Optimal Job Scheduling with Day-ahead Price and Random Local Distributed Generation: A Two-stage Robust Approach

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Abstract

In this paper, we consider a job scheduling problem with random local generation, in which some jobs must be scheduled day-ahead while the others can be scheduled in a real time fashion. To capture the randomness of the local distributed generation, we develop a two-stage robust optimization model by assuming an uncertainty set without probability information. Given that the problem is challenging, a nested primal cut algorithm is implemented to exactly solve it. A preliminary computational study, along with management insights, is presented to show the effectiveness of the proposed model.

NOMENCLATURE

Indices

n Time period, $n = 0, ..., N-1$
i Jobs, $i = 0, ..., I-1$
k Working mode, $k = 0, ..., K-1$
t Segments of job, $t = 0, ..., L_{ik}-1$

Parameters

C_n^{base} Base electricity price at time n
C_n^{extra} Non-base electricity price at time n
A_i Earliest start time for job i
B_i Due time for job i
P_n^G Local generation at time period n
L_{ik} Length of job i in mode k
D_{ikt} Workload of job i in mode k at segment t
P_n^{max} Consumption limit at base price at time n
R_i Whether job i is interruptible or not

Decision variables

θ_{iktn} Binary variable, 1 if the segment t in mode k of job i is assigned to period n
y_{ik} Binary variable, 1 if job i runs in mode k
x_i Integer variable, start time of job i
z_i Integer variable, completion time of job i
f_n The consumption at period n
p_{in} The consumption of job i at period n
w_n^{base} The billed consumption at base price at time n
w_n^{extra} The billed consumption at extra price at time n

I. INTRODUCTION

Restructuring of the power system and emergence of smart grid have enabled end-consumers to receive timely price information. With this information, consumers can take one of the two main approaches in *Demand Side Management* program to reduce their energy expenses: (i)*Energy Efficiency* and (ii)*Load Shifting* or *Demand Response (DR)*. In the first approach, consumers are encouraged to consume more conservatively, giving up some energy use in return for lower payment. In the other approach, the flexible applications will be transferred to *off-peak* hours which results in a more balanced consumption curve and less expenses [1].

Although providing more accurate and frequent price information allows for more *informed* decision makings about the energy consumption, it does not seem realistic to expect consumers to be willing and able to go through the information and make an optimal decision. According to [2] "while over 80% of consumers are very interested in learning how to cut their energy costs, less than one-half want to learn more about smart grids". So in order to benefit the most, consumers must be provided with the state-of-the-art hardware and software such as communication devices and technologies, modern platforms and fast and reliable scheduling algorithms to be employed in decision-support systems.

Over the past few years, many models and scheduling methods have been proposed to generate cost-effective schedules for both residential and industrial applications. Generally, there are three types of scheduling models. The first one is basic deterministic schedule models with exact or heuristic algorithms ([3], [4]). The second one is stochastic scheduling models, including simulation based models and/or random location generations ([5], [6]). The last one is control-based load control methods ([4], [7]).

In this paper, we consider an energy consumption scheduling problem with two sets of jobs. The first set includes the jobs that must be scheduled day-ahead, because of some considerations such as long setup time or external workforce. The second set of jobs are those which can be scheduled in real-time. Also, for each job, it could have multiple modes with different durations and energy demands. Also, we allow jobs to be interruptible or non-interruptible. So, the model can be applied in both residential and industrial settings to deal with various jobs. The random local generation is often described by probabilistic scenarios [8]. Nevertheless, it may be difficult to obtain the probability distribution or the obtained distribution is not reliable. Given an unreliable forecasting, the optimal schedule may lead to a significant electricity bill. So instead of relying on precise probabilistic estimation, it would be helpful to use robust optimization (RO) to derive a schedule with the least risk. In this paper, we present a 2-stage robust optimization model where the first stage problem is the scheduling problem for the first set of jobs, a cardinality-constrained box uncertainty set is used to describe random local generation, and the recourse problem is the scheduling problem of the second set of jobs. Comparing to existing stochastic programming models, the schedule derived from 2-stage RO is guaranteed to be reliable with respect to the predefined uncertainty set. It is worth mentioning that this 2-stage RO is novel as the recourse problem is an MIP problem, for which most existing algorithms are not applicable ([9], [10], [11]) as strong duality does not hold. Using a recent algorithm, the nested primal cut algorithm [12], we will be able to solve the problem exactly. To the best of our knowledge, it is the first 2-stage robust scheduling model with random local generation.

This paper is organized as follows: Section II describes the formulation, including both deterministic model and its robust counterpart, Section III describes the exact solution approach. The computational results are shown in section IV with some discussion. Section V concludes the paper.

II. Formulation

A. Deterministic Model

We assume that the local generation is from a renewable energy source, is of no cost but cannot sold back to the grid. In each time period n, a consumer will purchase the needed power if the local generation is not sufficient. We consider a step-wise price structure where the unit price is C_n^{base}, i.e. the base price, if the amount of power purchased is less than a predefined level P_n^{max}. Otherwise, the price will be C_n^{extra} with $C_n^{extra} > C_n^{base}$

The following characteristics of jobs are captured in our model: (i) The earliest start time A_i and due time B_i for each job i. (ii) Multiple modes have been considered for each job i. For example, a cooking appliance will have two modes, slow cooking or regular cooking, which will have different power intensity. Hence, the length of the job L_{ik} and the workload in each segment D_{ikt} varies according to mode k. For example, it may take longer but consume less in each time period. (iii) Some jobs are interruptible like HVAC, charging or ironing, while others should be completed without interruption like running a dish washer. (iv) The segments of a job i should be scheduled in the predefined order, that is, a job segment can only be executed if all the previous ones are finished. (v) Some jobs need to be scheduled day-ahead while the decisions for other jobs could be made during the daily operation. This feature enables the consumer to plan for some jobs that has to be done in a specific time, due to some considerations such as long setup time or a need to external technical support, while the second set of jobs have the freedom to be scheduled in real-time.

The deterministic scheduling problem is formulated as follows.

$$min \quad \sum_{n=0}^{N-1} (w_n^{base} C_n^{base} + w_n^{extra} C_n^{extra}) \tag{1}$$

$$st. \quad \sum_{t=0}^{L_{ik}-1} \theta_{iktn} \leq y_{ik}, \forall i, k, n \tag{2}$$

$$\sum_{n=0}^{N-1} \theta_{iktn} = y_{ik}, \forall i, k, t \tag{3}$$

$$\theta_{iktn} \leq \sum_{n'=n+1}^{N-1} \theta_{ikt'n'}, \forall i, k, t, t' \geq t+1, n \tag{4}$$

$$\sum_{k=0}^{K-1} y_{ik} = 1, \forall i \tag{5}$$

$$x_i = \sum_{n=0}^{N-1} n \sum_{k=0}^{K-1} \theta_{ik0n}, \forall i \tag{6}$$

$$x_i \geq A_i, \forall i \tag{7}$$

$$z_i = \sum_{n=0}^{N-1} n \sum_{k=0}^{K-1} \theta_{ik,L_{ik}-1,n}, \forall i \tag{8}$$

$$z_i \leq B_i, \forall i \tag{9}$$

$$z_i - x_i \leq \sum_{k=0}^{K-1} (L_{ik}-1) y_{ik} + R_i N, \forall i \tag{10}$$

$$p_{in} = \sum_{k=0}^{K-1} \sum_{t=0}^{L_{ik}-1} D_{ikt} \theta_{iktn}, \forall i, n \tag{11}$$

$$f_n = \sum_{i=0}^{I-1} p_{in}, \forall n \tag{12}$$

$$f_n \leq P_n^{max} + w_n^{extra} + P_n^G, \forall n \tag{13}$$

$$w_n^{base} \geq f_n - P_n^G - w_n^{extra}, \forall n \tag{14}$$

$$z_i, x_i \in \mathbb{Z}_+, f_n, p_{in}, w_n^{base}, w_n^{extra} \geq 0,$$

$$\theta_{iktn}, y_{ik} \in \{0, 1\}$$

The objective (1) is to minimize total consumption cost. Constraint (2) indicates that each time period n can only hold one segment of each job i with its k mode, that is, two segments of one job cannot be scheduled into the same time period n. Constraint (3) ensures that every segment of each job can only be scheduled once. Constraint (4) makes sure that a job segment can only be executed if all the previous ones are finished. For example, if the LHS of (4) is 1 (the segment t is assigned at period n), the RHS will be forced to be 1 (all the proceeding segments must be scheduled after n). Constraint (5) makes sure for each job i, only one mode is selected. Constraints (6)-(9) are used to restrict the earliest start time and due time of each job. Constraint (10) states whether the job i is interruptible, ie., $R_i = 1$, or not. If $R_i = 0$, the time periods processing a job are the same with its length, so the job will run without interruption. Constraint (11) computes the energy consumption of job i in time n while constraint (12) stands for the energy consumption of all jobs in period n. Constraints (13) and (14) capture the step price for different amounts of consumption. For example, (i) if the local generation is enough for the demand in time n ($f_n \leq p_n^G$), the w_n^{extra} will be forced to be 0 by objective function and w_n^{base} will be 0 since in (14) the RHS will get non-positive, so there will not be a charge; (ii) if $0 \leq f_n - p_n^G \leq P_n^{max}$, the w_n^{extra} will again be forced to be 0 and $w_n^{base} = f_n - P_n^G$ by (14); (iii) if $f_n - p_n^G \geq P_n^{max}$, constraint (13) will become active with $w_n^{extra} \geq 0$ and w_n^{base} will be forced to be P_n^{max}.

After substituting x_i, z_i, p_{in}, f_n with related RHS in (6)(8)(11)(12) respectively (the nonnegative integrality of x_i, z_i and nonnegativity of f_n, p_{in} is naturally guaranteed), the model is as follows.

$$min \quad (1)$$

$$st. \quad (2)-(5),$$

$$\sum_{n=0}^{N-1} n \sum_{k=0}^{K-1} \theta_{ik0n} \geq A_i, \forall i \tag{15}$$

$$\sum_{n=0}^{N-1} n \sum_{k=0}^{K-1} \theta_{ik,L_{ik}-1,n} \leq B_i, \forall i \tag{16}$$

$$\sum_{n=0}^{N-1} n \sum_{k=0}^{K-1} \theta_{ik,L_{ik}-1,n} - \sum_{n=0}^{N-1} n \sum_{k=0}^{K-1} \theta_{ik0n} \leq R_i N + \sum_{k=0}^{K-1} (L_{ik}-1) y_{ik}, \forall i \tag{17}$$

$$\sum_{i=0}^{I-1} \sum_{k=0}^{K-1} \sum_{t=0}^{L_{ik}-1} d_{ikt} \theta_{iktn} \leq P_n^{max} + w_n^{extra} + P_n^G, \forall n \tag{18}$$

$$w_n^{base} \geq \sum_{i=0}^{I-1}\sum_{k=0}^{K-1}\sum_{t=0}^{L_{ik}-1} d_{ikt}\theta_{iktn} - P_n^G - w_n^{extra}, \forall n \tag{19}$$
$$w_n^{base}, w_n^{extra} \geq 0, \theta_{iktn}, y_{ik} \in \{0,1\}$$

B. *Two-stage Robust Counterpart*

However, the distributed local generation, e.g. power from solar panel, is random and intermittent, and the probability information is often unreliable. We develop a robust optimization model where the random local generation is captured by a cardinality constrained box set based on the concept introduced by [13], [14]. Specifically, the uncertainty set is $\mathbb{P}^G = \{p_n^G : p_n^G = \overline{p}_n^G + s_n\tilde{p}_n^G, s_n \in \{0,1\}, \forall n, \sum_n s_n \geq \Gamma\}$ where Γ is an *uncertainty budget* which defines the decision maker's conservative level on the uncertainty set. The parameter Γ can take integer values in $[0, N]$. Now all jobs are partitioned into two sets due to their schedule determination times. The mode selection and schedule of each job j in set 1 needs to be determined day-ahead in the first stage. The decisions of the other jobs h define the *recourse problem* and are made in real-time fashion, i.e. after the first stage decision is made and the information of the uncertain distributed local generation is revealed. Let $\Psi^j = \{(2)-(5), (15)-(17), \theta_{jktn}, y_{jk} \in \{0,1\}\}$ for jobs j and $\Psi^h = \{(2)-(5), (15)-(17), \theta_{hktn}, y_{hk} \in \{0,1\}\}$ for jobs h, the two-stage robust counterpart, also called *adjustable* or *adaptable* robust optimization ([15], [10], [14]), is formulated as follows.

$$\min_{\{\theta_{jktn}, y_{jk}\}\in\Psi^j} \max_{p^G\in\mathbb{P}^G} \min_{\{\theta_{hktn}, y_{hk}, w^{base}, w^{extra}\}\in\Phi} (1) \tag{20}$$

where Φ is

$$\Phi = \Psi^h \bigcap \{(18)-(19)\} \tag{21}$$

III. Exact Solution Approach

The proposed two-stage robust model is challenging due to the presence of binary variables θ_{hktn}, y_{hk} in the second stage. Due to the two-stage nature, a few two-level algorithms are used to solve this problem ([11], [16], [9], [10]). However, the majority of the existing algorithms cannot be applied to solve it as those algorithms heavily depend on the strong duality of the recourse problem. A recent algorithm, called the nested primal cut algorithm [12], extends the primal cut algorithm ([16], [17]) for 2-stage RO and is proven to be effective to deal with MIP recourse problems. Therefore, it is adopted in this paper to solve the 2-stage robust scheduling problem. The detailed mathematical derivations and general algorithm description can be seen in [12].

A. *Outer Level Algorithm*

Assume for any given first stage decision we can find the corresponding worst case local generation $P^{G(m)}$, then extra variables $w_n^{base(m)}, w_n^{extra(m)}, \theta_{hktn}^{(m)}, y_{hk}^{(m)}$ and related constraints (23)-(32) are generated and added to the following algorithm to find a better first stage decision.

$$min \;\; \eta \tag{22}$$
$$st. \;\; \eta \geq \sum_{n=0}^{N-1}(w_n^{base(m)}C_n^{base} + w_n^{extra(m)}C_n^{extra}), \forall m \tag{23}$$
$$\sum_{t=0}^{L_{hk}-1}\theta_{hktn}^{(m)} \leq y_{hk}^{(m)}, \forall h,k,n,m \tag{24}$$
$$\sum_{n=0}^{N-1}\theta_{hktn}^{(m)} = y_{hk}^{(m)}, \forall h,k,t,m \tag{25}$$
$$\theta_{hktn}^{(m)} \leq \sum_{n'=n+1}^{N-1}\theta_{hkt'n'}^{(m)}, \forall h,k,m,t,t' \geq t+1, n \tag{26}$$
$$\sum_{k=0}^{K-1} y_{hk}^{(m)} = 1, \forall m,h \tag{27}$$
$$\sum_{n=0}^{N-1} n \sum_{k=0}^{K-1}\theta_{hk0n}^{(m)} \geq A_h, \forall m,h \tag{28}$$
$$\sum_{n=0}^{N-1} n \sum_{k=0}^{K-1}\theta_{hk,L_{ik}-1,n}^{(m)} \leq B_h, \forall m,h \tag{29}$$

$$\sum_{n=0}^{N-1} n \sum_{k=0}^{K-1} \theta_{hk,L_{ik}-1,n}^{(m)} - \sum_{n=0}^{N-1} n \sum_{k=0}^{K-1} \theta_{hk0n}^{(m)} \leq R_h N$$

$$+ \sum_{k=0}^{K-1} (L_{hk}-1) y_{hk}^{(m)}, \forall m, h \tag{30}$$

$$\sum_j \sum_{k=0}^{K-1} \sum_{t=0}^{L_{jk}-1} d_{jkt}\theta_{jktn} + \sum_h \sum_{k=0}^{K-1} \sum_{t=0}^{L_{hk}-1} d_{hkt}\theta_{hktn}^{(m)}$$

$$\leq P_n^{max} + w_n^{extra(m)} + P_n^{G(m)}, \forall m, n \tag{31}$$

$$w_n^{base(m)} \geq \sum_j \sum_{k=0}^{K-1} \sum_{t=0}^{L_{jk}-1} d_{jkt}\theta_{jktn} - P_n^{G(m)} - w_n^{extra(m)}$$

$$+ \sum_h \sum_{k=0}^{K-1} \sum_{t=0}^{L_{hk}-1} d_{hkt}\theta_{hktn}^{(m)}, \forall m, n \tag{32}$$

$$\theta_{jktn}, y_{jk} \in \Psi^j$$

$$w_n^{base(m)}, w_n^{extra(m)} \geq 0, \theta_{hktn}^{(m)}, y_{hk}^{(m)} \in \{0,1\} \tag{33}$$

B. Inner Level Algorithm

Given first stage decisions $\hat{\theta}_{jktn}, \hat{y}_{jk}$, the subproblem is

$$\max_{p^G \in \mathbb{P}^G} \min_{\{\theta, y, w^{base}, w^{extra}\}} (1) \tag{34}$$

$$st. \sum_j \sum_{k=0}^{K-1} \sum_{t=0}^{L_{jk}-1} d_{jkt}\hat{\theta}_{jktn} + \sum_h \sum_{k=0}^{K-1} \sum_{t=0}^{L_{hk}-1} d_{hkt}\theta_{hktn}$$

$$\leq P_n^{max} + w_n^{extra} + p_n^G, \forall n \tag{35}$$

$$w_n^{base(m)} \geq \sum_j \sum_{k=0}^{K-1} \sum_{t=0}^{L_{jk}-1} d_{jkt}\hat{\theta}_{jktn} - p_n^G - w_n^{extra}$$

$$+ \sum_h \sum_{k=0}^{K-1} \sum_{t=0}^{L_{hk}-1} d_{hkt}\theta_{hktn}, \forall n \tag{36}$$

$$\theta_{hktn}, y_{hk} \in \Psi^h, w_n^{base}, w_n^{extra} \geq 0 \tag{37}$$

After fixing binary variables θ_{hktn}, y_{hk} in the second stage, the strong duality holds in the recourse problem since it is now an LP with decision variables w_n^{base}, w_n^{extra}. Therefore by taking dual representation of this LP, the subproblem is equivalently converted into the following problem in the form of 2-stage RO.

$$\max_{p^G \in \mathbb{P}^G} \min_{\{\theta, y \in \Psi^h\}} \max_{\lambda, \pi} \sum_n \lambda_n (\sum_j \sum_{k=0}^{K-1} \sum_{t=0}^{L_{jk}-1} d_{jkt}\hat{\theta}_{jktn}$$

$$+ \sum_h \sum_{k=0}^{K-1} \sum_{t=0}^{L_{hk}-1} d_{hkt}\theta_{hktn} - p_n^{max} - P_n^G)$$

$$+ \sum_n \pi_n (\sum_j \sum_{k=0}^{K-1} \sum_{t=0}^{L_{jk}-1} d_{jkt}\hat{\theta}_{jktn}$$

$$+ \sum_h \sum_{k=0}^{K-1} \sum_{t=0}^{L_{hk}-1} d_{hkt}\theta_{hktn} - p_n^G) \tag{38}$$

$$st.\ c_n^{base} \geq \pi_n, \forall n \tag{39}$$

$$c_n^{extra} \geq \lambda_n + \pi_n, \forall n \tag{40}$$

$$\lambda_n, \pi_n \geq 0 \tag{41}$$

Since the $max-min$ subproblem is formulated into the two-stage robust optimization problem with the predefined *uncertainty* set Ψ^h, by linearizing $\lambda_n^{(l)} s_n = \mu_n^{(l)}$ and $\pi_n^{(l)} s_n = \delta_n^{(l)}$. Another primal cut algorithm can be used to exactly solve the subproblem formulated as (42)-(53).

$$max \ \ \eta' \tag{42}$$

$$\begin{aligned}\eta' \leq & \sum_n \lambda_n^{(l)} (\sum_j \sum_{k=0}^{K-1} \sum_{t=0}^{L_{jk}-1} d_{jkt}\hat{\theta}_{jktn} - P_n^{max} - \bar{p}_n^G \\ & + \sum_h \sum_{k=0}^{K-1} \sum_{t=0}^{L_{hk}-1} d_{hkt}\theta_{hktn}^{(l)}) \\ & + \sum_n \pi_n^{(l)} (\sum_j \sum_{k=0}^{K-1} \sum_{t=0}^{L_{jk}-1} d_{jkt}\hat{\theta}_{jktn} - \bar{p}_n^G \\ & + \sum_h \sum_{k=0}^{K-1} \sum_{t=0}^{L_{hk}-1} d_{hkt}\theta_{hktn}^{(l)}) - \sum_n \mu_n^{(l)} \tilde{p}_n^G \\ & - \sum_n \delta_n^{(l)} \tilde{p}_n^G, \forall l\end{aligned} \tag{43}$$

$$c_n^{base} \geq \pi_n^{(l)}, \forall n, l \tag{44}$$

$$c_n^{extra} \geq \lambda_n^{(l)} + \pi_n^{(l)}, \forall n, l \tag{45}$$

$$\mu_n^{(l)} \leq \lambda_n^{(l)}, \forall n, l \tag{46}$$

$$\mu_n^{(l)} \leq s_n c_n^{extra}, \forall n, l \tag{47}$$

$$\mu_n^{(l)} \geq \lambda_n^{(l)} - (1 - s_n) c_n^{extra}, \forall n, l \tag{48}$$

$$\delta_n^{(l)} \leq \pi_n^{(l)}, \forall n, l \tag{49}$$

$$\delta_n^{(l)} \leq s_n c_n^{base}, \forall n, l \tag{50}$$

$$\delta_n^{(l)} \geq \pi_n^{(l)} - (1 - s_n) c_n^{base}, \forall n, l \tag{51}$$

$$\lambda_n^{(l)}, \pi_n^{(l)} \geq 0 \tag{52}$$

$$\sum_n s_n \geq \Gamma \tag{53}$$

The algorithmic procedure is described as follows.

Algorithm 1. *(Nested Primal Cut Algorithm)*

Step 0

Set lower bound $LB = -\infty$, upper bound $UB = +\infty$, and $\{m\} = \emptyset$; Solve outer level problems (22)-(33). Let Y^1 denote the optimal first stage decision. Let $n = 1$ and go to step 1(a).

Step n. (a)

For given first stage decision Y^n, use primal cut algorithm to solve subproblem. Let $p^{G(n)}$ be the worst case uncertainty scenario and update $\{m\} = \{m\} \bigcup \{n\}$. Update $UB = min\{UB, \text{objective value of subproblem}\}$. Go to step n (b).

Step n. (b)

Create extra variables $w_n^{base(n)}, w_n^{extra(n)}, \theta_{hktn}^{(n)}, y_{hk}^{(n)}$ and add related constraints (23)-(32) with respect to $p^{G(n)}$ to outer level problem. Set $n = n+1$ and solve the outer level problem, let Y^n denote the optimal first stage decision and update LB be the objective value of outer level problems. Compute relative gap $\epsilon = (UB - LB)/UB$. If ϵ is less than or equal to a predefined tolerance value, then algorithm terminates; Otherwise go to step n (a).

IV. Preliminary Computational Results

We tested our model and algorithms on two sample experiments with $N = 12$ time periods, from 8AM to 8PM. The first experiment has four jobs where two jobs are scheduled day-ahead and the decisions of the other two jobs are made after uncertainty of local distributed generation is revealed, aptly named $(2+2)$ experiment. The second experiment has eight jobs with four jobs scheduled day-ahead and the others in real time fashion, similarly named $(4+4)$ experiment. The algorithm is implemented in C++ with CPLEX 12.2 as its MIP solver. All the computational experiments are performed on a PC desktop with Intel Core(TM) 2Duo 3.00GHz CPU and 3.25GB memory. The relative gap for terminating every problem (including subproblems in the nested primal cut algorithm) is set to be $1e-4$.

The conservative prediction of local generation, p_n^G, is randomly generated in $[10, 50]$KW, and the optimistic prediction is set to be 10% and 30% more, that is, $\tilde{p}_n^G = 10\% p_n^G$ and $\tilde{p}_n^G = 30\% p_n^G$ respectively. The consumption limit at the base price,

TABLE I
JOB CHARACTERISTICS

Job	1	2	3	4
Earliest Start Time	8AM	8AM	9AM	12PM
Due Time	7PM	7PM	6PM	8PM
Interruptible	Y	N	Y	N

TABLE II
WORK LOAD AND JOB LENGTH IN DIFFERENT MODES

Job. Mode	1.0	1.1	2.0	2.1	3.0	3.1	4.0	4.1
Total Load	181	184	80	84	260	254	314	299
Length	4	5	3	6	5	7	6	8
Work Load	53 40 42 46	35 30 35 42 42	25 26 29	11 14 15 16 15 13	54 51 52 52 51	41 35 42 41 46 23 26	50 52 50 55 55 52	38 36 37 39 37 37 40 35

P_n^{max}, is assumed to be 40 KW for 2+2 problem and 150 KW for 4+4 case. The base electricity price has been derived from [18] in a winter scenario. The price information corresponding to our time horizon is as follows: (i) 13.266(cent/KWh) for the *on-peak* periods (8AM-10AM), (ii) 7.5(cent/KWh) for the *mid-peak* periods (10AM-5PM), and (iii) 4.44(cent/KWh) for the *off-peak* periods (5PM-8PM). The non-base or extra electricity price is assumed to be three times of the base price, that is $c_n^{extra} = 3c_n^{base}$ for all n. Price information is assumed to be available day-ahead. The earliest starting time and due time of all jobs are randomly generated such that the feasibility of the problem holds. In each experiment, half of the jobs are assumed to be interruptible. Note that it is not necessary to keep this ratio in general. Job characteristics for the 2+2 sample problem are presented in Table I and Table II.

Four types of experiments have been conducted base on the number of jobs and prediction variation of $\tilde{p}_n^G$. Each experiment comprises five instances based on level of conservatism, Γ, from 1 to 5. The experimental results are presented in table III including the computation time and objective value in the worst cases. As seen in this table, total cost in the worst case decreases when Γ increases (more local generation is forecasted optimistically). Figure 1 depicts a typical convergence of the bounds over iterations, which indicates the optimal solution can be found in only a few iterations by using the nested primal cut algorithm. Note that although the problem size is small, the computation time is not negligible, which indicates the challenging level of 2-stage RO with discrete recourse problem. We mention that actual users may choose a larger relative gap to achieve an optimal trade off between the computation time and the quality of the solution.

TABLE III
THE EXPERIMENT RESULT

Experiment type	Γ	Time(s)	Obj Value in Worst Case
$2+2$, $\tilde{p}_n^G = 10\% p_n^G$	1 2 3 4 5	2.159 15.914 32.197 30.183 31.53	3094.14 3088.57 3060.82 3031.49 2995.49
$2+2$, $\tilde{p}_n^G = 30\% p_n^G$	1 2 3 4 5	2.546 29.774 58.014 71.039 85.64	3094.14 3035.49 2950.43 2862.68 2771.37
$4+4$, $\tilde{p}_n^G = 10\% p_n^G$	1 2 3 4 5	6.38 7.718 2.861 2.6 2.846	7957.96 7945.96 7928.2 7904.77 7878.23
$4+4$, $\tilde{p}_n^G = 30\% p_n^G$	1 2 3 4 5	6.708 7.645 15.48 15.63 23.86	7941.19 7911.89 7875.89 7822.61 7747.39

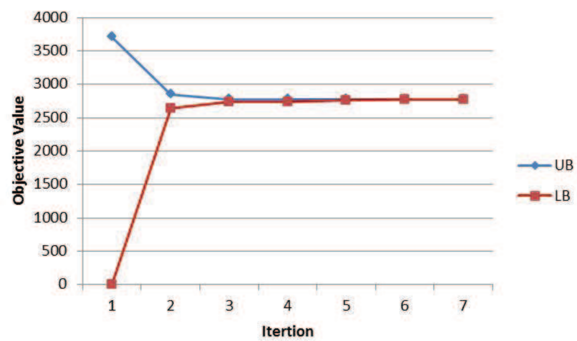

Fig. 1. Upper bound and bower bound vs. iteration

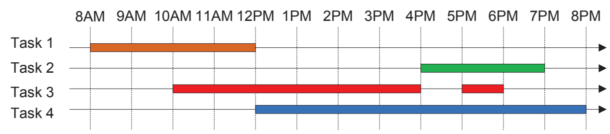

Fig. 2. Detailed schedule for case of 2+2, $\Gamma = 5$, and 30% unceraint‌y variation

The detailed scheduling for the case of $2 + 2$ with $\Gamma = 5$ and 30% variation can be seen in Figure 2, where jobs 1-2 are scheduled to be running in modes 0 and 0 respectively. The day-ahead optimal scheduling of jobs 1 and 2 considers all possible realizations of uncertainty local distributed generation. As seen in the Gantt chart, job 1 is scheduled during *on-peak* periods but with high local distributed generation, while job 2 is scheduled in *off-peak* periods. The schedule of jobs 3 and 4 shown in Figure 2 with modes 1 and 1 respectively is for the worst case realization. In fact, it is worth pointing out that the mode selection and scheduling of these two tasks can be adjusted according to actual uncertainty realizations.

Figure 3 demonstrates total load in each time period based on the optimal schedule and shows the contribution of local generation and purchased energy at base and extra price to meet the demand. From the consumption chart, it can be observed that in two time periods 9AM-10AM and 7PM-8PM, local generation is even more than the load from the scheduled jobs, which has made to-be-paid electricity bill to be zero. This observation highlights the role of local generation in reducing cost especially during *on-peak* hours. However, due to time restrictions of jobs, we may not be able to receive the benefit from the excess generation. Utilization of *Energy Storage Systems (ESS)* can prevent the loss of energy in these situations. We also noticed that more loads are scheduled around the peaks of local generation, i.e. from 10AM to 11Am and from 2PM to 3PM, to fully take advantage of the cost-free energy, which complies with our intuition.

V. Conclusion

In this paper, we present a two-stage robust optimization model to schedule jobs with a day-ahead price information and the random local generation. Such model is suitable for the situations where the probabilistic information for the random local generation is not available or not reliable. To the best of our knowledge, it is the first two-stage robust optimization scheduling model with random local generation. A preliminary computation study is performed to demonstrate the effectiveness of the

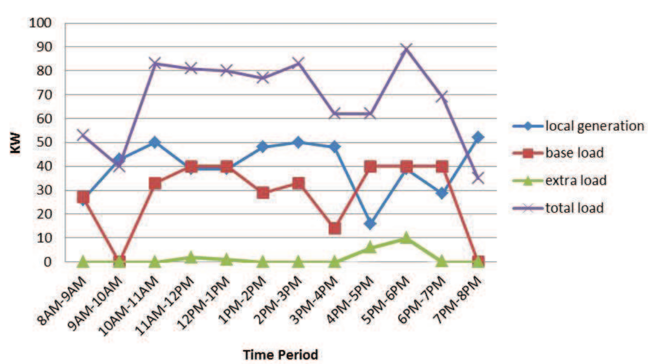

Fig. 3. Demand, local generation, base and non-base consumption in each time period

resulting schedules. One possible future research direction is to consider more general uncertainty sets ([14], [15]), which can provide a reliable description to capture the real randomness.

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