Planning Wireless Networks with Demand Uncertainty using Robust Optimization *

Grit Claßen, Arie M. C. A. Koster, and Anke Schmeink

Abstract An optimal planning of future wireless networks is fundamental to satisfy rising traffic demands jointly with the utilization of sophisticated techniques, such as OFDMA. Current methods for this task require a static model of the problem. However, uncertainty of data arises frequently in wireless networks, e. g., fluctuating bit rate requirements. In this paper, robust optimization is applied to deal with uncertainty in the framework of optimization models. We propose a mathematical formulation for the planning of wireless networks with demand uncertainy. Furthermore, computational results are presented to compare the robust formulation to its deterministic counterpart. The price of robustness is demonstrated regarding key parameters of networks that are subject to uncertainty.

1 Introduction

Future radio networks are designed to cope with drastically increasing user demand. These high user demands (compared to the requirements for ordinary telephony and short message services) result from traffic-intensive smartphone applications. Even though user demand and resource restrictions have been considered in the planning of third generation (3G) networks [2], these networks reach the limits of their capacity. To tackle the problem of insufficient network performance, future networks utilize a couple of advanced techniques, e. g., Orthogonal Frequency Division Mul-

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tiple Access (OFDMA). Nevertheless, an optimal planning of wireless networks is of utmost importance to fully utilize the gains of those techniques.

Current wireless network planning focuses on the maximization of the total traffic while minimizing the installation costs such that technological requirements and environmental aspects are taken into account [2, 12]. The planning requires a static model of the problem [6, 8, 14]. However, many factors of actual wireless networks are non-deterministic. Fluctuating bit rate requirements and channel conditions are just two of multiple examples for parameters that are subject to uncertainty in the actual problem. The robust optimization approach by Bertsimas and Sim [3, 4] deals with uncertainty by limiting the number of uncertainties by a robustness parameter. Varying the parameter results, to a greater or less extend, in robust solutions.

In this paper, we propose an integer linear program (ILP) based on [7] for the planning of static wireless networks (Section 2). We reformulate this problem by applying robust optimization to deal with demand uncertainty (Section 3). Computational results for a realistic wireless network scenario compare the robust ILP with its deterministic counterpart and demonstrate the price of robustness (Section 4).

2 System Model and Problem Formulation

First, we introduce the system model considering a future wireless network and downlink (DL) data transmission based on the model presented in [7]. Let S be a set of candidate sites for base stations (BSs). The deployment of BS $s \in S$ implicates a cost of c_s and a total DL bandwidth b_s. Furthermore, let T be a set of traffic nodes (TNs) to be assigned to the deployed BSs. Each $t \in T$ has a demand w_t and can be assigned to at most one BS, which applies, e. g., to 4G networks.

We furhter assume that no intra-cell interference occurs. This ideally holds, e. g., for wireless networks utilizing OFDMA for DL transmission. Moreover, inter-cell interference is limited by requiring the selected sites to constitute an *independent set* in a prespecified conflict graph $G = (S,E)$, i. e., a subset of S such that $ij \notin E \; \forall i, j$.

For each s,t pair, a value e_{st} called spectral efficiency is defined to incorporate, e. g., modulation and coding scheme that is supported by the associated signal-to-noise ratio (SNR). To establish a transmission link, it must exceed a certain threshold $e_{\min}$. If TN t is assigned to BS s, the established transmission link from s to t occupies a certain amount of the available DL bandwidth b_s at BS s. This amount can be computed by division of demand w_t by spectral efficiency e_{st}.

The presented system model is now used to propose an optimization model for the modulation of the planning problem as an ILP. The constraint on the spectral efficiency is included in the following auxiliary sets of indices:

$$S * T := \{(s,t) \in S \times T \; : e_{st} \geq e_{\min}\},$$
$$S_t := \{s \in S \; : (s,t) \in S * T\}, \; T_s := \{t \in T \; : (s,t) \in S * T\}.$$

Let $x_s \in \{0,1\}$ denote whether or not BS $s \in S$ is deployed and $z_{st} \in \{0,1\}$ whether $t \in T_s$ is assigned to s. The main objective of the optimization model is to maximize the total profit of the network operator. The profit is computed as a fixed revenue λ_{basic} for the coverage of a user plus a revenue λ_{volume} for the throughput per unit minus the monthly BS deployment costs. The optimization model is defined as follows:

$$\max \sum_{(s,t)\in S*T} (\lambda_{\text{basic}} + \lambda_{\text{volume}} w_t) z_{st} - \sum_{s\in S} c_s x_s \tag{1a}$$

$$\text{s.t.} \sum_{s\in S_t} z_{st} \leq 1 \qquad \forall t \in T \tag{1b}$$

$$z_{st} \leq x_s \qquad \forall (s,t) \in S*T \tag{1c}$$

$$x_i + x_j \leq 1 \qquad \forall ij \in E \tag{1d}$$

$$\sum_{t\in T_s} \frac{w_t}{e_{st}} z_{st} \leq b_s x_s \qquad \forall s \in S \tag{1e}$$

$$x_s \in \{0,1\},\ z_{st} \in \{0,1\} \qquad \forall s \in S,\ (s,t) \in S*T \tag{1f}$$

Constraints (1b) ensure that a TN is covered by at most one BS and by constraints (1c) a TN can be assigned to a BS if and only if this BS is deployed. Fulfilling constraints (1d) guarantees an independent set of deployed BSs. Constraints (1e) ensure that the total bandwidth (i. e., demand divided by spectral efficiency) allocated does not exceed the available DL bandwidth.

3 Robust Formulation

Although many parameters are uncertain, the only factors we consider to be subject to uncertainty in our model are the demand values. We apply the robust optimization approach presented in [4] to formulation (1). Thus, the demand values are now modeled as symmetric and bounded random variables that take values in $[\bar{w}_t - \hat{w}_t, \bar{w}_t + \hat{w}_t]$, where $\bar{w}_t$ denotes a default value and $\hat{w}_t$ its deviation. A robustness parameter $\Gamma \in \{0,\ldots,|T|\}$ limits the number of demand values w_t in (1e) deviating from their default value $\bar{w}_t$ (towards $\bar{w}_t + \hat{w}_t$). To ensure comparability of the results, the coefficients in the objective (1a) are set to the default values. By exploiting LP duality (introducing dual variables u_s and p_{st}), the robust optimization problem (1) can be reformulated as the following ILP:

$$\max \sum_{(s,t)\in S*T} (\lambda_{\text{basic}} + \lambda_{\text{volume}} \bar{w}_t) z_{st} - \sum_{s\in S} c_s x_s \tag{2a}$$

$$\text{s.t.} \quad (1b), (1c), (1d), (1f)$$

$$\sum_{t\in T_s} \frac{\bar{w}_t}{e_{st}} z_{st} + \Gamma u_s + \sum_{t\in T_s} p_{st} \leq b_s x_s \qquad \forall s \in S \tag{2b}$$

$$u_s + p_{st} \geq \hat{w}_t z_{st} \qquad \forall (s,t) \in S*T \tag{2c}$$

$$u_s \geq 0,\ p_{st} \geq 0 \qquad \forall s \in S,\ (s,t) \in S*T \tag{2d}$$

4 Computational Results

We apply branch and bound to solve the ILP. To strengthen the LP relaxation, two classes of valid inequalities are taken into account. First, we replace constraints (1d) by the *maximal clique* inequalities for the independent set polytope [13]:

$$\sum_{s \in U} x_s \leq 1 \qquad \forall U \subset S,\ U \text{ is a maximal clique in } G = (S, E).$$

Second, we adapt the concept of cover inequalities for the robust knapsack problem [10] to constraints (2b). A subset $C \subseteq T_s$ is a *cover* for BS $s \in S$ if

$$\sum_{t \in C} \frac{\bar{w}_t}{e_{st}} + \max_{I \subseteq C : |I| \leq \Gamma} \sum_{t \in I} \frac{\hat{w}_t}{e_{st}} > b_s.$$

The cover inequality

$$\sum_{t \in C} z_{st} \leq (|C| - 1) x_s \tag{3}$$

is valid. Since (3) has a variable right hand side, we modify and also strengthen the greedy heuristic for separating robust cover inequalities given in [10]. Finally, we extend the covers found to tighten the cover inequalities, see [10] and [5].

We generate a planning scenario based on signal propagation data for Munich, available at [1] with 40 candidate sites and 450 randomly distributed TNs. The signal prediction is done by a cube oriented ray launching algorithm presented in [11]. Two BSs are adjacent in G if and only if the distance between them is less or equal to 500 m. Furthermore, we use the following scenario parameters: $b_s = 10$ MHz, $c_s = 2000$ EUR $\forall s \in S$, $e_{\min} = 0.5$ bps/Hz, $\lambda_{\text{basic}} = 50$ EUR, $\lambda_{\text{volume}} = 0.0005$ EUR/bps.

Table 1 Profiles for TNs

service	percentage [%]	bit rate [kbps]
data	[10,20]	[512,2000]
web	[20,40]	[128,512]

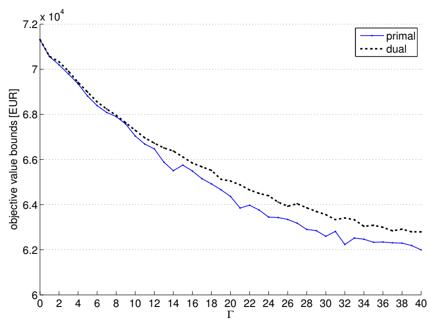
For each $t \in T$, $\bar{w}_t$ is computed by randomly generating user profiles from Table 1: For both data and web services a percentage and bit rate is uniformly drawn from the intervals. The remaining percentage is used for Voice-over-IP (VoIP) with a bit rate of 64 kbps. The deviation $\hat{w}_t$ is computed by taking a user profile with 40 % data services at 2000 kbps, 50 % web services at 512 kbps and 10 % VoIP at 64 kbps minus the default value $\bar{w}_t$.

We used a Linux machine with a 2.67 GHz Intel Xeon X5650 processor, 12 GB RAM, and CPLEX 12.2 [9]. A CPU time limit of 1 h is set.

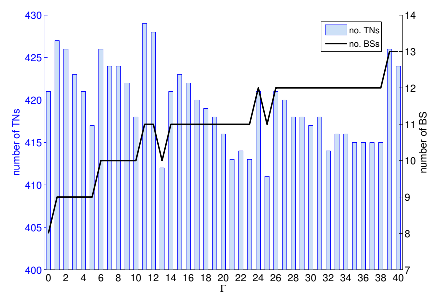
Fig. 1(a) shows the primal and dual bounds for $\Gamma \in \{0, 1, \ldots, 40\}$. We observe that the optimality gap increases with increasing Γ. The decrease in the objective value is the price of robustness, i. e., for larger values of Γ, the solution becomes more conservative and at the same time more robust against deviations. Note that non-robust wireless networks should not be planned with the default value only but with a higher value to ensure high quality of service. Fig. 1(b) shows that the number of served TNs decreases with increasing Γ as long as the number of deployed BSs remains the same.

To evaluate the robustness of the different solutions, we compute 1001 snapshots to simulate various traffic demand scenarios. In each snapshot, the demand of the TNs is binomially distributed between $\bar{w}_t$ and $\bar{w}_t + \hat{w}_t$ with a probability of 0.5. From this, we compute the (over)load for each deployed BS s in a solution: The allocated bandwidth divided by the DL bandwidth. Fig.2 shows the maximum load of the BSs averaged over the snapshots.

A value less than 1 means that there is no overload; the TNs can be served at their required bit rate and it is even possible to serve more than the assigned TNs.



(a) Bounds



(b) number of TNs and BSs

Fig. 1 Computational Results.

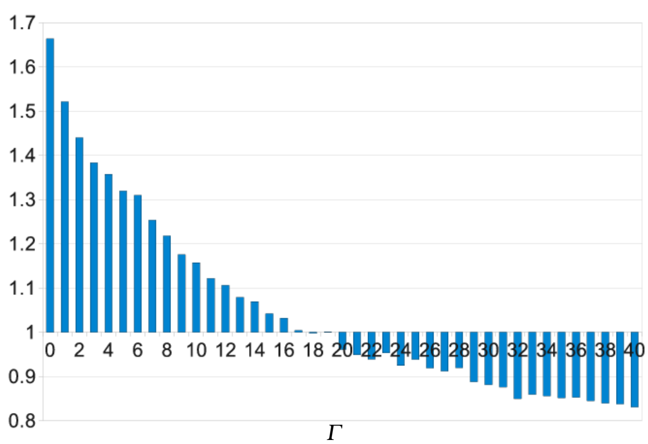


Fig. 2 Overload

Provided the snapshots are representative, Fig. 2 suggests to set $\Gamma = 20$ for a robust solution. This results in an increase of the objective value by 4 % compared to the common practice to plan the network suggesting that all traffic nodes take the peak demand.

5 Concluding Remarks

We have introduced an optimization model for the planning of future wireless networks which considers technical system characteristics such as OFDMA. Applying the approach by Bertsimas and Sim [3], we incorporated demand uncertainty. Network operators can assess the trade-off between robustness and profit by varying the robustness parameter which can save 4 % compared to a deterministic solution.

Future work will deal with further investigation of valid inequalities and primal heuristics for (2) to reduce the optimality gaps.

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