A STABILIZED MODEL AND AN EFFICIENT SOLUTION METHOD FOR THE YEARLY OPTIMAL POWER MANAGEMENT

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Abstract

We propose a stabilized model for the electricity generation management problem on a yearly scale. We also introduce an original and efficient solution method in a particular case. Our model is compared to other management methods and offers the best average cost while preserving a reasonable standard deviation of the cost over a set of testing scenarios.

Keywords: Uncertain linear programs; Stochastic programming; Dynamic programming; Electricity generation management.

Mathematics Subject Classification: 93E03; 49L20.

1. Introduction

Given a fixed mix of electric power plants (nuclear, thermal, hydroelectric and possibly peak demand management contracts modeled as virtual production units called EJP [for *Effacement Jours de Pointe* in French] contracts in France), the electricity generation management problem consists of minimizing the production cost over the management horizon while satisfying operational constraints for the production units and ensuring the balance between production and demand at each time step. In practice, the modelling approach is highly dependent on the time horizon of the optimization problem: for short time horizons, typically of one day or of one week, the problem is generally assumed to be deterministic (cf. [2],[6]), whether for longer management horizons, a special emphasis is put on the stochastic nature of data. In particular, on a yearly scale, reservoir inflows, demand, as well as availability of the plants cannot be considered deterministic: in France, for example, winter customer demand can vary up to one GW per degree Celsius while the peak loads are approximately 80 GW.

Production management problems (and particularly power management problems) have been widely studied both concerning the modelling and the solution methods ([5], [7], [8], [17], [12] [1], [11], [19], [16], [18]). Generally, the evolution of the uncertain parameters over the management period is modeled by a scenario tree and the goal is to minimize the expected production cost over this set of scenarios. Recent solution methods (see [1],[11],[19] for instance), use Lagrangian relaxation and various nondifferentiable optimization methods and tools to solve the master problem.

We present an alternative to the use of scenario trees. Our objective is to propose a stable management method, i.e., a method that provides relatively stable solutions in terms of cost (when the consumption but also the natural inflows and availability rates vary). Our contribution is to show, as in [14], that a stabilization of the dual function allows us to implement a stable management method. However, contrary to [14], the evolution of the uncertain parameters over the optimization period is not represented by a scenario tree. Moreover, we show that these modifications of the resulting dual problem preserve the price decomposition scheme. Dedicated solution methods for the electricity production management problem are also proposed.

The paper is organized as follows. We set up the physical model in Section 2 and we give a mathematical formulation of the deterministic formulation of the problem in Section 3. The modelling of the uncertainty is detailed in Section 4 and the stable counterpart of the problem is given in Section 5. An efficient and original method to solve the deterministic electricity generation management problem by dynamic programming when there is one hydro plant and an arbitrary number of thermal plants is provided in Section 6. In Section 7, solution methods using in particular the results of Section 6 are then proposed for the deterministic electricity generation management problem in the general case and for a specific model in the stochastic case. In this section, we also detail the solution methods and the management strategy used to implement the stabilized method introduced in Section 5. Finally, in the last section, we assess the efficiency of the stabilized method on numerical simulations.

2. A model for the electricity power generation problem

The goal is to decide the production levels of the plants comprising the mix in such a way that the demand is satisfied at each time step and the production cost is minimized. The physical model we consider is a stochastic dynamical system for which the uncertain parameters are the electricity consumption, the availability rates of the thermal plants and the quantity of inflows received by the different reservoirs of the hydroelectric power stations. The model uses some aspects of [7] and [12], introduces availability rates for the thermal units and value functions for the hydro units and the so-called EJP contracts, to be described in Section 2.3 below.
Let T be the number of subintervals obtained by partitioning the time horizon. This partition can be chosen uniformly (daily, weekly, monthly) or adaptively. Let $\mathcal{L} = \mathcal{L}_T \cup \mathcal{L}_H \cup \mathcal{L}_J$ be a partition of the set of production units in which $\mathcal{L}_T$ represents the set of thermal units, $\mathcal{L}_H$ the set of hydro units, and $\mathcal{L}_J$ the set of EJP contracts. We denote the duration (in hours) of time step t by $Duration(t)$. Also, for every $\ell \in \mathcal{L}$, the maximal theoretical power of production unit ℓ is denoted by $P^{\ell}_{\max}$ (in MW).

2.1. Thermal units.

For every thermal unit $\ell \in \mathcal{L}_T$, we denote the control for time step t by u^{ℓ}_t. It corresponds to the production level (in MWh) for thermal plant ℓ at time step t. Each thermal plant ℓ is made up of a certain number of thermal groups, some of which can be out of commission at time step t. Thus, for each time step t and each thermal plant ℓ, the maximal theoretical power $P^{\ell}_{\max}$ must be corrected taking into account the availability rate τ^{ℓ}_t of thermal plant ℓ at time step t. The real maximal power for time step t and thermal plant ℓ is then $\tau^{\ell}_t P^{\ell}_{\max}$. The constraints fixed on the production levels of the thermal plants may thus be expressed as

$$0 \leq u^{\ell}_t \leq \tau^{\ell}_t \, Duration(t) \, P^{\ell}_{\max}, \quad \ell \in \mathcal{L}_T, \quad t = 1, \ldots, T. \tag{1}$$

Finally, the production costs are linear functions of the production levels: the production cost for thermal plant ℓ at time step t is thus $c_\ell \, u^{\ell}_t$, where c_ℓ is a fixed positive unit cost. Nuclear power plants can be modeled similarly, see [18].

2.2. Hydroelectric power stations.

The hydroelectric network is made up of a set of interconnected hydro plants and reservoirs. Each plant can have one or several turbines and each turbine can receive water from different reservoirs. However, for simplicity, we suppose that each hydro plant only has one upstream reservoir and one downstream reservoir. Also, each reservoir only has one downstream hydroplant. The index of a station $\ell \in \mathcal{L}_H$ and of its upstream reservoir are the same. For every station $\ell \in \mathcal{L}_H$, let $u^{\ell}_t = (v^{\ell}_t, sp^{\ell}_t)$ be the control applied to station ℓ at time step t, where v^{ℓ}_t is the production (in MWh) of station ℓ at time step t and sp^{ℓ}_t the spillage at time step t for reservoir ℓ if there is an overflow. The state x^{ℓ}_t of hydro plant ℓ at time step t corresponds to the volume (in MWh) of reservoir ℓ at the beginning of this time step. Two kinds of inflows tend to increase the reservoir levels. On the one hand, there are the natural inflows due to rainwater: we denote the natural inflow (in MWh) of reservoir ℓ at time step t by $\mathcal{I}^{\ell}_t$. On the other hand, for each reservoir ℓ, water can come from an upstream hydro plant. For every station $\ell \in \mathcal{L}_H$, we denote the index of the reservoir downstream station ℓ (if it exists) by $f(\ell)$; the time (in time steps) for the water to go from station ℓ to reservoir $f(\ell)$ being $d(\ell, f(\ell))$. We assume that the stations are always available. The operational constraints of the stations are of two kinds: (i) box constraints on the production levels and the reservoir volumes and (ii) flow balance equations for each reservoir. These constraints read

$$
(2)\qquad
\begin{array}{ll}
x^\ell_{t+1} = x^\ell_t + \mathcal{I}^\ell_t - v^\ell_t + \displaystyle\sum_{m \,|\, f(m)=\ell} v^m_{t-d(m,f(m))} - sp^\ell_t, & \forall\, \ell \in \mathcal{L}_H, \forall\, t = 1,\ldots,T,\\
0 \le v^\ell_t \le Duration(t)\, P^\ell_{\max}, & \forall\, \ell \in \mathcal{L}_H, \forall\, t = 1,\ldots,T,\\
x^\ell_{\min} \le x^\ell_{t+1} \le x^\ell_{\max}, & \forall\, \ell \in \mathcal{L}_H, \forall\, t = 1,\ldots,T,
\end{array}
$$

where $x^\ell_{\min}$ and $x^\ell_{\max}$ are the minimal and maximal levels (in MWh) of reservoir ℓ volume. The initial storage of each hydro reservoir is known. To use the water in a parsimonious way (the more water we have at the end of the year, the more we can use during the next period), two strategies are frequently used. The first one ([7], [12], [8]) consists of constraining the level of the reservoir at the end of the management period to be greater than the level it had at the beginning of this period. An alternative choice consists of associating to each reservoir ℓ, a value function $V^\ell_H(\cdot)$ of the water storage at the last time step. This function associates to each admissible value of the water storage a value in Euros. It is a strictly increasing function which is quadratic in [5] and linear in [17]. We assume this function is concave and piecewise affine. Then, we wish to maximize the sum over all reservoirs ℓ of the values $V^\ell_H(x^\ell_{T+1})$.

2.3. **EJP contracts.** The EJP contracts can be seen as independent energy reservoirs having a limited production capacity. They cannot be used more than a certain number of time steps fixed by the contract and for each time step, either the full production capacity is used, or this capacity is not used at all. For every EJP contract $\ell \in \mathcal{L}_J$, let u^ℓ_t be the production (in MWh) or control applied to this contract ℓ for time step t. If the EJP contract is used for time step t, $u^\ell_t = Duration(t)P^\ell_{\max}$ and $u^\ell_t = 0$ otherwise. The state of EJP contract ℓ for time step t is represented by the variable x^ℓ_t giving the energy storage still available on this contract at the beginning of time step t. The energy storage x^ℓ_1 available on contract ℓ over the optimization period is given. The equations ruling the evolution of the controls and states is thus

$$(u^\ell_t - Duration(t)P^\ell_{\max})\; u^\ell_t = 0,\; x^\ell_{t+1} = x^\ell_t - u^\ell_t,\; x^\ell_{t+1} \ge 0,\; \ell \in \mathcal{L}_J,\; 1 \le t \le T.$$

In this article, we use flexible EJP contracts with constraints $u^\ell_t \in \{0, Duration(t)P^\ell_{\max}\}$ convexified in such a way that EJP contract states and controls satisfy

$$(3)\qquad 0 \le u^\ell_t \le Duration(t)\; P^\ell_{\max},\;\; x^\ell_{t+1} = x^\ell_t - u^\ell_t,\;\;\; x^\ell_{t+1} \ge 0,\;\; \ell \in \mathcal{L}_J,\;\; 1 \le t \le T.$$

This means that for each EJP contract, we have a certain amount of energy, which, every day, and as long as this reserve is not finished, can be used all or part of the day. This model amounts to considering that an EJP contract is a particular hydro reservoir without inflows. Indeed, similarly to hydro reservoirs, we associate to each contract ℓ a value function $V^\ell_J(\cdot)$ of its energy storage at the end of the horizon. This function is concave and piecewise affine. We then wish to maximize the sum over all contracts ℓ of the values $V^\ell_J(x^\ell_{T+1})$.

3. Deterministic formulation of the problem

In this section, we show that the optimal power management problem can be expressed as a linear optimization problem. We also prove a condition ensuring the feasibility of this problem. The production units have to be used in order to satisfy the demand. If $\mathcal{D}_t$ is the electricity consumption for time step t, the demand satisfaction constraints read

$$(4)\qquad \sum_{\ell \in \mathcal{L}_T} u^\ell_t + \sum_{\ell \in \mathcal{L}_J} u^\ell_t + \sum_{\ell \in \mathcal{L}_H} v^\ell_t \ge \mathcal{D}_t,\;\; t = 1,\ldots,T.$$

The electricity production management problem then consists of minimizing

$$(5)\qquad \sum_{\ell \in \mathcal{L}_T} \sum_{t=1}^{T} c_\ell\; u^\ell_t - \sum_{\ell \in \mathcal{L}_H} V^\ell_H(x^\ell_{T+1}) - \sum_{\ell \in \mathcal{L}_J} V^\ell_J(x^\ell_{T+1}),$$

under the constraints (1),(2), (3) and (4). The function $V^\ell_H(\cdot)$ can be expressed as $V^\ell_H(x) = \min_{0 \le k \le m^\ell_H - 1} f^\ell_{H,k}(x)$, where the functions $f^\ell_{H,k}(\cdot)$ are affine. More precisely, between values denoted by $g^\ell_{H,k}$ and $g^\ell_{H,k+1}$ ($k = 0,\ldots,m^\ell_H - 1$), the function $V^\ell_H(\cdot)$ coincides with the function $f^\ell_{H,k}(\cdot)$.

Thus, we can replace the contribution $-\sum_{\ell \in \mathcal{L}_H} V_H^\ell(x_{T+1}^\ell)$ of the hydroelectric power stations to the objective with $-\sum_{\ell \in \mathcal{L}_H} a_\ell$ adding the constraints

$$a_\ell \leq f_{H,k}^\ell(x_{T+1}^\ell), \;\; \ell \in \mathcal{L}_H, \;\; 0 \leq k \leq m_H^\ell - 1. \tag{6}$$

If we define

$$c_{H,k}^\ell = \frac{V_H^\ell(g_{H,k+1}^\ell) - V_H^\ell(g_{H,k}^\ell)}{g_{H,k+1}^\ell - g_{H,k}^\ell}, \quad d_{H,k}^\ell = V_H^\ell(g_{H,k}^\ell) - c_{H,k}^\ell \, g_{H,k}^\ell; \tag{7}$$

inequalities (6) become

$$a_\ell \leq c_{H,k}^\ell \, x_{T+1}^\ell + d_{H,k}^\ell, \;\; \ell \in \mathcal{L}_H, \;\; k = 0, \ldots, m_H^\ell - 1. \tag{8}$$

For each EJP contract $\ell \in \mathcal{L}_J$, the function $V_J^\ell(\cdot)$ is also concave, piecewise affine and can be written as $V_J^\ell(x) = \min_{0 \leq k \leq m_J^\ell - 1} f_{J,k}^\ell(x)$, where the functions $f_{J,k}^\ell(\cdot)$ are affine. We then introduce the quantities $c_{J,k}^\ell$ and $d_{J,k}^\ell$ by replacing H with J in (7). The electricity production management problem then boils down to the minimization of

$$\sum_{\ell \in \mathcal{L}_T} \sum_{t=1}^{T} c_\ell \, u_t^\ell - \sum_{\ell \in \mathcal{L}_H} a_\ell - \sum_{\ell \in \mathcal{L}_J} b_\ell,$$

under the constraints (1), (2), (3), (4), (8) and

$$b_\ell \leq c_{J,k}^\ell \, x_{T+1}^\ell + d_{J,k}^\ell, \;\; \ell \in \mathcal{L}_J, \;\; k = 0, \ldots, m_J^\ell - 1.$$

In what follows, we assume that the hydroelectric network is made of a set of pairs hydroelectric power station-reservoir not linked to each other. The flow balance equations for the hydro and EJP reservoirs can then be written

$$\forall\, t, \;\; x_{t+1}^\ell = x_1^\ell + \sum_{k=1}^{t} (\mathcal{I}_k^\ell - v_k^\ell - sp_k^\ell), \;\; \ell \in \mathcal{L}_H, \;\; x_{t+1}^\ell = x_1^\ell - \sum_{k=1}^{t} u_k^\ell, \;\; \ell \in \mathcal{L}_J.$$

Plugging these equality constraints into the constraints $x_{\min}^\ell \leq x_t^\ell \leq x_{\max}^\ell$, $\ell \in \mathcal{L}_H$, and $x_t^\ell \geq 0$, $\ell \in \mathcal{L}_J$, on the states of the reservoirs and contracts, we can express the electricity production management problem as

$$\left\{ \begin{array}{l} \min \;\; \displaystyle\sum_{t=1}^{T} \sum_{\ell \in \mathcal{L}_T} c_\ell \, u_t^\ell - \sum_{\ell \in \mathcal{L}_H} a_\ell - \sum_{\ell \in \mathcal{L}_J} b_\ell \\ a_\ell + c_{H,k}^\ell \displaystyle\sum_{t=1}^{T} (v_t^\ell + sp_t^\ell) \leq c_{H,k}^\ell \, (x_1^\ell + \sum_{t=1}^{T} \mathcal{I}_t^\ell) + d_{H,k}^\ell, \; \ell \in \mathcal{L}_H, 0 \leq k \leq m_H^\ell - 1, \\ b_\ell + c_{J,k}^\ell \displaystyle\sum_{t=1}^{T} u_t^\ell \leq c_{J,k}^\ell x_1^\ell + d_{J,k}^\ell, \;\; \ell \in \mathcal{L}_J, 0 \leq k \leq m_J^\ell - 1, \\ x_1^\ell + \displaystyle\sum_{k=1}^{t} \mathcal{I}_k^\ell - x_{\max}^\ell \leq \sum_{k=1}^{t} v_k^\ell + sp_k^\ell \leq x_1^\ell + \sum_{k=1}^{t} \mathcal{I}_k^\ell - x_{\min}^\ell, \; \ell \in \mathcal{L}_H, \; 2 \leq t \leq T, \\ 0 \leq v_t^\ell \leq Duration(t) \, P_{\max}^\ell, \;\; \ell \in \mathcal{L}_H, \; 1 \leq t \leq T, \\ 0 \leq sp_t^\ell, \;\; \ell \in \mathcal{L}_H, \; 1 \leq t \leq T, \\ 0 \leq u_t^\ell \leq Duration(t) \, P_{\max}^\ell, \;\; \ell \in \mathcal{L}_J, \; 1 \leq t \leq T, \\ 0 \leq u_t^\ell \leq \tau_t^\ell \, Duration(t) \, P_{\max}^\ell, \;\; \ell \in \mathcal{L}_T, \; 1 \leq t \leq T, \\ \displaystyle\sum_{k=1}^{T} u_k^\ell \leq x_1^\ell, \;\; \ell \in \mathcal{L}_J, \\ \mathcal{D}_t \leq \displaystyle\sum_{\ell \in \mathcal{L}_T} u_t^\ell + \sum_{\ell \in \mathcal{L}_H} v_t^\ell + \sum_{\ell \in \mathcal{L}_J} u_t^\ell, \;\; 1 \leq t \leq T. \end{array} \right. \tag{9}$$

In this problem, notice that the demand satisfaction constraints are active. The proposition below shows that the variables *sp* have been introduced to make the problem feasible (if the production capacity is important enough) whatever the natural inflows.

Proposition 3.1. *If for every hydro reservoir* $\ell \in \mathcal{L}_H$, $x_1^\ell \in [x_{\min}^\ell, x_{\max}^\ell]$ *and if*

$$\sum_{\ell \in \mathcal{L}_T} \tau_t^\ell \, Duration(t) \, P_{\max}^\ell \geq \mathcal{D}_t, \quad t = 1, \dots, T, \tag{10}$$

then the electricity production management problem (9) *is feasible.*

Proof. We show that the following production schedule is feasible: one uses only thermal production units in increasing order of their functioning cost until the demand is met. Indeed, the constraints on the control variables of such a production schedule are satisfied. On the other hand, we show, by recurrence, that for every hydro reservoir ℓ and for each time step t, $x_t^\ell \in [x_{\min}^\ell, x_{\max}^\ell]$. Let ℓ be an hydro reservoir. By hypothesis, the relation is well satisfied for this reservoir ℓ and for $t = 1$. Suppose now that for $1 \leq t \leq T-1$, we have $x_t^\ell \in [x_{\min}^\ell, x_{\max}^\ell]$. If $x_t^\ell + \mathcal{I}_t^\ell > x_{\max}^\ell$ then define $sp_t^\ell = x_t^\ell + \mathcal{I}_t^\ell - x_{\max}^\ell$. We have $sp_t^\ell \geq 0$, and since $v_t^\ell = 0$, $x_{t+1}^\ell = x_t^\ell + \mathcal{I}_t^\ell - sp_t^\ell = x_{\max}^\ell \in [x_{\min}^\ell, x_{\max}^\ell]$. Else if $x_t^\ell + \mathcal{I}_t^\ell \leq x_{\max}^\ell$, we define $sp_t^\ell = 0$, and $x_{t+1}^\ell = x_t^\ell + \mathcal{I}_t^\ell \geq x_t^\ell$ and $x_{t+1}^\ell \in [x_{\min}^\ell, x_{\max}^\ell]$. □

We could give other less restrictive but more complicated conditions on the feasibility of the problem. Notice, however, that condition (10) means that we can satisfy the demand using thermal and nuclear plants only which is generally the case (recall that, for mid-term production management, nuclear plants are in general modeled as thermal plants).
Besides, this model supposes that all the parameters of the system (the availability rates, the reservoir inflows and the electricity consumption) are known. In this case, the deterministic optimization problem (9) which we have just introduced indeed gives an optimal generation schedule. However, in practice, these parameters are uncertain. The objective is then to determine management strategies, that is to say adaptive production schedules which will depend on the realizations of the parameters that allow us to satisfy the consumption-whatever the value this consumption may be. We first explain how the uncertainty is modeled.

4. Modelling the uncertainty

4.1. **Availability rates.** Let ℓ be a thermal unit with n_ℓ thermal groups. Let $\alpha_{j,\ell}(t)$ be the probability that group j of unit ℓ works at time step t, and let $U_{j,\ell}^t$ be the random variable such that $U_{j,\ell}^t = 1$ if group j works at time step t, and $U_{j,\ell}^t = 0$ otherwise. We suppose that the groups are regularly tested every m_0 time steps. When broken, a group is repaired during a certain number of time steps. Between two consecutive testing dates, we assume that the availability of the units is not changing. This means that between two consecutive testing dates, a given group is either working or it is out of commission during the entire period. If $t_k = m_0 k + 1$ for $k \in \mathbb{N}$, then the probabilities $\alpha_{j,\ell}(t)$, for $t = t_k, \dots, t_{k+1} - 1$ are the same and we only need to evaluate $\alpha_{j,\ell}(t_k), k \geq 0$. These probabilities $\alpha_{j,\ell}(t_k)$ that a group j of unit ℓ works at time step t_k depend on the history of the availability of this group. If at time step t_{k-1}, the group was out of commission, there is a high probability (say $1 - \beta_1^\ell$ with β_1^ℓ small) that it works at time step t_k (the time between two testing dates is greater than the average time needed for repair) and a low probability β_1^ℓ that it is still out of commission at time step t_k. Now, if the group was working for the last m periods delimited by the last $m + 1$ testing dates, we can assume that the longer it has been working without failure (the larger m), the more likely it can break down at time step t_k. Thus, there is a decreasing function of m, $\beta_2^\ell(m)$ such that for any group j of unit ℓ

$$\mathbb{P}(U_{j,\ell}^{t_k} = 1 | \text{Group j was working from } t_{k-m} \text{ to } t_{k-1}) = \beta_2^\ell(m).$$

A particular case is when the state process of a given group is a homogeneous Markov chain where the state space is {F,W} where F stands for the failure state and W for the working state. In this case, $\beta_2^\ell(m) = \beta_2^\ell$ is fixed and corresponds to the probability for a group of unit ℓ to work on a given period knowing that it was working the period before. The transition matrix for the groups of unit ℓ is given by

$$P_\ell = \begin{pmatrix} \beta_1^\ell & 1-\beta_1^\ell \\ 1-\beta_2^\ell & \beta_2^\ell \end{pmatrix}.$$

The probability $\alpha_{j,\ell}(t_k)$ is then given for $k \geq 1$ by

$$\alpha_{j,\ell}(t_k) = p_F^\ell(j) P_\ell^k(1,2) + p_W^\ell(j) P_\ell^k(2,2)$$

where $p_W^\ell(j) = 1 - p_F^\ell(j)$ and $p_W^\ell(j)$ is the probability that group j of unit ℓ works at the first time step. For the simplicity of the exposure, we assume that for a given unit ℓ, either all the groups are working or all the groups are out of commission at the first time step. Thus, $\alpha_{j,\ell}(t_k)$ is j-independent and $\alpha_\ell(t_k)$ will denote the probability that a group of unit ℓ works at time t_k. Let $\bar{P}_{\max}^\ell$ be the maximal power of a group in unit ℓ. Then the theoretical maximal power available on thermal unit ℓ is given by $P_{\max}^\ell = n_\ell \bar{P}_{\max}^\ell$. The maximal available power of unit ℓ for the time steps $t = t_k, \ldots, t_{k+1} - 1$ is

$$\tilde{P}_{\max}^{\ell,k} = \sum_{j=1}^{n_\ell} U_{j,\ell}^{t_k} \bar{P}_{\max}^\ell = n_\ell \bar{P}_{\max}^\ell \frac{\sum_{j=1}^{n_\ell} U_{j,\ell}^{t_k}}{n_\ell} = P_{\max}^\ell \tau_{t_k}^\ell.$$

Notice that under the above hypotheses, the random variable $n_\ell \tau_{t_k}^\ell$ has binomial distribution $\mathcal{B}(n_\ell, \alpha_\ell(t_k))$. We then have $\mathbb{E}_\tau[\tau_{t_k}^\ell] = \alpha_\ell(t_k)$ and the variance of $\tau_{t_k}^\ell$ is $\mathrm{Var}_\tau(\tau_{t_k}^\ell) = \frac{\alpha_\ell(t_k)(1-\alpha_\ell(t_k))}{n_\ell}$.

4.2. **Electricity demand.** We suppose the demand $\mathcal{D}$ over the year is a random vector with known mean and covariance matrix. More precisely, we write the demand $\mathcal{D}_t$ for time step t as

$$\mathcal{D}_t = \bar{\mathcal{D}}_t + \varepsilon_t, \;\; t = 1, \ldots, T,$$

where $\bar{\mathcal{D}}_t$ is the mean demand for time step t and $\varepsilon = (\varepsilon_t)_{t=1}^T$ is a centered noise with covariance matrix $Q_{\mathcal{D}} \succ 0$.

The uncertainty in the inflows is not taken into account in the stable model we introduce in the following section.

5. Stable counterpart of the model

To introduce our model, we first need to introduce a dualization of problem (9). Let x be the vector of decision variables of problem (9), let $\lambda \in \mathbb{R}^T$ be the Lagrange multiplier corresponding to the last constraint in (9) and let L be the corresponding Lagrangian:

$$L(x,\lambda) = \lambda^\top \mathcal{D} + \sum_{\ell \in \mathcal{L}_T} \sum_{t=1}^T (c_\ell - \lambda_t) u_t^\ell + \sum_{\ell \in \mathcal{L}_H} f_H^\ell(x,\lambda) + \sum_{\ell \in \mathcal{L}_J} f_J^\ell(x,\lambda),$$

with

$$f_H^\ell(x,\lambda) = -a_\ell - \sum_{t=1}^T \lambda_t \, v_t^\ell, \;\; \ell \in \mathcal{L}_H,$$

and

$$f_J^\ell(x,\lambda) = -b_\ell - \sum_{t=1}^T \lambda_t \, u_t^\ell, \;\; \ell \in \mathcal{L}_J.$$

If χ is the feasibility set of problem (9) with the demand satisfaction constraints removed, to solve primal optimization problem (9), we have to solve $\min_{x \in \chi} \max_{\lambda \in \mathbb{R}^T} L(x,\lambda)$. This problem is equivalent to the dual problem

$$\max_{\lambda \in \mathbb{R}^T} \theta(\lambda; \tau, \mathcal{D}) \tag{11}$$

with $\theta(\lambda;\tau,\mathcal{D}) = \min_{x \in \chi} L(x,\lambda)$ (we insist here on the dependency of this dual function θ with respect to the availability rates τ and the demand $\mathcal{D}$). Indeed, (9) is a linear optimization problem over a bounded set and in this case the primal and the dual are equivalent to each other (in the sense that they have the same optimal value).

Notice that the dual function θ is not differentiable, concave and separable with respect to the production units. Indeed, if χ_ℓ is the set of constraints on the variables used to describe the production unit ℓ, the dual function is $\theta(\lambda;\tau,\mathcal{D}) = \lambda^\top \mathcal{D} + \sum_{\ell\in\mathcal{L}} \theta^\ell(\lambda)$ with

$$\theta^\ell(\lambda) = \min_{u_\ell\in\chi_\ell} \sum_{t=1}^{T} (c_\ell - \lambda_t)\, u_t^\ell, \quad \text{if } \ell \in \mathcal{L}_T, \tag{12}$$

$$\theta^\ell(\lambda) = \min_{x_\ell\in\chi_\ell} f_H^\ell(x_\ell,\lambda), \quad \text{if } \ell \in \mathcal{L}_H, \tag{13}$$

and

$$\theta^\ell(\lambda) = \min_{x_\ell\in\chi_\ell} f_J^\ell(x_\ell,\lambda), \quad \text{if } \ell \in \mathcal{L}_J. \tag{14}$$

To tackle the stochastic case, a simple way of taking into account the uncertainty of τ and $\mathcal{D}$ would be to replace each uncertain parameter by an estimation of its mean over the management horizon. This approach is equivalent to the problem $\max_{\lambda\in\mathbb{R}^T} \mathbb{E}_{\tau,\mathcal{D}}[\theta(\lambda;\tau,\mathcal{D})]$. Instead, to take into account the variability of τ and $\mathcal{D}$, we propose to solve

$$\max_{\lambda\in\mathbb{R}^T} \mathbb{E}_{\tau,\mathcal{D}}[\theta(\lambda;\tau,\mathcal{D})] - \kappa(\varepsilon)\, \sqrt{Var_{\tau,\mathcal{D}}(\theta(\lambda;\tau,\mathcal{D}))}, \tag{15}$$

where $\kappa(\varepsilon) > 0$ is a fixed risk factor depending on a given confidence level $0 < \varepsilon < 1$.

Computation of the expectation and of the variance in (15)**.** The dual function $\theta(\lambda;\tau,\mathcal{D})$ can be decomposed as

$$\theta(\lambda;\tau,\mathcal{D}) = \lambda^\top \mathcal{D} + \theta_T(\lambda;\tau) + \theta_H(\lambda) + \theta_J(\lambda), \tag{16}$$

where $\theta_T(\lambda;\tau)$ is the contribution of the thermal plants to the dual problem, $\theta_H(\lambda) = \sum_{\ell\in\mathcal{L}_H} \theta^\ell(\lambda)$ is the contribution of the hydroelectric power stations to the dual problem and $\theta_J(\lambda) = \sum_{\ell\in\mathcal{L}_J} \theta^\ell(\lambda)$ is the contribution of the EJP contracts (recall that the nuclear plants are modeled as thermal plants). The function $\theta_T(\lambda;\tau)$ is explicit and the only one to depend on the availability rates:

$$\theta_T(\lambda;\tau) = \sum_{t=1}^{T} Duration(t) \sum_{\ell|c_\ell<\lambda_t} \tau_t^\ell P_{\max}^\ell (c_\ell - \lambda_t). \tag{17}$$

Remembering that for a given thermal unit, the availability rates are constant between time steps t_k and $t_{k+1}-1$ for every $k \geq 0$ and assuming to alleviate notation that T is multiple of m_0, we can express $\theta_T(\lambda;\tau)$ as

$$\begin{aligned} \theta_T(\lambda;\tau) &= \sum_{k=1}^{T/m_0} \sum_{i=t_{k-1}}^{t_k-1} \sum_{\ell|c_\ell<\lambda_i} \tau_{t_{k-1}}^\ell Duration(i) P_{\max}^\ell (c_\ell - \lambda_i), \\ &= \sum_{k=1}^{T/m_0} \sum_{\ell\in\mathcal{L}_T} f_{k,\ell}(\lambda)\tau_{t_{k-1}}^\ell, \end{aligned} \tag{18}$$

with

$$f_{k,\ell}(\lambda) = P_{\max}^\ell \sum_{i|t_{k-1}\leq i\leq t_k-1 \text{ and } c_\ell<\lambda_i} Duration(i)(c_\ell - \lambda_i). \tag{19}$$

The function $\theta_H(\lambda)$ neither depends on the demand nor on the availability rates and is easily computed by solving $|\mathcal{L}_H|$ linear problems with $2T+1$ variables. Similarly, the function $\theta_J(\lambda)$ associated to the EJP subproblems neither depends on the demand nor on the availability rates and is computed by solving $|\mathcal{L}_J|$ linear problems with $T+1$ variables. Using (16) and (17), we therefore have

$$\mathbb{E}_{\tau,\mathcal{D}}[\theta(\lambda;\tau,\mathcal{D})] = \theta_H(\lambda) + \theta_J(\lambda) + \lambda^\top \bar{\mathcal{D}} + \sum_{t=1}^{T} \sum_{\ell | c_\ell < \lambda_t} \alpha_\ell(t) Duration(t) P_{\max}^{\ell}(c_\ell - \lambda_t). \tag{20}$$

Further, assuming that the availability rates and the demands are independent and using (16) and (18), we obtain that

$$Var_{\tau,\mathcal{D}}(\theta(\lambda;\tau,\mathcal{D})) = \lambda^\top Q_{\mathcal{D}} \lambda + \sum_{k=1}^{T/m_0} \sum_{\ell \in \mathcal{L}_T} f_{k,\ell}^2(\lambda) \frac{\alpha_\ell(t_{k-1})(1-\alpha_\ell(t_{k-1}))}{n_\ell}.$$

We now explain what can motivate the introduction of model (15).

Justification of model (15)**.**

(1) **Value-at-Risk approach on the dual problem.** Model (15) can be obtained using a Value-at-Risk approach on the dual problem (11) in which τ and $\mathcal{D}$ are random (see [14] for more details). This observation allows us to choose $\kappa(\varepsilon) = \sqrt{\frac{1-\varepsilon}{\varepsilon}}$ as in [9]. In model (15), the objective function is penalized by the standard deviation of $\theta(\lambda;\tau,\mathcal{D})$. Another way of limiting this standard deviation is to add a constraint of the kind $\sqrt{Var_{\tau,\mathcal{D}}(\theta(\lambda;\tau,\mathcal{D}))} \leq \alpha$. The optimal Lagrange multiplier associated to the dualization of this constraint can provide another estimation of $\kappa(\varepsilon)$.

(2) **Robust counterpart of the dual problem.** Let us introduce the vectors τ and $f(\lambda)$ in $\mathbb{R}^{|\mathcal{L}_T|\frac{T}{m_0}}$ defined by $\tau((k-1)|\mathcal{L}_T|+\ell|) = \tau_{t_{k-1}}^{\ell}$ and $f(\lambda)((k-1)|\mathcal{L}_T|+\ell) = f_{k,\ell}(\lambda)$ for $1 \leq k \leq \frac{T}{m_0}$ and $1 \leq \ell \leq |\mathcal{L}_T|$. This way, using (16) and (18), we can write the dual function $\theta(\lambda;\tau,\mathcal{D})$ as $\theta(\lambda;\tau,\mathcal{D}) = \lambda^\top \mathcal{D} + f(\lambda)^\top \tau + \theta_H(\lambda) + \theta_J(\lambda)$. For introducing a robust counterpart of the dual problem, we first define the $\mathbb{R}^{|\mathcal{L}_T|\frac{T}{m_0}} \times \mathbb{R}^{|\mathcal{L}_T|\frac{T}{m_0}}$ diagonal matrix Q_τ whose diagonal elements are given by $Q_\tau((k-1)|\mathcal{L}_T|+\ell,(k-1)|\mathcal{L}_T|+\ell) = \frac{\alpha_\ell(t_{k-1})(1-\alpha_\ell(t_{k-1}))}{n_\ell}$ for $1 \leq k \leq \frac{T}{m_0}$ and $1 \leq \ell \leq |\mathcal{L}_T|$. We then define the ellipsoid

$$\mathcal{E} = \left\{ \begin{pmatrix} \tau \\ \mathcal{D} \end{pmatrix} \mid \begin{pmatrix} \tau & - & \mathbb{E}_\tau[\tau] \\ \mathcal{D} & - & \mathbb{E}_{\mathcal{D}}[\mathcal{D}] \end{pmatrix}^\top \begin{pmatrix} Q_\tau & 0 \\ 0 & Q_{\mathcal{D}} \end{pmatrix}^{-1} \begin{pmatrix} \tau & - & \mathbb{E}_\tau[\tau] \\ \mathcal{D} & - & \mathbb{E}_{\mathcal{D}}[\mathcal{D}] \end{pmatrix} \leq \kappa^2(\varepsilon) \right\} \tag{21}$$

with $\mathbb{E}_{\mathcal{D}}[\mathcal{D}] = \bar{\mathcal{D}}$. Taking as uncertainty set for $(\tau^\top, \mathcal{D}^\top)^\top$ the ellipsoid $\mathcal{E}$, we can form the following robust counterpart (in the sense of [3]) of the dual problem (11):

$$\max_{\lambda \in \mathbb{R}^T} \min_{(\tau^\top,\, \mathcal{D}^\top)^\top \in \mathcal{E}} \left[\theta(\lambda;\tau,\mathcal{D}) = \lambda^\top \mathcal{D} + f(\lambda)^\top \tau + \theta_H(\lambda) + \theta_J(\lambda)\right],$$

which can be expressed as

$$\max_{\lambda \in \mathbb{R}^T} [\theta_H(\lambda) + \theta_J(\lambda) + \lambda^\top \bar{\mathcal{D}} + f(\lambda)^\top \mathbb{E}_\tau[\tau] - \kappa(\varepsilon) \sqrt{\lambda^\top Q_{\mathcal{D}} \lambda + f(\lambda)^\top Q_\tau f(\lambda)}].$$

It is easily seen that the above problem is nothing but problem (15).

(3) **Stabilization of the dual solutions.** Solutions of a linear problem are known to be unstable. Another interest of model (15) is to introduce a term allowing for the regularization of the dual problem and the stabilization of the dual solutions on which the management strategies depend (see Subsection 7.1.2).

Primal problem whose dual problem is (15)**.** Let us consider the stochastic version of problem (9) where the parameters τ and $\mathcal{D}$ are no longer known but uncertain, any value for $(\tau^\top, \mathcal{D}^\top)^\top$ in the ellipsoid $\mathcal{E}$ given by (21) being possible. Further, let us consider the optimization problem (9)' obtained from optimization problem (9) with additional variables τ and $\mathcal{D}$, and additional constraints $(\tau^\top, \mathcal{D}^\top)^\top \in \mathcal{E}$. Recalling (ii) above, it is easily seen that the dual problem of problem (9)' is the optimization problem (15), i.e., a robust counterpart of the dual of the initial problem (9). When only the demand (resp. the availability rates) is (resp. are) uncertain, problem (15) is the dual problem of problem (9) where the constraints $\mathcal{D} \in \mathcal{E}_{\mathcal{D}}$ (resp. $\tau \in \mathcal{E}_\tau$) have been added with $\mathcal{E}_{\mathcal{D}} = \{\mathcal{D} \in \mathbb{R}^T \mid (\mathcal{D} - \bar{\mathcal{D}})^\top Q_{\mathcal{D}}^{-1}(\mathcal{D} - \bar{\mathcal{D}}) \leq \kappa(\varepsilon)^2\}$ (resp. $\mathcal{E}_\tau = \{\tau \in \mathbb{R}^{|\mathcal{L}_T|\frac{T}{m_0}} \mid (\tau - \mathbb{E}_\tau[\tau])^\top Q_\tau^{-1}(\tau - \mathbb{E}_\tau[\tau]) \leq \kappa(\varepsilon)^2\}$).

One can see that for the new primal problem (9)', at time step t, the optimal value of demand $\mathcal{D}_t$ (resp. of availability rate τ_t^ℓ) is its smallest value (resp. largest value) in the ellipsoid $\mathcal{E}$. Since the demand satisfaction constraints are active, this means that for primal problem (9)' the production schedule is based on the minimal demands. One interpretation of this production schedule is as follows: it allows us on every scenario of demand not to deliver to a client more than the demand he needs. Such constraints may appear naturally when robustifying the demand satisfaction constraints written with variables $v_t \geq 0$ for the non-satisfaction of the demand at time step t. Since for each time step t, the sum of the productions and of v_t equals the demand $\mathcal{D}_t$, eliminating the variables v_t yields demand satisfaction constraints expressing that the sum of the productions should be below the demand $\mathcal{D}_t$. When $\mathcal{D} \in \mathcal{E}_{\mathcal{D}}$, a robust version of these constraints is that the sum of the productions at time t should be below the minimal value of $\mathcal{D}_t$ in $\mathcal{E}_{\mathcal{D}}$. These constraints will be active if the production capacity allows us to satisfy these minimal demands.

However, the production schedule of this primal problem is of course not used. The reader should be aware that the management strategies (detailed in Subsection 7.1.2) use a solution of (15), in such a way that the demands of the clients are satisfied at each time step (if the production capacity is important enough), whatever their values, even if they are higher than the minimal demands.

The main objective of the methodology is thus to stabilize the volatility of the cost by stabilizing the dual solutions.

The next two sections focus on solution methods.

6. An efficient solution method in a particular case

In this section, we present an original and efficient solution method for the electricity generation management problem by dynamic programming in the deterministic case when we have one hydroelectric power station and a given number of thermal plants. We explain in the next section how the developments of this section can be used when there is an arbitrary number of thermal plants, hydroplants and EJP contracts.

First, suppose that only thermal plants are available and that the last thermal unit has an infinite production capacity: $P_{\max}^{|\mathcal{L}_T|} = +\infty$. Eventually after reordering the production means, we can always assume that the components of the vector of thermal costs $(c_1, \ldots, c_{|\mathcal{L}_T|})^\top$ are ordered, i.e., $0 < c_1 \leq c_2 \leq \ldots \leq c_{|\mathcal{L}_T|}$. Thus, for time step t, if we only use thermal plants, the optimal production cost satisfying demand $\mathcal{D}_t$, requires the computation of the following optimal cost $\psi_t(\mathcal{D}_t)$:

$$\psi_t(\mathcal{D}_t) = \min_{u_t^1, \ldots, u_t^{|\mathcal{L}_T|}} \{\sum_{\ell=1}^{|\mathcal{L}_T|} c_\ell u_t^\ell : \ 0 \leq u_t^\ell \leq Duration(t)\, \tau_t^\ell\, P_{\max}^\ell, \ \sum_{\ell=1}^{|\mathcal{L}_T|} u_t^\ell = \mathcal{D}_t\}.$$

Thus, for every $t = 1, \ldots, T$, $\psi_t(\cdot)$ is a convex piecewise affine and strictly increasing function, such that $\psi_t(0) = 0$. It has 'kinks'

$$\mathcal{T}_{t,k} = \sum_{\ell=1}^{k} \tau_t^\ell\, Duration(t)\, P_{\max}^\ell, \quad k = 0, \ldots, |\mathcal{L}_T| - 1.$$

For every $t = 1, \ldots, T$, let $\mathcal{T}_{t,|\mathcal{L}_T|} = +\infty$. The functions $\psi_t(\cdot), t = 1, \ldots, T$, are differentiable for every point other than a 'kink' with

$$\begin{array}{lcll} \psi_t^{'}(x) &=& 0, & \text{if } x < 0, \\ \psi_t^{'}(x) &=& c_{k+1}, & \text{if } \mathcal{T}_{t,k} < x < \mathcal{T}_{t,k+1}, \quad k = 0, \ldots, |\mathcal{L}_T| - 1. \end{array} \tag{22}$$

In what follows, we use the piecewise affine dual functions

$$\psi_t^*(y) = \max_z \{yz - \psi_t(z)\}.$$

Notice that dom $\psi_t^* = [0, c_{|\mathcal{L}_T|}]$ and that a simple computation gives

$$\psi_t^*(y) = \sum_{\ell=1}^{|\mathcal{L}_T|-1} \tau_t^\ell \, \mathit{Duration}(t) \, P_{\max}^\ell (y - c_\ell)^+. \tag{23}$$

We show how the electricity generation management problem can be solved by dynamic programming when we add a hydroelectric power station. Recall that the constraints on the hydro controls v_t^1 are given by $0 \leq v_t^1 \leq \mathit{Duration}(t)\, P_{\max}^1$, for $t = 1, \ldots, T$, and that x_1^1 is the starting volume of the reservoir of the hydroelectric power station. Also, for every $t = 1, \ldots, T$, the natural inflow of this reservoir for time step t is denoted by $\mathcal{I}_t^1$. We assume that the electricity consumption $\mathcal{D}_t$ is greater than or equal to $\mathit{Duration}(t)\, P_{\max}^1$ and that $x_{\max}^1$ is infinite (thus no variables sp are necessary). In the model presented in Section 3, the value of the water in reservoir 1 is taken into account using a value function V_H^1 at the end of the optimization period. This allows us to manage the reservoir in a prudent manner and in this model the constant $x_{\min}^1$ in (2) in general represents a physical lower bound on reservoir 1 volume. Another management model that we consider in this section consists of suppressing the value functions V_H^1 and replacing $x_{\min}^1$ in (2) by a time dependent lower bound, i.e., $x_t^1 \geq x_{\min}^1(t) \geq 0$, for $t = 2, \ldots, T+1$. We thus ensure the trajectory of the level of the reservoir to be above a reference trajectory. In particular, the reservoir level will be above a prescribed value $x_{\min}^1(T+1)$ at the end of the last time step T. The production cost to satisfy demand $\mathcal{D}_t$ for time step t is then $\psi_t(\mathcal{D}_t - v_t^1)$. If consumption $\mathcal{D}_t$ and inflows $\mathcal{I}_t^1$ are known, we thus have to solve the following optimization problem:

$$\begin{array}{lll} C^* & = & \displaystyle\min_{x^1, v^1 \in \mathbb{R}^T} \Big\{ \sum_{t=1}^T \psi_t(\mathcal{D}_t - v_t^1) \ : \ 0 \leq v_t^1 \leq \mathit{Duration}(t)\, P_{\max}^1, \ \ x^1 \geq x_{\min}^1, \\ & & \qquad\qquad x_{t+1}^1 = x_t^1 + \mathcal{I}_t^1 - v_t^1, \ \ t = 1, \ldots, T \ \Big\}. \end{array} \tag{24}$$

Next, we use a dual formulation of this problem introducing the Lagrange multipliers $\mu \in \mathbb{R}_+^T$ for the inequality constraints $x^1 \geq x_{\min}^1$:

$$\begin{array}{lll} C^* & = & \displaystyle\min_{x^1, v^1} \max_{\mu \in \mathbb{R}_+^T} \{\mu^\top (x_{\min}^1 - x^1) + \sum_{t=1}^T \psi_t(\mathcal{D}_t - v_t^1) : 0 \leq v_t^1 \leq \mathit{Duration}(t)\, P_{\max}^1, \\ & & x_{t+1}^1 = x_t^1 + \mathcal{I}_t^1 - v_t^1, \ t = 1, \ldots, T\}, \\ & = & \displaystyle\max_{\mu \in \mathbb{R}_+^T} \min_{x^1, v^1} \{\mu^\top (x_{\min}^1 - x^1) + \sum_{t=1}^T \psi_t(\mathcal{D}_t - v_t^1) : 0 \leq v_t^1 \leq \mathit{Duration}(t)\, P_{\max}^1, \\ & & x_{t+1}^1 = x_t^1 + \mathcal{I}_t^1 - v_t^1, \ t = 1, \ldots, T\}. \end{array} \tag{25}$$

As was done in Section 3, we then express the variables x^1 as functions of the variables v^1: $x_{k+1}^1 = x_1^1 + \sum_{t=1}^k \mathcal{I}_t^1 - v_t^1, \ \ k = 1, \ldots, T$. It follows that

$$\begin{array}{lll} \mu^\top (x_{\min}^1 - x^1) & = & \displaystyle\sum_{k=1}^T \mu_k \left(x_{\min}^1(k+1) - x_1^1 + \sum_{t=1}^k (v_t^1 - \mathcal{I}_t^1) \right), \\ & = & \displaystyle\mu^\top x_{\min}^1 - x_1^1 \sum_{k=1}^T \mu_k + \sum_{k=1}^T \mu_k \sum_{t=1}^k (v_t^1 - \mathcal{I}_t^1), \\ & = & \displaystyle\mu^\top x_{\min}^1 - x_1^1 \sum_{k=1}^T \mu_k + \sum_{t=1}^T (v_t^1 - \mathcal{I}_t^1) \sum_{k=t}^T \mu_k. \end{array} \tag{26}$$

If for every $t = 1, \ldots, T$, we set $s_t = \sum_{k=t}^T \mu_k$, we have

$$\mu_t = s_t - s_{t+1}, \quad t = 1, \ldots, T-1, \quad \text{ and } \mu_T = s_T.$$

Using the above relations, we then obtain

$$\begin{aligned} \mu^\top x^1_{\min} &= s_T\, x^1_{\min}(T+1) + \sum_{t=1}^{T-1} (s_t - s_{t+1}) x^1_{\min}(t+1), \\ &= s_1\, x^1_{\min}(2) + \sum_{t=2}^{T} s_t\, (x^1_{\min}(t+1) - x^1_{\min}(t)). \end{aligned} \tag{27}$$

Plugging (27) into (26) then gives

$$\mu^\top (x^1_{\min} - x^1) = s^\top \Delta x_{\min} + s^\top (v^1 - \mathcal{I}^1), \tag{28}$$

where $\Delta x_{\min}(t) = x^1_{\min}(t+1) - x^1_{\min}(t),\ t = 1, \ldots, T$, with $x^1_{\min}(1) = x^1_1$. Using (25) and (28), and observing that $s_1 \geq s_2 \geq \ldots, \geq s_T \geq 0$, we then get the following representation:

$$\begin{aligned} C^* &= \min_{0 \leq v^1_t \leq Duration(t) P^1_{\max}} \ \max_{0 \leq s_T \leq \ldots \leq s_1} \{ s^\top (\Delta x_{\min} + v^1 - \mathcal{I}^1) + \sum_{t=1}^{T} \psi_t(\mathcal{D}_t - v^1_t) \} \\ &= \max_{0 \leq s_T \leq \ldots \leq s_1} \{ s^\top (\Delta x_{\min} - \mathcal{I}^1) + \sum_{t=1}^{T} \min_{0 \leq v^1_t \leq Duration(t) P^1_{\max}} [\psi_t(\mathcal{D}_t - v^1_t) + v^1_t s_t] \}. \end{aligned} \tag{29}$$

For $t = 1, \ldots, T$, if we let

$$\phi_t(y) = \min_z \{ yz + \psi_t(\mathcal{D}_t - z) \ : \ 0 \leq z \leq Duration(t) P^1_{\max} \}, \tag{30}$$

we can express (29) as

$$C^* = \max_s \left(s^\top (\Delta x_{\min} - \mathcal{I}^1) + \sum_{t=1}^{T} \phi_t(s_t) \ : \ 0 \leq s_T \leq \ldots \leq s_1 \right). \tag{31}$$

The complexity of problem (31) depends on the structure of the concave functions $\phi_t(\cdot)$.

When $y \geq \psi'_t(\mathcal{D}_t)$, (where $\psi'_t(\mathcal{D}_t)$ represents the left directional derivative of ψ_t at $\mathcal{D}_t$), the argmin $z(y)$ in problem (30) is zero. It follows that when $y \geq \psi'_t(\mathcal{D}_t)$, we have

$$\phi_t(y) = \psi_t(\mathcal{D}_t), \ \ y \geq \psi'_t(\mathcal{D}_t).$$

Similarly, when $y \leq \psi'_t(\mathcal{D}_t - Duration(t) P^1_{\max})$, we obtain

$$\phi_t(y) = Duration(t) P^1_{\max} y + \psi_t(\mathcal{D}_t - Duration(t) P^1_{\max}),$$

where $\psi'_t(\mathcal{D}_t - Duration(t) P^1_{\max})$ here represents the right directional derivative of ψ_t at $\mathcal{D}_t - Duration(t) P^1_{\max}$.

For intermediate values of y, i.e., when $\psi'_t(\mathcal{D}_t - Duration(t) P^1_{\max}) \leq y \leq \psi'_t(\mathcal{D}_t)$, we have $y = \psi'_t(\mathcal{D}_t - z(y))$. Thus, using (23), we get,

$$\phi_t(y) \ = \ \mathcal{D}_t y - \psi^*_t(y) \ = \ \mathcal{D}_t y - \sum_{\ell=1}^{|\mathcal{L}_T|-1} Duration(t) \tau^\ell_t P^1_{\max} (y - c_\ell)^+.$$

The right and left directional derivatives introduced above are given by

$$\begin{aligned} \psi'_t(\mathcal{D}_t - Duration(t) P^1_{\max}) &= \max_\ell \{ c_{\ell+1} \ : \ \mathcal{T}_{t,\ell} \leq \mathcal{D}_t - Duration(t) P^1_{\max} \}, \\ \psi'_t(\mathcal{D}_t) &= \min_\ell \{ c_\ell \ : \ \mathcal{T}_{t,\ell} \geq \mathcal{D}_t \}. \end{aligned}$$

Thus, gathering the results above, we conclude that all functions $\phi_t(\cdot), t = 1, \ldots, T$, belong to $\mathcal{S}(c)$ (a convex cone), the class of real concave piecewise affine functions, possibly having kinks only at points $c_\ell,\ \ell = 1, \ldots, |\mathcal{L}_T|$.

For our development, it is convenient to introduce the functions $f_t(\cdot), t = 1, \ldots, T$, such that

$$f_t(y) = (\Delta x_{\min}(t) - \mathcal{I}^1_t) y + \phi_t(y), \ \ t = 1, \ldots, T.$$

Finally, we recursively define the two following sequences of univariate functions $\Phi_t(\cdot)$ and $\Psi_t(\cdot)$:

$$\begin{array}{lcl}\Phi_{T+1}(y) & = & 0,\\ \Psi_t(y) & = & \Phi_{t+1}(y) + f_t(y),\\ \Phi_t(y) & = & \max\limits_{0 \le z \le y} \Psi_t(z), \;\; t = T, \ldots, 1.\end{array} \tag{32}$$

Theorem 6.1. *For every k=T,...,1, we have*

$$\begin{array}{lcl}\Phi_k(y) & = & \max\limits_{s} \left(\sum\limits_{t=k}^{T} f_t(s_t) \;:\; 0 \le s_T \le \ldots \le s_k \le y \right),\\ \max\limits_{y \ge 0} \Psi_k(y) & = & \max\limits_{s} \left(\sum\limits_{t=k}^{T} f_t(s_t) \;:\; 0 \le s_T \le \ldots \le s_k \right).\end{array} \tag{33}$$

Proof. Let us show the first equality in (33) by induction. For $k = T$, it is valid by definition. Suppose it is true for some $k \in \{2, \ldots, T\}$. Then:

$$\begin{array}{lcl}\Phi_{k-1}(y) & = & \max\limits_{0 \le z \le y} [\Phi_k(z) + f_{k-1}(z)]\\ & = & \max\limits_{0 \le z \le y} \left[f_{k-1}(z) + \max\limits_{s} \left(\sum\limits_{t=k}^{T} f_t(s_t) \;:\; 0 \le s_T \le \ldots \le s_k \le z \right) \right]\\ & = & \max\limits_{s} \left(\sum\limits_{t=k-1}^{T} f_t(s_t) \;:\; 0 \le s_T \le \ldots \le s_k \le s_{k-1} \le y \right).\end{array}$$

Thus, the first equality in (33) is valid for $k = 1, \ldots, T$. Consequently:

$$\begin{array}{lcl}\max\limits_{y \ge 0} \Psi_k(y) & = & \max\limits_{y \ge 0} [\Phi_{k+1}(y) + f_k(y)]\\ & = & \max\limits_{y \ge 0} \left[f_k(y) + \max\limits_{s} \left(\sum\limits_{t=k+1}^{T} f_t(s_t) \;:\; 0 \le s_T \le \ldots \le s_{k+1} \le y \right) \right]\\ & = & \max\limits_{s} \left(\sum\limits_{t=k}^{T} f_t(s_t) \;:\; 0 \le s_T \le \ldots \le s_{k+1} \le s_k \right).\end{array}$$

□

Using Theorem 6.1, we then have $C^* = \max\limits_{y \ge 0} \Psi_1(y)$. We now introduce

$$y_k^* = \min \{ y \;:\; \Psi_k(y) \ge \Psi_k(z), \forall z \in \mathbb{R} \}, \; k = 1, \ldots, T.$$

An algorithm to compute $C^* = \Psi_1(y_1^*)$ can then be deduced from the following lemma:

Lemma 6.1. *All functions Ψ_k and Φ_k, $k = 1, \ldots, T$, belong to $\mathcal{S}(c)$.*

Proof. Notice that $f_t \in \mathcal{S}(c)$, $t = 1, \ldots, T$. We show the lemma by induction. Indeed, $\Phi_{T+1} \in \mathcal{S}(c)$. Suppose that $\Phi_{k+1} \in \mathcal{S}(c)$ for some $k \le T$. Then, $\Psi_k \in \mathcal{S}(c)$ and y_k^* coincides with one of the values c_ℓ, $\ell = 1, \ldots, |\mathcal{L}_T|$. Since

$$\Phi_k(y) = \max_{0 \le z \le y} \Psi_k(z) = \left\{ \begin{array}{ll} \Psi_k(y), & y \le y_k^*,\\ \Psi_k(y_k^*), & y > y_k^*, \end{array} \right.$$

we conclude that $\Phi_k \in \mathcal{S}(c)$. □

We can thus explicitly update the functions Φ_k and Ψ_k recursively. Each of the following steps needs $O(|\mathcal{L}_T|)$ operations:

- addition of two functions of $\mathcal{S}(c)$;
- computation of the maximal value of a function of $\mathcal{S}(c)$;
- computation of an explicit representation of the function Φ_k from an explicit representation of the function Ψ_k.

We then get the following conclusion:

Theorem 6.2. *The optimal value C^* of problem* (31) *can be computed by dynamic programming in $O(|\mathcal{L}_T|\, T)$ arithmetic operations.*

We now generate explicit solutions for problem (31).

Lemma 6.2. *Define the following sequence:*

$$s_1^* = y_1^*, \quad s_k^* = \min\{s_{k-1}^*, y_k^*\}, \quad k = 2, \ldots, T.$$

Then vector $s^* = (s_1^*, \ldots, s_T^*)$ provides an optimal solution for problem (31).

Proof. Let us show by induction that:

$$C^* = \Phi_{k+1}(s_k^*) + \sum_{t=1}^{k} f_t(s_t^*), \quad k = 1, \ldots, T. \tag{34}$$

From Theorem 6.1, we have:

$$C^* = \max_{y \geq 0} \Psi_1(y) = f_1(s_1^*) + \Phi_2(s_1^*).$$

Thus, (34) holds for $k = 1$. Next, suppose that it holds for some k with $1 \leq k \leq T - 1$. Then

$$C^* = \sum_{t=1}^{k} f_t(s_t^*) + \max_{0 \leq s_{k+1} \leq s_k^*} \Psi_{k+1}(s_{k+1}).$$

Consequently,

$$\begin{aligned} \max_{0 \leq s_{k+1} \leq s_k^*} \Psi_{k+1}(s_{k+1}) &= \begin{cases} \Psi_{k+1}(y_{k+1}^*), & \text{if } y_{k+1}^* \leq s_k^*, \\ \Psi_{k+1}(s_k^*), & \text{otherwise,} \end{cases} \\ &= \Psi_{k+1}(s_{k+1}^*) = \Phi_{k+2}(s_{k+1}^*) + f_{k+1}(s_{k+1}^*). \end{aligned}$$

We have thus proved (34) for every $k = 1, \ldots, T$. Notice that $\Phi_{T+1} = 0$. Since s^* is feasible for problem (31), we conclude that it is an optimal solution to this problem. □

We have just shown how to solve the electricity generation management problem by dynamic programming when a hydroelectric power station and an arbitrary number of thermal plants are taken into account and when all the parameters of the problem are known. In the next section, we focus on solution methods in the general case, when there are thermal units, EJP units and possibly several hydroplants.

7. Solution methods

7.1. **Stabilized model.** In this subsection, we explain how problem (15) is solved and how the management strategies which give the production schedules are determined for the stabilized model introduced in Section 5.

7.1.1. *Solving the problem.* Problem (15) is solved using a bundle method described in [15]. We thus have to build a black box which, for every $\lambda \in \mathbb{R}^T$, is able to compute the value of the objective function and to give an arbitrary subgradient of the objective function at λ. The objective function is separable with respect to the hydro and EJP units and the computation of this objective function at a given λ is thus done solving the different optimization problems associated with the hydro and EJP units. For each iteration of the bundle method, we thus have to solve $|\mathcal{L}_H| + |\mathcal{L}_J|$ local subproblems, the number of variables of a local subproblem being approximately the number of control and state variables of the production unit associated with this local subproblem. More precisely, for fixed λ, the contribution of the thermal units to the objective function in (15) is explicitly known, and $\theta_H(\lambda)$ and $\theta_J(\lambda)$ are computed by solving linear optimization problems, using an interior point method for instance.

Concerning the computation of a subgradient, let $(x_\ell^*(\lambda), u_\ell^*(\lambda))$ be an optimal solution of the subproblem associated with unit $\ell \in \mathcal{L}_H \cup \mathcal{L}_J$, where the decision vector has been split into state variables x and control variables u. Then the t-th component of a subgradient is given by

$$-\bar{\mathcal{D}}_t + \sum_{\ell \in \mathcal{L}_H \bigcup \mathcal{L}_J} u_\ell^*(\lambda)(t) + \sum_{\ell \in \mathcal{L}_T | c_\ell < \lambda_t} Duration(t) \alpha_\ell(t) P_{\max}^\ell + \frac{\kappa(\varepsilon)}{\sqrt{Var(\theta(\lambda; \tau, \mathcal{D}))}} \times$$
$$\left((Q_{\mathcal{D}} \lambda)(t) + Duration(t) \sum_{\ell | c_\ell < \lambda_t} P_{max}^\ell f_{k,\ell}(\lambda) \frac{\alpha_\ell(t_{k-1})(1 - \alpha_\ell(t_{k-1}))}{n_\ell} \right),$$

where k is the unique integer such that $t_{k-1} \leq t \leq t_k - 1$ and $f_{k,\ell}(\lambda)$ is defined in (19).

More precisely, referring to [4], a global resolution using an iterative scheme can be described in four steps starting from a reference price λ used to initialize the algorithm with index $k = 1$ and $\lambda_1 = \lambda$:

(1) At iteration k, computation of a solution of local subproblem $\ell \in \mathcal{L}_H \cup \mathcal{L}_J$: $y_\ell(\lambda_k)$;
(2) Evaluation of the objective function at λ_k and computation of a subgradient $s(\lambda_k)$;
(3) Update of the multipliers by the coordinator using a black box method (i.e., computation of λ_{k+1});
(4) Update of index: $k \leftarrow k + 1$ and return to Step (i).

7.1.2. *Management strategies.* Once problem (15) is solved, we have to determine a management strategy in order to satisfy the electricity consumption whatever its value over the management period.
After solving (15), we get a solution λ^* indexed by the time steps. The management strategy then takes the form of Bellman values for each time step of the management period, these functions being computed using the following version of Bellman principle. The Bellman values are known at time step $T + 1$, for every reservoir and for a set of stock steps. Between two stock steps, the value is an affine function of the stock. If ℓ is a hydro reservoir, the value of the Bellman function $V^\ell(x, t)$ for stock step x and time step $1 \leq t \leq T$ is given by:

$$V^\ell(x, t) = \left\{ \begin{array}{l} \max \ V^\ell(x + \mathcal{I}_t^\ell - v_t^\ell - sp_t^\ell, t + 1) + \lambda_t^* \ v_t^\ell \\ x + \mathcal{I}_t^\ell - x_{\max}^\ell \leq v_t^\ell + sp_t^\ell \leq x + \mathcal{I}_t^\ell - x_{\min}^\ell \\ 0 \leq v_t^\ell \leq Duration(t) \ P_{\max}^\ell, \quad 0 \leq sp_t^\ell; \end{array} \right. \tag{35}$$

with $V^\ell(\cdot, T + 1) = V_H^\ell(\cdot)$ and if ℓ is an EJP stock by

$$V^\ell(x, t) = \left\{ \begin{array}{l} \max \ V^\ell(x - u_t^\ell, t + 1) + \lambda_t^* \ u_t^\ell \\ 0 \leq u_t^\ell \leq Duration(t) \ P_{\max}^\ell, \quad x - u_t^\ell \geq 0, \end{array} \right. \tag{36}$$

with $V^\ell(\cdot, T + 1) = V_J^\ell(\cdot)$. Once these functions are computed, it is possible to do a Monte-Carlo simulation of the production schedule, using $\delta_x V^\ell(x, t)$ as the production cost of the energy stored in reservoir ℓ and using the production units in increasing order of their production cost.

7.2. **Dedicated solution methods in the deterministic case.** In this section, we discuss solution methods for the deterministic electricity production management problem when the management of the reservoirs is done using control trajectories (as in Section 6). In this case, we thus have to solve a linear programming. When the size of this problem is very large, it may not be possible or too time consuming to solve the problem directly. In this case, when there is one hydro plant and an arbitrary number of thermal plants, the efficient solution method proposed in Section 6 can be used for computing the optimal value. When there is an arbitrary number of thermal, hydro and EJP units, a price decomposition technique (see [11], [14] for instance) can be useful. It amounts to solve the dual problem obtained when dualizing the demand satisfaction constraints. The corresponding dual function is separable with respect to the production units and a maximum of this function is found using a bundle method. At iteration k of the bundle method, we have to compute the value of the dual function function at the current value λ_k of the multiplier and to provide a subgradient of this function at λ_k. This is done solving (using interior point methods for instance) the local subproblems associated to the different production units.

The subproblems associated to the thermal units have explicit solutions. In this section, we intend to show that we can use the results of Section 6 to efficiently compute the optimal value of the subproblems associated to the hydro and EJP units. We focus on hydro subproblems (similar developments can be made for the EJP subproblems). If λ is the value of the Lagrange multiplier, the subproblem associated with a hydro unit $\ell \in \mathcal{L}_H$ is

$$(37)\qquad \begin{cases} \min \; -\sum_{t=1}^{T} \lambda_t \, v_t^\ell \\ 0 \leq v_t^\ell \leq Duration(t) P_{\max}^\ell, \; t=1,\ldots,T, \\ x^\ell \geq x_{\min}^\ell, \;\; x_{t+1}^\ell = x_1^\ell + \sum_{k=1}^{t} \mathcal{I}_k^\ell - v_k^\ell, \; t=1,\ldots,T. \end{cases}$$

As was done in Section 6, we introduce a dualization of the above problem introducing Lagrange multipliers for the constraints $x^\ell \geq x_{\min}^\ell$. Following the computations of Section 6 and using in particular (28), we see that problem (37) has the same optimal value as the following optimization problem:

$$(38)\qquad \max_{0 \leq s_T^\ell \leq \ldots \leq s_1^\ell} \sum_{t=1}^{T} f_t^\ell(s_t^\ell)$$

with for $t = 1, \ldots, T$,

$$(39)\qquad f_t^\ell(y) = (\Delta x_{\min}^\ell(t) - \mathcal{I}_t^\ell) y - (\lambda_t - y)^+ Duration(t) P_{\max}^\ell,$$

where $\Delta x_{\min}^\ell(t) = x_{\min}^\ell(t+1) - x_{\min}^\ell(t)$. The recurrence relations (32) with f_t substituted by f_t^ℓ can then be used to solve problem (38) by dynamic programming.

In practice, the following hypotheses are satisfied:

H_1: For every $\ell \in \mathcal{L}_H, x_{\min}^\ell(t) = 0$, for $t = 2, \ldots, T$, and $x_{\min}^\ell(T+1) = x_1^\ell > 0$.

H_2: $Duration(t) P_{\max}^\ell > \mathcal{I}_t^\ell$ for every $\ell \in \mathcal{L}_H$ and $t = 1, \ldots, T$.

H_3: For every $\ell \in \mathcal{L}_H$, $x_1^\ell > \sum_{t=1}^{T} \mathcal{I}_t^\ell$.

We then have the following:

Lemma 7.1. *Let Assumptions H_1, H_2, and H_3 hold and for every $\ell \in \mathcal{L}_H$, and $t = 1, \ldots, T$, let Ψ_t^ℓ and Φ_t^ℓ be the functions defined by* (32) *with f_t substituted by f_t^ℓ given in* (39)*. Then for every time step t such that $2 \leq t \leq T$:*

- $\forall y \geq 0, \Psi_t^\ell(y) = \Phi_t^\ell(y)$; *functions Ψ_t^ℓ and Φ_t^ℓ are concave piecewise affine and have kinks at $(\lambda_t, \lambda_{t+1}, \ldots, \lambda_T)$.*
- *If $y \geq \max(\lambda_t, \lambda_{t+1}, \ldots, \lambda_T)$, then $\Psi_t^\ell(y) = (x_1^\ell - \sum_{j=t}^{T} \mathcal{I}_j^\ell) y$.*
- *If $y \leq \min(\lambda_t, \lambda_{t+1}, \ldots, \lambda_T)$, Ψ_t^ℓ is affine with slope greater than $x_1^\ell > 0$.*

Proof. Let us fix $\ell \in \mathcal{L}_H$ and let us show the result by induction. For $t = T$, $\Psi_t^\ell(y) = f_T^\ell(y) = (x_1^\ell - \mathcal{I}_T^\ell) y - (\lambda_T - y)^+ Duration(T) P_{\max}^\ell$. If $y \leq \lambda_T$, $\Psi_t^\ell(y)$ is thus affine with slope $x_1^\ell + Duration(T) P_{\max}^\ell - \mathcal{I}_T^\ell > x_1^\ell$ and if $y \geq \lambda_T$, then $\Psi_t^\ell(y) = (x_1^\ell - \mathcal{I}_T^\ell) y$. So Ψ_T^ℓ is concave, piecewise affine, strictly increasing, having a kink at λ_T. Consequently, $\Phi_T^\ell(y) = \Psi_T^\ell(y)$, for every $y \geq 0$. Let us assume that the result is valid for some t such that $3 \leq t \leq T$. We have

$$\begin{aligned} \Psi_{t-1}^\ell(y) &= \Phi_t^\ell(y) + f_{t-1}^\ell(y) \\ &= \Phi_t^\ell(y) - \mathcal{I}_{t-1}^\ell y - (\lambda_{t-1} - y)^+ \, Duration(t-1) P_{\max}^\ell. \end{aligned}$$

So if $y \geq \max(\lambda_{t-1}, \lambda_t, \dots, \lambda_T)$, since $\Phi_t^\ell(y) = (x_1^\ell - \sum_{j=t}^{T} \mathcal{I}_j^\ell)y$, we have $\Psi_t^\ell(y) = (x_1^\ell - \sum_{j=t-1}^{T} \mathcal{I}_j^\ell)y$, i.e., $\Psi_t^\ell(\cdot)$ is affine with positive slope. If $y \leq \min(\lambda_{t-1}, \lambda_t, \dots, \lambda_T)$, then Ψ_{t-1}^ℓ is affine with slope greater than $x_1^\ell - \mathcal{I}_{t-1}^\ell + Duration(t-1)P_{\max}^\ell > x_1^\ell$. The function $\Psi_{t-1}^\ell(\cdot)$ is thus piecewise affine, strictly increasing concave and has kinks at $(\lambda_{t-1}, \lambda_t, \dots, \lambda_T)$. Consequently, by definition of Φ_{t-1}^ℓ, for every $y \geq 0$, $\Phi_{t-1}^\ell(y) = \Psi_{t-1}^\ell(y)$. □

The next theorem then provides a solution of problem (38):

Theorem 7.1. *Let Assumptions H_1, H_2, and H_3 hold. Then an optimal solution of problem* (38) *satisfies*

$$s_1^{*\ell} = \dots = s_T^{*\ell},$$

and $s_1^{*\ell}$ is one of the T scalars $(\lambda_1, \dots, \lambda_T)$.

Proof. For every $y \geq 0$,

$$\Psi_1(y) = \Phi_2(y) - (x_1^\ell + \mathcal{I}_1^\ell)y - (\lambda_1 - y)^+ \, Duration(1)P_{\max}^\ell. \tag{40}$$

Using (40) and Lemma 7.1, if $0 \leq y \leq \min(\lambda_1, \dots, \lambda_T)$, we see that Ψ_1^ℓ is affine with slope greater than or equal to $x_1^\ell - x_1^\ell - \mathcal{I}_1^\ell + Duration(1)P_{\max}^\ell \geq Duration(1)P_{\max}^\ell - \mathcal{I}_1^\ell > 0$. On the other hand, if $y \geq \max(\lambda_1, \dots, \lambda_T)$, then $\Psi_1^\ell(y) = -\sum_{j=1}^{T} \mathcal{I}_j^\ell y$, and $\Psi_1^\ell(\cdot)$ is affine with negative slope. The function $\Psi_1^\ell(\cdot)$ is thus concave, piecewise affine, with slope changing at the points $(\lambda_1, \dots, \lambda_T)$. Consequently $y_1^{*\ell}$ is one of the scalars $(\lambda_1, \dots, \lambda_T)$. We are thus finished by using Lemma 6.2. □

To solve (38), it thus suffices to evaluate the cost function for the T candidate vectors provided by this theorem.

Remark 7.1. *The method presented above does not provide primal solutions but the optimal value for the local subproblems. Thus, it cannot be used to determine a subgradient of the dual function at a given λ. However, the problem solved to compute the dual function at a given λ is an approximate problem for the initial primal problem with a given price λ_i to pay for the violation of the ith dualized constraint. In this sense, the above method can be seen as an approximate solution method for the deterministic electricity generation management problem.*

7.3. **Dedicated solution methods in the stochastic case.** The developments of the previous section can be used in the stochastic case when the model chosen consists of minimizing the average cost on a set of scenarios. Indeed, in this case we have to minimize $\sum_n p_n f(x_n)$ subject to $x_n \in \mathcal{U}_n$ where p_n is the probability of scenario n, $f(x_n)$ the cost on this scenario and $x_n \in \mathcal{U}_n$ the constraints on the production schedule of this scenario. Such a model is not suitable for determining management strategies but is useful for determining an approximate distribution of the cost over the management horizon. Since this problem is clearly separable with respect to the scenarios, the remarks of the previous section apply to compute the optimal cost on each scenario.

8. Numerical simulations

We use the following capacity production (as in [14] and [13]) given by EDF (the company producing electricity in France):

- Eleven thermal plants. Each thermal production unit ℓ is described by its unit production cost, its maximal power, the number of thermal groups and the probability α_ℓ that a thermal group works ($\alpha_\ell(\cdot)$ is constant in this simulation).
- Two independent hydroelectric power stations. Each station is connected to a different reservoir. We know the maximal storage (in GWh) and the starting storage of each reservoir and the maximal power (in MW) of each hydroelectric power station.
- An EJP contract of 22 days.

Different management strategies are compared. The model introduced in Section 5 is denoted by 'Dual. Stab'. The robust methods introduced in [13] are denoted (as in [13]) by ARC, $AARC_2$ and $AARC'_2$. Finally as in [14], we denote by 'Tree' the scenario tree based optimization method detailed in [14]. The duration of a time step is two weeks. Notice that the optimization method 'Tree' was used in [14] for a management horizon of one year but with a daily time step. We aggregated the tree used in [14] and adapted the optimization methods in order to use a two week time step.
We have 456 scenarios which are seen as possible evolutions of the demands and availability rates over the investment period. They are used to calibrate the model parameters. The management methods are then tested on these scenarios. The matrix $Q_{\mathcal{D}}$ is estimated by the demand empirical covariance matrix obtained from the 456 scenarios. We naturally choose for $\bar{\mathcal{D}}$ the empirical mean of the demands over the 456 scenarios.

We compare the mean and standard deviation of the cost on the 456 scenarios using the different management methods. The methods are ranked according to the value of the sum mean plus standard deviation of the cost.

	ARC	$AARC_2$	Dual. Stab.	'Tree'	$AARC'_2$
Mean	4.54×10^8	4.70×10^8	4.49×10^8	4.51×10^8	4.75×10^8
Sd	1.01×10^6	3.57×10^6	2.92×10^7	3.26×10^7	1.2×10^7
Rank	1	2	3	4	5

Table 1. Comparison of the different management methods.

As explained in [13], the ARC method is only of theoretical interest. The other robust methods $AARC_2$ and $AARC'_2$ yield low standard deviations of the cost but at the expense of the average cost. Our 'Dual. Stab' method gives the lowest average cost, and in particular, provides both a lower mean and a lower standard deviation of the cost than the more traditional scenario tree based method 'Tree' used in [14].

9. Conclusion

We presented a new efficient model for the electricity generation management problem. A new solution method for this problem has also been proposed in a particular case. In our numerical experience, our model seems to be very competitive with respect to other models used to deal with uncertainty in the electricity generation management problem.

It remains to study the generalization of the method developed in Section 6 to solve the deterministic electricity generation management problem by dynamic programming when there is more than just one hydro plant. Finally, the uncertainty of the inflows should also be theoretically taken into account in the model in the stochastic case.

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