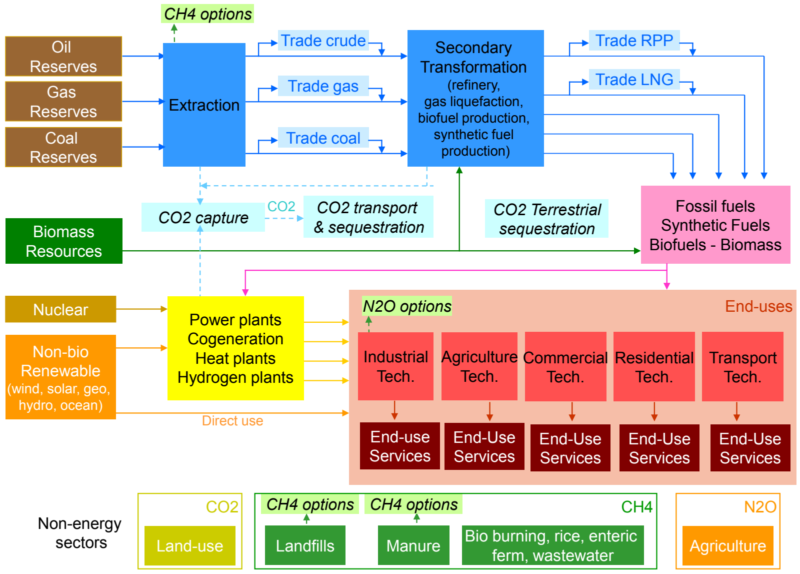

Figure 1: Reference energy system of TIAM

as well as upstream (from extraction to secondary transformation) are represented in a highly detailed mode, covering the 42 different service demands such as space heating, lighting, km driven by cars, by buses, production of iron and steel, of pulp and paper, etc. Such explicitness of the representation of technologies and fuels in all sectors allows precise tracking of capital turnover, and provides a detailed description of technological competition and sectoral and cross-sectoral energy-environmental policies.

TIAM also includes an endogenous climate module that allows the user to impose climate targets, such as upper bounds on concentrations, on atmospheric radiative forcing, or on temperature increase. The CO_2, CH_4 and N_2O emissions related to the energy sector are explicitly represented by the energy technologies included in the model. The non-energy-related CO_2, CH_4 and N_2O emissions (landfills, manure, rice paddies, enteric fermentation, wastewater, land-use) are also included in order to correctly represent the radiative forcing induced by them, but they are exogenously defined. Emissions from some Kyoto gases (CFC's, HFC's, SF_6) are not explicitly modeled, but a special radiative forcing term is added in the climate module. Emissions of chemically active gases such as NO_x, CO, VOC's are not modeled either, but their influence on the life cycles of GHG gases is implicitly accounted for in the concentration equations for the three main GHG's, but only through the calibration phase of the equations.

Greenhouse gas mitigation options available in the model are: energy substitutions, improved efficiency of installed devices, specific non-CO_2 abatement devices (e.g. suppression and/or combustion of fugitive CH_4 from landfills, thermal destruction of N_2O in the adipic acid industry, suppression of leakages at natural gas transmission level, anaerobic digestion of wastes with gas recovery, etc.), sequestration (CO_2 capture and underground storage, biological carbon sequestration), mitigation potential of up to 20% of the CO_2 and N_2O emitted by the agriculture sector, and reductions in energy service demands in reaction to increased carbon prices.

TIAM has been, and is being used in several European and international projects; some of these applications concern the stochastic analysis of climate policies [20, 23].

2.2 Uncertainty in the supply capacity

We first need to identify the TIAM constraints that contain random parameters of interest to the issue of supply security. In so doing, we wish to satisfy one important principle, namely to formulate the randomness of the situation as broadly as possible. By this we mean to avoid singling out one (or a few) import channel(s) for separate risk modeling, but rather to treat the entire set of energy import channels simultaneously when assessing the security of supply. This is justified by the fact that what matters is not that each importing corridor be reliable, but rather that the commodities being imported be delivered in the desired quantity, whatever the source(s). This principle allows the planner to diversify the sources in order to increase the reliability of the total supply.

In fact, this principle may be further extended to consider not only the channels carrying a given energy form, but all corridors carrying any energy form. If this viewpoint is favored, it implies that the importing region is able to substitute energy forms if need be. Experimentation is much needed in order to decide whether such an extension is a good modeling choice. In what follows, we simply ignore the index representing the energy form, so as to cover both cases.

In practice, the planners take strategic decisions in a first step, e.g. in order to increase the reliability of the supply, facing an imperfect view of the future. Then in a second step, once the uncertainty is resolved, one can use this knowledge to "optimally" exploit the system. In other words, the second step decisions are functions of the outcome of the uncertain parameters. There exist techniques to model such a dynamic feature[1] but unfortunately they introduce an order of complexity that leads for large scale model such as TIAM to intractable formulation. We thus omit it in this study and we refer the reader to the appendix for a more detailed discussion on uncertainty in dynamic models.

In TIAM, each channel is modeled as a technology that accepts as input some energy forms from a given region, and the same energy form as output to another region. An obvious candidate for the random parameter of energy supply is the availability factor of the technology representing an import channel. We denote each import channel by index k, k running from 1 to K. The availability factor is denoted $AF_{k,t}$. The standard TIAM constraint involving $AF_{k,t}$ is as follows:

$$ACT_{k,t} - AF_{k,t} \times CAP_{k,t} \leq 0, \text{ for } t = 1, \dots, T \text{ and } k = 1, \dots, K, \tag{1}$$

where $CAP_{k,t}$ is the capacity variable of channel k at period t, and $ACT_{k,t}$ is the variable denoting the quantity of energy carried through it (also called the activity of the channel). AF has a *nominal* value that is rather high (perhaps even equal to 1).

As mentioned above, we are interested in protecting the total energy supply of EU, not that of each channel separately. We therefore create an aggregate constraint by summing the K constraints (1). We obtain:

$$\sum_k (ACT_{k,t} - AF_{k,t} \times CAP_{k,t}) \leq 0, \text{ for } t = 1, \dots, T. \tag{2}$$

Constraint (2) now concerns the total energy imports. It is not an existing TIMES equation and must therefore be introduced as a new constraint. We rewrite constraint (2) as constraint (3), by dropping the time index, for simpler notation.

$$\sum_k (ACT_k - AF_k \times CAP_k) \leq 0. \tag{3}$$

The random AF_k coefficients are written as follows:

$$AF_k = 1 - d_k \xi_k$$

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[1] Refer to [20] where Stochastic Programming has been used to model uncertainty on climate sensitivity and economic growth. In this study, the uncertainty can be represented by a very small event tree.

where, for each channel k, $0 \leq d_k \leq 1$ is the maximum failure rate, $[1 - d_k, 1]$ is the range of uncertainty of the AF_k , and ξ_k is a set of independent random variables with support [0,1]. A small d_k means that the corridor has little variability, and conversely when $d_k = 1$, there is the possibility of a complete corridor shutdown. We may now rewrite (3) as follows:

$$\sum_K (ACT_k - (1 - d_k \xi_k) \times CAP_k) \leq 0.$$

And, regrouping the terms differently, as:

$$\underbrace{\sum_k (ACT_k - CAP_k)}_{\text{certain}} \quad + \quad \underbrace{\sum_k d_k \cdot CAP_k \cdot \xi_k}_{\text{uncertain}} \quad \leq \quad 0. \tag{4}$$

The first summation of constraint (4) is a linear deterministic expression. The second summation is a random variable.

2.3 Methods for handling constraints with uncertain coefficients

The requirement that constraint (4) is to be satisfied for all possible realizations of its random component implies that the certain part should be chosen so as to match the worst possible case. This corresponds to the simultaneous failure of all corridors, i.e., $\xi_k = 1$ for all k. This drastic requirement would exclude most solutions but the more conservative ones; it would not be deemed reasonable by the planners. Rather, one would like to have the uncertain constraint satisfied most of the time, at the risk of having it violated on rare occasions. The big issue is how to make this qualitative and vague requirement as a quantified and tractable entity.

The natural formulation for this requirement is to fix a lower bound on the probability that the constraint be satisfied. This idea was proposed as early as 1958 by Charnes and co-authors in [7], and further discussed in [24] and [25], under the name of Chance Constrained Programming (CCP). Unfortunately, this approach turned out to lead to untractable numerical issues in all situations, but a very few special cases [3]. This formulation is not directly implementable in an optimization program such as TIAM.

Robust Opimization (RO) is an alternative proposal which essentially aims at overcoming the numerical issues raised by the computation of probabilities. The idea, similar to CCP, is to make sure that the constraint remains feasible for a set of “relevant” realizations of the random factors, at the risk of possible failure in some “exceptional” cases. But, contrary to CCP, RO defines the set of relevant realizations in an explicit way, e.g., as a polyhedron, rather than implicitly by means of a condition on a probability. The paradigm of Robust Linear Optimization goes back to [26], but the field remained almost idle until the idea was revived circa 1997, independently and essentially simultaneously, in the frameworks of both Integer Programming [19] and Convex Programming [4] and [12]. The salient feature of RO is that it reformulates the uncertain constraint into plain inequalities, named the equivalent robust counterpart, that can be efficiently handled by convex optimization codes.

The more recent views on RO, as presented in the extensive monograph [3], reconciles RO and CCP under the concept of Distributionally Robust [6], or Ambiguous [17] Chance Constrained Programming, in short ACCP. ACCP shares with RO the goal of leading to implementable and tractable formulations. To this end, it modifies in the CCP formulation as follows: the probability of satisfaction is not measured with respect to a specific probability distribution for each random factor ξ_k, but with respect to a class that is described by few parameters (e.g., independent random variables with common support and known means). It turns out that this idea reconciles the concept of uncertainty set that underlies RO and the probabilistic statement in CCP. We shall briefly present RO from the viewpoint of ACCP and show how it can be implemented in our problem of interest.

2.4 Robust Optimization for TIAM

We state first the main result for our analysis of the uncertain constraint (4). We introduce the following weak assumption on the random factors.

Assumption 1 *The random factors ξ_k are independent random variables, with common support $[0,1]$ and known means $E(\xi_k) = \mu_k \leq \frac{1}{2}$, for $k = 1, \ldots, K$.*

The main result is that there exists a bona fide mathematical programming set of constraints whose solutions enforce (4) with probability at least $(1-\epsilon)$. In this formulation, $\epsilon > 0$ is a user-defined safety threshold.

Proposition 1 *Under Assumption 1, the condition that there exist variables $u \in \mathbb{R}^k$ and $v \in \mathbb{R}$ such that the deterministic set of inequalities*

$$\sum_k (ACT_k - (1 - d_k\mu_k)CAP_k) + \sum_k (1-\mu_k)u_k + \sqrt{\frac{K}{2}\ln\frac{1}{\epsilon}} \cdot v \leq 0 \tag{5a}$$

$$u_k + v - CAP_k \cdot d_k \geq 0, \; k = 1, \ldots, K \tag{5b}$$

$$u_k \geq 0, \; v \geq 0, \; k = 1, \ldots, K \tag{5c}$$

is satisfied guarantees that constraint (4) is satisfied with probability at least $(1-\epsilon)$.

Proposition 1 is a special case of a more general theorem in RO that is given and proved in the appendix. Note that (5) can be rewritten as

$$\sum_k (ACT_k - CAP_k) + \kappa(CAP) \leq 0,$$

where $\kappa(CAP)$ is a deterministic safety factor that replaces the uncertain component in (4).

In TIAM, there are 67 channels ($K = 67$) into EU, so that in order to insure a reliability of 95% for constraint (4), one must use a coefficient of variable v equal to 10.02 in (5a). For a lower reliability of 90%, the coefficient of variable v is equal to 8.78. Of course, the larger the reliability, the larger the extra cost incurred by the energy system as a whole, since the LHS of the first constraint in (5a) is increasing with ϵ.

The choice of the d_k parameters depends on our estimation of the range of uncertainty of the random availability factors. In particular, if $d_k = 1$ for all channels k, then the ACCP approach akin to pure diversification of the supply of energy.

To conclude this section, we show that the safety factor is increasing with μ, so the worst case occurs when the $\mu_k = 1/2$. Since u and v are free variables in Proposition 1, they can be chosen so as to minimize the safety factor κ and we have

$$\begin{aligned} \kappa(CAP) = \min_{u,v} \quad & \sum_k (\mu_k(d_kCAP_k - u_k) + u_k) + \sqrt{\frac{K}{2}\ln\frac{1}{\epsilon}} \cdot v \\ & u_k + v - CAP_k \cdot d_k \geq 0, \; k = 1, \ldots, K \\ & u_k \geq 0, \; v \geq 0, \; k = 1, \ldots, K. \end{aligned}$$

We claim that $(d_kCAP_k - u_k) \geq 0$ at the optimum. Suppose to the contrary that $d_{k'}CAP_{k'} - u_{k'} < 0$ for some k'. Let $u'_{k'} = d_{k'}CAP_{k'} < u_{k'}$ and $u'_k = u_k$ for $k \neq k'$. We easily check that u' is feasible and strictly improves the objective, a contradiction to optimality. Hence the coefficients of μ in κ are all nonnegative, which proves that k is nondecreasing with μ.

3 Case Study: security of the E.U. energy supply

3.1 The case study

In the TIAM model, 67 energy channels are available to import gas, coal, and oil products into EU, from several regions in the ROW. Each such channel is described via one trade process with the usual attribute of a TIMES technology: efficiency, technical life, investment cost, annual maintenance cost, availability factor. Each such process is represented at each time period by two variables (activity and capacity) in the TIMES Linear Program, and these two variables are linked by constraint (1) of Section 2.

As explained in Section 2, the availability factors are assumed uncertain. We make the (somewhat arbitrary) assumption that all availability factors have a range of uncertainty from 0 to 1 (a very unfavorable situation), which means that $d_k = 1,\ k = 1, \ldots, 67$. Finally, in order to make use of Proposition 1 of Section 2, we need to know the means of the availability factors, which we assume to be all equal, and set at two alternative values: 0.6 and 0.8. These assumptions mean that the corresponding values of μ_k are 0.4 and 0.2 respectively, both values satisfying the assumption required for the Proposition. Note that a mean of 0.6 is a pessimistic assumption implying that AF_k may become quite low with sizable probability.

With these assumptions, we made eight runs of the model, by combining two alternative values for the μ_k and four values for the guaranteed probability of satisfying the random capacity constraint (4). We call this probability the *reliability of energy supply.*

The two selected μ_k values are 0.4 and 0.2. The four reliability levels are: 0 (reference scenario[2]), 0.72, 0.90, and 0.95. (Note that the 90% reliability has a failure probability double that of the 95% reliability)

Surprisingly enough, there was little difference in the results when the value of μ_k was changed from 0.4 to 0.2. Therefore, we shall present only the four runs with $\mu_k = 0.4$, (this value of μ_k poses a larger challenge to the method, as it is closer to the upper bound of 0.5 assumed in the Proposition). Values larger than 0.5 could be envisioned at the expense of a modification of Proposition 1. We do not feel that it is worth considering such an unreliable situation.

Table 1 shows the raw results for the four main runs, consisting of the quantities of energy imported via the 67 channels at each period (2025, 2035, 2045, 2055). Note that many channels are never used in any of the runs. Only 29 channels are used in at least one run. However, the statistics discussed below are calculated for the entire set of imports, with values at 0 for the unused channels.

3.2 Cost-reliability trade-offs

The first and most important result is shown in Figure 2 depicting the trade-off between system cost for EU and the overall reliability of the EU energy supply, defined above as the probability that random constraint (4) is satisfied. The extra costs for improving reliability range from 175 B\$ to 230 B\$, i.e. from 0.52% to 0.68% of total EU cost. A more naïve way to improve reliability – such as an across-the-board increase of all channel capacities, would have resulted in larger costs for the same level of reliability. This constitutes a strong argument in favor of the ACCP method used in this research. The slow growth of the extra cost is of course linked to the fact that the constraint (5a) involves the expression $\sqrt{\ln 1/\epsilon}$, itself a very slowly growing function of $\ln 1/\epsilon$.

It is however worth noting that a reliability of 1 cannot be achieved by the approach taken, since, when ϵ approaches 0, the above expression grows to infinity, however slowly

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[2] In the Reference scenario, the true probability of satisfying the random constraint is not known. It may be strictly positive, but there is no obvious way to bound this probability. By convention we assign a 0 reliability level for the reference scenario.

Period		2025				2035				2045				2055			
Channel	$1-\epsilon$	0	0.72	0.90	0.95	0	0.72	0.90	0.95	0	0.72	0.90	0.95	0	0.72	0.90	0.95
Coal From AFR		596.6	303	303	303	0	0	543.4	424.3	0	0	0	0	0	0	0	0
Coal From AUS		136.5	136.5	136.5	136.5	0	0	0	0	0	0	0	0	0	0	0	0
Coal From CAC		527.1	2582.9	1795.4	1456.5	1836	1516.1	543.4	424.3	1836	1282.5	759.8	677.8	1603	1143.7	765.6	685.2
Coal From CSA		141.8	141.8	141.8	141.8	0	0	429.3	424.3	0	0	522.7	677.8	0	17.9	765.6	685.2
Coal From ODA		83.5	83.5	83.5	83.5	0	0	0	0	0	0	0	0	0	0	0	0
Coal From OEE		106.5	106.5	106.5	106.5	0	0	0	0	0	0	0	0	0	0	0	0
Coal From RUS		230	230	230	230	0	0	0	0	0	0	0	0	0	0	0	0
Coal From USA		73.2	73.2	73.2	73.2	0	0	0	243.3	0	0	0	56	0	0	0	160.8
Natural Gas From AFR		2637.6	2637.9	2613.4	2613.4	2560.2	2443.9	2419.4	2419.4	2366.2	1024.9	1024.6	1025.1	1816.6	552.8	532.4	533.3
Natural Gas From CAC		5605.8	5605.5	5630	5630	2984.4	0	0	0	0	0	0	0	0	0	0	0
Natural Gas From OEE		718.1	718.1	718.1	718.1	0	0	0	0	0	0	0	0	0	0	0	0
Natural Gas From RUS		1049	1049	1049	1049	0	0	0	0	0	0	0	0	0	0	0	0
Crude Oil From AFR		370.6	991.2	991.2	991.2	0	206.8	488	378.7	0	1433.4	759.8	677.8	0	1143.7	765.6	685.2
Crude Oil From CAC		0	0	0	0	0	0	543.4	424.3	0	0	0	0	0	0	765.6	685.2
Crude Oil From CAN		0	0	0	0	0	0	0	424.3	0	0	0	0	0	0	0	0
Crude Oil From CSA		0	0	0	0	0	0	52.4	52.4	0	0	759.8	677.8	0	1143.7	765.6	685.2
Crude Oil From MEA		0	0	0	0	0	0	0	0	0	0	0	13	0	698.2	698.2	685.2
Crude Oil From RUS		2227.9	2227.9	2227.9	2227.9	0	0	543.4	424.3	0	0	0	205.8	0	0	524	685.2
Crude Oil From USA		0	0	0	0	0	684.1	543.4	424.3	0	0	759.8	677.8	0	542.9	765.6	685.2
Gasoline From AFR		0	0	0	0	0	0	0	0	0	0	0	0	877.5	1091.9	765.6	685.2
Gasoline From CSA		0	0	0	0	0	0	0	0	742	517	671.2	677.8	742	522.1	571.7	685.2
Nat Gas liquids From AFR		80.4	180.2	180.2	180.2	0	0	0	0	0	0	0	0	0	0	0	0
Nat Gas liquids From RUS		70.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude Oil From AFR		2544.4	1923.8	1923.8	1923.8	1749	1542.2	1261	1370.3	1749	1433.4	1261	1370.3	874.5	1143.7	765.6	685.2
Crude Oil From CSA		0	544.8	544.8	544.8	0	1542.2	543.4	424.3	0	1433.4	759.8	677.8	0	1143.7	765.6	685.2
Crude Oil From MEA		3812.2	3812.2	3812.2	3812.2	4519.2	2287.3	2287.3	2287.3	4854	2287.3	2287.3	2287.3	6083.9	1143.7	1143.7	1143.7
Gasoline From AFR		0	0	0	0	0	0	0	0	867.3	867.3	759.8	677.8	867.3	872.1	765.6	685.2
Gasoline From CSA		0	0	0	0	0	0	0	0	0	0	520.1	677.8	0	0	557.5	685.2
Gasoline From MEA		0	0	0	0	936.8	38.3	38.3	38.3	1326.5	1326.8	759.8	677.8	939.1	939.7	765.6	685.2

Table 1: Imports of energy by EU.

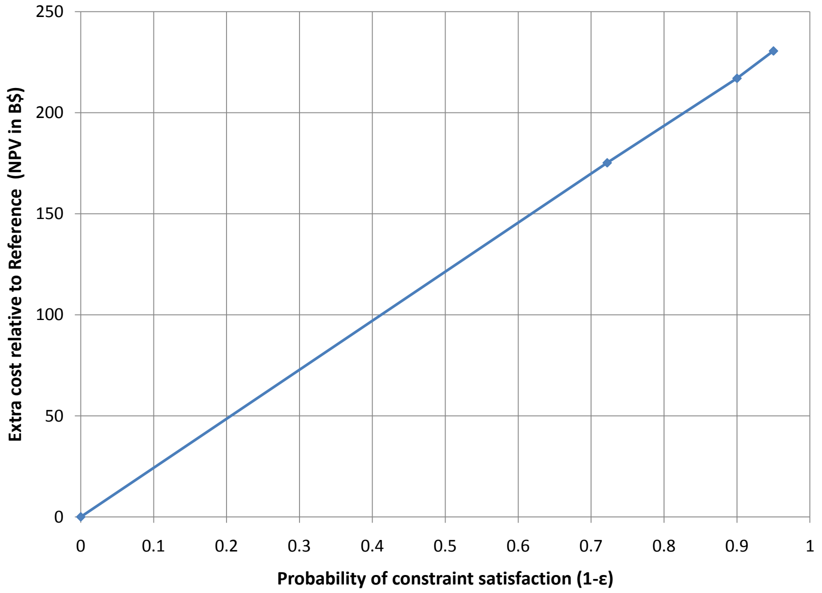

Figure 2: Cost-reliability trade-off for 4 runs

that may happen. In order to further explore the cost-reliability trade-off issue, we made two additional runs, with reliabilities equal to 0.99 and 0.999, respectively. The resulting costs are depicted in figure 3. For those reliability values that are very close to 1, the cost rises faster than linearly, but still very moderately.

3.3 Impact of extra investments on channel utilization

In order to further appreciate how the method chooses to achieve the desired reliability, we observe in Figure 4 the overall channel utilization (averaged over all channels). The expected drop in utilization compared to Reference, is apparent in Figure 4, and is caused by extra investments in channel capacities in order to insure reliability. The interesting fact is that large extra investments occur as soon as the mildest reliability is enforced ($1-\epsilon = 0.72$) but increasing reliability does not induce much extra investments in capacities. This observation is fully congruent with the slowly growing extra cost observed above.

3.4 Impact on energy imports and on primary energy

Another useful and interesting result concerns the impact of increasing reliability on the total amount of energy imported by EU, shown in Figure 5. We observe that energy imports slightly increase in 2025 and decrease significantly at later periods when reliability increases. The decrease ranges from 30% in 2030 to about 20% in 2055. This is an important result showing the resilience of the EU energy system in the medium and long terms

We also wanted to check if increasing the reliability of supply had an impact on total primary energy consumed by EU. These results are shown in the four Figure 6. Here however, we see that the total quantity of primary energy consumed in EU is little affected by the level of supply security. We conclude that the increase in reliability is mainly achieved via shifts from imported to locally produced energy. In a sense this is a reassuring result, which indicates that the solution provided by ACCP is indeed

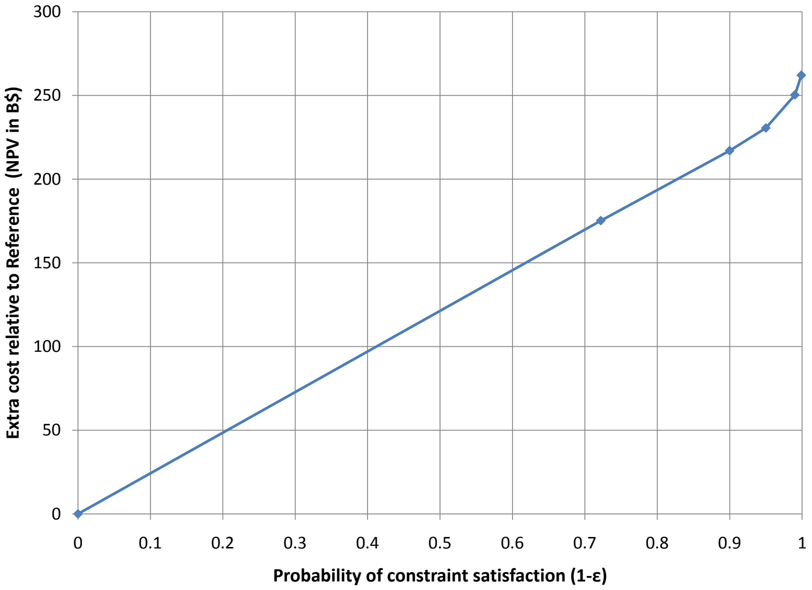

Figure 3: Cost-reliability trade-off for 6 runs

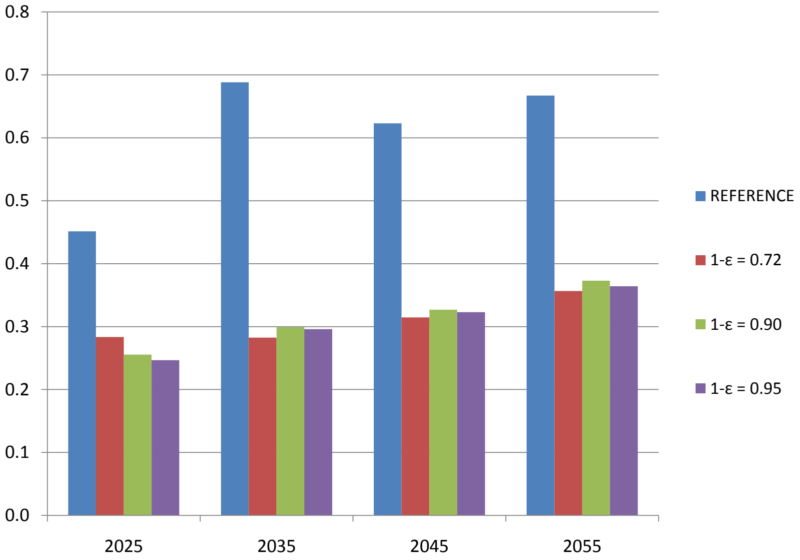

Figure 4: Average utilization (over all channels) of energy import channels

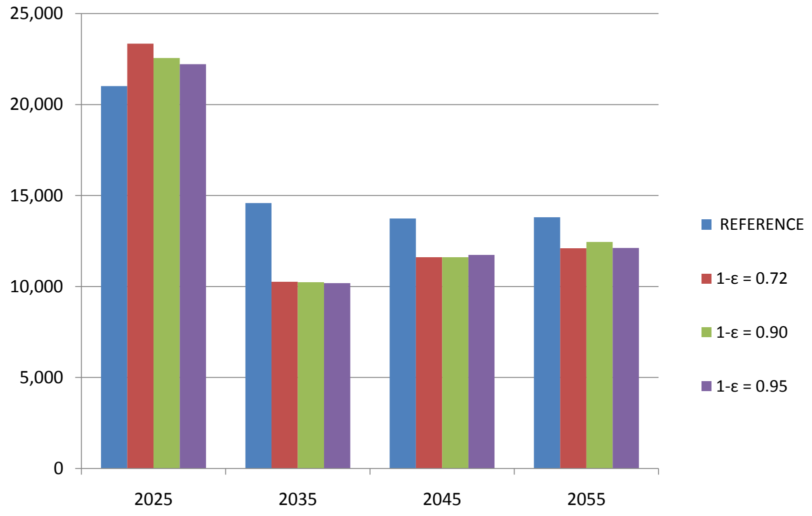

Figure 5: EU energy imports (PJ/yr)

implementable.

3.5 Other measures of security

Several indexes of security use the concept of diversification of supply as a proxy for security. This approach is based on the intuitively appealing notion that “all eggs should not be put in the same basket”. The literature also confirms that the concentration of supply in the hands of one or a few suppliers entails market power and thus the risk of creating quasi-monopolies, cartels, and increases the risk of price control and/or interruptions of supply.

We use four indexes of concentration and evaluate the impact of the robust solutions on these indexes:

- The first and simplest index is the maximum flow carried by the 67 channels. When the Max flow is reduced, flows tend to equalize and market dominance is reduced.
- The second index is the observed coefficient of variation (ratio of standard deviation over mean) of the 67 flows.
- The third index is the Hirschmann-Herfindahl index [18] which is the sum of squares of the channels’ market shares, as per the following formula:

$$H\text{-}H = \sum_{k=1}^{K} MS_k^2$$

where

$$MS_k = ACT_k / \sum_j ACT_j .$$

The H-H index has minimum value equal to $1/K$ when all market shares are equal, and maximum value of 1 when there is a single supplier.

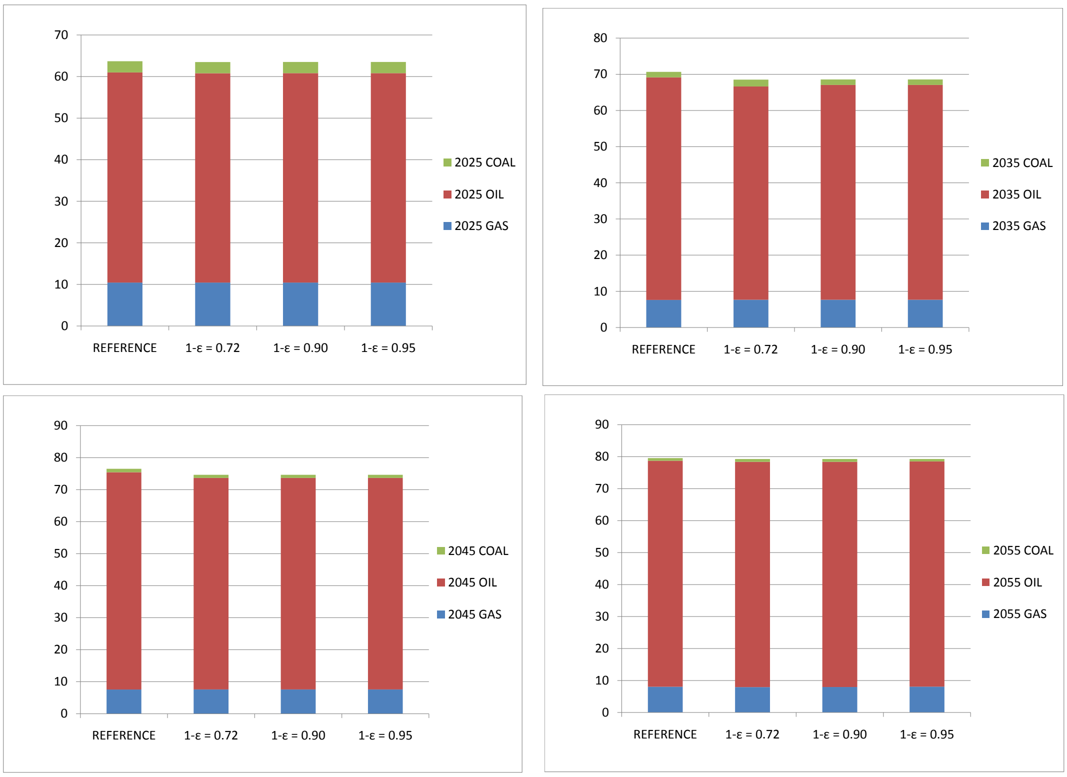

Figure 6: Total primary fossil energy in EU (EJ/yr)

- The fourth index is the Shannon-Wiener index [18, 15], defined by the following formula:

  $$S\text{-}W = 1 + \frac{\sum_{k=1}^{K} MS_k \times \ln(MS_k)}{\ln(K)}.$$

  The S-W index has minimum value of 0 when all market shares are equal, and maximum value of 1 when there is a single supplier.

Figure 7 shows the maximum flow value for each reliability level and period, Figure 8 shows the values of the coefficient of variation for each reliability level and period, Figure 9 shows the values of the H-H index for each reliability level and period, and Figure 10 shows the values of the S-W index for each reliability level and period.

These four indexes are affected in very much the same way when the reliability of inequality (4) increases: in 2025, not much impact is observed, due to the system's inertia. As time goes on, the four indexes show dramatic decreases compared to REFERENCE, and the decrease is more significant when the reliability level $(1-\epsilon)$ increases. The H-H index reaches a low level of 6% in 2055, for $\epsilon = 0.05$, close to its minimum possible level of 1.5% if all market shares were equal. The S-W index reaches 13% for $\epsilon = 0.05$ in 2055. At earlier periods such as 2035 and 2045, the decreases of these two indexes are less dramatic but still quite large.

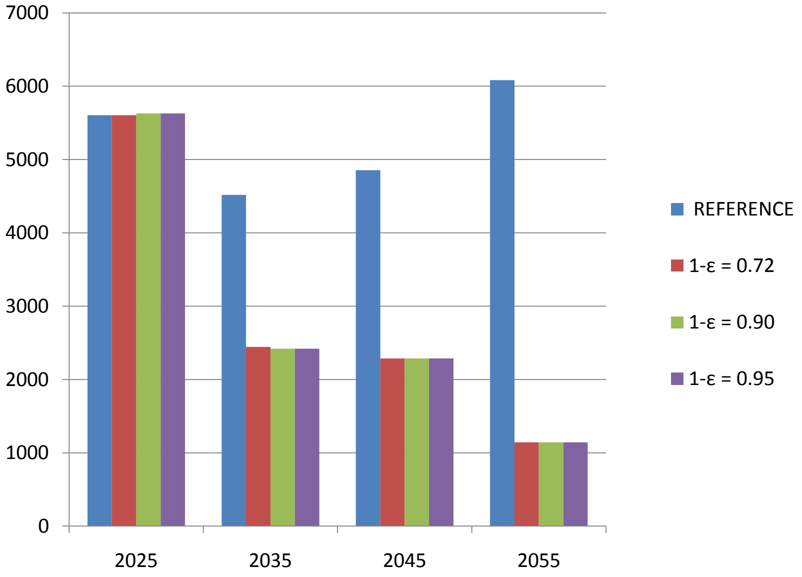

Figure 7: Value of the maximum flow through channels (PJ/yr)

We complete these observations on the alternative measures of concentration by showing the trade-offs between EU extra cost and the last two indexes (Figure 11). The H-H index decreases almost linearly when the EU cost increases, whereas the S-W index decreases at first slowly, and then more rapidly as cost increases. In view of the small additional cost of guaranteeing a 95% reliability of supply (compared to a 90% reliability), it seems recommendable to adopt the higher value since the 95% case is vastly superior in term of reliability.

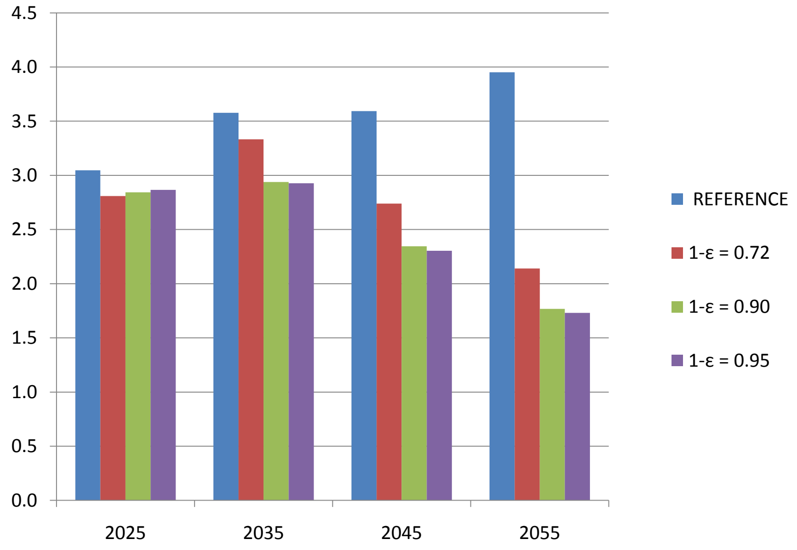

Figure 8: Values of the coefficient of variation of channel flows

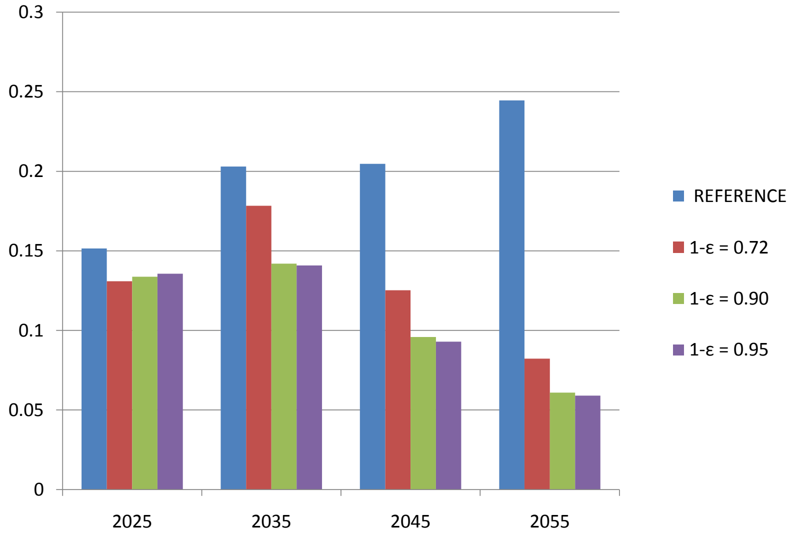

Figure 9: Values of the Hirschmann-Herfindahl Index of concentration

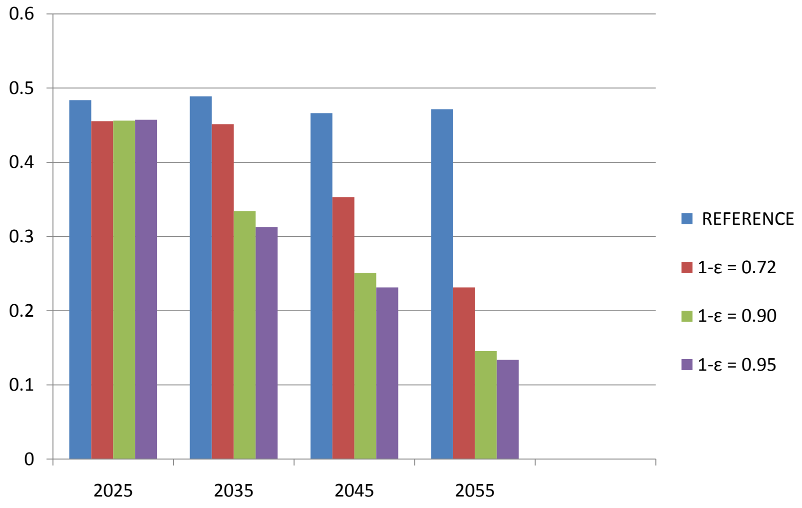

Figure 10: Values of the Shannon-Wiener Index of concentration

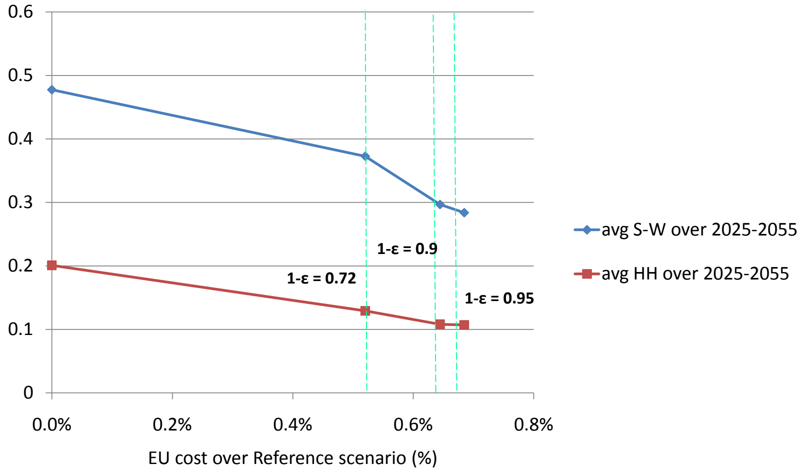

Figure 11: Trade-offs between EU cost and two indexes of concentration

4 Discussion and Conclusion

4.1 Summary of findings from the case study

The results obtained for the case study exhibit several interesting features regarding the security of EU energy supply. First, it appears that the supply of energy can be guaranteed with a known probability, under the very mild assumption that the mean of the random availability factors be known, or bounded at some level higher than half of the range. Second, such reliability is achieved at what may be considered moderate an extra cost, not exceeding 0.7% of the total EU energy cost. Moreover, the results, in addition to ensuring a degree of reliability, contribute very significantly to reduce the concentration of supply sources, a feature that is desirable in itself. The four indexes of concentration used in the article all decrease quite dramatically when the ACCP solution is used. Finally, the method is easy to formulate and apply and does not increase the computational effort in any significant manner.

4.2 Conclusions on the Robust Optimisation approach

As mentioned above the RO approach from the viewpoint of ACCP has allowed to model uncertainty on energy supply making only minimalist assumptions on probability distributions but ensuring a high level of reliability. The resulting formulation has also the advantage, and not the least, to be numerically tractable. Let us review some directions for future research. First we believe that the method can be used to model other important sources of uncertainty in TIAM concerning, e.g., the endogenous climate module with the uncertainty on the climate sensitivity parameter or the deployment of future backstop technologies such that carbon capture and sequestration. Second for numerical considerations the dynamic issue of the uncertainty has not been treated in this study. In [1], the authors show the way to implement adaptive decisions efficiently with respect to the resolved uncertainty on a small dynamic energy model. We believe that we could extend the ideas presented in [1] to the larger TIAM model to enrich our understanding of the impacts of uncertainty on those planning models.

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Appendix

A Proof of Proposition 1

Proposition 1 is a special case of a more general theorem in RO, by taking into account that the variables CAP_k's are nonnegative. For the interested readers, we state now the general theorem and prove Proposition 1 as a corollary. In order to show the derivation we shall use the concise notation

$$z_0 + \sum_k z_k \xi_k \leq 0 \tag{6}$$

to represent the inequality (4). The coefficients $\hat{z}$ are easily identified as

$$\begin{aligned} z_0 &= \sum_k (ACT_k - CAP_k) \\ z_k &= d_k \cdot CAP_k. \end{aligned}$$

The main result can be formulated as

Theorem 1 *Let η_k be K independent random variables with common support $[-1,1]$ and known means $E(\eta_k) = \nu_k$. The probabilistic inequality $\hat{z}_0 + \sum_k \hat{z}_k \eta_k \leq 0$ is satisfied with probability at least $(1-\epsilon)$ if there exists a vector $w \in \mathbb{R}^k$ such that the deterministic inequality*

$$\hat{z}_0 + \sum_k \hat{z}_k \nu_k + \sum_k (|w_k| - w_k \nu_k) + \sqrt{2K \ln \frac{1}{\epsilon}} \max_k |\hat{z}_k - w_k| \leq 0 \tag{7}$$

is satisfied.

Note that the range of the random factors is now $[-1,1]$. The above theorem is the formal statement of the theory for inequalities with random factors having known means ν_k and common range $[-1,1]$ as discussed in [3, example 2.4.9, p. 55]. For the sake of completeness we propose a proof of Theorem 1. This technical derivation is given in the next section.

We show now how to prove Proposition 1 as a corollary of Theorem 1.

Proof: [Proposition 1]

Let us start with (6) and define the variables $\eta_k = 2\xi_k - 1$. In view of Assumption 1 the range of η_k is $[-1,1]$ and $E(\eta_k) = \nu_k = 2\mu_k - 1 \leq 0$. Inequality (6) becomes

$$z_0 + \frac{1}{2}\sum_k z_k + \frac{1}{2}\sum_k z_k \eta_k \leq 0.$$

Let $\hat{z}_0 = z_0 + \frac{1}{2}\sum_k z_k$ and $\hat{z}_k = z_k/2$. The hypotheses of Theorem 1 for the inequality $\hat{z}_0 + \sum_k \hat{z}_k \eta_k \leq 0$ are verified. Hence,

$$\hat{z}_0 + \sum_k \hat{z}_k \nu_k + \sum_k (|w_k| - w_k \nu_k) + \sqrt{2K \ln \frac{1}{\epsilon}} \max_k |\hat{z}_k - w_k| \leq 0$$

is a sufficient condition to ensure the constraint satisfaction with probability at least $(1-\epsilon)$. If we substitute ν_k, $\hat{z}_0$ and $\hat{z}_k$ by their values, we obtain the condition

$$z_0 + \sum_k \mu_k z_k + \sum_k (|w_k| + w_k - 2\mu_k w_k) + \sqrt{\frac{K}{2} \ln \frac{1}{\epsilon}} \max_k |z_k - 2w_k| \leq 0. \tag{8}$$

Recall that $z_k = d_k CAP_k \geq 0$. We claim that only positive values $w \geq 0$ need to be considered. Indeed, the theorem does not specify the value it should take. In particular, we can choose w so as to have to minimize the right-most component $\sum_k(|w_k| + w_k - 2\mu_k w_k) + \sqrt{\frac{K}{2}\ln\frac{1}{\epsilon}}\max_k|z_k - 2w_k|$. If for some k', $w_{k'} < 0$, then $|w_{k'}| + w_{k'} = 0$ and the contribution of term k' in the summation is $-2\mu_{k'}w_{k'} + \max\{z_{k'} - 2w_{k'}, \max_{k\neq k'}|z_k - w_k|\}$. Clearly this term can be made smaller by taking $w_{k'} = 0$. Hence, we can assume $w_{k'} \geq 0$.

With $w \geq 0$ inequality (8) becomes

$$z_0 + \sum_k \mu_k z_k + \sum_k (1-\mu_k)2w_k + \sqrt{\frac{K}{2}\ln\frac{1}{\epsilon}}\max_k|z_k - 2w_k| \leq 0.$$

Writing u for $2w$ in the above inequality, we obtain the condition

$$z_0 + \sum_k \mu_k z_k + \sum_k (1-\mu_k)u_k + \sqrt{\frac{K}{2}\ln\frac{1}{\epsilon}}\max_k|z_k - u_k| \leq 0.$$

Using the same argument as before, we easily prove that we can restrict our choice of u to $u \leq z$. Hence, $|z_k - u_k| = z_k - u_k \geq 0$ and using the additional scalar variable $v \geq 0$ we can transform our inequality into

$$\begin{array}{rcl} z_0 + \sum_k \mu_k z_k + \sum_k (1-\mu_k)u_k + \sqrt{\frac{K}{2}\ln\frac{1}{\epsilon}}\cdot v & \leq & 0 \\ z_k - u_k & \leq & v,\ k = 1,\ldots,K \\ u \geq 0, v \geq 0. & & \end{array}$$

This concludes the proof of the proposition. ■

B Proof of the main theorem

The scheme is based on an immediate consequence of Markov inequality.

Lemma 1 *Let X be a real random variable. Then*

$$Prob(X > 0) \leq \inf_{t>0} E(\exp(tX))dF_X. \tag{9}$$

Proof: The proof is based on the following observations. First, for all $t > 0$, $\text{Prob}(X > 0) = \text{Prob}(tX > 0)$. Second, $\int \mathbb{I}(tX > 0)dF \leq \int \exp(tX)dF$ where $\mathbb{I}(a \geq 0)$ is the indicator function with $\mathbb{I}(a \geq 0) = 1$ is $a \geq 0$ and 0 otherwise. ■

The problem of interest is to bound the probability of $\hat{z}_0 + \sum_k \hat{z}_k\eta_k > 0$ when the η_k are independent random variables with support $[-1, 1]$ and mean $E(\eta_k) = \nu_k$. Setting $X = \hat{z}_0 + \sum_k \hat{z}_k\eta_k$ we get in the right-hand side of (9)

$$E(\exp(tX)) = E(\exp(t\hat{z}_0 + t\sum_k \hat{z}_k\eta_k)) = \exp(t\hat{z}_0)\prod_k E(\exp(t\hat{z}_k\eta_k)).$$

Here, we used, as announced, the property that the generating function of a sum of independent variables is the product of the individual generating functions.

Let us now turn our attention to the worst case distribution. To this end, let us perform changes in notation $\tau_k = t\hat{z}_k$ and $s_k = \eta_k$. The next result holds the same for all indices k. So we drop for a while the subscript k in the proposition below.

Proposition 2 *The worst distribution in $\mathcal{F}$, $\bar{F} = \arg\max_{F\in\mathcal{F}}\int \exp(ts)dF(s)$, is the two-point distribution, with $s = 1$ with probability $p = \frac{1+\nu}{2}$ and $s = -1$ with probability $q = 1 - p = \frac{1-\nu}{2}$.*

The proposition is rather intuitive, but not so easy to prove rigorously. We refer to [3] for a proof.

In view of the above proposition

$$E(\exp(\tau s)) \leq g_\nu(\tau) = pe^{\tau} + qe^{-\tau} = \frac{1+\nu}{2}e^{\tau} + \frac{1-\nu}{2}e^{-\tau}.$$

It is easier to work with the logarithm $h_\nu(\tau) = \log g_\nu(\tau)$.

$$\begin{aligned} h_\nu(\tau) &= \log(pe^{\tau} + qe^{-\tau}) \\ h'_\nu(\tau) &= \frac{pe^{\tau} - qe^{-\tau}}{pe^{\tau} + qe^{-\tau}} \\ h''_\nu(\tau) &= 1 - \left(\frac{pe^{\tau} - qe^{-\tau}}{pe^{\tau} + qe^{-\tau}}\right)^2. \end{aligned}$$

Taylor expansion of order 2 yields

$$h_\nu(\tau) = h_\nu(0) + h'_\nu(0)\tau + h''_\nu(\bar{\tau})\frac{\tau^2}{2}$$

for some $\bar{\tau}$ between 0 et τ. One easily check that $h_\nu(0) = \log(1) = 0$, $h'_\nu(0) = p - q = \nu$ and for all t, $0 \leq h''_\nu(\tau) \leq 1$. Hence

$$h_\nu(\tau) \leq \nu\tau + \frac{\tau^2}{2}.$$

Coming back to (9), we replace τ_k by its value and get

$$\inf_{t>0} E(\exp(tX))dF_X \leq \inf_t \exp\left(t(\hat{z}_0 + \sum_k \nu_k \hat{z}_k) + \frac{t^2}{2}\sum_k \hat{z}_k^2\right).$$

If $\hat{z}_0 + \sum_k \nu_k \hat{z}_k < 0$, the infimum in t is a minimum and is achieved at

$$t_{opt} = -\frac{\hat{z}_0 + \sum_k \nu_k \hat{z}_k}{\sum_k \hat{z}_k^2} > 0.$$

The hypothesis $\hat{z}_0 + \sum_k \nu_k \hat{z}_k < 0$ just means that the constraint is strictly satisfied in the average. We have thus

$$\text{Prob}(\hat{z}_0 + \sum_k z_k \hat{\xi}_k > 0) \leq \exp\left(-\frac{(\hat{z}_0 + \sum_k \nu_k \hat{z}_k)^2}{2\sum_k \hat{z}_k^2}\right).$$

A sufficient condition to have the right-hand side bounded by ϵ is

$$(\hat{z}_0 + \sum_k \nu_k \hat{z}_k)^2 \geq 2\ln\frac{1}{\epsilon}||\hat{z}||_2^2.$$

Since $\hat{z}_0 + \sum_k \nu_k \hat{z}_k \leq 0$, the above condition becomes

$$\hat{z}_0 + \hat{z}^T\nu + \sqrt{2\ln\frac{1}{\epsilon}}\ ||\hat{z}||_2 \leq 0.$$

When this deterministic constraint is satisfied, we can guarantee

$$\text{Prob}(\hat{z}_0 + \sum_k \hat{z}_k \eta_k > 0) \leq \epsilon.$$

We just proved

Proposition 3 *The deterministic constraint* $\hat{z}_0+\hat{z}^T\nu+\sqrt{2\ln\frac{1}{\epsilon}}\;||\hat{z}||_2 \leq 0$ *implies* $Prob(\hat{z}_0+\sum_k \hat{z}_k\eta_k > 0) \leq \epsilon$.

The proposition can be strengthened if we take advantage of the known range of η. For any w and in view of $-\mathbf{1} \leq \eta \leq \mathbf{1}$

$$\hat{z}_0 + \hat{z}^T\eta = \hat{z}_0 + w^T\eta + (\hat{z}-w)^T\eta \leq \hat{z}_0 + ||w||_1 + (\hat{z}-w)^T\eta.$$

Set $\bar{z}_0 = \hat{z}_0 + ||w||_1$ and $\bar{z} = \hat{z} - w$, we have from Proposition 3

$$\bar{z}_0 + \bar{z}^T\nu + \sqrt{2\ln\frac{1}{\epsilon}}\;||\bar{z}||_2 \leq 0 \;\Rightarrow\; \text{Prob}(\bar{z}_0 + \sum_k \bar{z}_k\eta_k > 0) \leq \epsilon.$$

Replacing $\bar{z}_0$ and $\bar{z}$ by their value, we have

$$\hat{z}_0 + \hat{z}^T\nu - w^T\nu + ||w||_1 + \sqrt{2\ln\frac{1}{\epsilon}}\;||\hat{z}-w||_2 \leq 0 \;\Rightarrow\; \text{Prob}(\hat{z}_0 + \sum_k \hat{z}_k\eta_k > 0) \leq \epsilon.$$

Finally by bounding the ℓ_2-norm such that $||a||_2 \leq \sqrt{K}\max_k |a_k|$ we prove Theorem 1

$$\hat{z}_0 + \hat{z}^T\nu - w^T\nu + ||w||_1 + \sqrt{2K\ln\frac{1}{\epsilon}}\;\max_k |\hat{z}_k - w_k| \leq 0 \;\Rightarrow\; \text{Prob}(\hat{z}_0 + \sum_k \hat{z}_k\eta_k > 0) \leq \epsilon.$$

C Discussion on the dynamic issue

In the sketch of methods to handle uncertainty, we did not pay attention to the dynamic feature of the TIAM model. In a multistage problem as this one, decisions are taken sequentially. Decisions at stage t need not be fixed prior to that stage, and at stage t the past history of realizations of the uncertain factors is known. This information should be used to design "optimal decisions". Conceptually, this can be done by replacing actual decisions by contingent decisions, often named "recourses". The framework of Stochastic Programming [8] is perfectly designed to handle this situation. Unfortunately, solving multistage uncertain linear programs even with moderate precision is already a challenge beyond all available optimization techniques. We refer to [3, pp. 411–413] for an enlightening discussion on the "dramatic theoretical gap between what Multi-Stage Programming intends to achieve and what, if any, it achieves".

A possible remedy to this dramatic state of affair is to restrict recourses to the class of affine decision rules. In this formulation, future decisions are affine functions of the past realizations of the uncertain coefficients. In so doing, the new variables in the mathematical programming formulation are not any more the actual decisions, but the coefficients of the affine functions. When embedded in a robust optimization framework, this approach leads to numerically tractable problems. Affine decision rules is not the panacea, because it is in essence sub-optimal (recourses are restricted to a very small subset of all possible functions), but it offers an attractive alternative that has proved very useful in solving practical problems.

Clearly, there is a need in our problem of interest to say what is to be done in the event of a corridor failure. In the light of the above discussion, this is definitely a major challenge. Robust Optimization with affine decision rules could be used. Some works on similar problems has been done in [1], but on instances of considerably smaller dimensions. In the present problem, introducing affine decision rules to decide on the supply routes and levels to match the actual realizations of capacities would spread uncertainty throughout the model. Straightforward application of the Robust Optimization machinery would convert the new uncertain program into a far too large model to be treated by existing computers and commercial solvers. Our choice has been to ignore the adjustment feature that is intrinsic to multistage problems under uncertainty. Therefore, we shall be content to use RO on various time periods, in order to reveal robust configurations of the corridors but falling short of indicating remedial actions for each outcome of the random events.