Informational validity of Fechtner's experiments outcomes

David Kisets
National Physical Laboratory, Israel

Abstract. All manifestations of dimensional harmony in nature and human practice are being always characterized by deviations from golden ratio that often makes their acceptance problematic. On the example of Fechner's experiments the paper discusses the way of solving this problem, based on informational approach, according to which the informatively optimal permissible deviation from dimensional harmony has been applied. The using of information optimization reveals that in experiment's data along with the phenomenon of the conformity of aesthetically preferred rectangle's sides' ratio with golden ratio the statistical majority attributed to the three most preferable rectangles should be regarded as the manifestation of the harmony in full accord with the permissible deviation from the golden ratio. Eventually the confidence to the harmony manifestation in regard to Fechner's experiments is being significantly increased. The paper also focuses on the significance of the used investigation method and its influence onto research results.

Keywords: Informational optimality; Mathematical harmony; Fechner's rectangles.

1. Introduction

The human perception of dimensional harmony is associated with the fundamental mathematical constant f_o = 0.618 which is often called golden ratio. There is a great number of examples of the embodiment of golden ratio in nature and various spheres of human activity [1]. At the same time, any of practical manifestations of dimensional harmony somewhat differs from this constant, that is why we are always faced with the confidence problem to such phenomena. Apparently the problem is also important in science, in human's art, technological and other activity for achieving sufficiently harmonious decisions.

The problem may be well exemplified and considered applying to the phenomenon of harmony in the classical psychological experiments that were conducted by Gustav Theodor Fechner in the 1860s in studying the feeling of beauty and harmony for adult people. It is known the experiments were later repeated by other scientists and consisted in the following: the 10 white rectangles with ratios of sides from 1:1 to 2:5 were presented to all the participants of “Fechner’s experiments” to evaluate their aesthetic feelings. The experiments embraced hundreds participants, and statistical results corresponding to graphical data, borrowed from [2], are presented in Table 1.

Table 1: Data of Fechner's experiment

Rectangle number	1	2	3	4	5	6	7	8	9	10
Ratio of sides (decimal fraction)	1:1 (1.0)	5:6 (0.833)	4:5 (0.8)	3:4 (0.75)	20:29 (0.690)	2:3 (0.667)	21:34 (0.618)	13:23 (0.565)	1:2 (0.5)	2:5 (0.4)
Approxim. weight (%)	3.5	0.5	2.5	3.0	7.5	20	35	19	7.5	1.5

Among compared rectangles the number 7 with the ratio 21:34, with rather high accuracy representing golden ratio, is the highly favorable and the most preferable from aesthetical point of view estimated on the 35% level of statistical weight. Despite the coincidence of statistical estimation for the number 7 rectangle with golden ratio, the comparatively high estimation rates of adjacent rectangles 6 and 8 may at first glance lower a confidence to the phenomenon which Fechner championed [3]. For instance, stressing the moderately good evidence for the phenomenon, at the same time it is acknowledged the obtained result as a whole regarding the above mentioned three most preferable rectangles is very unclear [4].

Evidently such phenomena need some further analysis to find out in what extent the experiments data meet the requirements of dimensional harmony. For this purpose we will discuss the applicability of simple non-trivial method of the analysis based on informational approach in terms of Shannon's information theory [5], rather than statistically, and demonstrate the problem of clarifying the results of Fechner’s experiments can be successfully resolved with using this method. Clearly the paper is aiming to a further widest coverage in studying the harmony in nature, studying and providing the harmony in human's practice and researches; thus the Fechner's experiments ought to be considered merely as a proper example in considering the pressing problem in toto.

2. Estimation criteria

The informational approach, in this instance, mainly rests on the assumption of permissible deviations from the numerical constants related to informational optimality (ρ_o), harmony (f_o), and balance ($\lambda = 0.5$), and that all the deviations are being determined basing upon the principle of informational optimality [6, 7].

The peculiarity of informational optimality expressed as the approximate fraction $\rho_o = 1/2\pi = 0.159$ - the optimum accuracy coefficient or (in our case) the optimum classification coefficient, as distinct from two other information constants, i.e. f_o and λ is in its universality regarding the determination of informatively permissible deviation from any of information constants in real usages of the conceptions concerned. In accordance with this approach the permissible range (R) of each informational constant is within ($1 \pm 0.5\rho_o$) of the constant. In so doing, irrespective of a system whose components represent or have been simulated as dimensional parts, the harmony exists within the following range:

$$R(f_o) = f_o\,(1 \pm 0.5\rho_o) = f_o\,(1 \pm 1/4\pi) \approx 0.618 \pm 0.049 \qquad (1)$$

The details of numerical proof of this range are dealt with in the Appendix.

Expression (1) represents the main quantitative estimation criterion in studies aimed at ascertaining whether the harmonious relation may be attributed to various phenomena in the nature and human practice.

In cases of statistical studies (like Fechner’s experiments) along with expression (1) one may apply the additional estimation criterion, involving statistical data of experiment as normalized relative statistical weights (K) as follows:

$$(1/K_m n)(1 - \textstyle\sum_{(R)} K_j\,) \leq \rho_o = 1/2\pi, \qquad (2)$$

where $\Sigma_{(R)}$ K_j and K_m - the sum of weights (as parts of 1) related to statistically estimated dimensional parts within the range *R*, and the weight related to the most preferable component respectively, being calculated for *n* components undergoing consideration. The ratio $1/K_m n$ represents the so-called form factor of weights diagram [8] of the components.

The criterion analogous to (2) was originally suggested and substantiated [8] for analyzing diagrams of weights. In the present consideration it is intended to establish whether the usage of statistical data in determining the experimental deviation from golden ratio possesses informational reliability.

3. **Results of analyzing**

The usage of criterion (1) amounts to the simple comparison of components with $R(f_o)$ margins. Graphical illustration of such comparison regarding Fechner's rectangles by the ratios of their sides in the scale of decimal fractions is presented on Fig. 1, where one can also find the range $R(f_o)$ of harmonious relation.

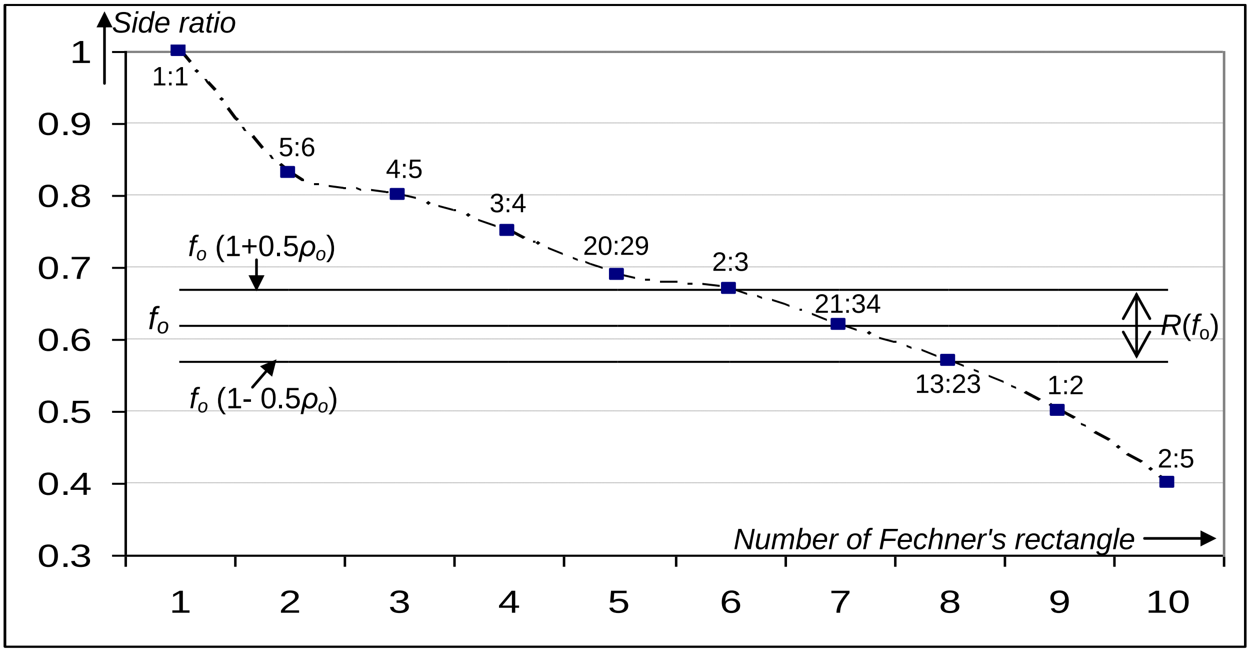

Fig.1: Fechner's rectangles and the range of harmonious relation

The figure demonstrates the distribution of three statistically preferable Fechner's rectangles with side ratios 21:34, 2:3 and 13:23 (which are characterized by sum of weights = 74%) are within the permissible range of harmonious relation and, what is most importantly, the practically precise location of adjacent to 21:34 rectangles 2:3 and 13:23 on the margins of $R(f_o)$. On the other hand, such remarkable coincidence may virtually be valued as an experimental substantiation of theoretical $R(f_o)$ as well.

Applying to the criterion (2) and taking into account that $K_m = 0.35$, $n = 10$, $\Sigma_{(R)}$ $K_j = 0.74$, the calculation results in the following: $(1/K_m n)(1- \Sigma_{(R)} K_j) = 0.074$ that is significantly less than $1/2\pi$ and thus meets the requirement of the criterion.

4. **Method influence**

It should be noted that the observed good coincidence of practically obtained and theoretical permissible ranges cannot be expected often. It eventually depends on the evolutionary stage of the object being studied (assuming that a perfection, including the harmony, is one of aims and results of the evolution in nature), of the statistical reliability of experimental data used (in case of experimental investigations) and, very importantly, depends also on methods of investigation and analysis.

In Fechner's experiments (and in similar others) the influence of used method onto research results is being traced also that, in particular, involves also a shape of the object being chosen for the investigation.

Studying analogous aesthetic preferences for ellipses, Fechner found that of nine ellipses with minor-to-major axis ratios ranging from 1:1 to 1:2.5, 42% of his subjects preferred the 1:1.5 ellipse, while 16.7% preferred the adjacent "golden", and that all other ellipses were estimated by even smaller percentages [9]. Clearly, the most preferable ellipse yet satisfies the condition (1) although is located on the boundary of permissible range. Hypothetically such displacement from "ideal" harmony, intrinsic to "golden" rectangle, can be explained by the visual illusion of elongation of an ellipse with the same axis ratio as the sides' ratio of rectangle and, seemingly, has relevance to methodical inaccuracy in choosing the dimensional ratio for estimating.

Non-adequate method may completely distort experiment's results; this, in our opinion, took place in the attempt to extend the Fechner's investigations onto triangles. When doing this, it has been found that the most preferred isosceles triangle is characterized by height-to-base ratio of 0.41:1, considerably less than the golden ratio [9, 10]. At the first glance this may be the reason for general criticism that, however, is wrong in principle. In this experiment if changing the estimation by taking the side-to-base ratios as the method of judging the degree of harmony of aesthetic feelings, the simple calculation results in the fact that a side-to-base ratio of 0.647:1 (that for an isosceles triangle corresponds to a height-to-base ratio of 0.41:1) totally meets the requirements of criterion (1). Thus, the expected phenomenon takes place in the experiment with triangles also; and as a whole, the proper judgment over the results of experiments definitely depends on the estimation method used.

5. **Final discussion**

Summarizing concerning the used approach, over the principle of informational optimality the signs of informational perfection (the harmony, the optimality and the balance) in nature and human practice, as distinct from mathematical abstractions, manifest themselves within permissible ranges close to respective mathematical constants, i.e. $\pm 1/4\pi$ (or about ±7.9%) of each constant.

Being based on this approach, the analysis of the Fechner's experiment data, demonstrates that along with the widely known phenomenon of the conformity of aesthetically preferred rectangle's sides ratio with golden ratio there is the remarkable evidence about the permissible information range of mathematical harmony that can be attributed to the phenomenon too. Apparently the usage of the range is applicable as a simple instrument for more deep analysis and identification of similar phenomena detected on the basis of statistical and non-statistical estimations. The analysis results not only in the complete proof of belonging the Fechner's experiment to golden ratio, but also demonstrates (in frame of the experiment) the remarkable practical coincidence with theoretically defined permissible range of harmony.

Correct results of the discussed and similar studies significantly depend on research methods used that has been convincingly illustrated on the examples with different shapes of investigated objects (of ellipses and triangles).

As a whole, the outcomes of treating Fechner's experiments data, discussed in the paper, allow expecting and hoping for the wide usage of the proposed approach in analogous studies (including phenomena related to informational optimality and balance as conceptions of mathematical perfection). Apparently the usage of the range is applicable as a simple instrument for more deep analysis and (or) identification of similar phenomena detected on the basis of statistical and non-statistical estimations.

Appendix: The proof of permissible range for the mathematical harmony

Dimensionally the estimation of mathematical harmony in the range of estimation error (δ) is presented in Fig.2.

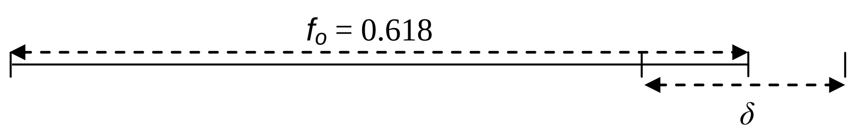

Fig.2: Mathematical harmony in the range of estimation error

In terms of information theory regarding dimensional ratios we can demonstrate that the permissible range of estimation error (δ_o) corresponds to the ratio $\delta_o / f_o = 1/2\pi$, which represents the so-called optimum classification coefficient. For this purpose we shall proceed to normalized classification weights (W) in return for probabilities, ordinarily used in information theory, and consider the following equations system:

$$\begin{cases} W_1 = (f_o - 0.5\delta) / (f_o + 0.5\delta); & (3) \\ W_2 = \delta / (f_o + 0.5\delta); & (4) \\ \varphi_o = exp\,(-W_1 \ln W_1 - W_2 \ln W_2) = 1.5, & (5) \end{cases}$$

where φ_o = the informatively optimal (necessary and sufficient) number of components that (in case of equal 1.5) is true for the most uncertain classification situation (50% confidence) about allowing or ignoring the lesser one of two components.

The solution of these equations system regarding the sought permissible range of estimation error $\delta_o = arg\,[\varphi_o(\delta) = 1.5]$ results in $\delta_o = 0.098$, and correspondingly with high estimation accuracy the equality $\delta_o / f_o = 1/2\pi$ is true.

Another approach in identifying the permissible range $R(f_o)$ is based on analyzing Fibonacci numbers (F_n = 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …). Along with the close connection of the golden ratio with the Fibonacci series, the Fibonacci numbers have an interesting property: in the sequence of ratios F_n / F_{n+1} there is a single one that absolutely exactly expresses the mathematical balance, i.e. $\lambda = 0.5$, and the ratio within the range $0 < F_n / F_{n+1} < \lambda$ does not exist. Noting the principal difference between dimensional conceptions of balance and harmony, this can be used for the condition of determining the maximum deviation from f_o on the base of Fibonacci numbers as follows:

$$\max \left| (F_n / F_{n+1}) - f_o \right| < (f_o - \lambda) \qquad (6)$$

The only one solution satisfies to this condition, namely: $max \left| (F_n / F_{n+1}) - f_o \right| = (2/3 - 0.618) = 0.049$. Clearly this result completely coincides with that obtained for the range $R(f_o)$ by means of informational approach.

By the way, the existence of λ in the Fibonacci series completely conforms with the so-called "Law of phyllotaxis" [11] that once again stresses the significance of both the balance and harmony as conceptions of informational perfection in nature.

References

[1] Stakhov A.P., Sluchenkova A. A. (2001). Web site “Museum of Harmony and Golden Section”, http://www.goldenmuseum.com/

[2] Dreyfus Tommy, Eisenberg Theodor. (2007). On Symmetry in School Mathematics, http://www.mi.sanu.ac.yu/vismath/drei/index.html

[3] Livio Mario. (2002). The golden ratio and aesthetics, http://plus.maths.org/issue22/features/golden/2pdf/index.html/op.pdf

[4] McManus I.C. (1980). The aesthetics of simple figures. *British Journal of Psychology, (1980), 71,* 505-524.].

[5] Shannon C. E. (1948). A mathematical theory of communication, *Bell Syst. Tech.J. 27*, 1947, 379–423.

[6] Kisets D. (2006). Unique features of the first perfect number, *Academic Open Internet Journal*, www.acadjournal.com, 2006, *Volume 17*.

[7] JD 10.0.3-1: Informatively optimal combining, expanding, and establishing traceability in evaluating measurement uncertainties. Report on scientific work. Researched and prepared by Dr. David Kisets. INPL, 2011, Jerusalem.

[8] Kisets D. (2002). Unified model of weights for the selection of informative factors, *Academic Open Internet Journal* www.acadjournal.com, 2002, *Volume 7*.

[9] Christopher D. Green. (2005). All That Glitters: A Review of Psychological Research on the Aesthetics of the Golden Section,http://htpprints.yorku.ca/archive/00000003/00/goldrev3.htm

[10] Witmer, L. (1894). Zur experimentellen Aesthetik einfacher räumlicher Formverhältnisse [On the experimental aesthetics of simple spatial relations]. *Philosophische Studien*, *9*, 96-144; 209-263.

[11] Agassiz, Louis. (1962). Essay on Classification, Ed. E. Lurie, *Belknap Press, Cambridge*, 1962:131.