

# MILP formulation for islanding of power networks

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## Abstract

In this paper, a mathematical formulation for the islanding of power networks is presented. Given an area of uncertainty in the network, the proposed approach uses mixed integer linear programming to isolate uncertain components and create islands, by intentionally (i) cutting lines, (ii) shedding loads and (iii) switching generators, while maximizing load supply. A key feature of the new method is that network constraints are explicitly included in the MILP problem, resulting in balanced, steady-state feasible DC solutions. A subsequent AC optimal load shedding optimization on the islanded network model provides a feasible AC solution. Numerical simulations on the 24-bus IEEE reliability test system and larger systems demonstrate the effectiveness of the method.

## 1 Introduction

In recent years, there has been an increase in the occurrence of wide-area blackouts of power networks. In 2003, separate blackouts in Italy [1], Sweden/Denmark [2] and USA/Canada [3] affected millions of customers. The wide-area disturbance in 2006 to the European system caused the system to split in an uncontrollable way [4], forming three islands. More recently, the UK network experienced a system-wide disturbance caused by an unexpected loss of generation; blackout was avoided by local load shedding [5].

While the exact causes of wide-area blackouts differ from case to case, some common driving factors emerge. Modern power systems are being operated closer to limits: liberalization of the markets, and the subsequent increased commercial pressures and change in expenditure priorities, has led to a reduction in security margins [6–8]. A more recently occurring factor is increased penetration of variable distributed generation, notably from wind power, which brings significant challenges to secure system operation [9].

For several large disturbance events, *e.g.*, [3], studies have shown that wide-area blackout could have been prevented by intentionally splitting the system into islands [10]. By isolating the faulty part of the network, the total load

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disconnected in the event of a cascading failure is reduced. *Controlled* islanding or system splitting is therefore attracting an increasing amount of attention. The problem is how to efficiently split the network into islands that are balanced in load and generation, have stable steady-state operating points within voltage and line limits, and so that the action of splitting does not cause transient instability. This is a considerable challenge, since the search space of line cutsets grows combinatorially with network size, and is exacerbated by the requirement for strategies that obey non-linear power flow equations and satisfy operating constraints.

Approaches in the literature broadly differ according to the motive of islanding, and within that the search method employed to determine the splitting boundary. The simplest example of the former is forming islands in which load and generation are balanced. In [11], a three-phase ordered binary decision diagram (OBDD) method is proposed that determines a set of islanding strategies. The approach uses a reduced graph-theoretical model of the network to minimize the search space for islanding; power flow analyses are subsequently executed on islands to exclude strategies that violate operating constraints, *e.g.*, line limits.

An alternative motive seeks to split the network into electromechanically stable islands, commonly by splitting so that generators with coherent oscillatory modes are grouped. If the system can be split along boundaries of coherent generator groups while not causing excessive imbalance between load and generation, then the system is less likely to lose stability. Determining the optimal cutset of lines involves considerations of load-generation balance and other constraints; algorithms include exhaustive search [12], minimal-flow minimal-cutset determination using breadth-/depth-first search [13], and graph simplification and partitioning [14]. The authors of [15] propose a framework that, iteratively, identifies the controlling group of machines and the contingencies that most severely impact system stability, and uses a heuristic method to search for a splitting strategy that maintains a desired stability margin. Wang et al. [16] employed a power flow tracing algorithm to first determine the domain of each generator, *i.e.*, the set of load buses that ‘belong’ to each generator. Subsequently, the network is coarsely split along domain intersections before refinement of boundaries to minimize imbalances.

While several useful strategies exist for splitting a network into synchronous balanced islands, little attention has focused on islanding in response to particular contingencies. If, for example, a line failure occurs and subsequent cascading failures are likely, it may be desirable to isolate a small part of the network—the impacted area—from the rest. A method that does not take the impacted area into account when designing islands may leave this area within an arbitrary large section of the network, all of which may become insecure as a result.

In this paper, an optimization-based approach to system islanding and load shedding is proposed. Given some uncertain set of buses and/or lines, solving an optimization determines (i) the optimal set of lines to cut, (ii) how to adjust the outputs of generators, and (iii) which loads to shed. It is assumed that this is done intentionally under central control and not left to automatic safety devices. The solution isolates the suspected parts from the rest of the network while minimizing the load shed. A key feature of the method is that any islands created satisfy power flow equations and operating constraints. Therefore, if a transiently stable path is followed from a pre-islanding state to the post-islanding operating point, the islanded network will be balanced and with minimal disrupt-

tion to load. While network dynamics or transient stability constraints are not modelled in the optimization, the transient response to the actions of islanding is modelled by time-domain simulation. It is shown that when suitable penalties are imposed on the cutting of high-flow lines and disconnection of generating units—so that the disruption to the network is small—the immediate actions of system splitting do not lead to transient instability in any of the test examples.

The proposed approach uses two stages: solving a mixed-integer linear programming (MILP) islanding problem, which includes the linear DC flow equations, determines a DC-feasible solution, and an AC optimal load shedding optimization subsequently provides an AC-feasible steady-state post-islanding operating point. Integer programming has many applications in power systems, but its use in network splitting and blackout prevention is limited. Bienstock and Mattia [17] proposed an IP-based approach to the problem of designing networks that are robust to sets of cascading failures and thus avoid blackouts; whether to upgrade a line’s capacity is a binary decision. Fisher et al. [18] propose a method for optimal transmission switching for the problem of minimizing the cost of generation dispatch by selecting a network topology to suit a particular load. In common with the formulation presented here, binary variables represent switches that open or close each line and the DC power flow model is used, resulting in a MILP problem. However, in this paper sectioning constraints are present, and the problem is to design balanced islands while minimizing load shed.

The organization of the paper is as follows. The next section outlines the motivation and assumptions that underpin the approach. The islanding formulation is developed in Section 3. The AC optimal load shedding problem is described in Section 4. In Section 5, computational results are presented. Finally, conclusions are drawn in Section 6.

## 2 Motivation and assumptions

The motivation for the formulation is stated as follows. Following some failure, it is assumed that limited information is available about the network and its exact state is uncertain; there are parts of the network that are suspected of having a fault, or are close to failure, and some assumed to have no faults. It is assumed that, in such a case, a robust solution to prevent cascading failures is to isolate the uncertain part of the network from the certain part, by forming one or more stable islands. Fig. 1(a) depicts such a situation; uncertain lines and buses are indicated by a “?”.

The aim is to split the network into disconnected sections so that the possible faults are all in one section. It is desirable that this section be small, since it may be prone to failure, and that the other section is able to operate with little load shedding. It is also desirable that the problem section has as little load shed as possible. Fig. 1(b) shows a possible islanding solution for this network, where all uncertain buses have been placed in a section 0 and all uncertain lines with at least one end in section 1 are disconnected. The following distinction is made between *sections* and *islands*.

- The optimized network consists of two sections, an “unhealthy” section 0 and a “healthy” section 1. No lines connect the two sections. On the other hand, neither section is required to be a single, connected component.

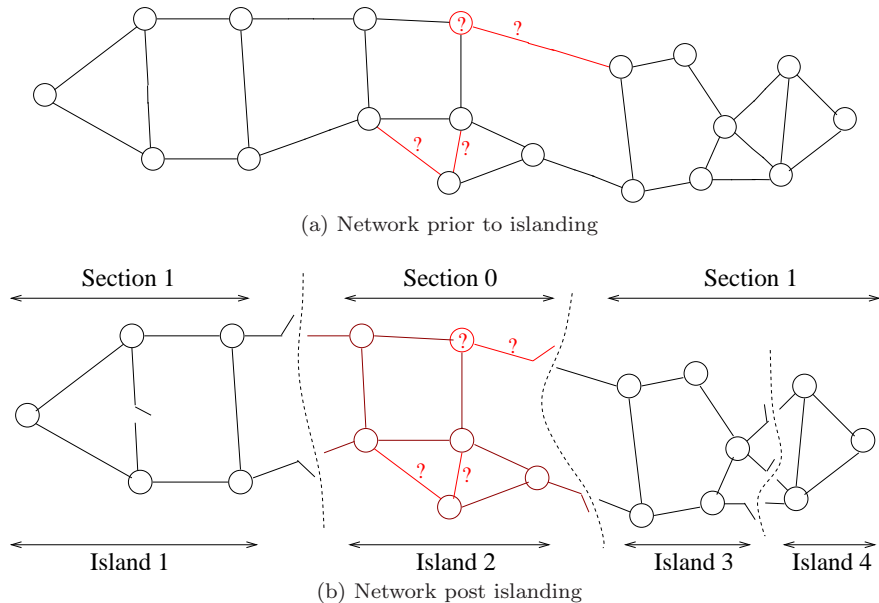


Figure 1: (a) Illustration of a network with uncertain buses and lines, and (b) the islanding of that network by disconnecting lines.

- An island is a connected component of the network.

Thus, either section may contain a number of islands, as in Fig. 1(b), where section 1 comprises islands 1, 3 and 4, while section 0 is a single island. The boundaries of sections and the number of islands formed will depend on the optimization.

It is assumed that generator outputs and load levels immediately after an initial fault are known. Centralized control of generation, load shedding and line breakers is assumed; demand may be reduced and lines may be disconnected instantaneously. Furthermore, a limited degree of control over a generator's output is assumed. It is required that, after these adjustments, the power flow equations and operating constraints are satisfied in each island of the network.

### 3 MILP islanding formulation

In this section, a MILP formulation is presented for islanding and minimizing the load shed in a network under stress.

Consider a network that comprises a set of buses  $\mathcal{B} = \{1, 2, \dots, n^{\mathcal{B}}\}$  and a set of lines  $\mathcal{L} = \{1, 2, \dots, n^{\mathcal{L}}\}$ . The two vectors  $F$  and  $T$  describe the connection topology of the network: a line  $l \in \mathcal{L}$  connects bus  $F_l$  to bus  $T_l$ . There exists a set of generators  $\mathcal{G} = \{1, 2, \dots, n^{\mathcal{G}}\}$  and a set of loads  $\mathcal{D} = \{1, 2, \dots, n^{\mathcal{D}}\}$ . A subset  $\mathcal{G}_b$  of generators is attached to bus  $b \in \mathcal{B}$ ; similarly,  $\mathcal{D}_b$  contains the subset of loads present at bus  $b \in \mathcal{B}$ .

### 3.1 Sectioning constraints

Motivated by the previous section, the intention is to allocate buses and lines into the two sections 0 and 1. It is suspected that some subset  $\mathcal{B}^0 \subseteq \mathcal{B}$  of buses and some subset  $\mathcal{L}^0 \subseteq \mathcal{L}$  of lines have a possible fault. These subsets thus contain all “uncertain” buses and lines, while the remainder of buses/lines are defined as “certain”. It is the uncertain components that are to be confined to section 0.

A binary decision variable  $\gamma_b$  is defined for each bus  $b \in \mathcal{B}$ ;  $\gamma_b$  shall be set equal to 0 if  $b$  is placed in section 0 and  $\gamma_b = 1$  otherwise. To partition the network in such a way, lines are to be disconnected. Accordingly, a binary decision variable  $\rho_l$  is defined for each  $l \in \mathcal{L}$ ;  $\rho_l = 0$  if line  $l$  is disconnected and  $\rho_l = 1$  otherwise.

Constraints (1a) and (1b) apply to each line  $l$  not assigned to  $\mathcal{L}^0$ . The line is cut if its two end buses are in different sections (*i.e.*  $\gamma_{F_l} = 0$  and  $\gamma_{T_l} = 1$ , or  $\gamma_{F_l} = 1$  and  $\gamma_{T_l} = 0$ ). Otherwise, if the two end buses are in the same section then  $\rho_l \leq 1$ , and the line may or may not be disconnected. Thus, these constraints enforce the requirement that any certain line between sections 0 and 1 shall be disconnected.

$$\rho_l \leq 1 + \gamma_{F_l} - \gamma_{T_l}, \forall l \in \mathcal{L} \setminus \mathcal{L}^0, \quad (1a)$$

$$\rho_l \leq 1 - \gamma_{F_l} + \gamma_{T_l}, \forall l \in \mathcal{L} \setminus \mathcal{L}^0. \quad (1b)$$

Constraints (1c) and (1d) apply to lines assigned to  $\mathcal{L}^0$ . A line  $l \in \mathcal{L}^0$  is disconnected if at least one of the ends is in section 1. Thus, an uncertain line either (i) shall be disconnected if entirely in section 1, (ii) shall be disconnected if between sections 0 and 1, or (iii) may remain connected if entirely in section 0.

$$\rho_l \leq 1 - \gamma_{F_l}, \forall l \in \mathcal{L}^0, \quad (1c)$$

$$\rho_l \leq 1 - \gamma_{T_l}, \forall l \in \mathcal{L}^0, \quad (1d)$$

Constraints (1e) and (1f) set the value of  $\gamma_b$  for a bus  $b$  depending on what section that bus was assigned to.  $\mathcal{B}^1$  is defined as the set of buses that are desired to remain in section 1. It may be desirable to exclude buses from the “unhealthy” section, and such an assignment will in general reduce computation time.

$$\gamma_b = 0, \forall b \in \mathcal{B}^0, \quad (1e)$$

$$\gamma_b = 1, \forall b \in \mathcal{B}^1. \quad (1f)$$

Given some assignments to  $\mathcal{B}^0, \mathcal{B}^1$  and  $\mathcal{L}^0$ , the optimization will disconnect lines and place buses in sections 0 or 1, hence partitioning the network into sections 0 and 1. What else is placed in section 0, what other lines are cut, and which loads and generators are adjusted, are degrees of freedom for the optimization, and will depend on the objective function.

### 3.2 DC power flow model

The power flow model employed is a variant of the “DC” model, assuming unit voltage at each bus and small phase angle differences, but accounting for line

losses. Kirchhoff's current law is applied at each bus  $b \in \mathcal{B}$ :

$$\sum_{g \in \mathcal{G}_b} p_g^G = \sum_{d \in \mathcal{D}_b} p_d^D + \sum_{l \in \mathcal{L}: F_l=b} p_l^L - \sum_{l \in \mathcal{L}: T_l=b} (p_l^L - \bar{h}_l^L), \quad (2)$$

where  $p_g^G$  is the real power output of generator  $g \in \mathcal{G}_b$  at bus  $b$ ,  $p_d^D$  is the real power demand from load  $d \in \mathcal{D}_b$ . The variable  $p_l^L$  is the real power flow into the first end (bus  $F_l$ ) of line  $l$ , and  $p_l^L - \bar{h}_l^L$  is the flow into the second end, reduced by the loss  $\bar{h}_l^L$ . Loss modelling is described later in this section.

When a line  $l$  is connected, Kirchhoff's voltage law (KVL) demands that a flow of real power is established depending only on the difference in phase angle across the line. However,  $p_l^L$  may not be equated directly to this flow, since if a line is disconnected by the optimization, zero power will flow through that line. In this case, different phase angles must be allowed at each end of the line. To achieve this, the KVL expression is equated to a variable  $\hat{p}_l^L$ .

$$\hat{p}_l^L = -B_l^L (\delta_{F_l} - \delta_{T_l}), \quad (3)$$

where  $B_l^L$  is the susceptance of line  $l$ . Then, when line  $l$  is connected, it is required that  $p_l^L = \hat{p}_l^L$ , but when  $l$  is disconnected  $p_l^L = 0$ . This is modelled as follows.

Assume the maximum possible magnitude of real power flow through a line  $l$  is  $P_l^{L \max}$ . Then

$$-\rho_l P_l^{L \max} \leq p_l^L \leq P_l^{L \max} \rho_l, \quad (4a)$$

$$-(1 - \rho_l) \hat{P}_l^{L \max} \leq \hat{p}_l^L - p_l^L \leq \hat{P}_l^{L \max} (1 - \rho_l). \quad (4b)$$

When the sectioning constraints set a particular  $\rho_l = 0$ , then  $p_l^L = 0$  but  $\hat{p}_l^L$  may take whatever value necessary to satisfy the KVL constraint (3). Conversely, if  $\rho_l = 1$  then  $p_l^L = \hat{p}_l^L$ .

Line limits  $P_l^{L \max}$  may be expressed either directly as MW ratings on real power for each line, or as a limit on the phase angle difference across a line. Since in the model the real power through a line is just a simple scaling of the phase difference across it, then any phase angle limit may be expressed as a corresponding MW limit. Note that at the very minimum  $\hat{P}_l^{L \max} \geq P_l^{L \max}$ , but these limits should be large enough to allow two buses across a disconnected line to maintain sufficiently different phase angles.

### 3.3 Loss modelling

While the DC power flow model allows the islanding problem to remain linear, one disadvantage is that real power losses in the network are assumed to be zero. The lossless DC model will under-estimate the amount of load that needs to be shed when forming islands, and thus could lead to poor islanding decisions. The actual loss function,  $h_l^L$ , is derived from the AC real power flows, and is nonlinear. While this may be approximated by a piecewise linear function, investigations have shown that this offers little or no performance benefit over a simple constant-loss approximation, but adversely affects computation [19]. In this paper, then, a constant loss model is employed. The loss for line  $l$  is given by

$$\bar{h}_l^L = \rho_l h_l^{L*}, \quad (5)$$

where  $h_l^{L*} = h_l^L(p_l^{L*}, v_{F_l}^*, v_{T_l}^*)$  is the loss as determined from the current operating point of the network, (*e.g.*, the solution of an AC optimal power flow problem), in which line  $l$  has a flow  $p_l^{L*}$ , voltages  $v_{F_l}^*$  and  $v_{T_l}^*$ , and a corresponding loss. The inclusion of  $\rho_l$  drives the loss to zero if the islanding optimization cuts the line.

### 3.4 Generation constraints

In situations where there is a need to react quickly to an unplanned contingency, to prevent cascading failures the time available to island the network and adjust loads and generators will be short. Therefore, it is assumed that full re-scheduling of generators and/or the addition of new units to the network will not be possible. On the other hand, a certain amount of spinning reserve will be available in the network for small-scale changes. For any unit, it is assumed that a new setpoint, close to the current operating point, may be commanded. This setpoint should be reachable within a short time period, and also must not violate limits.

A further assumption made is that a generator obeys a binary regime: either it operates near its previous real power output, or it may have its output switched to zero. That is,

$$p_g^G \in [P_g^{G-}, P_g^{G+}] \cup \{0\}.$$

This latter case models the removal of the source of mechanical input power; it is assumed that electrical power will fall to zero within the timeframe of islanding. Although the switched-off generating unit contributes no power in steady state to the network, it remains electrically connected to the network.

To model this disjoint set constraint, define a binary variable  $\zeta_d \in \{0, 1\}$  for each generator.

$$\zeta_g P_g^{G-} \leq p_g^G \leq \zeta_g P_g^{G+}, \quad (6)$$

for all  $g \in \mathcal{G}$ . If  $\zeta_g = 0$  then generator  $g$  is switched off; otherwise it outputs  $p_g^G \in [P_g^{G-}, P_g^{G+}]$ . These limits depend on the ramp and output limits of the generator, and the amount of reserve available to the unit.

### 3.5 Load shedding

Following separation of the network into islands, and given the limits on generator power outputs, it follows that it may not be possible to fully supply all loads. However, the optimization is to determine a feasible steady-state for the islanded network, and thus it is necessary to permit some shedding of loads. Note that this is *intentional* load shedding, not automatic shedding as a result of low voltages or frequency, and as such will require appropriate control hardware in the network.

Suppose that a load  $d \in \mathcal{D}$  has a constant real power demand  $P_d^D$ . It is assumed that this load may be reduced by disconnecting a proportion  $1 - \alpha_d$ . For all  $d \in \mathcal{D}$ :

$$p_d^D = \alpha_d P_d^D, \quad (7)$$

where  $0 \leq \alpha_d \leq 1$ . In determining a feasible islanded network, it is desirable to promote full load supply, and so load shedding is minimized in the objective function.

### 3.6 Objective function

The overall objective of islanding is to minimize the risk of system failure. The motivation for islanding assumed that there is some uncertainty associated with a particular subset of buses and/or lines; a fault may occur and so these components are to be isolated from the rest of the network.

Suppose a reward  $M_d$  is associated with a the per unit supply of load  $d$ . In islanding the uncertain components, the objective is to maximize the total value of supplied load. However, in placing *any* load in section 0, a risk of not being able to supply power to that load is assumed, since that section contains “unhealthy” components and may fail. Accordingly, a load loss penalty  $0 \leq \beta_d < 1$  is defined, which may be interpreted as the probability of being able to supply a load  $d$  if placed in section 0. If  $d$  is placed in section 1, a reward  $M_d$  is realized per unit supply, but if  $d$  is placed in section 0, with the uncertain components, a reward of  $\beta_d M_d < M_d$  is realized. The objective is to maximize the expected load supplied,  $J^*$ :

$$J_{\text{DC}}^* = \max \sum_{d \in \mathcal{D}} M_d P_d (\beta_d \alpha_{0d} + \alpha_{1d}), \quad (8)$$

where,

$$\alpha_d = \alpha_{0d} + \alpha_{1d}, \forall d \in \mathcal{D}, \quad (9a)$$

$$0 \leq \alpha_{0d} \leq 1, \forall d \in \mathcal{D}, \quad (9b)$$

$$0 \leq \alpha_{1d} \leq \gamma_b, \forall b \in \mathcal{B}, d \in \mathcal{D}_b. \quad (9c)$$

Here a new variable  $\alpha_{sd}$  is introduced for the load  $d$  delivered in section  $s \in \{0, 1\}$ . If  $\gamma_b = 0$ , (and so the load at bus  $b$  is in section 0), then  $\alpha_{1d} = 0, \alpha_{0d} = \alpha_d$  and the reward is  $\beta_d M_d P_d \alpha_d$ . On the other hand, if  $\gamma_b = 1$  then  $\alpha_{1d} = \alpha_d$  and  $\alpha_{0d} = 0$ , giving a higher reward  $M_d P_d \alpha_d$ . Thus the objective has a preference for  $\gamma_b = 1$  and a smaller section 0.

### 3.7 Overall formulation

The overall formulation for islanding is to maximize (8) subject to (1)–(9). The resulting problem is a MILP.

**Remark 1 (Penalizing line cuts and generator switching)** *While the sectioning constraints force certain lines to be cut, it may also be desirable to penalize the unnecessary disconnection of other, healthy lines in the network. To do has the computational advantage of encouraging binary variables  $\rho_l$  to take on integer values in the LP relaxations of the problem. This may be achieved by including a small additive reward in the objective for non-zero values of  $\rho_l$ :*

$$\epsilon_1 \sum_{l \in \mathcal{L} \setminus \mathcal{L}^0} W_l \rho_l \quad (10)$$

where  $W_l$  is some weight. A uniform weight, e.g.,  $W_l = 1, \forall l$  will equally discourage line cuts, while cuts to high-flow lines may be more heavily discouraged with  $W_l = s_l^{L*}$ , where  $s_l^{L*}$  is the pre-islanding apparent power flow through the line.



For similar reasons, it may be desirable to penalize the switching-off of generators in the objective by rewarding non-zero values of  $\zeta_g$

$$\epsilon_2 \sum_{g \in \mathcal{G}} W_g \zeta_g, \quad (11)$$

where  $W_g$  is some weight. A uniform weight, e.g.,  $W_g = 1, \forall g$ , will encourage large generators to switch off, rather than several small units, for any given decrease in total generation. Generation disconnection can be more evenly penalized by instead setting  $W_g$  equal to the generator's capacity  $P_g^{G+}$ .

## 4 Post-islanding AC optimal load shedding

The solution of the DC islanding optimization includes a set of lines to disconnect, new generation levels, and the proportions of loads to be shed. In general, however, the predictions of the DC model will not match reality, and no consideration is given to reactive power and voltage. Therefore, to determine a feasible AC solution for the islanded network, an AC optimal load shedding (OLS) problem is solved immediately after the islanding optimization.

The AC-OLS optimization problem is a standard OPF problem albeit with load shedding. The AC-OLS is solved for the network in its islanded state. That is, the set  $\mathcal{L}$  is modified by removing lines for which  $\rho_l = 0$ . Furthermore, any generator for which  $\zeta_g = 0$  has its upper and lower bounds on real power set to zero; others are free to vary real power output within a restricted region, as described previously.

This problem also maximizes the value of total real power supplied to loads:

$$J_{\text{AC}}^* = \max \sum_{d \in \mathcal{D}} R_d \alpha_d P_d, \quad (12)$$

subject to,

$$f(x) = 0, \quad (13a)$$

$$g(x) \leq 0, \quad (13b)$$

$$(p_g^G, q_g^G) \in \mathcal{O}_g, \forall g \in \mathcal{G}, \quad (13c)$$

$$(p_d^D, q_d^D) = \alpha_d (P_d^D, Q_d^D), \forall d \in \mathcal{D}. \quad (13d)$$

Here,  $R_d$  is the reward for supplying load  $d$ , and is equal to  $M_d$  if the load has been placed in section 1 and  $\beta_d M_d$  if placed in section 0. The equality constraint (13a) captures Kirchhoff's current and voltage laws in a compact form;  $x$  denotes the collection of bus voltages, angles, and real/reactive power injections across the islanded network. The inequality constraint (13b) captures line limits and bus voltage limits.

The set  $\mathcal{O}_g$  is the post-islanding region of operation for generator  $g$ , and depends on the solution of the islanding optimization and pre-islanded outputs of the generator. If  $\zeta_d = 1$  the unit remains fully operational, and its output may vary within some region around the pre-islanded operating point; most generally  $(p_g^G, q_g^G) \in \mathcal{O}_g(p_g^{G*}, q_g^{G*})$ , where  $(p_g^{G*}, q_g^{G*})$  is the pre-islanding operating point and  $\mathcal{O}_g$  is defined by the output capabilities of the generating unit. If real and reactive power are independent,  $p_g^G \in [P_g^{G-}, P_g^{G+}]$  and  $q_g^G \in [Q_g^{G-}, Q_g^{G+}]$ . If,

conversely, the islanding optimization has set  $\zeta_g = 0$ , then real power output is set to zero:  $p_g^G = 0$ . In that case, the unit may remain electrically connected to the network, with reactive power output free to vary within some specified interval  $[Q_g^{G-}, Q_g^{G+}]$ . Loads are assumed to be homogeneous; real and reactive components are shed in equal proportions.

The AC-OLS is a nonlinear programming (NLP) problem and may be solved efficiently by interior point methods.

**Remark 2 (Transient response to islanding)** *Solving the MILP islanding problem and, subsequently, the AC-OLS provides a feasible steady-state operating point for the network in its post-islanding configuration. Since the objective minimizes the load shed and the constraints limit the changes to generator outputs, the proposed solution will naturally limit, to some extent, the disruption to the network. Nevertheless, it is possible that the islanding actions may lead to transient instability, since the transient response is not modelled when designing islands.*

*A full dynamic simulation of a large system is time consuming. This is not a problem if islanding is being investigated as part of an off-line contingency analysis. Then if the dynamic simulation shows that the first islanding solution is unstable, alternative islanding solutions (gathered during the optimization) can be simulated until a stable one is found. However, if islanding is being done on-line after an unexpected contingency, then there is not likely to be sufficient time for full dynamic simulation, and in this emergency situation it is necessary to have some confidence that the output of the optimization is dynamically stable. Fortunately, it turns out that for all of the cases described in the next section, increasing the penalty for cutting high-flow lines results in transiently stable islands.*

## 5 Computational results

This section presents computational results using the above islanding formulation. Computation times and results on optimality of the DC-based solutions are presented for a range of test networks. In Sec. 5.2, transient stability in response to islanding is investigated, and the results of time-domain simulations are presented. In Sec. 5.3, the problem of ensuring a “normal” voltage profile across the network, following islanding, is discussed.

### 5.1 Computation times and optimality

The speed with which islanding decisions have to be made depends on whether the decision is being made *before* a fault has occurred, as part of contingency planning within secure OPF, or *after*, in which case the time scale depends on the cause of the contingency. Especially in the second case it is important to be able to produce feasible solutions within short time periods even if these are not necessarily optimal.

Fig. 2 shows the times required to find obtain feasible islanding solutions to varying proven levels of optimality. Times are recorded for different networks ranging from a 9-bus system to a 300-bus system. For a network with  $n^B$  buses,  $n^B$  scenarios were generated by assigning in turn each single bus to  $\mathcal{B}^0$ . No

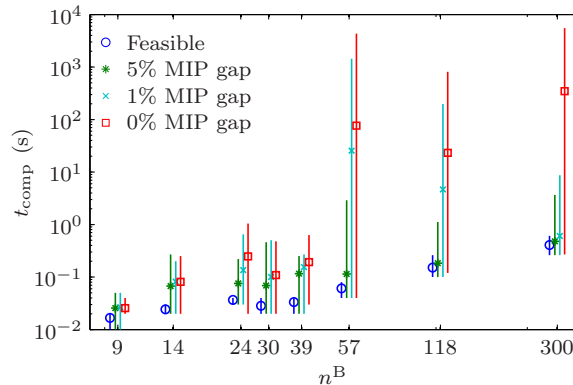


Figure 2: Mean, max and min times for finding, to different levels of optimality, islanding solutions for different test networks.

assignments were made to  $\mathcal{B}^1$ . For the networks with no ramp rates or spinning reserve data available, it is assumed that each generator may vary its output by  $\pm 5\%$  of the pre-islanding level. Where no line limits are present for a network, a maximum phase angle difference of 0.4 rad is imposed for each line. In the objective function, the values of  $\epsilon_1$  and  $\epsilon_2$  in (10) and (11)—the penalties on line cuts and generator disconnection respectively—are 0.1 and 0.0001, with  $W_g = P_g^{G+}$  in the latter. This penalizes line disconnection more heavily.

Problems are solved on a dual quad-core 64-bit Linux machine with 8 GiB RAM, using AMPL 11.0 with parallel CPLEX 12.3 to formulate and solve MILP problems. Computation times quoted include only the time taken to solve the islanding optimization to the required level of optimality, and not the AC-OLS, and are obtained as total elapsed seconds used by CPLEX during the `solve` command. A time limit of 5000 seconds is imposed. The required levels of optimality for each problem are ‘feasible’—an integer feasible solution—and relative MIP gaps of 5%, 1% and 0%.

Examining first the times required to find a feasible islanding solution, the results in Fig. 2 show that all problems are solved to feasibility well within 1 s. In all cases, a feasible solution was found at the root node, aided by CPLEX’s cut generation, without requiring branching; thus, the rise in computation time is largely owing to the increasing size of the LP relaxation problem.

For a MILP problem solved by branching, the optimal integer solution is bounded from below (for maximization) by the highest integer objective value found so far during the solution process, and from above by an objective value deduced from all node sub-problems solved so far. The relative MIP gap is the relative error between these two bounds. Fig. 2 indicates the progress made by the CPLEX solver, in terms of the times required to reach relative MIP gaps of 5%, 1% and 0% (*i.e.*, optimality) respectively. Performance is very promising for solving to 5%, with all problems solved to this tolerance within five seconds. Times to 1% and 0% gaps are of the same order for the smaller networks (up to 39 buses), but the 57-, 118- and 300-bus networks can take significantly longer to solve to these tolerances.

Table 1 shows the means of the relative errors between the solution value returned at termination of the solver and the actual optimum, where known.

	Feasible	5% gap	1% gap
Min	0.00	0.00	0.00
Mean	8.57	0.42	0.04
Max	25.00	3.86	0.80

Table 1: Relative errors (%) between optimal and returned solutions.

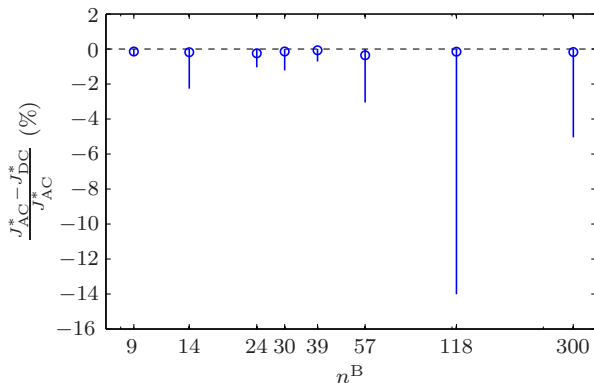


Figure 3: Mean, max and min relative errors between DC and AC objective values.

The ‘real’ gaps between early termination solutions and the true optima are nearer zero than the 5% or 1% bounds. Therefore, good islanding solutions—at least with respect to the DC model—can be provided even when the solver is terminated early. Moreover, these solutions can be found quickly. For practical application in real time—with the network in a stressed condition—a good, but possibly sub-optimal, integer solution may be acceptable, given that islanding is a last resort and fast decision making is required. Moreover, because the DC model is an approximation of the AC model, it may make little sense to pursue proven optimal DC solutions.

Fig. 3 shows the differences between the objectives as predicted by the DC islanding optimization and the post-islanding AC-OLS. The adopted islanding solution in each case is that from solving the problem to full optimality. Since a small number of problems require a long time to solve, only those cases solved to optimality within 5000 seconds are included. Furthermore, a number of islanding solutions were found to be AC infeasible, and so were removed from the comparison. (This problem is discussed in a later section). The number of cases included in the comparison for each network is given in Tab. 2.

The comparison of DC and AC objective values illustrates that the ability of the DC model to predict the AC objective. The measure indicates an over-estimation of the load supplied if negative, and an under-estimation if positive. It is clear that although on average the objective values are well within 1%, a number of instances exist of significant over-estimation of the objective. This indicates the limited ability of the DC-based MILP optimization as a predictor of optimal AC islanding solutions, and suggests that undue effort should not be taken in finding solutions to full optimality, when ‘good’ DC solutions can be found quickly by terminating the solver early. The remaining issues are transient

$n^B$	9	14	24	30	39	57	118	300
MIP gap > 0%	0	0	0	0	0	1	1	17
AC infeasible	0	2	7	7	9	6	17	72
Cases compared	9	12	17	23	30	50	100	211

Table 2: Number of unique problems included in the comparisons.

stability and AC feasibility of the network after islanding, which are discussed in the remainder of this paper.

## 5.2 Transient stability

This subsection investigates the transient response to islanding and presents the results of time-domain simulations. Firstly, a case study of the 24-bus IEEE RTS [20] is described. Subsequently, results of time-domain simulations are presented for the 9- to 300-bus networks, and the effects of line and generator penalties are investigated.

### 5.2.1 IEEE 24-bus Reliability Test System

The IEEE RTS [20] comprises 24 buses and 38 lines. Of the buses, 17 have loads attached. All loads are assumed to be constant, and total load demand is 2850 MW. Total generation capacity is 3405 MW from 32 synchronous generators.

The failure scenario simulated is as follows. Starting with the network operating at a steady state corresponding to an OPF solution, bus 15 experiences a three-phase-to-ground fault at a time  $t = 0.5$  s, which is subsequently cleared by tripping line (15, 24) after a further 0.295 s. Time-domain simulations of the scenario were performed using PSAT [21] and second-order non-linear models of synchronous machine dynamics (with machine parameters taken from [22] and damping  $D = 0.5$ ). Note that this is a conservative simulation, since in practice the damper windings and the control systems (turbine governor, AVR) would act to dampen the oscillations. Without remedial action, the disturbance to the network causes severe generator swings, leading to multi-swing instability and a loss of synchronism within four seconds (Fig. 4(a)).

The network is considered immediately after the line trip, while still operational, and the objective is to avoid total network failure by using controlled islanding. Buses 1, 3, 15 and 24 are assigned  $\mathcal{B}^0$ . In determining a post-islanding steady state, the generator limits are set to allow a small movement from the pre-islanding operating point,  $p_g^{G*}$ . Ramp rates,  $R_g^G$  (MW/min), for the generators may be found in [22]. A time limit of two minutes is assumed for ramping to any new real power level. Thus, limits are set as

$$P_g^{G+} = \min\{p_g^{G*} + 2R_g^G, P_g^{\max}\}, \quad (14a)$$

$$P_g^{G-} = \max\{p_g^{G*} - 2R_g^G, P_g^{\min}\}. \quad (14b)$$

The optimal islanding solution for  $\beta_d = 0.5$ , with a penalty  $\epsilon_1 = 0.1$  on line cuts, isolates buses 1, 3, 15 and 24 by cutting lines (1, 2), (1, 5), (3, 9), (15, 16), and (15, 21). Line (3, 24) within the isolated section has also been cut.

	$\sum_g p_g^G$	$\sum_d p_d^D$	$\sum_l h_l^L$	$J^*$
Post-islanding DC	2650.0	2613.9	36.1	2429.4
Post-islanding AC	2666.1	2610.1	56.0	2427.5

Table 3: DC and AC solution data for the islanded network.

Tab. 3 shows, for the post-islanding DC and AC steady-state solutions, the total generation, load supplied, line losses, and expected load supplied (the objective value). To achieve a steady-state balance after islanding, 240 MW of load is to be shed.

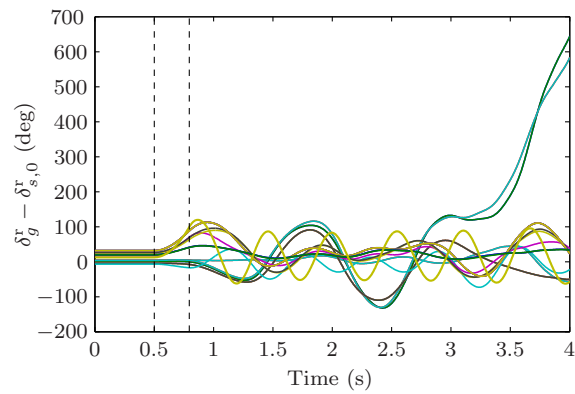
Fig. 4(b) shows the response of synchronous machines in the network to the following events.

- For  $0 \leq t < 0.5$  s, the network is operating at its original AC-OPF solution.
- At  $t = 0.5$  s, line bus 15 experiences a three-phase fault.
- At  $t = 0.795$  s, the fault is cleared by tripping line (15, 24).
- At  $t = 2$  s, the network is split by instantaneously cutting lines and shedding loads.

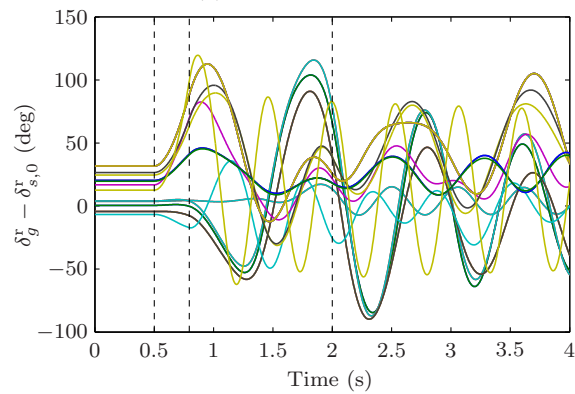
The rotor angle of each generator is shown relative to the rotor angle of the slack generator in the same island. Thus, although some of these angle differences are over 100 degrees, the maximum instantaneous difference between any pair of *adjacent* machines was observed to be 95.8 degrees, immediately before the splitting of the network. After splitting, all adjacent angles are less than 50 degrees, and the generators in each island remain in synchronism throughout the simulation. Note that the oscillations do not decay quickly, owing to the very light damping, yet simulation beyond  $t = 4$  s confirms that the rotor angles settle without loss of stability. No generator adjustments are made, since these require longer timescales and are subject to the control systems present in the network. Load shedding is controlled and by design, and not a result of low voltages or frequency. While this simulation is therefore a simplification of the real situation, omitting load dynamics, control system models and the actions of protective relays, it does indicate that, in this case, the initial actions of islanding—cutting lines and intentionally shedding loads—prevent what would otherwise be a loss of synchronism.

### 5.2.2 Other test networks

Table 4 reports the results of time-domain simulations of the islanding scenarios described in Sec. 5.1, indicating the number of instances of islanding leading to transient instability. The simulations were as described in the previous subsection, but with the islanding starting from an undisturbed pre-islanding operating point, at which all generators have an angular frequency of 1 p.u. and the network is balanced. In real situations, islanding is a measure of last resort and will commence with the network in a state of emergency, following a number of outages and trips. Therefore, these simulations to assess stability are only an approximation. However, if an islanding solution is unstable from such an operating point then this is a strong indication of instability when starting with the network in a stressed condition, and that the solution is not suitable.



(a) Without islanding



(b) With islanding

Figure 4: Response of machine relative rotor angles to disturbance and network splitting. Note the different rotor angle scales. The damping used in this example is lower than would be used in practice, but even with this low level of damping the system is stable in the islanding case.

$n^B$	9	14	24	30	39	57	118	300
Cases compared	9	12	17	23	30	50	100	211
Unstable	0	0	1	0	0	0	1	12
Stable	9	12	16	23	30	50	99	199

Table 4: Results of time-domain simulations with original line-cut and generator penalties.

$n^B$	9	14	24	30	39	57	118	300
Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Mean	0.16	0.78	0.59	0.03	0.34	0.06	0.20	0.46
Max	0.48	10.85	2.87	0.68	5.09	1.00	2.70	7.03

Table 5: Decrease in objective  $J_{DC}^*$  (%) for increased line-cut and generator penalties.

In total, 14 out of 452 islanding solutions were found to lead to instability. Investigation of the individual cases found that, in all cases, severe transients were caused by cuts to high-flow lines. However, re-solving the islanding optimization with increased penalties on high-flow lines ( $W_l = s_l^{L*}$  and  $\epsilon_1 = 10^{-4} \sum_d P_d^D$ ) and switching-off of generators ( $W_g = P_g^{G+}$  and  $\epsilon_2 = 1$ ) led, in every unstable case, to a transiently-stable alternative solution, and no unstable solutions for any other problems.

As these larger penalty coefficients are no longer insignificant compared to the order of magnitude of the primary objective, a fall in the level of optimality (with respect to the primary objective) of solutions is expected. Tab. 5 reports the falls observed in DC objective values,  $J_{DC}^*$ , for each network. This degradation in the objective is small, and is an acceptable trade-off for removing all of the transiently unstable cases. As an added advantage, as Fig. 5 shows, solve times for the larger networks are shorter with the heavier penalties.

### 5.3 Network voltage profile

A number of islanding solutions obtained by solving the MILP problem were subsequently found to be AC infeasible; that is, there was no solution to the AC-OLS lying within normal voltage bounds. In fact, by softening the normal voltage bounds a solution was found to all of the ‘infeasible’ instances in Tab. 2. Inspection of these cases found that though there was sufficient real and reactive power generation capacity in each island, local shortages or surpluses of reactive power in the re-configured networks led to out-of-bounds voltages.

Therefore, an islanding solution formed by considering only real power—even if network constraints are included—is not always satisfactory. However, these results also show that even if a global reactive power balance is achieved, local shortages or surpluses of reactive power can lead to an abnormal voltage profile. Many of the IEEE test networks are prone to the same problem, as observed from the results in this paper. The conclusion is that designing network partitions by consideration of real and reactive power balances in each island is not sufficient to guarantee solutions with a good voltage profile.



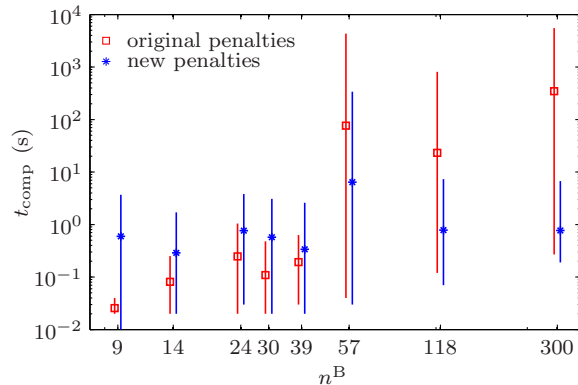


Figure 5: Mean, max and min solve times to optimality with original penalties ( $\epsilon_1 = 0.1$ ,  $W_l = 1$ ,  $\epsilon_2 = 10^{-4}$ ,  $W_g = P_g^{G+}$ ) and new penalties ( $\epsilon_1 = 10^{-4} \sum_d P_d^D$ ,  $W_l = s_l^{L*}$ ,  $\epsilon_2 = 1$ ,  $W_g = P_g^{G+}$ ).

## 6 Conclusions

In this paper, an optimization-based approach to controlled islanding and intentional load shedding has been presented. The proposed method uses MILP to determine which lines to cut, loads to shed, and generators to switch in order to isolate an uncertain or failure-prone region of the network. The optimization framework allows linear network constraints—a loss-modified DC power flow model, line limits, generator outputs—to be explicitly included in decision making, and produces balanced, steady-state feasible DC islands. AC islanding solutions are found via the subsequent solving of an AC optimal load shedding problem. The transient stability of resulting islanding solutions is assessed via time-domain simulation.

The approach has been demonstrated through examples on a range of test networks, and the practicality of the method in terms of computation time has been demonstrated. Time-domain simulations of the islanding solutions have indicated that transient instability is avoided, for our example scenarios, by appropriate choices of penalties on cuts to high-flow lines and disconnections of generating units, both of which discourage disruption to the network.

Outstanding issues include those of ensuring—without resorting to *a-posteriori* time-domain simulation—transient stability, and also a healthy voltage profile in all parts of the network after islanding. Future research will address these issues by investigating the inclusion of constraints for transient stability in the problem, and the modelling of reactive power in problem areas. In addition, techniques for improving computation times for islanding will be investigated.

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