Robust Unit Commitment Problem with Demand Response and Wind Energy

Long Zhao, Bo Zeng

Department of Industrial and Management Systems Engineering, University of South Florida, Tampa, FL 33620

Email: longzhao@mail.usf.edu, bozeng@usf.edu

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Abstract

To improve the efficiency in power generation and to reduce the greenhouse gas emission, both Demand Response (DR) strategy and intermittent renewable energy have been proposed or applied in electric power systems. However, the uncertainty and the generation pattern in wind farms and the complexity of demand side management pose huge challenges in power system operations. In this paper, we analytically investigate how to integrate DR and wind energy with fossil fuel generators to (i) minimize power generation cost; (2) fully take advantage wind energy with managed demand to reduce greenhouse emission. We first build a two-stage robust unit commitment model to obtain day-ahead generator schedules where wind uncertainty is captured by a polyhedron. Then, we extend our model to include DR strategy such that both price levels and generator schedule will be derived for the next day. For these two NP-hard problems, we derive their mathematical properties and develop a novel and analytical solution method. Our computational study on a IEEE 118 system with 36 units shows that (i) the robust unit commitment model can significantly reduce total cost and fully make use of wind energy; (ii) the cutting plane method is computationally superior to known algorithms.

Key Words: unit commitment, demand response, wind energy, robust optimization, cutting plane

1 Introduction

To supply high-quality electric power to customers in a secured manner is becoming more and more challenging due to that electricity demand in U.S. is increasing rapidly year by year in response to population growth, economic growth, and warmer global weather. In traditional power systems, electric load is principally met by thermal power units that use coal, gas, or other fossil fuels. Usually the operation cost of running thermal generators, which consists of start up cost, fixed cost, fuel cost, and market cost(in case of purchasing power from market pool), is very high in power industry. Therefore based on economic criteria, to use an optimum strategy to schedule the on/off status and generation level of those generators is so important that millions of dollars can be saved per year for large utilities if the cost is reduced only by 0.5%(Baldick (1995)). However, the scheduling operation which is known as the classical unit commitment problem (UC) is hard to solve because complex and critical technical constraints are to be subjected such as (i) generation capacity limit of each unit, (ii) minimum time period a unit should be on(off) once it is start up(shut down), (iii) maximum rate that the generation level can ramp up or down from this time period to the next. Many deterministic and stochastic UC problems as the extension of the classical UC problem have been solved by various approaches during the past two decades: Frangioni and Gentile (2006) solved the deterministic single unit commitment problem using dynamic programming approach; While Takriti et al. (2000) presented a stochastic model that incorporates fuel constraints and electricity spot prices and used Lagrangian relaxation and benders decomposition to solve it; More previous research about UC problems could be referred to a survey by Padhy (2004).

However, due to the fast-paced policies and emerging techniques in power market, there is an urgent need to pursue the optimum scheduling strategy of new unit commitment problems that will bloom in the future and could be quite different from classical ones.

On one hand, the usage of renewable and low-carbon sources such as hydro, wind, wave-power, and solar are expected to increase a lot in future years in order to reduce or avoid the environmental impacts of fossil-fueled electricity. Among those resources wind energy is special importance which is expected to provide 20% of U.S. electricity market by 2030(DoE (2008)) starting from 2.4% in 2009(Flowers (2010)). But unlike the conventional thermal generation sources which are stable and controllable, wind power is uncontrollable and highly intermittent. Therefore it is risky to introduce large amount of wind energy into current power systems because the intermittency nature of wind can raise costs for regulation and hamper the reliability of the power system. In order to model its uncertainty, stochastic programming has been widely used by assuming wind output scenarios and related probabilities(Wang et al. (2009), Constantinescu et al. (accepted), Sioshansi and Short (2009), Tuohy et al. (2009)). However, this kind of assumption may not be realistic as in most cases the exact wind distribution is rarely available day-ahead as predictability of wind output remains low for short-term operation. Since the growth of wind resource usage is envisaged before and after 2030, how to integrate this intermittent energy into current power generation with soft prediction that no probability information is provided needs to be addressed in an effective way.

On the other hand, demand response, which enables customers to respond to variable prices at different time periods, is emerging in the demand side management of power market. Because many utilities have daily demand patterns which vary between peak and off-peak hours in a day, ie., people use less electricity in the noon that in the morning, and more electricity is usually used in the afternoon than in the evening. The demand response provide a possibility to reduce peak time load by allowing customers to decide whether and when to curtail or shift their electric consumption based on retail rate designs that charge higher prices during high-demand hours and lower prices at other times. That is, to some reasonable extent the power system operators can control the demand increase/decrease at different times by adjusting the electricity price to customers. See Figure 1 as an example, (b) shows the fixed price and variable prices and the demand before/after response is shown in (c). From Figure 1 we can also notice that demand response can increase the wind usage in high penetration by driving demand into high wind generation periods. Since late 1990s, more and more technical reports in various states have shown its effectiveness based on results of experiments or simulations. Faruqui and Sergici (2009) shows that an experimental program induces a drop in peak demand that ranges between 3% to 6% and a drop by 13% to 20% in critical-peak hours. Neenan et al. (2003) reports that in an experiment in New York 2002, the customers reduced their hourly electricity usage by an average of 34% compared to the baseline and the reliability benefits were estimated to range between $1.697 and $16.9 million. Since early 2000s, demand response resources have significantly increased their market share in organized markets by providing day-ahead or real-time services(Kathan et al. (2010)). Although many experiments have been conducted, there are few research about using mathematical programming method to evaluate the impact demand response has on unit commitment problems and to pursue optimal price-assigning strategy. The previous analytical results of demand response based unit commitment problem are mainly in Su (2007) where no uncertainty exists and deterministic mixed-integer programming models are used, and in Su and Kirschen (2009) but critical ramping constraints and

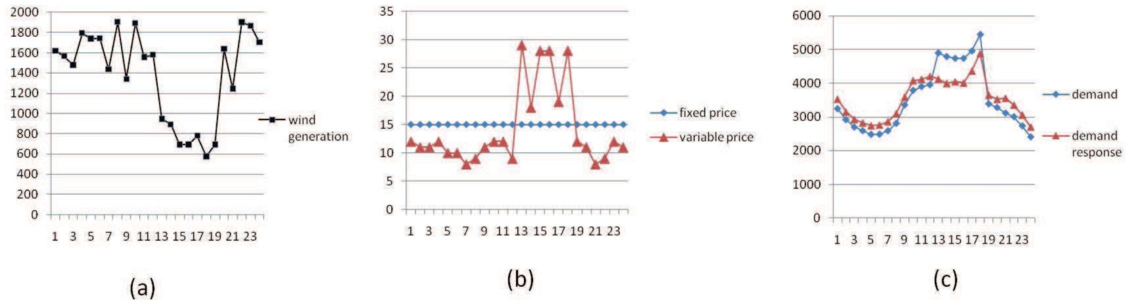

Figure 1: The effect of combining wind and demand response. (a) wind generation, (b) fixed price vs. variable prices, (c) demand vs. demand after price response

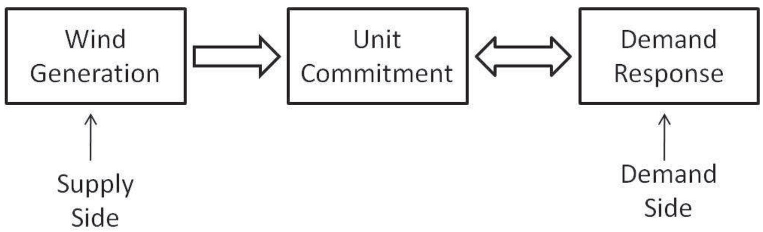

Figure 2: Supply and demand management to unit commitment system.

minimum up/down constraints in UC problems are not considered.

In our paper, we first consider day-ahead unit commitment problem based on intermittent wind energy which is captured by assuming polyhedrons consisting of bound constraints and multiple uncertainty budget constraints $V = \{\underline{v}_t \leq v_t \leq \overline{v}_t, \sum_{t \in T_1} \theta_t v_t \geq V'_1, \sum_{t \in T_2} \theta_t v_t \geq V'_2, ..., \sum_{t \in T_{seg}} \theta_t v_t \geq V'_{seg}\}$. After that a set of price levels $p_l, l = 0, 1, ..., L-1$ and related demand increase/decrease percentage $d_l\%, l = 0, 1, ..., L-1$ is predefined for each hour to seek optimal price-assigning strategy in the context of demand response. The integration of the systems is shown in Figure 2. Both models are NP-hard two-stage robust optimization problems. Robust optimization is a recent methodology to deal with mathematical programming problems affected by uncertain data where no probability distribution is available. Robust optimization theory dates back to Ben-Tal and Nemirovski (1998, 1999, 2000) who considered continuous robust problems and Bertsimas and Sim (2003, 2004) who focused on discrete cases. Ben-Tal and Nemirovski (1998, 1999, 2000, 2008, 2002) showed that given *ellipsoidal* uncertainty assumption, the robust convex program corresponding to some of the most important generic convex optimization problems is tractable and can be exactly or approximately solved via interior point methods, though the robust counterpart of an LP becomes an SOCP and that of an SOCP turns out to be an SDP. However, they didn't extend robust programming approach to discrete problems. Bertsimas and Sim (2003, 2004) proposed a different approach to control the level of robustness in discrete problems which leads to linear optimization problems but the uncertainty data are assumed to be independent, ie., the uncertainty entries in constraint matrix or cost vector are independent variables. Based on those above theories, robust optimization has been applied to several kinds of classical problems in very recent years. Atamturk and Zhang (2007) and Takriti and Ahmed (2004) analyzed robust two-stage network problems; Adida and Perakis (2006), See and Sim (2010), Bertsimas and Thiele (2006),Bienstock et al. (2004) applied robust optimization to inventory control or supply chain management problems; Ghaoui et al. (2003) and Lu (2009) alleviated portfolio problems in a robust perspective; Robust capacitated vehicle routing problem was solved by Fukasawa et al. (2006) using branch-and-cut-and-price approach, and etc.

Our contributions include: (i) To the best of our knowledge, it is the first time that the two-stage robust optimization model is applied to the next generation unit commitment problem with uncertain wind energy supply and demand response; (2) A novel and analytical solution algorithm has been developed to solve this challenging two-stage robust unit commitment model. In fact, its finite convergence is proven and can be applied to solve general two-stage robust optimization problems within reasonable times.

The paper is organized as follows. We first present the two-stage robust UC model with wind uncertainty in Section 2. In Section 3, this model is studied and a novel solution algorithm is developed and its theoretical analysis is performed. In Section 4, UC-wind model is extended to incorporate DR strategy. The computational results and management discussion are presented in Section 5. Section 6 with discussion of future research directions.

2 Wind-UC Model

Traditionally, day-head UC problems involve two sets of decisions that need to be determined in two stages. In the first stage, the generators on/off status need to be determined for the next day such that the resulting plan for those generators meets their physical restrictions. Then, for each particular period, the generation level of each spinning generator will be determined, which could be performed in a real time or nearly real-time environment. Given the penetration of wind energy, such a working fashion gives us a chance to integrate wind energy supply in the second stage so that the partial demand can be met by wind energy and the expensive fossil fuel power generation can be reduced. However, two prominent issues need to be considered: (*i*) wind energy generation is random; (*ii*) power supply must be very reliable while purchasing power from spot market to cover the unsatisfied demand (i.e. energy deficit) is typically very expensive. Such as situation motivates us to build a two-stage robust optimization model for unit commitment with uncertain wind energy supply. In this section, we first describe the classical deterministic unit commitment model which is also the nominal model in this paper. Then, we introduce the uncertainty set to capture the randomness of wind farm output and formulate the two-stage robust optimization counterpart. We finally derive some structural properties of the aforementioned robust optimization model.

2.1 Formulations

We consider the day-ahead unit commitment problem with I thermal units for T time periods. In the remainder of this paper, we follow the convention that one period stands for 60 mins and therefore T equals to 24 while our models and solution method are applicable to any time scale as well. To minimize the operating cost and to meet physical requirements, the following decisions should be made for each time period $t \in T$: (i) The on/off status $y_{it} \in \{0, 1\}$ of each unit $i \in I$. If unit $i \in I$ is on (i.e. $y_{it} = 1$), the running cost will be r_i per hour; (ii) The turn on operation $z_{it} \in \{0, 1\}$. If unit $i \in I$ is turned on at the beginning of period t (i.e. $z_{it} = 1$), the start up cost will be a_i; (iii) The power generation level $x_{it} \geq 0$ of unit i in period t which incurs $g_i(x_{it})$ generation cost; (iv) The amount of power for sale (purchase) $s_t \geq 0$ ($b_t \geq 0$) if the power generated is more (less) than customer demand and the predicted sale(purchase) price is $q_t(e_t)$ in period t in spot market.

Assuming that precise wind energy generation in the next T periods $v_i, i = 1, \ldots, T$ is known, we present next Wind-UC Model by integrating wind energy into classical day-ahead UC-model(Cerisola et al. (2009),Frangioni and Gentile (2006),Takriti et al. (2000)), which also serves as the nominal model to its robust counterpart.

Wind-UC

$$min \sum_{t=0}^{T-1}\sum_{i=0}^{N-1}(g_i(x_{it}) + r_i y_{it} + a_i z_{it}) + \sum_{t=0}^{T-1}(e_t b_t - q_t s_t); \tag{1}$$

$$st. \quad - y_{i(t-1)} + y_{it} - y_{ih} \leq 0, \forall i, t \geq 1, t \leq h \leq min(m_+^i + t - 1, T-1); \tag{2}$$

$$y_{i(t-1)} - y_{it} + y_{ih} \leq 1, \forall i, t \geq 1, t \leq h \leq min(m_-^i + t - 1, T-1); \tag{3}$$

$$- y_{i(t-1)} + y_{it} - z_{it} \leq 0, \forall i, t \geq 1; \tag{4}$$

$$l_i y_{it} \leq x_{it} \leq u_i y_{it}, \forall i, t; \tag{5}$$

$$\sum_{i=0}^{N-1} x_{it} + v_t + b_t - s_t = d_t, \forall t; \tag{6}$$

$$x_{i,t+1} \leq x_{it} + y_{it}\Delta_+^i + (1 - y_{it})u_i, \forall i, t = 0, 1, ..., T-2 \tag{7}$$

$$x_{it} \leq x_{i,t+1} + y_{i,t+1}\Delta_-^i + (1 - y_{i,t+1})u_i, \forall i, t = 0, 1, ..., T-2 \tag{8}$$

$$y_{it}, z_{it} = \{0, 1\}, x_{it}, b_t, s_t \geq 0, \forall i, t;$$

The objective function of Wind-UC model is to minimize total operating cost consisting of start up cost, running cost, fuel cost, and market cost(the cost is positive if buying power from spot market and negative if selling power to spot market). Constraints (2) and (3) are minimum up/down constraints(Takriti et al. (2000)). If the unit $i \in I$ is turned on(off) in one period, it has to stay in the on(off) status for a minimum number of periods, denoted by $m_+^i(m_-^i)$. Constraints (4) stands for start up operation(Cerisola et al. (2009)), that is, unit i is started up at the beginning of period t if its status is off at time $t-1$ and is on at time t. Constraint (5) illustrates the generation capacity of each unit(Frangioni and Gentile (2006)), where l_i and u_i stand for the minimum and maximum output of unit i respectively. Constraint (6) ensures that the customer demand d_t should be satisfied. Constraints (7) and (8) are ramping up/down limits in unit commitment system(Frangioni and Gentile (2006)). These constraints require that the maximum increase in generation level of unit i from one period to the next cannot be more than Δ_+^i. Similarly, Δ_-^i is introduced to restrict the maximum decrease of unit i from period to period.

As discussed earlier, wind energy generation for next T periods in general cannot be precisely estimated. To describe its randomness in our derivation of reliable schedules for generators, we introduce a polyhedral uncertainty set for wind energy generation. Specifically, each individual v_t is bounded by lower bound $\underline{v}_t$ and upper bound $\overline{v}_t$. Aggregated effect over multiple periods are modeled by a *budget* constraint such that the overall wind generation over these periods are greater than or equal to a specific value. To capture the intermittent nature of wind energy generation, we introduce multiple budget constraints over disjoint segments of the planning horizon consisting of consecutive periods. For example, the uncertainty set $\mathbf{V}$ is defined as $v_t \in \mathbf{V} = \{\underline{v}_t \leq v_t \leq \overline{v}_t, \sum_{t\in T_1} \theta_t v_t \geq \mathbf{V}'_1, \sum_{t\in T_2} \theta_t v_t \geq \mathbf{V}'_2, \sum_{t\in T_3} \theta_t v_t \geq \mathbf{V}'_3\}$, where $T_1 = \{0, 1, ..., 7\}$, $T_2 = \{7, 8, ..., 15\}$, and $T_3 = \{16, 17, ..., 23\}$.

Previously we captured the wind uncertainty with cardinality constraints introduced by Bertsimas and Sim Bertsimas and Sim (2003). But we found that this kind of constraint is not sophisticated enough to describe wind uncertainty and actually it could be linearized very easily and is also not computational demanding.

Next, we present the robust counterpart of Wind-UC model as the following semi-infinite programming problem. The most significant difference is that first stage decision variables $\{y_{it}, z_{it}\}$ should be made day-ahead considering uncertain wind energy supply, while the second stage decision variables $\{x_{it}, b_t, s_t\}$ should be made after wind energy supply is completely known.

Robust Wind-UC

$$min_{\{y_{it},z_{it}\in Y\}} \sum_{t=0}^{T-1}\sum_{i=0}^{N-1}(a_i z_{it} + r_i y_{it}) + max_{\{v_t\in\mathbf{V}\}} min_{\{x_{it}\in X, b_t, s_t\geq 0,\}} \sum_{t=0}^{T-1}\sum_{i=0}^{N-1} g_i(x_{it}) + \sum_{t=0}^{T-1}(e_t b_t - q_t s_t); \tag{9}$$

$$st. \; Y = \{(2) - (4); y_{it}, z_{it} = \{0,1\}, \forall i, t;\}$$

$$X = \{x_{it} : (5) - (8), x_{it} \geq 0, \forall i, t;\}$$

$$\mathbf{V} = \{(v_1, \ldots, v_T) : \underline{v}_t \leq v_t \leq \overline{v}_t, \sum_{t\in T_1}\theta_t v_t \geq V'_1, \sum_{t\in T_2}\theta_t v_t \geq V'_2, ..., \sum_{t\in T_N}\theta_t v_t \geq V'_N, T = \bigcup_n T_n\}$$

As Robust Wind-UC reduces to the classical UC problem if no wind power generation is present, the next result directly follows from the fact that a well-known NP-hard problem (Tseng (1996)).

Proposition 1. *Robust Wind-UC problem is NP-hard.*

Note that if the fuel cost function $g_i(x)$ takes the linear function form $g_i(x) = c_{i0} + c_i x$, the inner most min problem becomes a linear programming problem for any given y_{it}, z_{it}, v_t for $i \in I$ and $t \in T$. With this linear fuel cost function, we propose dualize the inner most min problem to drop the min operator and gain better structural insights. Let $\lambda_{it}, \pi_{it}, \varphi_t, \forall i \in I, t \in T$, $\rho_{it}, \delta_{it} \forall i \in I, t \in T/\{T-1\}$, and $\mu_t, \gamma_t, \forall t \in T$ be the dual variables for the inner most min problem, the inner $\max\min$ problem is reformulated as the following bi-linear programming problem.

Bi-linear Form of the Inner Max-min Problem

$$\sum_{i=0}^{I-1}\sum_{t=0}^{T-1}(l_i\hat{y}_{it}\lambda_{it} - u_i\hat{y}_{it}\pi_{it}) + \sum_{t=0}^{T-1}(d_t - v_t)\varphi_t$$
$$- \sum_{i=0}^{I-1}\sum_{t=0}^{T-2}\rho_{it}(\hat{y}_{it}\Delta_+^i + (1-\hat{y}_{it})u_i)$$
$$- \sum_{i=0}^{I-1}\sum_{t=0}^{T-2}\delta_{it}(\hat{y}_{i,t+1}\Delta_-^i + (1-\hat{y}_{i,t+1})u_i) \tag{10}$$

$$st.\ c_i \geq \lambda_{i0} - \pi_{i0} + \varphi_0 + \rho_{i0} - \delta_{i0}, \forall i \in I, t = 0 \tag{11}$$

$$c_i \geq \lambda_{it} - \pi_{it} + \varphi_t - \rho_{i,t-1} + \delta_{i,t-1} + \rho_{it} - \delta_{it}, \forall i \in I, t \in \{1, ..., T-2\} \tag{12}$$

$$c_i \geq \lambda_{i,T-1} - \pi_{i,T-1} + \varphi_{T-1} - \rho_{i,T-2} + \delta_{i,T-2}, \forall i \in I, t = T-1 \tag{13}$$

$$\mu_t + \varphi_t \leq e_t, \forall t \in T \tag{14}$$

$$\gamma_t - \varphi_t \leq -q_t, \forall t \in T \tag{15}$$

$$\lambda_{it}, \pi_{it}, \rho_{it}, \delta_{it}, \mu_t, \gamma_t \geq 0, \varphi_t\ unsigned; (v_1, \ldots, v_T) \in \mathbf{V}$$

As one term in the objective function, $v_t\varphi_t$, is the product of two decision variables, this optimization problem is a typical bi-linear programming problem. With this bilinear programming maximization problem to represent the inner max min problem, the original two-stage Wind-UC problem can be treated as a min max problem. In fact, given the fact that the first stage decision variables only appear in the objective function of the bi-linear problem, we observe that if it can be solved for any given y_{it}, z_{it} for $i \in I$ and $t \in T$, the classical Benders decomposition approach can be adopted to solve the overall min max problem. However, a prominent challenge needs to be addressed: how to efficiently solve this particular bilinear programming problem given the fact that bilinear programming problems are NP-hard in general and currently there is no efficient algorithm. So, it is critical to develop a fast algorithm that explores the underling structure and generate optimal solution in a short time. Also, it is unclear that how effective the cutting plane from Benders approach could be in the solution process? In the next section, we discuss analyze the structure of this bilinear programming problem and present some answers to these questions.

We mention that in the case where the fuel cost function is of the quadratic form $g_i(x) = c_{i2}x^2 + c_{i1}x + c_{0i}$ and is increasing with x, it can always be approximated with multiple linear functions without introducing binary variables and therefore the aforementioned dualization technique still works.

3 Exact Algorithms for Two-stage Robust Wind-UC

In this section, we first identify some important property of the optimal solutions for the bilinear programming problem and derive a reformulation so that it can solved efficiently using any commercial solver. Then, in addition to the naturally obtained Benders cut from the optimal solution to the bilinear programming problem, we develop a novel cutting plane algorithm with analysis on its convergence. We mention that, to the best our knowledge, such a algorithmic procedure has not been reported and it provides an effective strategy to solve the difficult multi-stage robust optimization problem. Our computational experiment in Section 5 confirms its superior performance in comparison to classical algorithms based on Benders cuts.

3.1 Optimal Solution and Benders-dual Cutting Planes

We take the master-subproblem strategy to solve the two-stage robust optimization problems. We first build the master problem based on the first stage min problem as follows.

Master problem of Wind-UC Model

$$min_{\{y_{it},z_{it}\}}\ \vartheta \qquad (16)$$
$$st.(2)-(4);\ \text{(cuts from subproblems)}$$
$$y_{it}, z_{it} \in \{0,1\};$$

Then, we study how to generate valid inequalities from the second stage max and min problems, i.e. the bi-linear programming problem. Because the structure of our bilinear programming problem, we can easily obtain the following result. Recall that the only bilinear terms in the objective function are $v_t\varphi_t$ for $t \in T$.

Proposition 2. *For any given feasible $\{y_{it}, z_{it}\} \in Y$ in Robust Wind-UC problem, one optimal (worst) wind output is a vertex of $\mathbf{V}$. Furthermore, for this vertex, v_t takes value at either $\underline{v}_t$ or $\overline{v}_t$ for $t \in T$, except (at most) one in each segment taking any values between $[\underline{v}_t, \overline{v}_t]$.*

Proof. Note that for a fixed set of dual variables for the inner most min problem, the bi-linear programming problem reduces to a linear programming on $\vec{v} \in \mathbf{V}$. So, the first statement follows directly (Falk (1973)). Moreover, given the optimal dual variables φ_t^*, optimal value of $\vec{v}$ can be determined by a knapsack problem with simple constraints, $\max\{-\sum_{t=1}^{T}\varphi_t^* v_t : \vec{v} \in \mathbf{V}\}$. Because of the structure of $\mathbf{V}$, the second statement follows immediately. $\square$

Based on the result in Proposition 2, the bi-linear term $-\sum_t v_t\varphi_t$ in the bi-linear programming problem in (10-15) can be linearized to be a mixed-integer programming problem using a set of binary variables. The main idea is to use the fact that the order of φ_t/θ_t determines the worst case extreme point of V. So we use binary variables ψ_{ij} to capture the relationship between $\frac{\varphi_i}{\theta_i}$ and $\frac{\varphi_j}{\theta_j}, \forall i, j \in T$. And let $v_t = \underline{v}_t + \varpi_t(\overline{v}_t - \underline{v}_t)$, where ϖ_t is binary variable. In addition, let α_t denote $o_t\varphi_t$, β_{ij} denote $o_i w_j \varphi_i$ and ζ_t denote $\varpi_t\varphi_t$, we have the following MIP formulation corresponding to the bi-linear term.

Linearization technique for bilinear term.

$$-\sum_t v_t\varphi_t = -\sum_t \underline{v}_t\varphi_t - \sum_t \zeta_t(\overline{v}_t - \underline{v}_t) + \sum_{n=1}^{N}\eta_n \tag{17}$$

$$st.\ \ \eta_n \le (\sum_{t\in T_n}\theta_t\underline{v}_t - V')(\sum_{t\in T_n}\frac{\alpha_t}{\theta_t}) + \sum_{i\in T_n}\sum_{j\in T_n}\beta_{ij}(\overline{v}_j - \underline{v}_j), \forall n = 1,2,...,N \tag{18}$$

$$\sum_{t\in T_n}\theta_t\underline{v}_t + \sum_{t\in T_n}\theta_t\varpi_t(\overline{v}_t - \underline{v}_t) - V'_n \ge 0, \forall n = 1,2,...,N \tag{19}$$

$$\sum_{t\in T_n}\theta_t\underline{v}_t + \sum_{t\in T_n}\theta_t\varpi_t(\overline{v}_t - \underline{v}_t) - V'_n \le \sum_{t\in T_n} o_t(\overline{v}_t - \underline{v}_t), \forall n = 1,2,...,N \tag{20}$$

$$o_t \le \varpi_t, \forall t \in T \tag{21}$$

$$\sum_{t\in T_n} o_t = 1, \forall n = 1,2,...,N \tag{22}$$

$$\frac{e_i}{\theta_i}\psi_{ij} \ge \frac{\varphi_i}{\theta_i} - \frac{\varphi_j}{\theta_j}, \forall i,j \in T_n, \forall n = 1,2,...,N \tag{23}$$

$$\psi_{ij} + \psi_{ji} = 1, \forall i \ne j \in T_n, \forall n = 1,2,...,N \tag{24}$$

$$\psi_{ii} = 0, \forall i \in T_n, \forall n = 1,2,...,N \tag{25}$$

$$\varpi_i \ge \varpi_j - \psi_{ij}, \forall i,j \in T_n, \forall n = 1,2,...,N \tag{26}$$

$$\alpha_t \le \varphi_t, \forall t \tag{27}$$

$$\alpha_t \ge \varphi_t - (1 - o_t)e_t, \forall t \tag{28}$$

$$\alpha_t \le o_t e_t, \forall t \tag{29}$$

$$\beta_{ij} \le \varphi_i, \forall i,j \in T \tag{30}$$

$$\beta_{ij} \ge \varphi_i + (o_i + w_j - 2)e_i, \forall i,j \in T \tag{31}$$

$$\beta_{ij} \le o_i e_i, \forall i,j \in T \tag{32}$$

$$\beta_{ij} \le w_j e_i, \forall i,j \in T \tag{33}$$

$$\zeta_t \le \varphi_t, \forall t \tag{34}$$

$$\zeta_t \ge \varphi_t - (1 - \varpi_t)e_t, \forall t \tag{35}$$

$$\zeta_t \le \varpi_t e_t, \forall t \tag{36}$$

$$o_t, \varpi_t, \psi_{ij}, m_{ij} \in \{0,1\}; \alpha_t, \beta_{ij}, \zeta_t \ge 0; \eta_n\ \ unsigned$$

As a result, for any given feasible $\{y_{it}, z_{it}\} \in Y$ in problem (9), the derivation of the solution to the bi-linear programming problem in (10-15) reduces to a solution to a MIP problem that can be done by any MIP solver.

Proposition 3. *One optimal solution to the bilinear programming problem in (10-15) can be obtained by solving the following MIP problem.*

$$max \sum_{i=0}^{I-1}\sum_{t=0}^{T-1}(l_i\hat{y}_{it}\lambda_{it} - u_i\hat{y}_{it}\pi_{it}) + \sum_{t=0}^{T-1}(d_t)\varphi_t - \sum_{i=0}^{I-1}\sum_{t=0}^{T-2}\rho_{it}(\hat{y}_{it}\Delta_+^i + (1-\hat{y}_{it})u_i)$$
$$-\sum_{i=0}^{I-1}\sum_{t=0}^{T-2}\delta_{it}(\hat{y}_{i,t+1}\Delta_-^i + (1-\hat{y}_{i,t+1})u_i) - \sum_t \underline{v}_t\varphi_t - \sum_t \zeta_t(\overline{v}_t - \underline{v}_t) + \sum_{n=1}^{N}\eta_n \qquad (37)$$

$st.(11)-(15),(18)-(36)$

$\lambda_{it}, \pi_{it}, \rho_{it}, \delta_{it}, \mu_t, \gamma_t \geq 0, \varphi_t$ *unsigned*; $o_t, \varpi_t, \psi_{ij}, m_{ij} \in \{0,1\}; \alpha_t, \beta_{ij}, \zeta_t \geq 0; \eta_n$ *unsigned.*

Given MIP problems can be solved to optimality using commercial solvers, we can easily convert the optimal solution to a valid inequality which is similar to cutting planes generated in Benders decomposition procedures. In fact, because the min max natural of the original problem, any feasible to the bi-linear programming problem provides a valid inequality to the first min problem while the inequality from the optimal solution is of the best quality.

Theorem 4. *Given a optimal solution,* $\lambda_{it}^k, \pi_{it}^k, v_t^k, \varphi_t^k, \delta_{it}^k$*, to the bi-linear programming problem (or its associated MIP problem), a valid inequality in the form of*

$$\vartheta \geq \sum_{i=0}^{I-1}\sum_{t=0}^{T-1}(l_iy_{it}\lambda_{it}^k - u_iy_{it}\pi_{it}^k) + \sum_{t=0}^{T-1}(d_t - v_t^k)\varphi_t^k - \sum_{i=0}^{I-1}\sum_{t=0}^{T-2}\rho_{it}^k(y_{it}\Delta_+^i + (1-y_{it})u_i)$$
$$-\sum_{i=0}^{I-1}\sum_{t=0}^{T-2}\delta_{it}^k(y_{i,t+1}\Delta_-^i + (1-y_{i,t+1})u_i) + \sum_{t=0}^{T-1}\sum_{i=0}^{N-1}(a_iz_{it} + r_iy_{it}) \qquad (38)$$

can be supplied to the master problem.

Actually, the cutting plane algorithm based on this result guarantees to converge to one optimal solution to the two-stage robust optimization Wind-UC problem.

Recall that Benders decomposition method decomposes a MIP (or LP) problem into two problems, the master problem and subproblem(s). By making use of the dual(s) of subproblem(s) and supplying valid cuts from optimal solutions of the dual(s) to the master problem, the master problem will approximate the original MIP (LP) problem. Provided enough iterations on generating valid inequalities, solutions to the master problem finally converges to an optimal solution to the original problem. Although the cutting plane in the form of (38) is not generated in the same fashion as to those of Benders decomposition, information of max and min problems in the second stage are obtained through dualizing the second stage min problem and passed to the first stage min problem in the form of the optimal dual variables. Given those similarity, we classify these two types of cutting planes as Benders-dual cutting planes. In Section 3.2, we describe a novel cutting plane generation procedure that does not require any information from dual problems.

We mention that the linearization technique that converts the derivation of the optimal solution for the bi-linear programming problem into solving a MIP problem gives us many advantages. First, this framework just requires the linear programming structure of the second stage min problem. In fact, it also works for some nonlinear programs. Second, it is applicable to any budget constrained or cardinality constrained uncertainty sets. Third, MIP problems are deeply studied and many commercial solvers or algorithms can be applied to solve large-scale instances. Applications to robust optimization models for practical instances, including network design and scheduling problems, with more complicated second stage min problems as well as complex uncertainty set are investigated in Zhao and Zeng (2010).

3.2 A Novel Cutting Plane Algorithm

In this section, we describe a novel cutting plane algorithm with a different valid inequality. We also gives the proof for its finite convergence to one optimal solution to the two-stage robust optimization Wind-UC problem. Next, we introduce a class of valid inequalities. Its validity directly follows from the definition of two-stage robust optimization model. Although we independently discover this group of valid inequalities, we recently note that Takeda et al. (2008) also identified this group of valid inequalities. However, to the best of our knowledge, no cutting plane algorithm has been developed for these inequalities and their computational strength has been investigated.

Theorem 5. *For a given wind output $\vec{v}^k$, the inequalities presented in* (39)-(44) *are valid* *for the original two-stage robust optimization Wind-UC problem.*

$$\vartheta \geq \sum_{t=0}^{T-1} \sum_{i=0}^{N-1} (a_i z_{it} + r_i y_{it}) + \sum_{t=0}^{T-1} \sum_{i=0}^{N-1} c_i x_{it}^k + \sum_{t=0}^{T-1} (e_t b_t^k - q_t s_t^k); \tag{39}$$

$$l_i y_{it} \leq x_{it}^k \leq u_i y_{it}, \forall i, t; \tag{40}$$

$$\sum_{i=0}^{N-1} x_{it}^k + v_t^k + b_t^k - s_t^k = d_t, \forall t, k; \tag{41}$$

$$x_{i,t+1}^k \leq x_{it}^k + y_{it} \Delta_+^i + (1 - y_{it}) u_i, \forall i, \forall t = 0, 1, ..., T-2 \tag{42}$$

$$x_{it}^k \leq x_{i,t+1}^k + y_{i,t+1} \Delta_-^i + (1 - y_{i,t+1}) u_i, \forall i, \forall t = 0, 1, ..., T-2 \tag{43}$$

$$x_{it}^k, b_t^k, s_t^k \geq 0 \tag{44}$$

We emphasize that this type of cutting planes is essentially different from Benders-dual type cutting planes. Several critical observations are listed as follows.

- First, this type of cutting planes does not involve or require any information from any dual problem.
- Second, it links the decision variables for both the first stage and the second stage min problems through a fixed wind energy supply.
- Third, this type of cutting planes only needs a concrete wind energy output in its generation. And any point in the uncertainty set could lead to a valid inequality to the master problem.

As a consequence, a cutting plane algorithmic procedure to solve the two stage robust optimization Wind-UC problem is described as following.

Cutting Plane Algorithm.

Initialization. Assign feasible values of first stage decision variables y_{it}^1, z_{it}^1 in Wind-UC model, set $UB = \infty$, $LB = -\infty$, $k = 1$.

Iteration k(a). Solve the subproblem (37) given y_{it}^k, z_{it}^k, obtain worst case wind output v^k, update $UB = min\{UB, obj^* + \sum_{t=0}^{T-1} \sum_{i=0}^{N-1} (r_i y_{it}^k + a_i z_{it}^k)\}$ where obj^* is the optimal objective value of the subproblem in this iteration. Add inequalities (39)-(44) to the master problem.

Iteration k(b). Solve the updated master problem (16), let $y_{it}^{k+1} = y_{it}^*, z_{it}^{k+1} = z_{it}^*$, where y_{it}^* and z_{it}^* are optimal solutions to the master problem. Update LB to be the optimal objective value of the master problem in this iteration.

Stopping. If $UB - LB \leq \varepsilon$, then stop. Otherwise $k = k + 1$, go to next iteration.

Proposition 6. *The proposed algorithm converges to one optimal solution with finite steps.*

Proof. Proof. Suppose $y_1, ...y_{k-1} \in Y$ have been checked and the cuts corresponding to worst case wind output $v_1, ..., v_{k-1} \in V$(at least we can have one initial point y_1 and its worst case v_1) have been added to master problem. Let problem (9) be denoted by $min\ cy + max\ f(y, v)$. The next y_k obtained by solving the relaxed problem satisfies that $y_k \notin \{y_1, ...y_{k-1}\}$ unless y_k is optimal to problem (9). That is, if one point in the first stage is repeated, then it is optimal. Assume $y_k \in \{y_1, ...y_{k-1}\}$ and without less of generality we assume $y_k = y_1$. Since y_1 is the optimal solution to this relaxed problem, we have $LB = \{cy_1 + f(y_1, v_1), cy_1 + f(y_1, v_2), ..., cy_1 + f(y_1, v_{k-1})\}^+ \leq \{cy + f(y, v_0), cy + f(y, v_1), ..., cy + f(y, v_{k-1})\}^+, \forall y \in Y$. Therefore $LB \geq cy_1 + f(y_1, v_1)$. On the other hand, since any feasible solution is an upper bound, we have $UB = (cy_1 + f(y_1, v_1), cy_2 + f(y_2, v_2), ...cy_m + f(y_m, v_m))^-$ and $UB \leq cy_1 + f(y_1, v_1)$. $UB = LB$ indicates optimal. Similarly, we can prove that $v_k \notin \{v_1, v_2, ...v_{k-1}\}$ unless y_k is optimal. In the worst case all feasible points $y \in Y$ whose number is finite and whose corresponding optimal wind outputs have been added to the cuts, then in next step no matter what y we obtained it is replication of a feasible point in Y, therefore the algorithm will stop and convergent to optimality. □

4 Wind-UC-DR Model

If demand response is considered, the price level which should be selected as the rate at each time is the decision to be made, where L levels price p_l and demand increase/decrease percentage $d_l\%$ are pre-defined for each time period. Therefore we have the following Wind-UC-DR model with additional decision variables $w_{tl} \in \{0, 1\}$. Since the power rate in each time period can be different, the objective function is to maximize the profit rather than minimizing operations cost. The right-hand side of constraint (46) is the predicted demand after demand response; Constraint (47) guarantees that after applying demand response the bill of ratepayers will not increase; Constraint (48) states that only one price level can be selected in each time.

$$min \sum_{t=0}^{T-1}\sum_{i=0}^{N-1}(a_i z_{it} + r_i y_{it} + c_i x_{it}) + \sum_{t=0}^{T-1}(e_t b_t - q_t s_t) - \sum_t d_t \sum_{l=0}^{L-1} w_{tl} p_l (1 + d_l\%) \tag{45}$$

$$st. \ (2) - (5), (7) - (8);$$

$$\sum_{i=0}^{N-1} x_{it} + v_t + b_t - s_t = d_t \sum_{l=0}^{L-1} w_{tl}(1 + d_l\%), \forall t; \tag{46}$$

$$\sum_t d_t \sum_{l=0}^{L-1} w_{tl} p_l (1 + d_l\%) \leq bill_1; \tag{47}$$

$$\sum_{l=0}^{L-1} w_{tl} = 1, \forall t; \tag{48}$$

$$x_{it}, b_t, s_t \geq 0; y_{it}, z_{it}, w_{tl} = \{0, 1\}, \forall i, t, l; v_t \in V.$$

Problem defined in (45)-(48) is the corresponding robust counterpart of Wind-UC-DR model. And the inner max-min problem can also be formulated as a bi-linear programming problem similar to that of Wind-UC model. Analogously, the first stage decision variables $\{y_{it}, z_{it}, w_{tl}\}$ should be made day-ahead while the second stage decision variables $\{x_{it}, b_t, s_t\}$ should be made after wind uncertainty is realizing. The master problems and subproblems of this model are similar to those of Wind-UC model, and problem (49) can be solved by the cutting plane algorithm proposed in previous section.

$$\begin{aligned} min_{\{y_{it}, z_{it}, w_{tl}\}} \ & \sum_{t=0}^{T-1}\sum_{i=0}^{N-1}(a_i z_{it} + r_i y_{it}) + max_{\{v_t \in V\}} min_{\{x_{it} \in X, b_t, s_t \geq 0,\}} \sum_{t=0}^{T-1}\sum_{i=0}^{N-1} c_i x_{it} \\ & + \sum_{t=0}^{T-1}(e_t b_t - q_t s_t) - \sum_t d_t \sum_{l=0}^{L-1} w_{tl} p_l (1 + d_l\%); \end{aligned} \tag{49}$$

$$st. \ (2) - (4), (47 - 48);$$

$$y_{it}, z_{it}, w_{tl} = \{0, 1\}, \forall i, t, l;$$

$$X = \{x_{it} : (5), (7) - (8), (46), x_{it} \geq 0, \forall i, t;\}$$

5 Computational results

In this section, we perform a set of computational study to demonstrate the benefits of two-stage robust Wind-UC model and Wind-UC-DR model and to show that our cutting plane algorithm is computationally superior to existing methods and CPLEX solvers. A set of $I = 36$ thermal units from an IEEE 118 system(Ma and Shahidehpour (1999)) are used in all experiments and some parameters are adjusted for our models: (i) the fuel cost is linear in stead of quadratic form, and (ii) the ramping up/down parameters are adjusted according to the rule in Frangioni and Gentile (2006). The fixed price before response is 15 and the same $L = 10$ levels price and demand increase/decrease are pre-defined at each time (the discrete increase/decrease levels are sampled from experiment results in Faruqui and Sergici (2009)). For low level wind penetration purpose, the lower bound of v_t is randomly generated between $0 - 100$ and upper bound of v_t is randomly generated between $100 - 200$ for each t. The experiment platform is CPLEX12.1 on Dell OPTIPLEX 760 with 3.00GHz CPU and 3GB of RAM. The baseline of the profit, 558616, is the profit without wind energy and demand response.

5.1 Algorithms based on Benders-dual Cuts vs. CP

The performances of Benders decomposition with pareto-optimal cut(Magnanti and Wong (1981)), and proposed cutting plane method are compared based on Wind-UC model. Stopping gap is set to be within 0.5%, one budget constraint covering $T = 24$ hours is used. A simpler linearization technique can be used here by decomposing the subproblem into $T = 24$ small independent problems as follows. Each problem stands for the case that wind takes values between bounds at one time, ie., at time k. To be simplicity, all θ_t in budget constraint $\sum_{t \in T} \theta_t v_t \geq V'\}$ is assigned to be 1. We use ξ in budget constraints to capture the wind budget uncertainty limit in T hours, like $V' = \xi \sum_t \underline{v}_t + (1 - \xi) \sum_t \underline{v}_t$. The cases 1-5 are corresponding to $\xi = 0.1, 0.3, 0.5, 0.7, 0.9$ respectively. The computation results are shown in Table 1. We can see that Benders decomposition spends at least thousands of seconds to solve different cases while the proposed cutting plane only needs hundreds of seconds. Also, the number of iterations in cutting plane is much less than that of Benders decomposition.

$$max_{k\in T} \sum_{i=0}^{I-1}\sum_{t=0}^{T-1}(l_i\hat{y}_{it}\lambda_{it} - u_i\hat{y}_{it}\pi_{it}) + \sum_{t=0}^{T-1}(d_t)\varphi_t - \sum_{i=0}^{I-1}\sum_{t=0}^{T-2}\rho_{it}(\hat{y}_{it}\Delta_+^i + (1-\hat{y}_{it})u_i)$$
$$- \sum_{i=0}^{I-1}\sum_{t=0}^{T-2}\delta_{it}(\hat{y}_{i,t+1}\Delta_-^i + (1-\hat{y}_{i,t+1})u_i) + \sum_{t\in T/\{k\}}\underline{v}_t(\varphi_k - \varphi_t) - \varphi_k V' - \sum_{t\in T/\{k\}}(\overline{v}_t - \underline{v}_t)(\alpha_t - \beta_t) \quad (50)$$

$st.(11) - (15)$

$$\theta_k\underline{v}_k \le V' - \sum_{t\in T/\{k\}}\theta_t\underline{v}_t - \sum_{t\in T/\{k\}}\theta_t(\overline{v}_t - \underline{v}_t)\varpi_t \le \theta_k\overline{v}_k; \quad (51)$$

$$\alpha_t \le \varpi_t e_t, \forall t \quad (52)$$

$$\alpha_t \ge \varphi_t + (\varpi_t - 1)e_t, \forall t \quad (53)$$

$$\alpha_t \le \varphi_t, \forall t \quad (54)$$

$$\beta_t \le \varpi_t e_k, \forall t \quad (55)$$

$$\beta_t \ge \varphi_k + (\varpi_t - 1)e_k, \forall t \quad (56)$$

$$\beta_t \le \varphi_k, \forall t \quad (57)$$

$$\lambda_{it}, \pi_{it}, \rho_{it}, \delta_{it}, \mu_t, \gamma_t \ge 0, \varphi_t\ unsigned; \alpha_t, \beta_t \ge 0, \varpi_t \in \{0,1\}$$

5.2 One uncertainty budget constraint

Please note that linearization technique that decomposes the subproblem into $T = 24$ small independent problems is more powerful than the linearization technique in Section 3.1 when there is only one budget constraint. The performance of the proposed cutting plane method is investigated by setting lower gap. All the parameters are the same with previous subsection. The result is shown in Table 2. Note that in CPLEX the relative gap of the master problem of Wind-UC-DR model is set to be 0.001 when we tried to stop within

Table 1: The computational comparison of different approaches

	Benders-like			CP		
cases	profit	time	iterations	profit	time	iterations
case1	596669	2239	80	594674	50	3
case2	589931	4619	70	589478	243	3
case3	581290*	>20000*	120*	583293	803	3
case4	578362	12670	59	575876	324	2
case5	572166*	>20000*	70*	571181	27	2

0.1% and 0.5% overall gap, and to be default gap when we want to find optimal solution. Experiment experience shows that solving the master problems in CPLEX takes most of the computation time, especially when the default gap is required and CPLEX will be out of memory even with few cuts.

5.3 Multiple uncertainty budget constraints

The performance of the proposed cutting plane method to deal with multiple budget constraints is investigated. We use four budget constraints with each covering 6 hours, so $\vec{v} \in \mathbf{V} = \{\vec{v} : \underline{v}_t \leq v_t \leq \overline{v}_t, \sum_{t\in T_1} \theta_t v_t \geq V_1', \sum_{t\in T_2} \theta_t v_t \geq V_2', \sum_{t\in T_3} \theta_t v_t \geq V_3', \sum_{t\in T_4} \theta_t v_t \geq V_4'\}$. In multiple budget constraints case, linearization technique in Section 3.1 is much better. For example, when 4 budget constraints divide the 24 hours into 4 segments with each segment has 6 hours, there will be $6 \times 6 \times 6 \times 6 > 1000$ small problems if decomposition linearization is used. In this experiment, $\xi_1, \xi_2, \xi_3, \xi_4$ are used to capture the wind budget uncertainty. $(\xi_1, \xi_2, \xi_3, \xi_4)$ are set to be $(0.9, 0.8, 0.7, 0.6)$, $(0.8, 0.3, 0.7, 0.6)$, $(0.5, 0.9, 0.1, 0.7)$, $(0.4, 0.8, 0.2, 0.6)$, and $(0.7, 0.8, 0.3, 0.5)$ in five cases. All the other parameters including the relative gap in master problems are the same with previous experiments. Table 3 shows the results.

Table 2: The computational result of cutting plane method with one budget constraint.

		Wind-UC				Wind-UC-DR			
cases	gap	profit	time	iterations	increase	profit	time	iterations	increase
case1	0.5%	594674	50	3	6.45%	610479	212	4	9.28%
	0.1%	596215	383	5	6.73%	611379	2106	9	9.45%
	optimal	596669	2160	8	6.81%	NA	NA	NA	NA
case2	0.5%	589478	243	3	5.52%	604162	404	3	8.15%
	0.1%	589478	245	3	5.52%	604993	1391	6	8.30%
	optimal	589931	1293	5	5.61%	NA	NA	NA	NA
case3	0.5%	583293	803	3	4.42%	598270	1383	3	7.10%
	0.1%	583293	803	3	4.42%	598870	8646	8	7.21%
	optimal	583811	1427	4	4.51%	NA	NA	NA	NA
case4	0.5%	575876	324	2	3.09%	593255	1104	3	6.20%
	0.1%	578513	759	3	3.56%	593311	1844	4	6.21%
	optimal	578513	759	3	3.56%	NA	NA	NA	NA
case5	0.5%	571181	27	2	2.25%	587606	85	2	5.19%
	0.1%	573279	55	3	2.62%	588110	165	3	5.28%
	optimal	573279	55	3	2.62%	588165	1838	3	5.29%

Table 3: The computational result of cutting plane method with multiple budget wind constraints.

		Wind-UC				Wind-UC-DR			
cases	gap	profit	time	iterations	increase	profit	time	iterations	increase
case1	0.5%	575010	206	2	2.93%	590076	237	2	5.63%
	0.1%	577648	420	3	3.41%	592339	595	3	6.04%
	optimal	577648	420	3	3.41%	NA	NA	NA	NA
case2	0.5%	582337	1141	3	4.25%	597313	839	3	6.93%
	0.1%	582337	1141	3	4.25%	597313	1323	4	6.93%
	optimal	582505	2430	5	4.28%	NA	NA	NA	NA
case3	0.5%	581942	117	2	4.18%	600869	127	2	7.56%
	0.1%	584579	257	3	4.65%	601391	339	4	7.66%
	optimal	584579	257	3	4.65%	601441	8318	8	7.67%
case4	0.5%	582720	491	2	4.31%	599721	265	2	7.36%
	0.1%	585357	873	3	4.79%	599896	480	3	7.39%
	optimal	585357	873	3	4.79%	NA	NA	NA	NA
case5	0.5%	580419	622	2	3.90%	597016	970	2	6.87%
	0.1%	583057	1237	3	4.38%	597398	1698	3	6.94%
	optimal	583057	1237	3	4.38%	NA	NA	NA	NA

6 Conclusion and Future research

By introducing wind output and demand response, we construct NP-hard two-stage robust optimization problems and derive mathematical properties of optimal solutions. We develop a novel cutting plane method to this problem. Our study shows that the robust unit commitment model with wind and demand response can significantly reduce total operation cost from thermal units and the novel cutting plane method can dramatically decrease the computation time compared with traditional Benders decomposition or commercial solvers. In future (i) more general uncertainty polyhedron will be considered, and (ii) multiple uncertain factors, such as those from multiple renewable energy sources, will be considered.

References

Elodie Adida and Georgia Perakis. Dynamic pricing and inventory control: Uncertainty and competition. *Operations Research*, 58(2):289–302, 2006.

Alper Atamturk and Muhong Zhang. Two-stage robust network flow and design under demand uncertainty. *Operations Research*, 55(4):662–673, 2007.

Ross Baldick. The generalized unit commitment problem. *Power Systems, IEEE Transactions on*, 10(1):465–475, 1995.

Aharon Ben-Tal and Arkadi Nemirovski. Robust convex optimization. *Mathematics of Operations Research*, 23(4):769–805, 1998.

Aharon Ben-Tal and Arkadi Nemirovski. Robust solutions of uncertain linear programs. *Operations Research Letters*, 25(1):1–14, 1999.

Aharon Ben-Tal and Arkadi Nemirovski. Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical Programming*, 88(3):411–424, 2000. 0025-5610.

Aharon Ben-Tal and Arkadi Nemirovski. Robust optimization methodology and applications. *Mathematical Programming*, 92(3):453–480, 2002. 0025-5610.

Aharon Ben-Tal and Arkadi Nemirovski. Selected topics in robust convex optimization. *Mathematical Programming*, 112(1):125–158, 2008.

Dimitris Bertsimas and Melvyn Sim. Robust discrete optimization and network flows. *Mathematical Programming*, 98(1):49–71, 2003. 0025-5610.

Dimitris Bertsimas and Melvyn Sim. The price of robustness. *Operations Research*, 52(1): 35–53, 2004.

Dimitris Bertsimas and Aurelie Thiele. A robust optimization approach to inventory theory. *Operations Research*, 54(1):150–168, 2006.

Daniel Bienstock, George Nemhauser, Dimitris Bertsimas, and Aurlie Thiele. A robust optimization approach to supply chain management. In *Integer Programming and Combinatorial Optimization*, Lecture Notes in Computer Science, pages 145–156. Springer Berlin / Heidelberg, 2004.

Santiago Cerisola, Alvaro Baillo, Jose M. Fernandez-Lopez, Andres Ramos, and Ralf Gollmer. Stochastic power generation unit commitment in electricity markets: A novel formulation and a comparison of solution methods. 57(1):32–46, 2009.

Emil M. Constantinescu, Victor M. Zavala, Matthew Rocklin, Sangmin Lee, and Mihai Anitescu. A computational framework for uncertainty quantification and stochastic optimization in unit commitment with wind power generation. *IEEE transactions on power systems*, accepted.

DoE. 20u.s. electricity supply. Technical report, 2008.

James E. Falk. A linear max-min problem. *Mathematical Programming*, 5(1):169–188, 1973.

Ahmad Faruqui and Sanem Sergici. Household response to dynamic pricing of electricity-a survey of experiment evidence. Technical report, 2009.

Larry Flowers. Wind powering america update, 2010.

Antonio Frangioni and Claudio Gentile. Solving nonlinear single-unit commitment problems with ramping constraints. *Operations Research*, 54(4):767–775, 2006.

Ricardo Fukasawa, Humberto Longo, Jens Lysgaard, Marcus Poggi de Arago, Marcelo Reis, Eduardo Uchoa, and Renato F. Werneck. Robust branch-and-cut-and-price for the capacitated vehicle routing problem. *Mathematical Programming*, 106(3):491–511, 2006.

Laurent El Ghaoui, Maksim Oks, and Francois Oustry. Worst-case value-at-risk and robust portfolio optimization: A conic programming approach. *Operations Research*, 51(4): 543–556, 2003.

David Kathan, Caroline Daly, Eric Eversole, Maria Farinella, Jignasa Gadani, Ryan Irwin, Cory Lankford, Adam Pan, Christina Switzer, and Dean Wright. National action plan on demand response. Technical report, 2010.

Zhaosong Lu. A computational study on robust portfolio selection based on a joint ellipsoidal uncertainty set. *Mathematical Programming*, pages 1–9, 2009.

H. Ma and S. M. Shahidehpour. Unit commitment with transmission security and voltage constraints. *Power Systems, IEEE Transactions on*, 14(2):757–764, 1999.

Thomas L. Magnanti and Richard T. Wong. Accelerating benders decomposition: Algorithmic enhancement and model selection criteria. *Operations Research*, 29(3):464–484, 1981.

Bernie Neenan, Donna Pratt, Peter Cappers, James Doane, Jeremey Anderson, Richard Boisvert, Charles Goldman, Osman Sezgen, Galen Barbose, Ranjit Bharvirkar, Michael K. Meyer, Steve Shankle, and Derrick Bates. How and why customers respond to electricity price variability: A study of nyiso and nyserda 2002 prl program performance. Technical report, 2003.

Narayana P. Padhy. Unit commitment-a bibliographical survey. *Power Systems, IEEE Transactions on*, 19(2):1196–1205, 2004.

Chuen-Teck See and Melvyn Sim. Robust approximation to multiperiod inventory management. *Operations Research*, 58(3):583–594, 2010.

Ramteen Sioshansi and Walter Short. Evaluating the impacts of real-time pricing on the usage of wind generation. *Power Systems, IEEE Transactions on*, 24(2):516–524, 2009.

Chua-Liang Su. *Optimal Demand-Side Participation in Day-Ahead Electricity Markets.* PhD thesis, University of Mathester, 2007.

Chua-Liang Su and Daniel Kirschen. Quantifying the effect of demand response on electricity markets. *Power Systems, IEEE Transactions on*, 24(3):1199–1207, 2009.

A. Takeda, S. Taguchi, and RH Tutuncu. Adjustable robust optimization models for a nonlinear two-period system. *Journal of Optimization Theory and Applications*, 136 (2):275–295, 2008. ISSN 0022-3239.

Samer Takriti and Shabbir Ahmed. On robust optimization of two-stage systems. *Mathematical Programming*, 99(1):109–126, 2004.

Samer Takriti, Benedikt Krasenbrink, and Lilian S. Y. Wu. Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem. 48(2):268–280, 2000.

Chuang-Li Tseng. *On power system generation unit commitment problems.* Ph.d., University of California, Berkeley, 1996.

Aidan Tuohy, Peter Meibom, Eleanor Denny, and Mark O'Malley. Unit commitment for systems with significant wind penetration. *Power Systems, IEEE Transactions on*, 24 (2):592–601, 2009.

Jianhui Wang, Audun Botterud, Vladimiro Miranda, Claudio Monteiro, and Gerald Sheble. Impact of wind power forecasting on unit commitment and dispatch. Technical report, 2009.

Long Zhao and Bo Zeng. An effective cutting plane procedure for two-stage robust optimization problems. Working paper, University of South Florida, 2010.