A Chance-Constrained Model & Cutting Planes for Fixed Broadband Wireless Networks *

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Abstract In this paper, we propose a chance-constrained mathematical program for fixed broadband wireless networks under unreliable channel conditions. The model is reformulated as integer linear program and valid inequalities are derived for the corresponding polytope. Computational results show that by an exact separation approach the optimality gap is closed by 42 % on average.

1 Introduction

Fixed broadband wireless communications is a promising technology for delivering private high-speed data connections by means of microwave radio transmission [2]. Microwave, in the context of this work, refers to terrestrial point-to-point digital radio communications, usually employing highly directional antennas in clear line-of-sight and operating in licensed frequency bands. The rapid and relatively cheap

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deployment is in particular interesting for emerging countries and remote locations as well as for private and isolated networks in urban areas (e. g., connected hospitals, parts of a harbour) where classical copper/fiber lines are too costly [8]. In contrast to wired networks, the capacity of a microwave link is not constant, but depends on the used modulation scheme, which on its turn depends on the condition of channel. Varying channel conditions result in varying link capacities.

In this paper, we study the planning of fixed broadband wireless networks under unreliable channel conditions. We develop a chance-constrained optimization model and, for the case where the outage probabilities of the microwave links are independent, derive an integer linear programming (ILP) formulation (Section 2). Further, we derive two classes of cutset-based valid inequalities to strengthen this formulation (Section 3). Preliminary computational results confirm the importance of the cutting planes (Section 4).

2 Mathematical formulation

The minimum cost design of a fixed broadband wireless network can be formally stated as follows, cf. [3] for technical details. The network's topology is modeled as a digraph $G = (V, E)$ with V denoting the set of radio base stations and E the set of directional microwave radio links. The traffic requirements are modeled by a set K. For each pair $k \in K$, s^k denotes the origin, t^k the destination and $d^k \geq 0$ the expected demand.

For each microwave link $uv \in E$, the capacity is basically determined by the channel bandwidth (e.g., 7 MHz, 28 MHz) and the modulation scheme (e.g., 16-QAM, 128-QAM) used to transmit data. Where exactly one channel bandwidth has to be chosen at design stage, adaptive modulation is performed at runtime, depending on the channel conditions, i.e., if the receiving base station observes a deterioration in signal quality, the modulation scheme is lowered to avoid outage of the link.

Let W_{uv} be the set of bandwidth choices available for arc $uv \in E$. The choice to operate link $uv \in E$ at bandwidth b^w_{uv}, $w \in W_{uv}$, implies a cost c^w_{uv}. The modulation scheme is modeled with a random variable η^w_{uv} with (known) discrete probability, representing the number of bits per symbol of the current modulation scheme. The capacity of a microwave link is basically given by the product of b^w_{uv} and η^w_{uv}.

Given an infeasibility tolerance $\varepsilon > 0$, our aim is to design a minimum cost network such that its capacity is sufficient with a probability of at least $1 - \varepsilon$. This joint chance constraint reads

$$\mathscr{P}\left(\sum_{k\in K} d^k f^k_{uv} \leq \sum_{w\in W_{uv}} \eta^w_{uv} b^w_{uv} y^w_{uv} \qquad \forall uv \in E\right) \geq 1 - \varepsilon \tag{1}$$

with binary decision variables y^w_{uv} indicating whether bandwidth $w \in W_{uv}$ is chosen for arc $uv \in E$ and flow variables f^k_{uv} denoting the fraction of demand d^k, $k \in K$, routed on arc $uv \in E$.

In case the random variables η_{uv}^{w} are independent, we can reformulate the left hand side of (1) as the product of probabilities introducing the following notation: For arc $uv \in E$, let M_{uv}^{w} be the set of modulations in case of bandwidth choice $w \in W_{uv}$ with, for $m \in M_{uv}^{w}$, b_{uv}^{wm} the resulting capacity. Given $uv \in E$, $w \in W_{uv}$, and $m \in M_{uv}^{w}$, let ρ_{uv}^{wm} be the probability that the link is operated at modulation m or higher.

Now, we may assume that each link is operated at a chosen modulation (or higher) as long as the overall probability of the assumptions is at least $1-\varepsilon$. For this, the binary decision variables y obtain a new index m. The minimum cost fixed broadband wireless network design problem then reads:

$$\min \sum_{uv\in E}\sum_{w\in W_{uv}}\sum_{m\in M_{uv}^{w}} c_{uv}^{w} y_{uv}^{wm} \tag{2a}$$

$$s.t. \sum_{u\in V: vu\in E} f_{vu}^{k} - \sum_{u\in V: uv\in E} f_{uv}^{k} = \begin{cases} 1, & \text{if } v = s^{k}, \\ -1, & \text{if } v = t^{k}, \\ 0, & \text{otherwise} \end{cases} \qquad \forall v\in V, k\in K \tag{2b}$$

$$\sum_{w\in W_{uv}}\sum_{m\in M_{uv}^{w}} y_{uv}^{wm} = 1 \qquad \forall uv\in E \tag{2c}$$

$$\sum_{k\in K} d^{k} f_{uv}^{k} \le \sum_{w\in W_{uv}}\sum_{m\in M_{uv}^{w}} b_{uv}^{wm} y_{uv}^{wm} \qquad \forall uv\in E \tag{2d}$$

$$\prod_{uv\in E}\Big(\sum_{w\in W_{uv}}\sum_{m\in M_{uv}^{w}} \rho_{uv}^{wm} y_{uv}^{wm}\Big) \ge 1-\varepsilon \tag{2e}$$

$$f_{uv}^{k} \in [0,1], y_{uv}^{wm} \in \{0,1\} \tag{2f}$$

Besides the total bandwidth cost function (2a), the flow conservation constraints (2b), and the link capacity constraints (2d), constraints (2c) ensure that exactly one bandwidth-modulation pair is chosen. The joint chance constraint (1) is now equivalently modeled in (2e) as the product of all links of the cumulative probabilities.

Note that now, we assume explicitly a hypothesis on the modulation scheme in the capacity constraints (2d). Obviously, for a given link and bandwidth, the lower the modulation scheme is, the lower the assumed capacity and the higher the probability that the effective capacity supports the routed traffic. In other words, more conservative hypotheses on the modulation schemes leads to more reliable solutions.

Constraint (2e) can be easily linearized: By employing monotonicity of logarithmic functions and because the logarithm of a product equals to the sum of the logarithms, (2e) is equivalent to

$$\sum_{uv\in E}\log\left(\sum_{w\in W_{uv}}\sum_{m\in M_{uv}^{w}} \rho_{uv}^{wm} y_{uv}^{wm}\right) \ge \log(1-\varepsilon) \tag{3}$$

Now, note that by constraints (2c) exactly one of the sum elements within each logarithmic function will be nonzero. Hence, (3) is equivalent to

$$\sum_{uv\in E}\sum_{w\in W_{uv}}\sum_{m\in M_{uv}^{w}} \log(\rho_{uv}^{wm}) y_{uv}^{wm} \ge \log(1-\varepsilon) \tag{4}$$

3 Valid inequalities

Constraints (2b), (2c), and (2d) define a classical network design problem studied intensively in the literature, see [9] and the references therein. In particular, *cut-based inequalities* have been proven to be effective to enhance the performance of ILP solvers [1]. Let $S \subset V$ be a proper and nonempty subset of the nodes V and $\overline{S} = V \setminus S$ its complement. The set $E(S,\overline{S}) := \{uv \in E : u \in S, v \in \overline{S}\}$, i. e., the set of arcs from S to $\overline{S}$ defines a *cutset*. Similarly, let $K(S,\overline{S}) := \{k \in K : s^k \in S, t^k \in \overline{S}\}$ be the set of demands having their origin in S and their destination in $\overline{S}$. Finally, let $d(S,\overline{S}) := \sum_{k \in K(S,\overline{S})} d^k$. An appropriate aggregation of constraints (2b), (2d), and nonnegativity of the variables results in the following *base cutset inequality*:

$$\sum_{uv \in E(S,\overline{S})} \sum_{w \in W_{uv}} \sum_{m \in M^w_{uv}} b^{wm}_{uv} y^{wm}_{uv} \geq d(S,\overline{S}) \tag{5}$$

Chvátal-Gomory (CG) rounding yields two classes of valid inequalities.

Cutset Inequalities By dividing (5) by $a \in \{b^{wm}_{uv} : uv \in (S,\overline{S}), w \in W_{uv}, m \in M^w_{uv}\}$ and rounding up both sides, the well-known *cutset inequalities* [9] are obtained:

$$\sum_{uv \in E(S,\overline{S})} \sum_{w \in W_{uv}} \sum_{m \in M^w_{uv}} \left\lceil \frac{b^{wm}_{uv}}{a} \right\rceil y^{wm}_{uv} \geq \left\lceil \frac{d(S,\overline{S})}{a} \right\rceil \tag{6}$$

Shifted Cutset Inequalities Instead of applying CG-rounding directly, we can first shift the coefficients of (5). Given a cutset $(S,\overline{S})$, let $a_{uv} = \min_{w \in W_{uv}} \min_{m \in M^w_{uv}} b^{wm}_{uv}$ for $uv \in E(S,\overline{S})$. By (2c) and $a(S,\overline{S}) := \sum_{uv \in E(S,\overline{S})} a_{uv}$, (5) can be rewritten as:

$$\sum_{uv \in E(S,\overline{S})} \sum_{w \in W_{uv}} \sum_{m \in M^w_{uv}} (b^{wm}_{uv} - a_{uv}) y^{wm}_{uv} \geq d(S,\overline{S}) - a(S,\overline{S}) \tag{7}$$

Now, let $a' \in \{b^{wm}_{uv} - a_{uv} : uv \in (S,\overline{S}), w \in W_{uv}, m \in M^w_{uv}\}$. By Chvátal-Gomory rounding, we obtain the following *shifted cutset inequalities*:

$$\sum_{uv \in E(S,\overline{S})} \sum_{w \in W_{uv}} \sum_{m \in M^w_{uv}} \left\lceil \frac{b^{wm}_{uv} - a_{uv}}{a'} \right\rceil y^{wm}_{uv} \geq \left\lceil \frac{d(S,\overline{S}) - a(S,\overline{S})}{a'} \right\rceil \tag{8}$$

It can be shown that (6) and (8) define facets of the convex hull of feasible solutions under certain conditions (beyond the scope of this paper).

4 Computational Results

Setting We have performed preliminary computational experiments on a 5×5 grid network ($|V| = 25$, $|E| = 80$, $|K| = 50$) based on [7]. We consider two bandwidth choices for each link: 7 MHz (28 MHz) with cost 1000 (6000) using the 128-QAM

(256-QAM) scheme, with an availability of 99.9 %. In fading conditions, these links will use the 16-QAM (32-QAM) scheme (with 100 % availability).

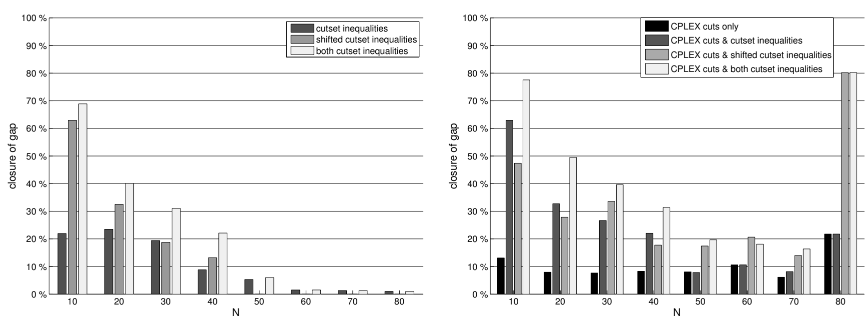
By assuming the same availability for radio links using the highest modulation scheme and under the hypothesis that the lowest modulation scheme guarantees an availability of 100 % (independent of the bandwidth), we can replace (4) by

$$\sum_{uv\in E}\sum_{w=1}^{W_{uv}} y_{uv}^{w2} \leq \left\lfloor \frac{\log(1-\varepsilon)}{\log(\rho)} \right\rfloor =: N \tag{9}$$

where ρ is the availablity probability of the highest modulation scheme. Note that a larger infeasibility tolerance ε implies a larger value N, i.e., the reliability of the solutions decreases. We consider $N = 10$ $(\varepsilon = 0.01),\ 20, \ldots,\ 80$ (no reliability).

All computations are performed with CPLEX 12.2 [5] on a Linux machine with 2.67 GHz Intel Xeon X5650 processor and 12 GB RAM.

Optimality gap closed In this study, we limit ourselves to a comparison of the optimality gap with/without separation of violated cutset inequalities (6) and/or shifted cutset inequalities (8). To this end, the separation of these inequalities is done exactly by an auxiliary ILP (details omitted, cf. e. g., [4, 6]). The cutset inequalities are separated only in the root node of branch and bound.



(a) Cutset inequalities only

(b) CPLEX cuts and cutset inequalities

Fig. 1 Optimality gap closed.

As reference value, we consider the optimality gap, i. e., the difference between LP relaxation and best known solution (computed by CPLEX with a time limit of 12 h, optimal for $N = 10, 60, 70, 80$). Fig. 1 shows the optimality gap closed, i. e., the percental reduction of the optimality gap at the end of the root node, for two types of computations, with/without internal CPLEX cuts. For the results in Fig.1(a), we disabled the internal cuts of CPLEX and separated (i) cutset inequalities (6), (ii) shifted cutset inequalities (8), and (iii) both. Inequalities (8) close the gap with 16 % on average, whereas inequalities (6) close only 10 %.

Obviously, the optimality gap is most closed by the combination of both cutset inequalities (up to 69 % for $N = 10$). With increasing N, the closure of the optimality gap decreases with hardly any closure from $N = 60$. We conjecture that inequalities (6) and (8) are less likely violated since the capacity constraints (2d) are less restrictive. On average, the optimality gap is closed by 21 % for the combination of both types of inequalities.

In Fig. 1(b), we enabled the internal cuts of CPLEX. The optimality gap closed by internal cuts is only 10 % on average compared to 42 % by the combination of internal cuts and cutset inequalities (6) and (8). Note that also CPLEX can separate cutset inequalities [1]: only for $N = 80$, some are found. In case both types are separated, on average 54 violated inequalities are found (17 of type (6) and 37 of type (8)). Again, for increasing N the optimality gap closed decreases, except for $N = 80$ where 80 % of the gap is closed. For $N = 60$, the optimality gap is closed less by the combination of (6) and (8) than only by the shifted cutset inequalities (8). Such a phenomenon can occur due to varying internal CPLEX cuts.

5 Concluding Remarks

In this paper, we have presented a chance-constrained programming approach for the assignment of bandwidth in reliable fixed broadband wireless networks. We have proposed cutset inequalities and shifted cutset inequalities to enhance the computability of this problem. In our computational studies, we have discussed the optimality gap closed and compared the performance of the different cutset inequalities with and without internal CPLEX cuts. The results show that by the combination of the cutset and the shifted cutset inequalities, the optimality gap is closed by 41 % on average if the internal cuts for CPLEX are enabled.

As future work, we intend to investigate more realistic network topologies, different probability models and the reliability regarding traffic fluctuations.

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