Robust mid-term power generation management

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Abstract

We consider robust formulations of the mid-term optimal power management problem. For this type of problems, classical approaches minimize the expected generation cost over a horizon of one year, and model the uncertain future by means of scenario trees. In this setting, extreme scenarios -with low probability in the scenario tree- may fail to be well represented. More precisely, when extreme events occur, strategies devised with the classical approach can result in significant financial losses. By contrast, robust techniques can handle well extreme cases. We consider two robust formulations that preserve the separable structure of the original problem, a fundamental issue when solving real-life problems. Numerical results assess the validity and practicality of the approaches.

Keywords: robust counterpart; stochastic optimization; duality and space decomposition; electricity power management.

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1 Introduction

We are interested in optimization problems arising from power generation management on a yearly scale. We consider a diversified set of utilities that are subject to random events, such as customer demand, availability of nuclear and thermal generators, and water inflows for hydroelectric power generators. In such a setting, the goal is to achieve minimal production cost while satisfying the customer demand and the operational constraints of the units. The operational impact of the volatility of the demand can be drastic if handled improperly. For instance, in France the variation of customer demand in winter can be up to one GW per degree Celsius while peak loads are around 80 GW. So it is critical to ensure robustness of the computed optimal production plans and associated marginal values; faced with uncertainties like customer demand, water inflows and plants availability.

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There are three key ingredients for a systematic approach to this problem: (a) an accurate representation of uncertain events and inputs, (b) the choice of the decision function, and (c) the algorithm employed for numerically solving the problem. A useful tool to address item (a) is scenario tree generation explained in [GKHR03]. This is indeed a well-suited approach when emphasis is set not only on generation scheduling but also on management strategies in view of subsequent Monte Carlo analysis. With respect to (c), Lagrangian relaxation and decomposition methods [LPRS96] are powerful approaches to overcome eventual computational burden because they can be combined with efficient non-differentiable optimization methods (see e.g. [BLRS01], [GKKN$^+$02], [NR00], [KTW00]).

In practice, once the solution to the optimization problem solved by Lagrangian relaxation is available one can use the marginal costs computed on the scenario tree to obtain local feedbacks as in [CCC95]. Specifically, a stochastic dynamic programming technique is used to compute Bellman values of each reservoir, and thus the overall resolution scheme could be thought of as a stochastic dual dynamic programming method [PP91]. This scheme is known to be numerically efficient. Still, the price to pay for obtaining such local feedback is the high volatility of the value function. Robust techniques aim at reducing such volatility, for instance by means of polyhedral risk measures as in [ERW04] and [ERW05].

In this paper, we introduce a new approach that can be applied to general stochastic programming problems, consisting of two main steps. The first step incorporates uncertainty at each node of the scenario tree representing the evolution of the uncertain parameters over the optimization period. The second one proposes a new robust optimization methodology, adapted to this scenario tree. More precisely, in this second step, we aim at improving the robustness of the management strategies by an appropriate combination of primal and dual relaxations that preserve the separable structure of the problem. In a sense, these relaxations may be viewed as the answers to the following questions. For the primal relaxation: how to control the sales turnover faced with uncertainty of the demand? And for the dual relaxation: how to control the production costs given random availability of plants? Among the concrete benefits of robust feedback laws, one can outline a significant reduction of the variance of simulated cost - up to 38% of reduction for a comparable average cost-; a parsimonious use of water in reservoirs; and a reduction of very high cost strategies. Finally, we point out that our robust counterparts of local feedbacks can be interpreted as Value-at-Risk approaches to suitably chosen problems.

The paper is organized as follows. Section 2 describes the nominal optimization model with emphasis on the representation of uncertainties. In Section 3, we introduce the robust approach and outline the connection between dual regularization and Value-at-Risk. Section 4 describes the numerical implementation and provides some simulation results. Some additional modeling details are given in the appendix.

2 The physical model

The physical model is a stochastic dynamic system where the random inputs are the customer demand, the availability of the thermal units and the quantity of natural water inflows. We introduce in this section a standard modeling for this problem that we call nominal model.

If we represent random inputs and events as a scenario tree, we can formulate the optimization problem as a finite horizon discrete time stochastic control problem on this tree.

2.1 Model setting

We aim at minimizing the average production cost along the scenario tree while satisfying the demand and the operating constraints of the system at each node of the tree. This stochastic control problem may be formulated as [Rom00]:

$$\left\{\begin{array}{ll} \min\limits_{u} \sum\limits_{n\in\mathcal{O}} \sum\limits_{p\in\mathcal{P}_n} \sum\limits_{\ell\in\mathcal{L}_T\cup\mathcal{L}_N} \pi_n\, c_\ell\, u^\ell(n,p) + \sum\limits_{n\in\mathcal{O}_T} \sum\limits_{\ell\in\mathcal{L}_H\cup\mathcal{L}_E} \pi_n\, {c'_\ell}^\top\, x^\ell(n) & \\ \forall n\in\mathcal{O},\ \forall p\in\mathcal{P}_n,\ \sum\limits_{\ell\in\mathcal{L}} u^\ell(n,p) = \mathcal{D}_0(n,p), & (a) \\ \forall \ell\in\mathcal{L}, \quad (x^\ell, u^\ell)\in\chi_\ell, & (b) \end{array}\right. \tag{1}$$

where

- $\mathcal{L}_T, \mathcal{L}_N, \mathcal{L}_H$ are the sets of thermal, nuclear and hydroelectric plants; $\mathcal{L}_E$ is the set of EJP contracts (*Effacement Jours de Pointe* in French or demand side management contracts in English; considered to be virtual plants)"; and $\mathcal{L}$ is the set of all power utilities;
- $\mathcal{O}$ is the set of nodes of the tree and $\mathcal{O}_T$ the set of leaves, π_n is the probability of being at node n and $\mathcal{P}_n$ is the set of time subdivisions associated to node n;
- $x^\ell(n)$ is the state of plant $\ell\in\mathcal{L}_H\cup\mathcal{L}_E$ at node n at the beginning of the time step associated to this node;
- $u^\ell(n,p)$ is the control variable of plant ℓ at node n and time subdivision p;
- $\mathcal{D}_0(n,p)$ is the customer demand at node n and time subdivision p;
- c'_ℓ is a vector of costs and $c_\ell > 0$ is unit ℓ (thermal or nuclear) unit production cost;
- χ_ℓ is the functional set of constraints on the control and state variables of plant ℓ.

Some comments. Constraints (a) are the demand satisfaction constraints. For $\ell\in\mathcal{L}_T\cup\mathcal{L}_N$, constraints (b) are box constraints on the production levels. For $\ell\in\mathcal{L}_H\cup\mathcal{L}_E$, an energy reservoir is available at the beginning of the management period. The level of this reservoir can decrease when this energy is used to satisfy the demand or possibly increase if $\ell\in\mathcal{L}_H$, when water inflows arrive at this reservoir. For $\ell\in\mathcal{L}_H\cup\mathcal{L}_E$, the equations of evolution of the energy reservoir associated to ℓ and the box constraints on the state and control variables of this unit ℓ are gathered in constraints (b).

For all the units $\ell\in\mathcal{L}_H\cup\mathcal{L}_E$, the production cost is null. However, for such a unit, we wish to keep a value as high as possible for its energy reservoir at the end of the optimization period. The value functions of the energy reservoirs are piecewise affine and concave. We then use slack variables (that we incorporate in the vector of states), to write the contributions of the hydro and EJP units to the objective function as in (1).

We then consider a problem dual to (1), obtained when dualizing the demand satisfaction constraints (a). A corresponding dual solution is used afterwards in the simulation

phase to compute Bellman values. In this way, we can define adaptive production plans for any given realization of the uncertain parameters.

The appendix provides details on operational constraints (b), the dual solution method, the Bellman values computation, and their use in the simulation phase.

Due to the autonomy of the plants, model (1) can be formulated as an optimization problem with separated domains of constraints, linear objective and one coupling constraint (production/demand equilibrium). Each domain of constraint is describing a plant process. To write this optimization problem in a compact form, we partition the vector of states as $x = (x_{\mathtt{H}}, x_{\mathtt{E}})$ where $x_{\mathtt{H}} = (x^{\ell}(n))_{\ell \in \mathcal{L}_H, n \in \mathcal{O}}$ (hydroelectric plants) and $x_{\mathtt{E}} = (x^{\ell}(n))_{\ell \in \mathcal{L}_E, n \in \mathcal{O}}$ (virtual plants). We also partition the vector of controls accordingly to the utilities as $u = (u_{\mathtt{T}}, u_{\mathtt{N}}, u_{\mathtt{H}}, u_{\mathtt{E}})$ where:

- $u_{\mathtt{T}} = (u^{\ell}(n,p))_{\ell \in \mathcal{L}_T, n \in \mathcal{O}, p \in \mathcal{P}_n}$ (thermal plants);
- $u_{\mathtt{N}} = (u^{\ell}(n,p))_{\ell \in \mathcal{L}_N, n \in \mathcal{O}, p \in \mathcal{P}_n}$(nuclear);
- $u_{\mathtt{H}} = (u^{\ell}(n,p))_{\ell \in \mathcal{L}_H, n \in \mathcal{O}, p \in \mathcal{P}_n}$ (hydroelectric);
- $u_{\mathtt{E}} = (u^{\ell}(n,p))_{\ell \in \mathcal{L}_E, n \in \mathcal{O}, p \in \mathcal{P}_n}$ (virtual).

Introducing cost vectors c_i and coupling matrices $\mathcal{A}_i$ with ad hoc sizes for $i \in \{1, ..., 4\}$, (1) can be formally described in the following way:

$$(MP) \quad \begin{cases} \min\limits_{u \in \mathcal{U}} f(u, x_{\mathtt{H}}, x_{\mathtt{E}}) \\ \mathcal{A}u = \mathcal{D}_0 \text{ given } \in \mathbb{R}^D, \end{cases} \tag{2}$$

when setting

$$\begin{array}{lcl} f(u, x_{\mathtt{H}}, x_{\mathtt{E}}) & = & c_1^{\top} u_{\mathtt{T}} + c_2^{\top} u_{\mathtt{N}} + c_3^{\top} x_{\mathtt{H}} + c_4^{\top} x_{\mathtt{E}}, \\ \mathcal{A}u & = & \mathcal{A}_1 u_{\mathtt{T}} + \mathcal{A}_2 u_{\mathtt{N}} + \mathcal{A}_3 u_{\mathtt{H}} + \mathcal{A}_4 u_{\mathtt{E}}, \end{array}$$

and $\mathcal{U} = \mathcal{T} \times \mathcal{N} \times \mathcal{H}(x_{\mathtt{H}}) \times \mathcal{E}(x_{\mathtt{E}})$ where:

- $\mathcal{T}$ is the set of constraints describing the thermal generators;
- $\mathcal{N}$ is the set of constraints for the nuclear generators;
- $\mathcal{H}(x_{\mathtt{H}})$ describes the dynamics of the hydroelectric plants;
- $\mathcal{E}(x_{\mathtt{E}})$ describes the dynamics of the virtual plants.

2.2 Model analysis

The efficiency of this model is assessed using a set of independent scenarios representing different evolutions of the demand, the inflows for water reservoirs and the availability for thermal units. For each scenario, an optimal generation schedule is determined (see the appendix) and thus the Bellman functions obtained by stochastic dynamic programming on marginal values give only a local optimum. Using local feedbacks as global strategies could result in a high volatility of cost: extreme scenarios -with low probability in the scenario tree- may fail to be well represented. More precisely, when extreme events occur, strategies devised with the classical approach can produce significant financial losses. So it is desirable to strengthen this model by including a more reliable model of uncertainty

on the scenarios at the earliest stages of the problem setting. The goal is to ensure some regularity of the optimal strategies with respect to the inputs of the optimization problem. In other words, we would like to find a robust counterpart with the following properties:

(i) reduce the volatility of the simulated cost over a set of scenarios;

(ii) reduce the number of extreme case optimal strategies (parsimonious use of water reservoir that might not be nearly empty for a long period);

(iii) reduce the number of very high cost strategies.

We will see in the next section that such an objective of variance reduction may be easy to formulate in a Value-at-Risk setting.

3 Robust counterpart of the decision model

3.1 A robustification for stochastic optimization problems

Let $X \subset \mathbb{R}^n$, let Y be a random vector in $\mathbb{R}^n$ on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and let $\mathbf{O}$ be the family of uncertain optimization problems

$$\mathbf{O} = \{\mathbf{O}(\omega)\}_{\omega\in\Omega} = \left\{ \begin{array}{l} \max x^\top Y(\omega) \\ x \in X, \quad g(x) \geq 0 \end{array} \right\}_{\omega\in\Omega}. \tag{3}$$

Assuming that we have some knowledge of the trajectories of Y, say for instance $\{Y(\omega), \omega \in \Omega\} \subset \mathcal{U}$, a well-known subset with nice geometrical properties (typically convexity), then we can solve the worst case problem

$$\left\{ \begin{array}{l} \max \phi_{\mathcal{U}}(x) \\ x \in X, \quad g(x) \geq 0, \end{array} \right.$$

with $\phi_{\mathcal{U}}(x) = \min\limits_{y\in\mathcal{U}} x^\top y$. To be more specific, if $\mathbb{E}_Y[Y]$, the mean of Y, and its covariance matrix $Q \succ 0$ are well-defined[1], and as we are concerned with volatility reduction, we can specify the uncertainty set $\mathcal{U}$ as

$$\mathcal{U} = \{y \in \mathbb{R}^n, (y - \mathbb{E}_Y[Y])^\top Q^{-1}(y - \mathbb{E}_Y[Y]) \leq \kappa^2\}$$

for κ^2 well chosen. Then we notice that $\phi_{\mathcal{U}}(x) = x^\top \mathbb{E}_Y[Y] - \kappa\sqrt{x^\top Q x}$. Therefore, a robust counterpart in the sense of [BTN98] of the uncertain problem (3) simply comes as:

$$\left\{ \begin{array}{l} \max \; x^\top \mathbb{E}_Y[Y] - \kappa\sqrt{x^\top Q x}, \\ x \in X, \quad g(x) \geq 0, \end{array} \right.$$

where κ, the volume of the uncertainties can be chosen depending on the assumptions about the distribution of Y. Indeed, let us consider more generally a random income function $h(x, Y)$ that is concave with respect to deterministic variable x, let Φ be the

[1] $Y \in L^1_{\mathbb{R}} \cap L^2_{\mathbb{R}}$ where $L^p_{\mathbb{R}}$ is the space of Lebesgue p-integrable real functions.

cumulative distribution function of the Gaussian $\mathcal{N}(0,1)$ density and let $\varepsilon \in (0,1)$ be a confidence level. An upper bound on the risk-averse problem

$$\begin{cases} \max \gamma \\ \mathbb{P}(h(x,Y) \geq \gamma) \geq 1-\varepsilon, \\ x \in X, \quad g(x) \geq 0, \end{cases}$$

can be derived from probabilistic risk control techniques ([BP05] and [Smi95]) as

$$(VaR_\varepsilon) \begin{cases} \max \mathbb{E}_Y[h(x,Y)] - \kappa(\varepsilon)\, \sigma_Y[h(x,Y)] \\ x \in X, \quad g(x) \geq 0, \end{cases}$$

where $\sigma_Y[h(x,Y)]$ is the volatility of $h(x,Y)$ and $\kappa(\varepsilon)$ is a risk factor depending on the assumptions about the distribution:

$$\kappa(\varepsilon) = \begin{cases} \Phi^{-1}(1-\varepsilon) > 0, & \text{if } h(x,Y) \text{ is Gaussian,} \\ \sqrt{\frac{1}{\varepsilon} - 1}, & \text{if } h(x,Y) \in L^1_{\mathbb{R}} \cap L^2_{\mathbb{R}} \text{ and is not Gaussian.} \end{cases}$$

A volatility penalization - in other words the Value-at-Risk approach - introduced that way allows us to control the behaviour of a Value (or Risk) function with a given level of confidence. This level of confidence ε can be linked to the views on the volume of uncertainties: for instance, to $\kappa = 3$ corresponds a confidence level of 90% in the non-Gaussian case and a confidence level of 99.9% in the Gaussian case. In the case of power generation management, doing so allows us to find the best compromise between mean production cost and volatility of strategies if we accept to deal with some sub-optimality on the most favorable scenarios.

3.2 Application to the power generation model

In order to define robust management strategies, we need to compute a robust dual solution. Towards this aim, we introduce a new approach that can be described as a two step process.

In the first step, we build a scenario tree of random variables instead of a scenario tree of realizations of random variables. At each node of the tree, the uncertain parameters are not fixed as is usually the case, but they are modeled by random variables with values in a given set that has to be specified. To construct this tree, one has to take into account the characteristics of the underlying processes. For instance, the availability rates possibly change only every two weeks so this structure has to be preserved in the resulting tree. This will be discussed in greater detail later. The second step introduces a robust optimization approach adapted to the modified scenario tree by application of Section 3.1 to the dual optimization problem. The advantage of this approach is that we preserve the separable structure of the dual optimization problem. In practice, we will apply this methodology to the optimal power management problem to modify partial dual functions so we can face uncertainties in the demand and/or plant availability rates.

Before dealing with robustification issues, recall that in the deterministic setting where we consider the demand $\mathcal{D}_0$ perfectly known in the tree and not random, the dual problem of (2) is $\max_{\lambda \in \mathbb{R}^D} \theta(\lambda)$ where:

$$\theta(\lambda) = \tilde{\theta}(\lambda) + \theta_{\mathcal{D}_0}(\lambda)$$

once we have introduced the partial dual functions:

$$\left\{\begin{array}{lcl} \theta_{\mathcal{D}_0}(\lambda) & = & \lambda^\top \mathcal{D}_0, \tilde{\theta} = \theta_T + \theta_N + \theta_H + \theta_E, \\ \theta_T(\lambda) & = & \inf\limits_{u_{\mathtt{T}} \in \mathcal{T}} (c_1 - \mathcal{A}_1^\top \lambda)^\top \, u_{\mathtt{T}}, \\ \theta_N(\lambda) & = & \inf\limits_{u_{\mathtt{N}} \in \mathcal{N}} (c_2 - \mathcal{A}_2^\top \lambda)^\top \, u_{\mathtt{N}}, \\ \theta_H(\lambda) & = & \inf\limits_{u_{\mathtt{H}} \in \mathcal{H}(x_{\mathtt{H}})} c_3^\top x_{\mathtt{H}} - \lambda^\top \mathcal{A}_3 u_{\mathtt{H}}, \\ \theta_E(\lambda) & = & \inf\limits_{u_{\mathtt{E}} \in \mathcal{E}(x_{\mathtt{E}})} c_4^\top x_{\mathtt{E}} - \lambda^\top \mathcal{A}_4 u_{\mathtt{E}}. \end{array}\right. \tag{4}$$

Now it is natural to robustify (2) by formulating a VaR problem on the subsystem that might be submitted to random information or state: $\theta_{\mathcal{D}_0}$ for the customer demand $\mathcal{D}$ and (θ_T, θ_N) for the states of the thermal and nuclear plants.

3.2.1 Primal relaxation of the demand and dual regularization

The goal is to obtain robust versions of partial dual functions. This will be achievable with a primal relaxation on the predictions made for the demands on the different scenarios that are prone to errors. Rather than consider that the demands at each node and each time subdivision of the tree are known exactly to be $\mathcal{D}_0$, we suppose that we have a vector $\mathcal{D}$ that belongs to a given uncertainty set $\mathcal{E}$ which is the ellipsoid given by:

$$\mathcal{E} = \mathcal{E}(\bar{\mathcal{D}}, Q, \kappa) = \{\, y \in \mathbb{R}^D \;\mid\; (y - \bar{\mathcal{D}})^\top Q^{-1} (y - \bar{\mathcal{D}}) \leq \kappa^2(\varepsilon) \,\};$$

where $\bar{\mathcal{D}} = \mathbb{E}_{\mathcal{D}}[\mathcal{D}]$, the covariance matrix Q is given by $Q(i,j) = \mathrm{Cov}(\mathcal{D}_i, \mathcal{D}_j)$ and $\kappa(\varepsilon)$ depends on the assumptions made about the distribution of the vector of demands $\mathcal{D}$. Then our new optimization problem is reformulated as:

$$(MP_{\mathcal{R}}) \quad \left\{\begin{array}{l} \min\limits_{u \in \mathcal{U}} f(u, x_{\mathtt{H}}, x_{\mathtt{E}}) \\ \mathcal{A}u = \mathcal{D} \in \mathcal{E}. \end{array}\right. \tag{5}$$

The dual function associated to (5) is $\max\limits_{\lambda} \theta_{\mathcal{R}}(\lambda)$ with

$$\begin{array}{rcl} \theta_{\mathcal{R}}(\lambda) & = & \min\limits_{u \in \mathcal{U},\, \mathcal{D} \in \mathcal{E}} f(u, x_{\mathtt{H}}, x_{\mathtt{E}}) + \lambda^\top (\mathcal{D} - \mathcal{A}u) \\ & = & \tilde{\theta}(\lambda) + \phi_{\mathcal{E}}(\lambda) \end{array}$$

where $\phi_{\mathcal{E}}(\lambda) = \min\limits_{\mathcal{D} \in \mathcal{E}} \lambda^\top \mathcal{D} = \lambda^\top \bar{\mathcal{D}} - \kappa(\varepsilon)\sqrt{\lambda^\top Q \lambda}$ is the support function of $\mathcal{E}$. So the robustification just turns out to replace $\theta_{\mathcal{D}_0}(\lambda)$ in the original dual function θ by $\phi_{\mathcal{E}}(\lambda) = \theta_{\bar{\mathcal{D}}}(\lambda) - \kappa(\varepsilon)\sqrt{\lambda^\top Q \lambda}$, where $\bar{\mathcal{D}} = \mathbb{E}_{\mathcal{D}}[\mathcal{D}]$. We can also point out that the relaxation of the demand in the ellipsoid $\mathcal{E}$ in problem (5) can be interpreted as a VaR approach on the dual of problem (MP). Indeed, it is clear from Section 3.1 that an alternative formulation for the robust partial dual function associated to the demand is

$$\left\{\begin{array}{l} \max \;\; \gamma \\ \mathbb{P}(\lambda^\top \mathcal{D} \geq \gamma) \geq 1 - \varepsilon. \end{array}\right. \tag{6}$$

Economical Interpretation. In (6), we are looking for the maximal global revenue that can be ensured with an arbitrary degree of confidence. So, we call this method VaR_{Rev}.

3.2.2 Regularization of the dual thermal problem

In this subsection, we intend to exploit a stochastic model of the availability of thermal plants in order to formulate a Value-at-Risk problem on the costs of thermal power generation. Let n_ℓ be the number of thermal groups of unit ℓ. For thermal unit ℓ, let $\alpha_{j,\ell}(t)$ be the probability that group j works at time step t and let $U^t_{j,\ell}$ be the random variable such that $U^t_{j,\ell} = 1$ if group j works at time step t and $U^t_{j,\ell} = 0$ otherwise. We suppose that the groups are regularly tested and, if necessary, repaired every m_0 time steps. Between two consecutive testing dates, we assume that the availability of the units is not changing. The probability $\alpha_{j,\ell}(t_k)$ that a group j of unit ℓ works at time step $t_k = m_0k + 1$ with $k \in \mathbb{N}$, depends on the past evolution of the availability of this group. If at time step t_{k-1}, the group was out of commission, there is high probability (say $1 - \beta^\ell_1$ with β^ℓ_1 small) that it works at time step t_k (the time between two testing dates is greater than the mean time to repair) and a low probability β^ℓ_1 that it is still out of commission at time step t_k. Now if the group was working for the last m periods, we can assume that the longer it has been working without failure (the larger m) the more likely it can break down at time step t_k. Thus, there is a decreasing function of m, $\beta^\ell_2(m)$ such that for any group j of unit ℓ,

$$\mathbb{P}(U^{t_k}_{j,\ell} = 1|\text{Group j was working from } t_{k-m} \text{ to } t_{k-1}) = \beta^\ell_2(m).$$

A particular case is when the state process of a given group is a homogeneous Markov chain where the state space is {F,W} where F stands for the failure state and W for the working state. In this case, $\beta^\ell_2(m) = \beta^\ell_2$ is fixed and corresponds to the probability that a group of unit ℓ works in a given period knowing that it was working the period before. The transition matrix for the groups of unit ℓ is given by:

$$P_\ell = \begin{pmatrix} \beta^\ell_1 & 1 - \beta^\ell_1 \\ 1 - \beta^\ell_2 & \beta^\ell_2 \end{pmatrix}.$$

The probability $\alpha_{j,\ell}(t_k)$ is then given for $k \geq 1$ by:

$$\alpha_{j,\ell}(t_k) = p^\ell_F(j)P^k_\ell(1, 2) + p^\ell_W(j)P^k_\ell(2, 2);$$

where $p^\ell_W(j) = 1 - p^\ell_F(j)$ and $p^\ell_W(j)$ is the probability that group j of unit ℓ works at the first time step. For the simplicity of the exposure, we assume that for given unit ℓ, either all the groups are working or all the groups are out of commission at the first time step. Thus, $\alpha_{j,\ell}(t_k)$ is j-independent and $\alpha_\ell(t_k)$ will denote the probability that a group of unit ℓ works at time t_k. Further, we can partition the scenario tree in subtrees such that the root node and the leaves of a given subtree respectively correspond to time steps t_k and t_{k+1}-1 for some $k \in \mathbb{N}$. Thus the availability rates at the different nodes of any subtree of this partition are the same for a given unit. Let $\mathcal{O} = \cup^m_{k=1} \mathcal{O}_k$ be such that $\mathcal{O}_k$ are the nodes of the k-th subtree $\mathcal{S}_k$ in this partition. Let $T_k = \{(j, p) \mid j \in \mathcal{O}_k, p \in \mathcal{P}_j\}$. The dual thermal subproblem then reads:

$$\begin{cases} \min \displaystyle\sum_{\ell\in\mathcal{L}_T}\sum_{k=1}^{m}\sum_{(j,p)\in T_k} (\pi_j c_\ell - \lambda(j, p))u^\ell(j, p) \\ 0 \leq u^\ell(j, p) \leq \tau^\ell(k)\tau^\ell_T(j)P^\ell_{\max}d(j, p), \end{cases}$$

where $P^\ell_{\max}$ is the maximal available power for thermal unit ℓ, $d(j, p)$ is the duration of time subdivision p for node j, $\tau^\ell_T(j)$ gives the programmed deterministic availability rate

for unit ℓ and node j and $\tau^\ell(k)$ is the random availability rate of unit ℓ for the nodes of the set $\mathcal{O}_k$. The problem is reformulated as follows:

$$\begin{cases} \min \displaystyle\sum_{\ell\in\mathcal{L}_T}\sum_{k=1}^{m}\sum_{(j,p)\in T_k} \tau^\ell(k)(\pi_j c_\ell - \lambda(j,p))\tilde{u}^\ell(j,p) \\ 0 \le \tilde{u}^\ell(j,p) \le \tau_T^\ell(j) P^\ell_{\max} d(j,p), \end{cases}$$

by setting $\tilde{u}^\ell(j,p) = \frac{u^\ell(j,p)}{\tau^\ell(k)}$. Now, we can refer to Subsection 3.1 and compute robust solutions for a given confidence level $0 < \varepsilon < 1$ as:

$$(VaR_T) \begin{cases} \min \ \gamma \\ \mathbb{P}(\displaystyle\sum_{\ell}\sum_{k}\sum_{(j,p)\in T_k} \tau^\ell(k)(\pi_j c_\ell - \lambda(j,p))u^\ell(j,p) \le \gamma) \ge 1-\varepsilon, \\ 0 \le u^\ell(j,p) \le \tau_T^\ell(j) P^\ell_{\max} d(j,p). \end{cases} \tag{7}$$

This is a problem of **maximization of the income** of the thermal utilities or equivalently a problem of **minimization of the losses**.

We now need to study the modeling of the availability rates $\tau^\ell(k)$ to give an explicit form for problem (7).

Modeling of the availability rates $\tau^\ell(k)$. Let $\bar{P}^\ell_{\max}$ be the maximal power of a group in unit ℓ. Then the theoretical maximal power available for thermal unit ℓ is given by $P^\ell_{\max} = n_\ell \bar{P}^\ell_{\max}$. Let $t(k)$ be the time step associated to the root of the subtree $\mathcal{S}_k$. The maximal power available for unit ℓ for the nodes of the set $\mathcal{O}_k$ is then:

$$\tilde{P}^{\ell,k}_{\max} = \sum_{j=1}^{n_\ell} U^{t(k)}_{j,\ell} \bar{P}^\ell_{\max} = n_\ell \bar{P}^\ell_{\max} \frac{\sum_{j=1}^{n_\ell} U^{t(k)}_{j,\ell}}{n_\ell} = P^\ell_{\max}\tau^\ell(k).$$

Notice that under the above hypotheses, the random variable $n_\ell\tau^\ell(k)$ follows the binomial law $\mathcal{B}(n_\ell, \alpha_\ell(t(k)))$. We then have $\mathbb{E}[\tau^\ell(k)] = \alpha_\ell(t(k))$ and the variance of $\tau^\ell(k)$, $\mathrm{Var}(\tau^\ell(k)) = \frac{\alpha_\ell(t(k))(1-\alpha_\ell(t(k)))}{n_\ell}$. Now let Y be the random variable $\displaystyle\sum_{\ell,k} \tau^\ell(k)\ v^\ell(k)$ with $v^\ell(k) = \displaystyle\sum_{(j,p)\in T_k} (\pi_j c_\ell - \lambda(j,p))u^\ell(j,p) = \tilde{c}^\top_{\ell,k} u_{\ell,k}$ with $\tilde{c}_{\ell,k} = (\pi_j c_\ell - \lambda(j,p))_{(j,p)\in T_k}$ and $u_{\ell,k} = (u^\ell(j,p))_{(j,p)\in T_k}$. Since the random variables $(\tau^\ell(k))_{\ell,k}$ are independent, we have

$$\mathbb{E}[Y] = \sum_{\ell,k} \alpha_\ell(t(k))\tilde{c}^\top_{\ell,k} u_{\ell,k} \ \text{ and } \ Var[Y] = \sum_{\ell,k} u^\top_{\ell,k}\, Q_{\ell k}\, u_{\ell,k},$$

where the matrix $Q_{\ell k}$ is defined by $Q_{\ell k} = \frac{\alpha_\ell(t(k))(1-\alpha_\ell(t(k)))}{n_\ell}\tilde{c}_{\ell,k}\tilde{c}^\top_{\ell,k}$. From Subsection 3.1, (7) amounts to solving:

$$\begin{cases} \min \ \displaystyle\sum_{\ell,k} \alpha_\ell(t(k))\ \tilde{c}^\top_{\ell,k} u_{\ell,k} + \kappa(\varepsilon)\sqrt{\sum_{\ell,k} u^\top_{\ell,k} Q_{\ell k} u_{\ell,k}} \\ 0 \le u_{\ell,k}(j,p) \le \tau_T^\ell(j) P^\ell_{\max} d(j,p), \ \text{for all } (\ell,k,j,p), \end{cases} \tag{8}$$

where $\kappa(\varepsilon) = \sqrt{\frac{1-\varepsilon}{\varepsilon}}$. Notice that this is a second order cone optimization problem whose dimension is high in practice (the number of nodes of the tree times the number of time

subdivisions). Another more conservative approach would be to use the same regularization approach for each time subdivision of the nodes. This is possible since the dual thermal subproblem is separable with respect to the time subdivisions of the nodes. The advantage of this approach is that we have an explicit solution for the new dual thermal subproblem and the new dual thermal subproblem is again separable with respect to both the thermal units and the time subdivisions. This allows us to solve problems of large size. Moreover, the availability rate of unit ℓ for each time subdivision follows a binomial law which can be approximated by a Gaussian law if $(n_\ell\,\alpha_\ell(t(k)) \geq 10$ and $n_\ell\,(1-\alpha_\ell(t(k))) \geq 10)$ or n_ℓ large enough, say $n_\ell \geq 6$. Finally, we could also envisage a simpler model for the availability rates process and consider that $\alpha_\ell(t_k)$ does not depend on k. In what follows it is both assumed that $\alpha_\ell(t_k) = \alpha_\ell$ does not depend on k and that the regularization of the thermal subproblem is done at each time subdivision. This will allow us to check how this approach on the thermal subproblems gives an immunization with respect to the uncertainty we have on the availability of the units.

3.2.3 Intermediate summary

So far we have introduced two different regularizations to the original dual optimization problem that was initially formulated as :

$$(DP) \qquad \max_{\lambda} \theta_{\mathcal{D}_0}(\lambda) + (\theta_T + \theta_N + \theta_H + \theta_E)\,(\lambda) \tag{9}$$

where the θ_j are given by (4) and where in the nominal scenario tree the demand is $\mathcal{D}_0$ so that $\theta_{\mathcal{D}_0}(\lambda) = \lambda^\top \mathcal{D}_0$.

1. From a relaxation of the demand, we formulate a first dual regularized problem

$$(VaR_{Rev}) \qquad \max_{\lambda} \theta^{\mathcal{R}}_{\mathcal{D}}(\lambda) + (\theta_T + \theta_N + \theta_H + \theta_E)\,(\lambda)$$

   with

$$\theta^{\mathcal{R}}_{\mathcal{D}}(\lambda) = \theta_{\bar{\mathcal{D}}}(\lambda) - \kappa(\varepsilon_1)\sqrt{\lambda^\top Q \lambda},$$

   where ε_1 is the confidence level chosen to implement VaR_{Rev}.

2. From a rewriting of the random availability of the thermal plants, we get

$$(VaR_T) \qquad \max_{\lambda} \theta_{\mathcal{D}_0}(\lambda) + \theta^{\mathcal{R}}_T(\lambda) + (\theta_N + \theta_H + \theta_E)\,(\lambda)$$

   with

$$\theta^{\mathcal{R}}_T(\lambda) = \sum_{\ell \in \mathcal{L}_T} \sum_{n,p} \left\{ \begin{array}{l} \min \left(\alpha_\ell(\pi_n c_\ell - \lambda(n,p)) + \kappa(\varepsilon_2)\sqrt{\frac{\alpha_\ell(1-\alpha_\ell)}{n_\ell}}|\pi_n c_\ell - \lambda(n,p)| \right) u, \\ 0 \leq u \leq \tau^\ell_T(n) P^\ell_{\max} d(n,p), \end{array} \right. \tag{10}$$

where ε_2 is the confidence level chosen to implement VaR_T. If we use this technique on the whole thermal subproblem then $\theta^{\mathcal{R}}_T(\lambda)$ is given by (8). Finally, combining the two previous regularizations, we come to the mixed problem (called (VaR_{Mixed})):

$$\max_{\lambda} \ \theta^{\mathcal{R}}_{Mixed}(\lambda) = \max_{\lambda} \theta^{\mathcal{R}}_{\mathcal{D}}(\lambda) + \theta^{\mathcal{R}}_T(\lambda) + (\theta_N + \theta_H + \theta_E)\,(\lambda). \tag{11}$$

4 Implementation and numerical simulations

4.1 Implementation and simulation protocol

To solve the primal optimization problem (2), we have to solve $\min_{u\in\mathcal{U}} \max_{\lambda} L(u, x_{\mathtt{H}}, x_{\mathtt{E}}, \lambda)$, where L is the usual Lagrangian. This will be equivalent to the dual problem $\max_{\lambda} \theta(\lambda)$ if only thermal and hydro units are considered. Indeed, in this case, problem (2) is a below bounded linear program and both the primal and the dual are equivalent to each other. If we take into account EJP contracts, the set of constraints is not convex (see the appendix) and the duality gap is strictly positive. However, the weak duality relationship still holds:

$$\min_{u\in\mathcal{U}} \max_{\lambda} L(u, x_{\mathtt{H}}, x_{\mathtt{E}}, \lambda) \geq \max_{\lambda}\ \theta(\lambda).$$

Moreover, numerical simulations have shown that the duality gap is generally quite small. The dual problem thus allows us to approximate primal solutions and estimate marginal prices.

4.1.1 Space decomposition for optimization

First, the space decomposition method for the dual function θ is described and next, we explain the adaptations necessary for the regularized problems. Let $f^\ell(u, x_{\mathtt{H}}, x_{\mathtt{E}})$ be the contribution of unit ℓ to the objective function $f(u, x_{\mathtt{H}}, x_{\mathtt{E}})$ in (2). The dual function θ is non-differentiable, concave and separable with respect to the units as it can be written $\theta(\lambda) = \lambda^\top \mathcal{D}_0 + \sum_{\ell\in\mathcal{L}} \theta^\ell(\lambda)$ with

$$\theta^\ell(\lambda) = \min_{(x^\ell,u^\ell)\in\chi_\ell} f^\ell(u, x_{\mathtt{H}}, x_{\mathtt{E}}) - \sum_{n\in\mathcal{O}} \sum_{p\in\mathcal{P}_n} \lambda(n,p) u^\ell(n,p), \tag{12}$$

the dual function of the subproblem associated with unit ℓ. This is especially of interest to treat problems of large size as is the case for our application. To maximize θ (or which is the same, to minimize the convex function $-\theta$) we use a bundle method described in [LS97]. This requires building a black box which, for any $\lambda \in \mathbb{R}^D$, is able to compute $-\theta(\lambda)$ and to give an arbitrary subgradient $s(\lambda) \in \partial(-\theta(\lambda))$. The computation of $-\theta(\lambda)$ is done solving the different optimization problems associated with the different production units. As for a computation of a subgradient, if $u^\ell(\lambda)$ is an optimal control in (12) then we can give the subgradient $s(\lambda) = -\mathcal{D}_0 + \sum_{\ell\in\mathcal{L}} u^\ell(\lambda)$.

The robustifications proposed can still be solved using a space decomposition framework. A bundle method is still used to compute the optimal Lagrange multipliers. The computations of the dual function and of a subgradient of this function are modified as follows assuming that $\bar{\mathcal{D}} = \mathcal{D}_0$.

- For the dual regularized problem (VaR_{Rev}), we have to increase the value of the dual function by $-\kappa(\varepsilon_1)\sqrt{\lambda^\top Q\lambda}$ and a subgradient by $-\kappa(\varepsilon_1)\dfrac{Q\lambda}{\sqrt{\lambda^\top Q\lambda}}$ where ε_1 is the confidence level used to implement (VaR_{Rev}). So the only extra cost of this model is the estimation of the matrix Q.
- Method VaR_T simply modifies the thermal dual problem which becomes problem (7) (if the regularization is done on the whole tree) or (10) otherwise. If (10) is used,

then the solution of the new thermal dual problem is still separable with respect to both the units ℓ and the time subdivisions p. The optimal control $u^\ell(n,p)$ for time subdivision p, node n and unit ℓ is then immediately given by the following formulas:

$$\begin{cases} u^\ell(n,p) = 0 & \text{if Cond,} \\ u^\ell(n,p) = \tau_T^\ell(n)\, P^\ell_{\max}\, d(n,p) & \text{otherwise;} \end{cases}$$

where the condition Cond is:

$$\left(\pi_n c_\ell \geq \lambda(n,p) \text{ or } (\pi_n c_\ell < \lambda(n,p) \text{ and } \alpha_\ell \leq \frac{\kappa(\varepsilon)^2}{\kappa(\varepsilon)^2 + n_\ell}) \right).$$

4.1.2 Data and simulation protocol

The data used for the simulations is inspired by real data and provided by EDF (the French company producing electricity in France). We use a daily time step and we divide each time step in three time subdivisions. Our generation strategy is tested on a set of 456 scenarios. To each scenario is associated a realization of the inflows in the reservoirs, of the availability rates of the thermal units and of the demand at each time subdivision of the year. From these scenarios, three different trees are built following the lines of [CCC95] (that are directly used by the nominal model). Each tree corresponds to a vision more or less difficult of the evolution of the inflows, of the availability rates and of the demand for the coming year. These trees are called Easy, Median and Difficult trees with evident interpretations of the predictions of the demands, availability rates and inflows on these trees. The scenario trees are trees of a depth of 364 days, with 5227 nodes. The following generation units are used:

- Eleven thermal units. Every thermal unit ℓ is described by its (unit) generation cost, its maximal and minimal power, the number of thermal groups and the probability α_ℓ that a group works.
- Two independent hydro plants. Each hydro plant is connected to a different reservoir. We know the maximal storage capacity (in GWh) of each reservoir, the initial level of each reservoir and the maximal power (in MW) of each plant. The maximal storage capacity of the biggest reservoir is around 30 times that of the other reservoir. This explains why we are essentially interested in the evolution of the biggest reservoir level over the year.
- An EJP contract of 22 days.

Before presenting the results, it remains to be seen how the covariance matrix Q involved in VaR_{Rev} method is chosen. We test a heuristic using the information in the nominal scenario tree.

4.1.3 Calibration of the covariance matrix Q

We suppose the demands at the different time subdivisions are uncorrelated. We thus have to deem the diagonal elements of Q corresponding to the variances $\sigma^2(n,p)$ of the demand for nodes n and time subdivisions p. The node n is associated to a time step $st(n)$. To estimate $\sigma(n,p)$, the demands for time step $st(n)$ in the nominal scenario

tree are sorted by increasing order. Let us denote by $d_{(i)}^{st(n)}, 1 \leq i \leq m_{st(n)}$, the sample of demands ordered by increasing order for time step $st(n)$. We choose for $\sigma(i)$ which determines the uncertainty we have on $d_{(i)}^{st(n)}$:

$$\sigma(i) = \min\left(\frac{d_{(i+1)}^{st(n)} - d_{(i)}^{st(n)}}{2}, \frac{d_{(i)}^{st(n)} - d_{(i-1)}^{st(n)}}{2}\right), \qquad i = 1, \ldots, m_{st(n)},$$

with the convention $d_{(0)}^{st(n)} = 0$ and $d_{(m_{st(n)}+1)}^{st(n)} = 2d_{(m_{st(n)})}^{st(n)} - d_{(m_{st(n)}-1)}^{st(n)}$.

We now compare the outputs obtained with the nominal model and with the robust approaches VaR_{Rev}, VaR_T and VaR_{Mixed}.

4.2 Numerical results

4.2.1 Central and dispersion characteristics of the costs

We compute the mean and the standard deviation of the simulated costs. The empirical quantile of order 0.95 (VaR 5%) and of order 0.99 (VaR 1%) of the distribution of these costs are also computed. The results are reported in Tables 1-3 using the three different trees (Easy, Median and Difficult tree) to solve the optimization problem and for all the methods. Notice that the costs are computed in Francs.[2]

Output	Nominal	VaR_{Rev}	VaR_T	Mixed
Mean	467.5M	488.3M	459.5M	460.0M
St. Dev.	47.9M	46.6M	31.8M	30.6M
VaR 1%	671.6M	672.3M	557.8M	558.7M
VaR 5%	543.8M	574.2M	517.8M	518.9M

Table 1: Central and dispersion characteristics of the empirical distribution of the simulated costs on the Easy tree.

Output	Nominal	VaR_{Rev}	VaR_T	Mixed
Mean	466.5M	486.6M	462.2M	462.3M
St. Dev	46.5M	47.1M	29.2M	28.9M
VaR 1%	689.4M	679.5M	557.8M	554.3M
VaR 5%	548.9M	573.4M	516.5M	515.2M

Table 2: Central and dispersion characteristics of the empirical distribution of the simulated costs on the Median tree.

[2] 1 Franc = 0.1524 Euro.

Output	Nominal	VaR_{Rev}	VaR_T	Mixed
Mean	464.7M	479.8M	465.9M	465.1M
St. Dev.	44.6M	47.2M	28.1M	28.3M
VaR 1%	667.5M	693.7M	556.6M	556.5M
VaR 5%	543.7M	562.0M	517.1M	518.1M

Table 3: Central and dispersion characteristics of the empirical distribution of the simulated costs on the Difficult tree.

We essentially note the following:

- The average management cost on all the scenarios and with each of the three trees are quite close for the four methods considered. For method VaR_{Rev}, the average cost is between 3.2% and 4.4% greater than for the nominal method. For method VaR_T and Mixed, the average cost can be greater or less than for the nominal method. Nevertheless, these costs only vary between 0.08% and 1.7% compared with those of the nominal model.
- Method VaR_{Rev} is not satisfactory for our purposes as the standard deviation of the costs as well as the quantiles of order 0.95 and 0.99 are greater than the same quantities computed for the nominal model. As for the methods VaR_T and Mixed, they lead in all the cases to reductions of the standard deviation of the costs (up to 38% of reduction on the Difficult tree) and of the 0.95 and 0.99 quantiles. One reason that could explain the bad results of method VaR_{Rev} would be that the relaxation of the demand constraint in an ellipsoid works as an opportunity for the system to have another reserve to perform its optimization. But this reserve does not exist in the Monte Carlo simulation and thus, the strategy reveals itself to be too optimistic. Also, the size of the ellipsoid should be reduced (our choice was conservative). The good results of VaR_T could be attributed to the fact that the total thermal costs are around 100 times greater than the maximal possible value of the biggest reservoir. It thus indeed seems interesting to envisage a robust approach which takes into account the only randomness involved in the thermal subproblem: the availability rates of the thermal plants.

4.2.2 Study of the trajectories of the biggest reservoir

For a given method, the management strategy of the biggest reservoir varies slightly when the scenario tree changes (see Figure 1 which follows). The nominal and VaR_{Rev} methods tend to empty more the reservoir whose level increases at the end of the year. Methods VaR_T and Mixed do not use the reservoir or very little at the beginning of the year. Globally, the reservoir has a higher level with these methods and is nearly full (the maximal storage capacity is 3500 GWh) at the end of the year. We say that the biggest reservoir is at a low level if it contains at most 5% of its maximal storage capacity. Table 4 which follows permits to see in detail tendencies already observed in Figure 1.

4.2.3 Comparison of the distribution of the costs

The empirical densities of the management cost (see Figure 2 which follows) for the nominal and VaR_{Rev} methods have tails of distribution bigger than those of methods

VaR_T and Mixed. A few scenarios are of very high cost for the nominal and VaR_{Rev} methods. On the contrary, the dispersion of the costs for models VaR_T and Mixed is smaller. This can be illustrated by a few figures:

- The scenario of highest cost corresponds to costs of $744.8M$, $823.2M$, $573.8M$ and $575.3M$ for respectively the nominal, VaR_{Rev}, VaR_T and Mixed methods.
- The scenarios of lowest cost for each of the models have close costs that are worth $382.2M$, $386.8M$, $399.7M$ and $399.1M$ for respectively the nominal method, VaR_{Rev}, VaR_T and the Mixed approach.

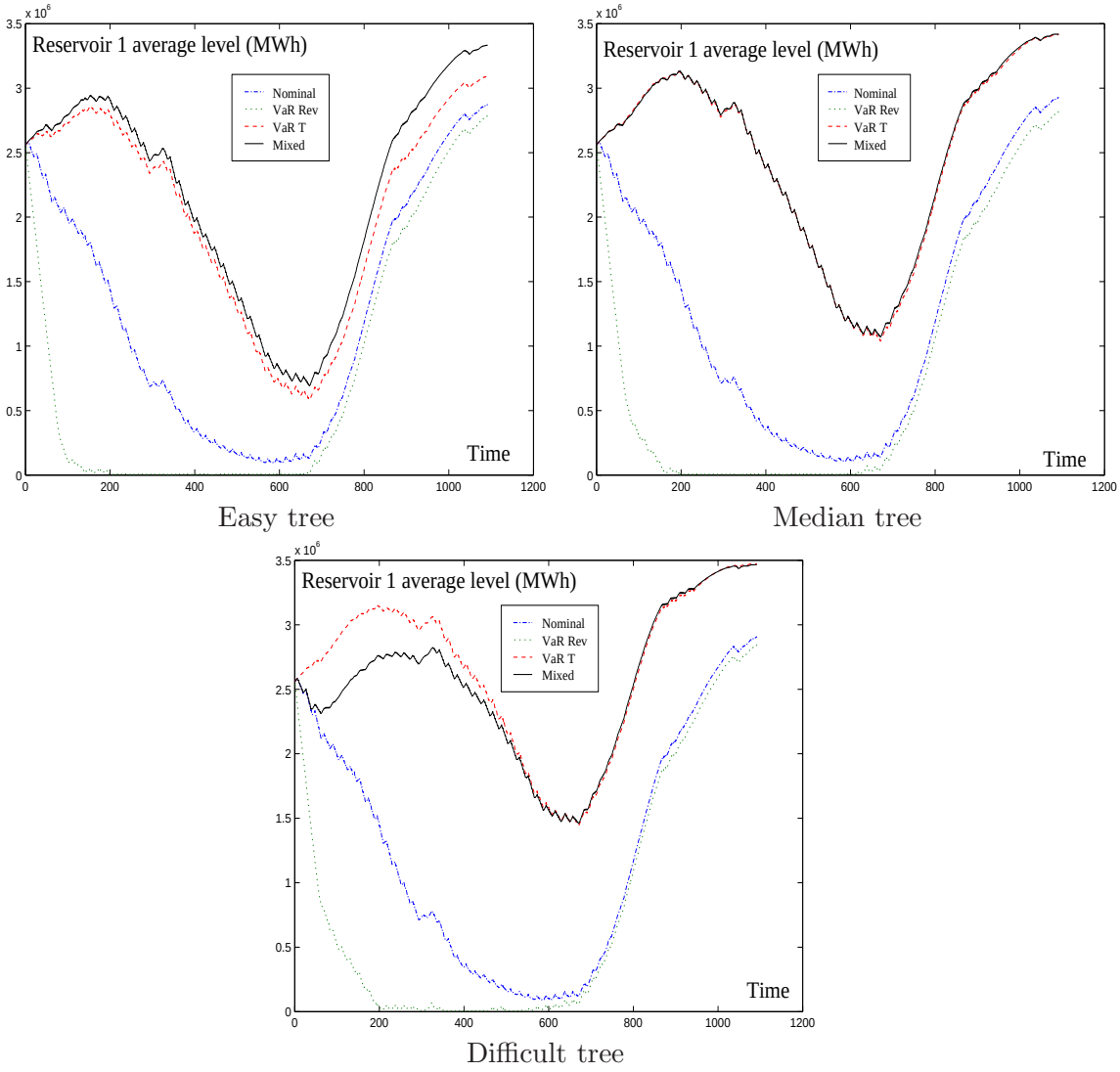


Figure 1: Evolution of the average level (in MWh and on the whole scenarios) of the biggest reservoir during the year for all the methods.

# of weeks X	Low level reservoir			
	Nominal	VaR_{Rev}	VaR_T	Mixed
1	426	456	5	9
2	423	456	4	4
3	423	456	3	3
4	417	456	3	2
5	412	456	3	2
10	365	456	0	0
15	256	456	0	0
20	92	453	0	0
25	14	257	0	0
30	0	25	0	0

Table 4: Number of scenarios among 456 for which the biggest reservoir is at least X weeks with a low level.

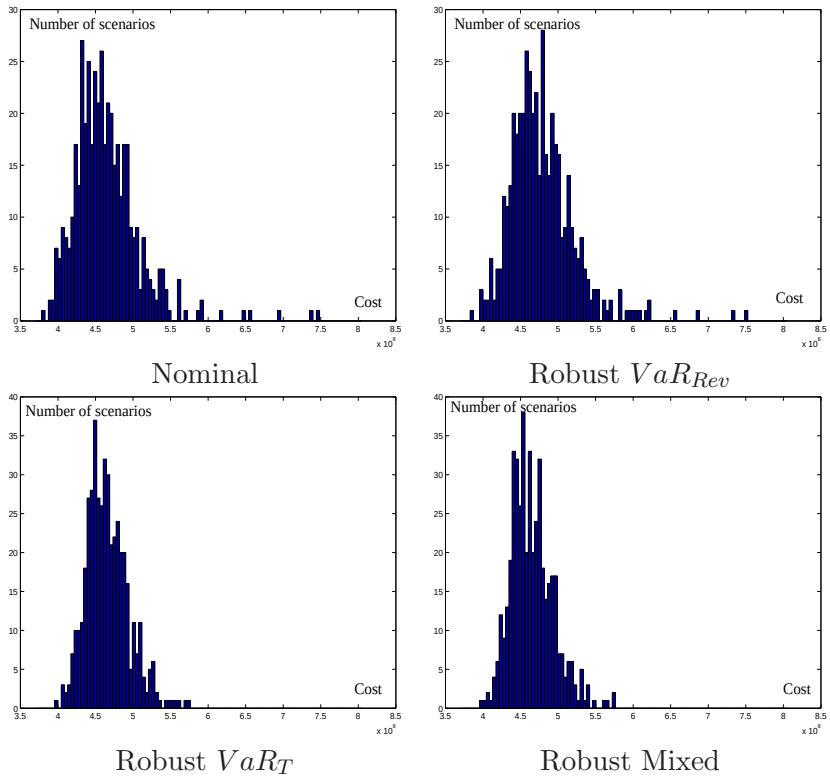
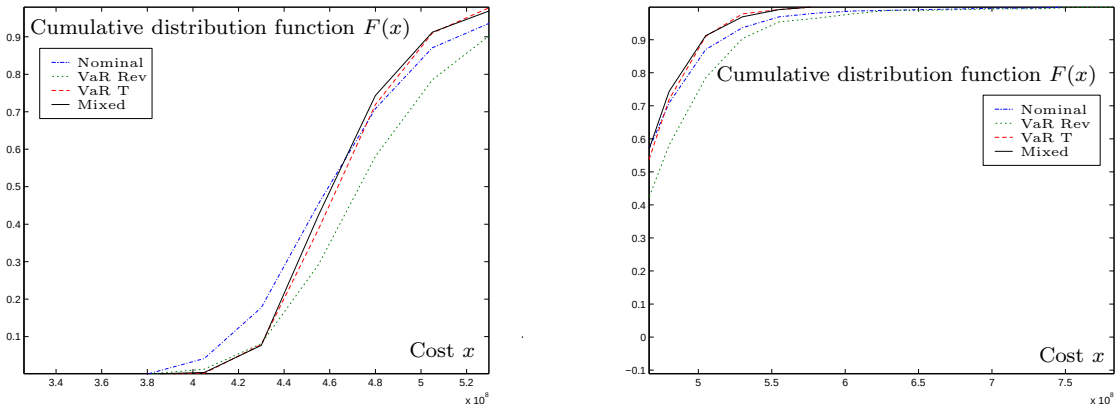


Figure 2: Empirical densities of the management cost using the Difficult tree.

In Figure 3 which follows, the shape of the empirical cumulative distribution function of the management cost confirms this tendency. The nominal method has more scenarios whose cost is less than $470M$. On the other hand, there exists a non-negligible number of scenarios of high costs. For VaR_{Rev}, it is worse as the number of scenarios whose cost is less than $470M$ is small.



Lower part Higher part

Figure 3: Empirical CDF of the costs using the Difficult tree for the four methods.

5 Conclusion

This paper has presented an application of Value-at-Risk method to robustify a stochastic optimization problem of yearly electricity generation management. The starting point of our investigation was a nominal model which led to a large standard deviation of the cost and which excessively emptied the hydro reservoirs. Two models have been proposed to reduce these drawbacks. The first one is a robustification of the dual problem which takes into account the uncertainty for the demand. From a theoretical point of view, it aims at stabilizing the Lagrange multipliers which correspond to electricity prices in our application and from a practical point of view, it can be seen as a robust model designed to secure the global revenue. However, this model VaR_{Rev} has shown some limitations when compared to the nominal one. The second model formulates a Value-at-Risk problem on the income of the thermal plants, and has revealed very good robustness properties: sharp diminution of the standard deviation of the simulated management cost and prudent use of the water reservoirs. We have also implemented a combination of these two methods to get an interesting trade-off between the reduction of the volatility of the management cost and the parsimonious use of water. We believe that the performance of VaR_{Rev} model could be improved with a more accurate estimation of the covariance matrix for the demand side.

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Appendix

In this appendix, we first detail the formulation of nominal problem (1). To this end, we explain how the evolution of the random variables is represented over the year. Three important models of power generation units (thermal, hydro and demand side management contracts called EJP) are then described. The resolution of the different dual subproblems is briefly commented. Finally, the way of determining a production schedule is outlined.

Prediction of random events and global problem

The evolution of the uncertain parameters over the optimization period is represented by a scenario tree. Each node of the tree corresponds to a one day period of time. Each day has three time subdivisions. We complete the notation introduced in Sections 2.1, 3.2, 3.2.2 and 4.1.3 introducing for each node n of the tree:

- $F(n)$, the father node of node n.
- $S(n)$, the set of son nodes of node n.
- $\pi_T(n)$, the probability to go from node $F(n)$ to node n.

Other variables attached to a node will be introduced in the description of the power generation units. Below is an example of a scenario tree. In this example we have $F(1) = 0$, $S(2) = \{4, 5\}, \ldots$

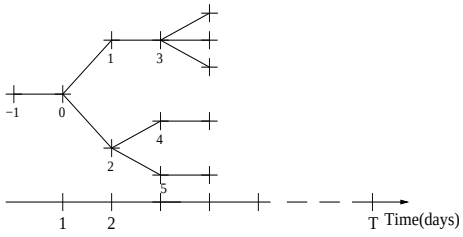

Figure 4: Tree representing the different scenarios.

A scenario in the tree is thus a path from the root node to a leaf node. The construction of the tree is based on aggregation procedures using historical data (demand, inflows,...). These problems are discussed in [CCC95],[Pfl01].
Given this representation of random events, the global problem of yearly power management scheduling (1) consists of minimizing the average generation cost over the random tree while satisfying the demand and the operating constraints of the generation units. The reader should be aware that the solution of this problem is indexed by the nodes of the scenario tree. If the scenario that occurs is represented in the tree, we get a generation schedule for this scenario. For a generation schedule that is not represented in the tree, see the last section of this appendix.

Modeling of power generation units

Three kinds of generation units are modeled : the thermal, hydro and EJP units.

Thermal Units. Let ℓ be the index of a thermal unit. The thermal units are completely described by two characteristics:

1. The generation levels: $u^\ell(n,p)$ must remain between 0 and $\tau_f^\ell(n)\,\tau_T^\ell(n)\,P_{\max}^\ell\,d(n,p)$ with
   - $\tau_f^\ell(n)$ the random availability rate, realization of a random variable τ^ℓ;
   - $\tau_T^\ell(n)$ the programmed availability rate (deterministic);
   - $P_{\max}^\ell$ the maximal power (in MW) of unit ℓ.
2. The generation costs: c_ℓ is the unit production cost for unit ℓ.

Hydro Units. A hydro valley is a set of interconnected plants and reservoirs with natural inflows in each reservoir. The constraints are of two kinds: (i) box constraints on the volume of each reservoir and on the discharged water and (ii) flow balance equations for each reservoir. Let ℓ be a hydro plant. The following notation is used:

- $y^\ell(n)$ is the content (in MWh) of the unique reservoir associated to ℓ at the beginning of time step $st(n)$. $y_{\min}^\ell$ and $y_{\max}^\ell$ are the lower and upper bounds (in MWh) on reservoir ℓ level.
- $\tau_H^\ell(n)$ is the programmed availability rate (deterministic) for unit ℓ and node n.
- $a^\ell(n,p)$ is the natural inflow (in MWh) for reservoir ℓ, node n and time subdivision p. We set $a^\ell(n) = \sum_{p\in\mathcal{P}_n} a^\ell(n,p)$.
- $P_{\max}^\ell$ is the maximal power of hydro plant ℓ.
- $dev^\ell(n,p)$ is the spillage (in MWh) and $u^\ell(n,p)$ is the discharge (control) for plant ℓ (in MWh) for node n, time subdivision p.
- $V_H^\ell(y)$ is the value of the water level y for reservoir ℓ at the last time step T. The operating costs for all reservoirs are null for $st(n) < T$. Hence, only the value of the water at time T is taken into account.

The hydro subproblem thus consists of minimizing

$$-\sum_{\ell\in\mathcal{L}_H}\sum_{n\in O_T}\pi_n\,V_H^\ell\left(y^\ell(n)+a^\ell(n)-\sum_{p\in\mathcal{P}_n}\left(u^\ell(n,p)+dev^\ell(n,p)\right)\right),$$

under the constraints

$$\begin{cases} y^\ell(n) = y^\ell(F(n)) + \displaystyle\sum_{p\in\mathcal{P}_n}\left(a^\ell(F(n),p) - u^\ell(F(n),p) - dev^\ell(F(n),p)\right),\ \forall(n,\ell), \\ y_{\min}^\ell \le y^\ell(n) + \displaystyle\sum_{p\in\mathcal{P}_n}\left(a^\ell(n,p) - u^\ell(n,p) - dev^\ell(n,p)\right) \le y_{\max}^\ell,\ \forall n\in\mathcal{O}_\mathcal{T},\ \forall\,\ell, \\ 0 \le u^\ell(n,p) \le \tau_H^\ell(n)\,P_{max}^\ell\,d(n,p),\ \forall(n,p,\ell), \\ 0 \le dev^\ell(n,p),\ \forall(n,p,\ell), \\ y_{\min}^\ell \le y^\ell(n) \le y_{\max}^\ell,\ \forall(n,\ell). \end{cases}$$

The state variables are the contents $(y^\ell(n))_n$ of the reservoirs, the spillage $(dev^\ell(n,p))_{n,p}$ and the slack variables used to write the hydro subproblem in a linear form.

EJP contracts. EJP contract ℓ is represented by a production unit with the following features:

- E^ℓ is the total number of days the contract can be used.
- The power linked to contract ℓ is P_E^ℓ (in MW).
- Each day, either the contract is used all day long (providing $24P_E^\ell$ MWh) or it is not used. A control variable $t^\ell(n)$ defined for every node n permits one to know whether contract ℓ is used at node n ($t^\ell(n) = 1$) or not ($t^\ell(n) = 0$).
- $s^\ell(n)$ is the number of days still available on the contract ℓ for node n at the beginning of time step $st(n)$.
- $V_E^\ell(\cdot)$ is the value function for EJP reservoir ℓ at the last time step T.

Given starting reservoir level E^ℓ on EJP contract ℓ, the value of the EJP reservoir at time step T is maximized, which yields the following EJP subproblem:

$$\begin{cases} \min \ -\sum_{\ell\in\mathcal{L}_E}\sum_{n\in\mathcal{O}_T} \pi_n V_E^\ell(s^\ell(n) - t^\ell(n)) & \\ s^\ell(n) = E^\ell, & \text{for the root node,} \\ t^\ell(n)(t^\ell(n) - 1) = 0, & \forall\ n \in \mathcal{O}, \\ s^\ell(F(n)) - t^\ell(F(n)) = s^\ell(n), & \forall\ (n,\ell), \\ s^\ell(n) \geq 0, & \forall\ (n,\ell), \\ s^\ell(n) - t^\ell(n) \geq 0, & \forall\ n \in \mathcal{O}_T,\ \forall \ell. \end{cases}$$

Thus if $\ell \in \mathcal{L}_E$, the EJP control $u^\ell(n,p)$ is given by $t^\ell(n)\, P_E^\ell\, d(n,p)$. Finally, let us mention that, in a first approximation, and for mid-term electricity production management, nuclear plants can be modeled as thermal plants [BR97].

Space decomposition

Following [BGLS97], the maximization of the dual function θ is done by an iterative scheme and can be described with 4 steps starting from a reference price λ used to initialize the algorithm with index $k = 1$ and $\lambda_1 = \lambda$.

1. At iteration k, decomposition in subproblems and computation of the local solutions of the subproblems;
2. Evaluation of the dual function θ at λ_k and computation of a subgradient $s(\lambda_k)$;
3. Update of the multipliers by the coordinator using a black box method (i.e., computation of λ_{k+1});
4. Update of the index $k \leftarrow k + 1$ and go to step 1.

Solving the different dual subproblems

The first step of the space decomposition algorithm given in the previous paragraph is briefly detailed. The nominal thermal dual subproblem has an evident solution and the hydro subproblem is a linear optimization problem of large size (around 36 000 variables in our case) solved using interior point methods. The EJP problem is an NP hard, non-convex optimization problem solved using stochastic dynamic programming. We know, for every contract ℓ, the values $V_E^\ell(y)$ at the last time step T for all possible values y (in days, having an energetic equivalent) of EJP reservoir ℓ. We deduce, using HJB (Hamilton-Jacobi-Bellman) equations (backward phase), the Bellman values $V^\ell(y,n)$ for all contract ℓ, every reservoir step y and every node n:

$$V^\ell(y,n) = \left\{ \begin{array}{l} \max \sum_{m\in S(n)} \pi_T(m) \left(V^\ell(y - t^\ell(n), m) + \sum_{p\in\mathcal{P}_n} d(n,p)\lambda(n,p) P_E^\ell t^\ell(n) \right) \\ y - t^\ell(n) \geq 0, \quad t^\ell(n) \in \{0,1\}. \end{array} \right.$$

Knowing the storage level of every contract ℓ at the beginning of the year, the forward phase consists of deducing the optimal EJP controls to apply using the Bellman values computed in the backward phase. Both kinds of methods used (stochastic dynamic programming and interior points methods) have a complexity that depends on the dimension of the dual space. A known drawback of dynamic programming is that its complexity grows exponentially with the state variable dimension.

The simulation process

The resolution of (9) (or of (11) with our robust methodology) provides optimal marginal prices $\lambda^*(n,p)$ that are useful to elaborate a strategy which, for any realization of the random variables on the time period, allows us to compute a generation schedule. This strategy uses the nominal model scenario tree where at each node of the tree we find realizations of the uncertain parameters. It has the form of Bellman functions that are computed with the following version of the Bellman principle. Let a reserve ℓ be given. For the last time step T, the Bellman function $V^\ell(y,n)$ is known for each reservoir level y and each leaf n. Between two grid points, the value is supposed to be affine. The Bellman values $V^\ell(y,n)$ for all the nodes n of the tree and all reservoir steps y are computed using the following recurrence formula:

$$\left\{ \begin{array}{l} V^\ell(y,n) = \max \displaystyle\sum_{m\in S(n)} \pi_T(m) \left[V^\ell \left(y + a^\ell(n) - \sum_{p\in\mathcal{P}_n} u^\ell(n,p) \right.\right. \\ \qquad\qquad \left.\left. - \displaystyle\sum_{p\in\mathcal{P}_n} dev^\ell(n,p), m \right) + \sum_{p\in\mathcal{P}_n} \lambda^*(n,p)\, u^\ell(n,p) \right] \\ y + a^\ell(n) - y^\ell_{\max} \leq \displaystyle\sum_{p\in\mathcal{P}_n} u^\ell(n,p) + dev^\ell(n,p) \leq y + a^\ell(n) - y^\ell_{\min} \\ 0 \leq u^\ell(n,p) \leq \tau_H^\ell(n)\, P^\ell_{max}\, d(n,p), \quad 0 \leq dev^\ell(n,p), \end{array} \right.$$

if ℓ stands for a hydro reservoir and

$$\left\{ \begin{array}{l} V^\ell(y,n) = \max \displaystyle\sum_{m\in S(n)} \pi_T(m) \left(V^\ell(y - t^\ell(n), m) + \sum_{p\in\mathcal{P}_n} d(n,p)\lambda^*(n,p) P_E^\ell t^\ell(n) \right) \\ t^\ell(n)(t^\ell(n) - 1) = 0, \quad y - t^\ell(n) \geq 0, \end{array} \right.$$

if ℓ stands for an EJP contract. The Bellman function for time step t is then given by $V^\ell(y,t) = \sum_{n\in\, t}\ \pi(n)V^\ell(y,n)$ for reservoir level y, unit ℓ.

This algorithm is only a stochastic dynamic programming (SDP) performed on marginal values. Once those Bellman functions are computed, it is possible to perform a Monte Carlo simulation of the generation scheduling using $\delta_y V^\ell(y,t)$ as a "fuel cost" of the energy kept in reservoir ℓ.

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