Economic and Environmental Analysis of Photovoltaic Energy Systems via Robust Optimization

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March 22, 2012

Abstract

This paper deals with the problem of determining the optimal size of a residential grid-connected photovoltaic system to meet a certain CO2 reduction target at a minimum cost. Ren et al. proposed a novel approach using a simple linear programming that minimizes the total energy costs for residential buildings in Japan. However, their approach is based on a specific net tariff system that was used in Japan until October 2009, and it is not applicable to the current Japanese net tariff system.

We propose a modified approach for the current Japanese tariff system. The mathematical formulation is general in the sense that it includes formulations for other tariff systems as special cases. Therefore, the approach is applicable not only to the Japanese system but also to other tariff systems (e.g., gross feed-in tariff system). We further extend this approach by using a robust optimization technique to cope with the uncertainty in photovoltaic power generation caused by weather variability. Numerical experiments show the minimum size requirements of solar photovoltaic systems for meeting CO2 reduction targets and its economic costs in nominal and robust cases.

keywords: photovoltaic energy system, optimal size, CO2 reduction target, robust optimization, irradiation uncertainty.

1 Introduction

Production of electricity by the burning of fossil fuels produces a lot of carbon dioxide (CO2), which contributes to the greenhouse effect. A global movement in promoting low- or zero-carbon energy production will be necessary to help meet the growing energy needs. Photovoltaic (PV) solar energy systems, which generate electricity through direct conversion of sunlight or solar irradiation into electrical energy, are excellent sources of 'zero-carbon' energy. PV energy systems are potentially excellent low-carbon energy technologies.

Feed-in tariff systems for renewable energy have been adopted by over 50 countries around the world. These systems promote the generation of renewable energy including PV. There are essentially two types of feed-in tariff scheme: net feed-in tariff and gross feed-in tariff. Net feed-in tariff provides a feed-in payment only for generated energy that is not consumed on site, that is, excess electricity that is fed back into the grid from its grid-connect PV system. On the other hand, gross feed-in tariff provides a payment for the whole PV system output. Japan,

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various Australian states, etc., have net feed-in tariffs, whereas Germany, Denmark and Spain, etc., have gross feed-in tariffs. Both sorts of feed-in tariff encourage people to install their own low-carbon energy generators. Indeed, such systems have given residents, small businesses, and community groups a great opportunity to install rooftop solar PV systems on existing private residences and other private buildings.

The purpose of this study is to estimate how large PV systems should be at the provincial, municipal, or community levels to reach CO2 reduction goals and how much money owners of PV systems will save or pay once the systems are installed. Numerous countries including Japan have agreed to the Kyoto protocol and have tried to meet their CO2 reduction targets. From the government's point of view, it is important to estimate the minimum size requirement of solar PV systems for meeting their CO2 reduction target and to evaluate economic costs borne by owners of PV systems. Such estimates would be help in reassessing green-tariff prices.

It is not easy to find the best size of a PV system intuitively. It is rather necessary to formulate an optimization problem for finding the optimal size of a PV system for a particular tariff system. Determining the best size of a PV system under a net feed-in tariff is more complicated compared with determining it under a gross feed-in tariff. Our formulation of the problem has the objective of minimizing the total cost related to electricity under a net or gross feed-in tariff scheme for residents in a community, and it includes constraints for achieving CO2 emission reductions and for satisfying daily demand for electricity. We need to consider a balance between revenues and costs when installing a grid-connected PV system. The total cost of a PV system consists of a high initial cost and a very low subsequent running cost (which we will regard as zero in this paper). The total revenue consists of subsidies for self-consumption and the additional feed-in tariff for excess electricity fed into the grid. To install a large-size PV system, one has to pay a high initial cost but one also can receive high subsidies for excess electricity. However, if one has no excess electricity, installation costs may not be recovered.

Some studies have devised ways of choosing the optimal size of a PV system under a feed-in tariff scheme. They primarily used the following method (except for [17]): calculate the economic costs of PV systems for various fixed PV sizes and choose the optimal one that has the minimum cost. For example, Hernandez et al. [9] performed such a parametric analysis to determine the optimal size of the PV system with different types of feed-in tariff. Gong and Kulkarni [6] described a methodology to optimize not only the array size but also the array surface tilt angle and configuration. Ren et al. [17] proposed a novel mathematical programming formulation to find the best size of a PV system. Their model minimizes the annual energy cost including the PV installation cost and utility electricity cost, and this cost is subtracted from the revenue gotten from selling the excess electricity and the revenue from the green tariff.

Our model is closely related to the model of Ren et al. [17]. There are mainly two differences between ours and theirs: Firstly, our model has an added CO2 emission reduction constraint CO2 emission reduction constraint. Secondly, the electricity price (yen/kWh) for selling electricity back to energy companies can be set to be higher than the price for buying electricity from the energy companies (in this case, regional electric power companies in Japan). Ren et al. [17] implicitly assume that surplus energy from the PV system is purchased at the same price as the electricity sales price of energy companies, which is according to old Japanese feed-in tariff, but under the new Japanese feed-in tariff (started in November 2009), surplus energy from the PV system is purchased by those companies at two times their electricity sales price. Under the new scheme, Ren et al.'s model provides an unrealistic solution such that the electrical energy bought from some energy company is sold to the company at the same time to make a profit. We modified Ren et al.'s model so that it gives us a realistic solution under the new Japanese feed-in tariff and it includes a CO2 emission reduction constraint. The resulting model is general in the sense that it includes models for other tariff systems as special cases. Therefore, the approach shown in this paper is applicable not only to the Japanese system but also to other tariff systems (e.g., gross feed-in tariff system).

Many studies have looked into the environmental cost as well as the economic cost of PV systems. For example, Bernal-Agustin and Dufo-Lopez [4] evaluated the environmental aspects of grid-connected PV systems by using life-cycle analysis. Ren et al. [17] calculated resulting CO2 emission reductions for optimally-sized PV systems of their cost minimization models. These studies evaluated CO2 emissions a-posteriori and did not consider CO2 reduction targets a-priori. On the other hand, such a CO2 reduction target can be taken into account in the framework of energy planning models that determine the optimal mix of generating stations such as fossil fuel, nuclear, hydroelectric, photovoltaic, and so on. Hashim and coworkers [8, 5, 14] devised mixed integer programming models that determine the optimal structure necessary to meet a given CO2 reduction target while maintaining or enhancing power to the grid. Linares and Romero [13] applied goal programming to a case study on energy planning in Spain. Their model involves minimizing several criteria, *e.g.*, the total cost of electricity generation, CO2 emissions, and so on.

We further extend our model to cope with the uncertainty in photovoltaic power generation. PV electricity varies depending on weather conditions, mainly the level of solar irradiation. The solar irradiation is highly unpredictable. Therefore, it is difficult to determine the best PV system size from a model constructed from only a small historical data set on solar irradiation in order to meet a CO2 reduction target in the future. Standard mathematical optimization models commonly assume that the data inputs are precisely known and ignore the influence of parameter uncertainties on the optimality and feasibility of the models. As a result, when the data differ from the assumed values, the generated optimal solution may violate critical constraints and perform poorly from an objective function point of view. Robust optimization [1, 2] addresses the issue of data uncertainties and provides us with a robust solution for uncertain constraints or/and objective function. We used robust optimization techniques in a model including uncertainty in solar irradiation data and found that we could make a robust decision on the best size of the PV system.

The paper is organized as follows. Section 2 describes Ren et al.'s mathematical formulation [17] for deciding the optimal size of a PV system under a net tariff system with equal sales and purchase prices. It also discusses the same problem under a gross tariff system. Section 3 extends Ren et al.'s formulation in order to fit to the current Japanese net tariff system and gives a solution satisfying electricity demand and a CO2 emission reduction target. Section 4 describes our new robust optimization formulation that decides the optimal PV size by taking the uncertainty in solar irradiation into consideration. Section 5 reports on the numerical results for the deterministic model and its robust variant, and Section 6 summarizes our contributions and future work.

2 Preliminaries

2.1 Net Tariff System with Equal Sales and Purchase Price

Ren et al. [17] presents a model for deciding the optimal size of the PV system in residential buildings from deterministic solar irradiation data on the area of Kitakyushu, Japan. To decide the optimal size, [17] uses mathematical optimization techniques that minimize the objective function under several constraints. Input data for the model are summarized in Table 1, and its variables are in Table 2.

The objective is to minimize the annual energy cost for owners of grid-connected PV systems. The energy cost consists of the annualized PV installation cost and purchased electricity cost, subtracted from revenue gotten from selling the excess electricity and revenue from the green tariff. There are four constraints. Constraint (a) implies that the electricity demand should be satisfied with PV electricity used on-site (self-consumption) and electricity purchased from the electricity grid. Constraint (b) implies that the sum of the electricity for sale to the electricity

Table 1: Input data (The upper table shows the notations used in the existing model of Ren et al. [17], and the lower table shows additional notations used in our model.)

p^S_{ij}	electricity sales price* at time j in the ith period† [yen/kWh]	$i = 1, \cdots, T,\ j = 1, \cdots, H_i$
p^P_{ij}	electricity purchase price* [yen/kWh]	$i = 1, \cdots, T,\ j = 1, \cdots, H_i$
p^G_{ij}	green tariff price* [yen/kWh]	$i = 1, \cdots, T,\ j = 1, \cdots, H_i$
D_{ij}	hourly electricity demand [kWh]	$i = 1, \cdots, T,\ j = 1, \cdots, H_i$
C	capital cost of a residential PV system [yen/year]‡	
E_{ij}	electricity generated by a PV system [kWh/kW]*	$i = 1, \cdots, T,\ j = 1, \cdots, H_i$
$\overline{z}$	an upper bound of the capacity size of a PV system [kW]	
K_i	The number of days in the ith period†	$i = 1, \cdots, T$
T	The number of decision making periods†	
H_i	The number of sets of hours	
F^P_{ij}	CO2 emission factor of purchased electricity [kg-CO_2/kWh]	$i = 1, \cdots, T,\ j = 1, \cdots, H_i$
F^M	CO2 emissions from production of silicon PV modules [kg-CO_2/(kW·year)]‡	
G	CO_2 reduction goal (Japan's target is a 25 % reduction from the 1990 level by 2020) [kg-CO_2]	

$\star$ p^S_{ij} means the sales price for electricity from a PV system to the grid, and p^P_{ij} means the purchase price for electricity from the grid. $p^G_{ij} < p^P_{ij} < p^S_{ij}$ currently holds in the Japanese net tariff system (see Table 3).

† "period" means a set of consecutive days.

‡ C and F^M are evaluated under the assumption that the PV system lifetime is 20 years.

$*$ We estimated the PV-generated electricity E_{ij} according to procedure in [10].

$$E_{ij} \text{ [kWh/kW]} = \text{solar irradiation on inclined surface [kWh/m}^2] \div \text{solar irradiation intensity [kW/m}^2] \times \text{temperature loss (\%)} \times \text{other loss(\%)}, \tag{1}$$

$$\text{solar irradiation on inclined surface [kWh/m}^2] = \text{global solar irradiation [kWh/m}^2] \times \text{correction coefficient with respect to surface inclination.}$$

Global solar irradiation is essential data for estimating E_{ij}. We obtained such data from Japan Meteorological Agency website [11]. We used the typical temperature loss and other loss values for Japan, and we got the correction coefficient data from [16]. We set solar irradiation intensity to 1 in all cases.

Table 2: Decision variables

x^S_{ij}	the amount of electricity sold to the grid [kWh]	$i = 1, \cdots, T,\ j = 1, \cdots, H_i$
x^P_{ij}	the amount of electricity purchased from the grid [kWh]	$i = 1, \cdots, T,\ j = 1, \cdots, H_i$
x^C_{ij}	the amount of self-consumed electricity [kWh]	$i = 1, \cdots, T,\ j = 1, \cdots, H_i$
z	capacity of a PV system [kW]	

grid and PV electricity used on-site can not exceed the amount of PV-generated electricity. Constraint (c) derives a non-negative value, and constraint (d) is a bounding constraint of the variable z. Suppose that one year is divided into T periods, each period consists of K_i days, and each day consists of H_i hours. The optimization model is formulated as follows:

$$
\begin{array}{lll}
\underset{\boldsymbol{x}^S,\boldsymbol{x}^P,\boldsymbol{x}^C,z}{\text{minimize}} & Cz + \displaystyle\sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i(p_{ij}^P x_{ij}^P - p_{ij}^S x_{ij}^S - p_{ij}^G x_{ij}^C) & \\
\text{subject to} & \text{(a)}: \ x_{ij}^P + x_{ij}^C \geq D_{ij} \ \ (i=1,\cdots,T,\ j=1,\cdots,H_i) & \\
 & \text{(b)}: \ x_{ij}^S + x_{ij}^C \leq E_{ij}z \ \ (i=1,\cdots,T,\ j=1,\cdots,H_i) & \\
 & \text{(c)}: \ x_{ij}^P, x_{ij}^S, x_{ij}^C \geq 0 \ \ (i=1,\cdots,T,\ j=1,\cdots,H_i) & \\
 & \text{(d)}: \ 0 \leq z \leq \overline{z}. &
\end{array}
\tag{2}
$$

Ren et al.'s model [17] assumes (T, K_i, H_i)=(365,1,24) and $p_{ij}^G = 0$, and furthermore, it appends the maintenance cost of the PV system to the objective in the above model. We estimate the PV-generated electricity E_{ij} as shown in Table 1 according to [10], which is different from the estimation of [17].

In the model setting of [17], the electricity sales price is almost equal to the electricity purchase price, i.e., $p_{ij}^S = p_{ij}^P$, as it was under the net tariff system before November 2009 in Japan. However, since November 2009, the electricity sales price has been about twice the purchase price in order to promote the introduction of residential PV systems. Under the current Japanese feed-in tariff system, we cannot get realistic solutions from (2). Indeed, when the electricity sales price is higher than the purchase price under the net tariff system, (2) gives an optimal solution with $x_{ij}^P > 0$ and $x_{ij}^S > 0$, which means that we should sell what we have purchased from the grid for twice the price. However, this is not actually allowed. We have to avoid optimal solutions such that sales and purchases to and from the grid happen at the same time and find optimal solutions which fit the current feed-in tariff system in Japan.

2.2 Gross Tariff System

We will modify the linear programming (LP) problem (2) defined for the old Japanese net tariff system in [17] to deal with the gross tariff system. In the case of the gross tariff system, the decision variables of electricity self-consumption x_{ij}^C equal zero, because all PV system outputs would be sold to the grid. Thus, electricity demand D_{ij} would be met with only purchased electricity x_{ij}^P. Therefore, the model for deciding the optimal size of the PV system under the gross tariff system is formulated as an LP:

$$
\begin{array}{ll}
\underset{\boldsymbol{x}^S,\boldsymbol{x}^P,z}{\text{minimize}} & Cz + \displaystyle\sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i(p_{ij}^P x_{ij}^P - p_{ij}^S x_{ij}^S) \\
\text{subject to} & x_{ij}^P \geq D_{ij} \ \ (i=1,\cdots,T,\ j=1,\cdots,H_i) \\
 & x_{ij}^S \leq E_{ij}z \ \ (i=1,\cdots,T,\ j=1,\cdots,H_i) \\
 & x_{ij}^P, x_{ij}^S \geq 0 \ \ (i=1,\cdots,T,\ j=1,\cdots,H_i) \\
 & 0 \leq z \leq \overline{z}.
\end{array}
$$

We can easily show that $x_{ij}^P = D_{ij}$ and $x_{ij}^S = E_{ij}z$ for the optimal solution to the above problem. We can transform the above problem into a simple problem having only one variable z:

$$
\begin{array}{ll}
\underset{z}{\text{minimize}} & Cz + \displaystyle\sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i(p_{ij}^P D_{ij} - p_{ij}^S E_{ij}z) \\
\text{subject to} & 0 \leq z \leq \overline{z}.
\end{array}
\tag{3}
$$

3 Optimal PV Size for the Japanese Net Tariff System

We shall deal with the current Japanese net feed-in tariff system where a premium rate is paid for renewable energy such as solar power electricity fed back into the electricity grid. The existing model (2) cannot output a reasonable and realistic optimal solution in the current net tariff system. Thus, we shall extend the existing model to make it work with the current feed-in tariff system.

3.1 Complementary Optimization Problem

We shall add a complementary constraint, $x_{ij}^P \times x_{ij}^S = 0$, to (2) in order to avoid selling and purchasing electricity to and from the grid simultaneously. The constraint makes sales or purchases of electricity 0, which means that sales and purchases will not happen at the same time. We also can formulate (2) with the additional complementary constraint, $x_{ij}^P \times x_{ij}^S = 0$, as a mixed 0-1 integer problem. However, we decided to modify (2) by adding the complementary constraint for simplicity and transformed the complementary problem into a simply solvable problem.

We shall also append a constraint about CO2 emission reduction to (2). Japan's target is a 25 % reduction from its 1990 level by 2020, so we can set the annual goal for CO_2 reduction as $G = 0.25\times$ (CO_2 amount from electricity production in 1990) kg. We also regard $\sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i F_{ij}^P (x_{ij}^S + x_{ij}^C)$ as the resulting annual reduction in CO_2 emissions, because electric power companies would have to generate additional $\sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i (x_{ij}^S + x_{ij}^C)$ if no PV power were available. Taking the life-cycle assessment of a PV system into consideration, we can construct a CO2 emission reduction constraint as follows:

$$\sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i F_{ij}^P (x_{ij}^S + x_{ij}^C) - F^M z \geq G.$$

Appending the CO2 emission reduction constraint to the above complementary problem with the current Japanese feed-in tariff system, we obtain the following optimization problem.

$$\begin{array}{ll} \underset{\boldsymbol{x}^S, \boldsymbol{x}^P, \boldsymbol{x}^C, z}{\text{minimize}} & Cz + \sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i (p_{ij}^P x_{ij}^P - p_{ij}^S x_{ij}^S - p_{ij}^G x_{ij}^C) \\ \text{subject to} & \text{(a)}, \text{(b)}, \text{(c)}, \text{(d)}, \\ & \text{(e)}: \ \ x_{ij}^P \times x_{ij}^S = 0 \ \ (i = 1, \cdots, T, \ j = 1, \cdots, H_i) \\ & \text{(f)}: \ \ \sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i F_{ij}^P (x_{ij}^S + x_{ij}^C) - F^M z \geq G \end{array} \tag{4}$$

Constraints (a)-(d) are those of problem (2).

3.2 Transformed Problem with One Variable

The following proposition enables us to deal with problem (4) easily.

Proposition 3.1 *Suppose that $p_{ij}^G < p_{ij}^S$ for $\forall i, j$ $(i = 1, \cdots, T,\ j = 1, \cdots, H_i)$. Then, the optimal solution $(x_{ij}^{C*}, x_{ij}^{P*}, x_{ij}^{S*})$ of (4) is determined by z as*

$$x_{ij}^{C*} = \min\{D_{ij}, E_{ij}z\}, \quad x_{ij}^{P*} = D_{ij} - x_{ij}^{C*}, \quad x_{ij}^{S*} = E_{ij}z - x_{ij}^{C*}. \tag{5}$$

Proof. (5) implies that

i) $x_{ij}^{C*} = E_{ij}z$, $x_{ij}^{P*} = D_{ij} - E_{ij}z$ and $x_{ij}^{S*} = 0$, if $D_{ij} > E_{ij}z$, and

ii) $x_{ij}^{C*} = D_{ij}$, $x_{ij}^{P*} = 0$ and $x_{ij}^{S*} = E_{ij}z - D_{ij}$, if $D_{ij} \leq E_{ij}z$.

The optimal solution of (4) must have $x_{ij}^{P*} = 0$ or $x_{ij}^{S*} = 0$ because of (e). In cases i) and ii), the solution (x_{ij}^{C*},x_{ij}^{P*}, x_{ij}^{S*}) satisfies constraints (a), (b), (c) and (e) in (4).

We will show that the solution $(x_{ij}^{C*}, x_{ij}^{P*}, x_{ij}^{S*})$ is optimal to (4) with a fixed value of z. First, let us consider the case of $D_{ij} > E_{ij}z$. x_{ij}^{S*} must be zero. Otherwise, $x_{ij}^{P*} = 0$ makes problem (4) infeasible. Minimizing x_{ij}^{P*} and maximizing x_{ij}^{C*} lead to an optimal solution with $x_{ij}^{S*} = 0$, which proves i). Second, let us consider the case of $D_{ij} \leq E_{ij}z$. When $x_{ij}^{P*} = 0$, we easily see that ii) gives an optimal solution of (4) with $p_{ij}^G < p_{ij}^S$. When $x_{ij}^{S*} = 0$, $x_{ij}^{C*} = E_{ij}z$ and $x_{ij}^{P*} = 0$ constitute an optimal solution. By comparing the objective function values of these two solutions, we find that ii) is optimal if $D_{ij} \leq E_{ij}z$.

In i) and ii), constraint (f) reduces to $(\sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i F_{ij}^P E_{ij} - F^M)z \geq G$, which is a constraint for z. Therefore, $(x_{ij}^{C*}, x_{ij}^{P*}, x_{ij}^{S*})$ of i) and ii) is an optimal solution to (4) with any fixed feasible value of z satisfying (d) and (f). □

Using Proposition 3.1, we can rewrite constraint (f) as

$$\sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i F_{ij}^P E_{ij} z - F^M z \geq G, \tag{6}$$

which can then transformed into

$$z \geq \frac{G}{\sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i F_{ij}^P E_{ij} - F^M} := \underline{z}$$

under the assumption that $\sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i F_{ij}^P E_{ij} - F^M > 0$ (this can be reasonably assumed). Therefore, problem (4) transforms into a problem with one variable z:

$$\begin{aligned} &\underset{z}{\text{minimize}} \quad Cz + \sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i \left((p_{ij}^S - p_{ij}^P - p_{ij}^G)\min\{D_{ij}, E_{ij}z\} - p_{ij}^S E_{ij}z + p_{ij}^P D_{ij}\right) \\ &\text{subject to} \quad \text{(d')}: \quad \underline{z} \leq z \leq \overline{z}. \end{aligned} \tag{7}$$

The following theorem proves that the objective function of (7) is concave when $p_{ij}^P + p_{ij}^G \leq p_{ij}^S$ for all i, j. We can easily solve such a problem by evaluating the objective function at $\underline{z}$ or $\overline{z}$ and selecting the one with a smaller function value, since (7) must have an optimal solution at $\underline{z}$ or $\overline{z}$. The objective function of (7) is convex when $p_{ij}^P + p_{ij}^G \geq p_{ij}^S$ for all i, j. Accordingly, the binary search algorithm easily finds an optimal solution z of (7).

In the previous Japanese net tariff system where $p_{ij}^S = p_{ij}^P$, it is possible that $p_{ij}^P + p_{ij}^G \geq p_{ij}^S$. On the other hand, in the current Japanese net tariff system where $p_{ij}^S = 2 \times p_{ij}^P$, we have $p_{ij}^P + p_{ij}^G \leq p_{ij}^S$.

Theorem 3.2 *If $p_{ij}^P + p_{ij}^G \leq p_{ij}^S$ for all i, j, the objective function of* (7) *is concave. On the other hand, if $p_{ij}^P + p_{ij}^G \geq p_{ij}^S$ for all i, j, the objective function is convex.*

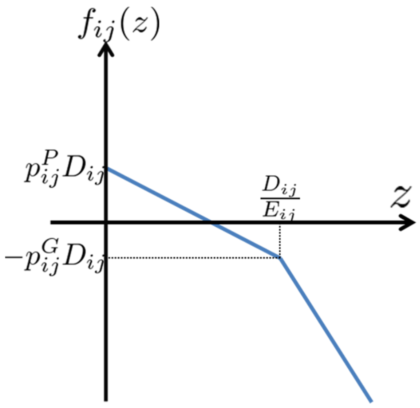

Fig 1: f_{ij} under $p^P_{ij} + p^G_{ij} \leq p^S_{ij}$

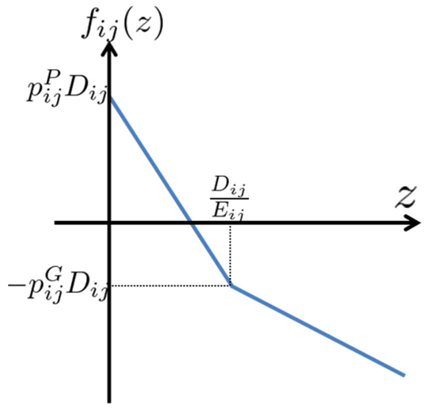

Fig 2: f_{ij} under $p^P_{ij} + p^G_{ij} \geq p^S_{ij}$

Proof. We can express the objective function of (7) as follows:

$$g(z) + \sum_{i=1}^{T} \sum_{j=1}^{H_i} K_i f_{ij}(z),$$

where $f_{ij}(z) = (p^S_{ij} - p^P_{ij} - p^G_{ij})\min\{D_{ij}, E_{ij}z\} - p^S_{ij}E_{ij}z + p^P_{ij}D_{ij}$ and $g(z) = Cz$. We will show that $f_{ij}(z)$, $i = 1, \cdots, T, j = 1, \cdots, H_i$, are convex or concave in z. When $D_{ij} > E_{ij}z$, $f_{ij}(z)$ and its derivative $f'_{ij}(z)$ are

$$f_{ij}(z) = -(p^P_{ij} + p^G_{ij})E_{ij}z + p^P_{ij}D_{ij}, \quad f'_{ij}(z) = -(p^P_{ij} + p^G_{ij})E_{ij},$$

and when $D_{ij} \leq E_{ij}z$, $f_{ij}(z)$ and $f'_{ij}(z)$ are

$$f_{ij}(z) = -p^S_{ij}E_{ij}z + (p^S_{ij} - p^G_{ij})D_{ij}, \quad f'_{ij}(z) = -p^S_{ij}E_{ij}.$$

Note that the derivative of $g(z)$, $g'(z)$, is a constant. When $p^P_{ij} + p^G_{ij} \leq p^S_{ij}$ for all i, j, $f'_{ij}(z)$ is non-increasing with respect to z and the objective function of (7) is concave. On the other hand, when $p^P_{ij} + p^G_{ij} \geq p^S_{ij}$ for all i, j, it is non-decreasing with respect to z and the objective function is convex. Fig 1 shows $f_{ij}(z)$ in the case that $p^P_{ij} + p^G_{ij} \leq p^S_{ij}$, and Fig 2 shows $f_{ij}(z)$ in the other case. $\square$

Remark 3.3 *We can regard optimization model* (3) *under a gross tariff system as a special case of optimization model* (7) *under a net tariff system. Indeed, assuming that* $p^S_{ij} = p^P_{ij} + p^G_{ij}$, (7) *becomes the same as* (3) *with an additional CO2 emission constraint* $\underline{z} \leq z$.

4 Robust Optimization Model for the Best PV Size

In constructing problem (7), we need to predict the PV electricity E_{ij}, electricity demand D_{ij}, and electricity prices of p^S_{ij}, p^P_{ij} and p^G_{ij} for some time in future. We may use the prior yearly data for predicting them. However, there is a large variation in the historical data for E_{ij} based on solar irradiation from year to year. In this section, we regard E_{ij} as uncertainty data and consider (7) as an uncertain optimization problem. We want to find a robust solution for the best size of the PV system that meets the CO2 emission constraint strictly and minimizes the cost under uncertainty. Robust optimization, proposed in [1, 2], is an appropriate approach for this purpose.

Robust optimization is a modeling methodology and set of computational tools to process optimization problems in which the data are uncertain but known to belong to some uncertainty sets. The problem setup is as follows. Let $\mathcal{U}$ denote the uncertainty set for uncertain parameter $\boldsymbol{u}$, *i.e.*, the set of all potentially realizable values of the uncertain parameter. We do not know which value will be realized. For a simple uncertain optimization problem including an uncertain parameter $\boldsymbol{u} \in \mathcal{U}$:

$$\min_{\boldsymbol{x}\in\mathcal{X}} f(\boldsymbol{x}, \boldsymbol{u}),$$

robust optimization deals with the worst-case objective function among $\boldsymbol{u} \in \mathcal{U}$ as follows:

$$\min_{\boldsymbol{x}\in\mathcal{X}} \max_{\boldsymbol{u}\in\mathcal{U}} \quad f(\boldsymbol{x}, \boldsymbol{u}).$$

The above robust optimization formulation assumes that the uncertain parameters will not be observed until all the decision variables are determined. This is not always the case for uncertain optimization problems. In particular, multi-stage decision models involve uncertain parameters some of which are revealed during the decision process. Therefore, a subset of the decision variables can be chosen after these parameters are observed in a way to correct the suboptimality of the decisions made with less information in the earlier stages. In the problem for finding the optimal PV size, $(x^S_{ij}, x^P_{ij}, x^C_{ij})$ can be chosen after observing the PV electricity E_{ij} values, whereas z must be decided before that. Adjustable robust optimization (ARO) formulations model these decision environments by allowing recourse actions to be taken. These models are related to the two-stage (or multi-stage) stochastic programming formulations with recourse. ARO models were recently introduced in [3, 7] for solving uncertain LPs.

We consider a two-stage decision-making environment. We let $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ represent the first and second-stage decision variables, respectively. We assume that parameter $\boldsymbol{u}$ will be realized and observed after the first-stage decision are made but before the second-stage decisions need to be made. Furthermore, we assume that the feasible set for the second-stage decisions depends on the choice of $\boldsymbol{x}_1$ as well as the observed values of the parameters $\boldsymbol{u}$. Therefore, the feasible set is denoted by $\mathcal{X}_2(\boldsymbol{x}_1, \boldsymbol{u})$. The following representation of the ARC problem is from [19]:

$$\min_{\boldsymbol{x}_1\in\mathcal{X}_1} \max_{\boldsymbol{u}\in\mathcal{U}} \min_{\boldsymbol{x}_2\in\mathcal{X}_2(\boldsymbol{x}_1,\boldsymbol{u})} \quad f(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}).$$

We shall formulate the ARC problem for choosing the optimal PV size in an uncertain environment and give a simple solution method for it.

4.1 Robust Optimization for a Net Tariff System

To choose the optimal PV size in an uncertain environment, we make the second-stage decision $(x^S_{ij}, x^P_{ij}, x^C_{ij})$ after observing the PV electricity values E_{ij}, whereas we make the first-stage decision z before that. The robust optimization requires an uncertainty set, *i.e.*, the set of all potentially realizable values of the uncertain parameter $\boldsymbol{E}_i := (E_{i1}, \cdots, E_{iH_i})^\mathsf{T}$. We will construct uncertainty set $\mathcal{U}_i \subset \mathcal{R}^{H_i}$ for PV-generated electricity $\boldsymbol{E}_i$ in the ith period for $i = 1, \ldots, T$. For example, when setting the model parameters as $(T, K_i, H_i) = (12, \text{the number of days in the } i\text{th month}, 24)$, $i = 1, \cdots, T$, we need to construct uncertainty sets $\mathcal{U}_i \in \mathcal{R}^{24}$ for each month, and using the uncertainty set, we can make hourly decisions on $(x^S_{ij}, x^P_{ij}, x^C_{ij})$, $j = 1, \ldots, H_i$.

The uncertainty set $\mathcal{U}_i$ must be appropriate to the situation. If we set $\mathcal{U}_i$ too large, the optimal decision will be very robust to uncertain data $\boldsymbol{E}_i$ but too conservative. Moreover, if we define $\mathcal{U}_i$ with complicated functions, we cannot easily solve the resulting problem. Many robust optimization studies have used polyhedral sets or ellipsoidal sets for $\mathcal{U}_i$ for the sake of computational tractability. We give examples for $\mathcal{U}_i$ later in this subsection.

For the sake of computational tractability, robust optimization problems often assume "constraint-wise uncertainty". This assumption means that the uncertain parameters included in each constraint and objective function do not overlap. However, in this problem setting, uncertain data $\boldsymbol{E}_i$ appear in constraints (b) and (f), and the assumption is violated, because (f) can be represented as (6). Therefore, to make our problem tractable, we assume the worst case for each of constraints (b) and (f).

We formulate a two-stage optimization model for (4) as

$$\min_{\underline{z}\le z\le \overline{z}} \max_{\substack{\boldsymbol{E}_i\in\mathcal{U}_i\\ i=1,\dots,T}} \min_{(\boldsymbol{x}^S,\boldsymbol{x}^P,\boldsymbol{x}^C)\in\mathcal{X}(z,\boldsymbol{E})} \left\{ Cz + \sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i(p_{ij}^P x_{ij}^P - p_{ij}^S x_{ij}^S - p_{ij}^G x_{ij}^C) \right\},$$

where

$$\mathcal{X}(z,\boldsymbol{E}) = \left\{ (\boldsymbol{x}^S,\boldsymbol{x}^P,\boldsymbol{x}^C) : \begin{array}{l} x_{ij}^P + x_{ij}^C \ge D_{ij}, \quad x_{ij}^S + x_{ij}^C \le E_{ij} z, \\ x_{ij}^P, x_{ij}^S, x_{ij}^C \ge 0, \quad x_{ij}^P \times x_{ij}^S = 0, \quad (i=1,\cdots,T,\ j=1,\cdots,H_i) \end{array} \right\}$$

and

$$\underline{z} = \frac{G}{\displaystyle\min_{\substack{\boldsymbol{E}_i\in\mathcal{U}_i\\ i=1,\dots,T}} \sum_{i=1}^{T}\sum_{j=1}^{H_i} K_i F_{ij}^P E_{ij} - F^M}.$$

We set the lower bound $\underline{z}$ to achieve the CO2 reduction target in the worst case. Note that the worst case to maximize the energy cost and the worst case to minimize the CO2 reduction are independent. Using the notation: $p_{ij}^R = p_{ij}^S - p_{ij}^P - p_{ij}^G$ and Proposition 3.1, we can transform the above problem into

$$\min_{\underline{z}\le z\le \overline{z}} \left\{ Cz + \sum_{i=1}^{T} \max_{\boldsymbol{E}_i\in\mathcal{U}_i} K_i \sum_{j=1}^{H_i} \left(p_{ij}^R \min\{D_{ij}, E_{ij}z\} - p_{ij}^S E_{ij} z + p_{ij}^P D_{ij} \right) \right\}. \tag{8}$$

We will focus on the problem in the ith period in (8) for some fixed z:

$$\max_{\boldsymbol{E}_i\in\mathcal{U}_i} K_i \sum_{j=1}^{H_i} \left(p_{ij}^R \min\{D_{ij}, E_{ij}z\} - p_{ij}^S E_{ij} z + p_{ij}^P D_{ij} \right). \tag{9}$$

Using the H_i-dimensional vectors,

$$\boldsymbol{p}_i^R = \begin{pmatrix} p_{i1}^R \\ \vdots \\ p_{iH_i}^R \end{pmatrix},\ \boldsymbol{p}_i^S = \begin{pmatrix} p_{i1}^S \\ \vdots \\ p_{iH_i}^S \end{pmatrix},\ \boldsymbol{p}_i^P = \begin{pmatrix} p_{i1}^P \\ \vdots \\ p_{iH_i}^P \end{pmatrix},\ \boldsymbol{D}_i = \begin{pmatrix} D_{i1} \\ \vdots \\ D_{iH_i} \end{pmatrix},$$

we can rewrite (9) in a more compact form:

$$\begin{array}{ll} \underset{x_{i0},\boldsymbol{x}_i,\boldsymbol{\xi}_i}{\text{maximize}} & K_i \left({\boldsymbol{p}_i^R}^\top \boldsymbol{\xi}_i - {\boldsymbol{p}_i^S}^\top \boldsymbol{x}_i + {\boldsymbol{p}_i^P}^\top \boldsymbol{D}_i \right) \\ \text{subject to} & \boldsymbol{\xi}_i \le \boldsymbol{D}_i, \quad \boldsymbol{\xi}_i \le \boldsymbol{x}_i, \quad x_{i0} = z, \quad (x_{i0},\boldsymbol{x}_i) \in \overline{\mathcal{U}_i}, \end{array} \tag{10}$$

where $\overline{\mathcal{U}_i} = \left\{ (x_{i0},\boldsymbol{x}_i) : \boldsymbol{x}_i = \boldsymbol{E}_i x_{i0},\ \boldsymbol{E}_i \in \mathcal{U}_i \right\}$. The equivalence between (9) and (10) is obvious, since at the optimality of (10), one has

$$\boldsymbol{x}_i^* = \begin{pmatrix} E_{i1}z \\ \vdots \\ E_{iH_i}z \end{pmatrix} \quad \text{and} \quad \boldsymbol{\xi}_i^* = \begin{pmatrix} \min\{D_{i1}, E_{i1}z\} \\ \vdots \\ \min\{D_{iH_i}, E_{iH_i}z\} \end{pmatrix}.$$

Note that $\overline{\mathcal{U}_i}$ is a convex cone as long as $\mathcal{U}_i$ is a convex set. Taking the dual of the cone programming problem (10) leads to

$$\begin{array}{ll} \underset{\boldsymbol{\alpha}_i,\boldsymbol{\beta}_i,\boldsymbol{\gamma}_i,\eta,\delta}{\text{minimize}} & -z\eta + \boldsymbol{D}_i^\top \boldsymbol{\alpha}_i + K_i {\boldsymbol{p}_i^P}^\top \boldsymbol{D}_i \\ \text{subject to} & \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i = K_i \boldsymbol{p}_i^R, \quad \boldsymbol{\beta}_i + \boldsymbol{\gamma}_i = K_i \boldsymbol{p}_i^S, \\ & \boldsymbol{\alpha}_i \geq \mathbf{0}, \quad \boldsymbol{\beta}_i \geq \mathbf{0}, \quad \delta + \eta = 0, \quad (\delta, \boldsymbol{\gamma}_i) \in \overline{\mathcal{U}_i}^*, \end{array} \tag{11}$$

where the dual cone $\overline{\mathcal{U}_i}^*$ of $\overline{\mathcal{U}_i}$ is

$$\overline{\mathcal{U}_i}^* = \Big\{(\delta, \boldsymbol{\gamma}_i) : x_{i0}\delta + \boldsymbol{x}_i^\top \boldsymbol{\gamma}_i \geq 0, (x_{i0}, \boldsymbol{x}_i) \in \overline{\mathcal{U}_i}\Big\}.$$

The constraint $(\delta, \boldsymbol{\gamma}_i) \in \overline{\mathcal{U}_i}^*$ can be replaced by $-\eta \geq \max_{\boldsymbol{E}_i \in \mathcal{U}_i} (-K_i p_i^S + \boldsymbol{\beta}_i)^\top \boldsymbol{E}_i$ by using the constraints $\boldsymbol{\gamma}_i = K_i p_i^S - \boldsymbol{\beta}_i$ and $\eta = -\delta$ of (11). Therefore, (11) transforms into

$$\begin{array}{ll} \underset{\boldsymbol{\alpha}_i,\boldsymbol{\beta}_i}{\text{minimize}} & \max_{\boldsymbol{E}_i \in \mathcal{U}_i} (-K_i \boldsymbol{p}_i^S + \boldsymbol{\beta}_i)^\top \boldsymbol{E}_i z + \boldsymbol{D}_i^\top \boldsymbol{\alpha}_i + K_i {\boldsymbol{p}_i^P}^\top \boldsymbol{D}_i \\ \text{subject to} & \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i = K_i \boldsymbol{p}_i^R, \quad \boldsymbol{\alpha}_i \geq \mathbf{0}, \quad \boldsymbol{\beta}_i \geq \mathbf{0}. \end{array}$$

Using the above equivalent transformation of problem (9), problem (8) results in

$$\begin{array}{ll} \underset{\boldsymbol{\alpha},\boldsymbol{\beta},\underline{z}\leq z\leq \overline{z}}{\text{minimize}} & Cz + \sum_{i=1}^{T} \Big\{ \max_{\boldsymbol{E}_i \in \mathcal{U}_i} (-K_i \boldsymbol{p}_i^S + \boldsymbol{\beta}_i)^\top \boldsymbol{E}_i z + \boldsymbol{D}_i^\top \boldsymbol{\alpha}_i + K_i {\boldsymbol{p}_i^P}^\top \boldsymbol{D}_i \Big\} \\ \text{subject to} & \mathcal{S} := \big\{(\boldsymbol{\alpha}, \boldsymbol{\beta}) : \ \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i = K_i \boldsymbol{p}_i^R, \ \boldsymbol{\alpha}_i \geq \mathbf{0}, \ \boldsymbol{\beta}_i \geq \mathbf{0}, \quad (i = 1, \cdots, T)\big\}. \end{array} \tag{12}$$

The optimal solution z of (12) is easily obtained, since it exists at $\underline{z}$ or $\overline{z}$ as shown later. In fact, it is enough to solve (12) with fixed $z = \underline{z}$ and $z = \overline{z}$ and choose the solution with the smaller objective function value as the optimal one. For a compact convex set $\mathcal{U}_i$, let $\phi_i(\boldsymbol{\beta}_i)$ be the optimal value of $\max_{\boldsymbol{E}_i \in \mathcal{U}_i} (-K_i \boldsymbol{p}_i^S + \boldsymbol{\beta}_i)^\top \boldsymbol{E}_i$, that is a coefficient of z in the objective function of (12). By denoting the objective function of (12) as $\Phi(\boldsymbol{\alpha}, \boldsymbol{\beta}, z)$, i.e.,

$$\Phi(\boldsymbol{\alpha}, \boldsymbol{\beta}, z) := Cz + \sum_{i=1}^{T} \Big\{ \phi_i(\boldsymbol{\beta}_i) z + \boldsymbol{D}_i^\top \boldsymbol{\alpha}_i + K_i {\boldsymbol{p}_i^P}^\top \boldsymbol{D}_i \Big\},$$

minimizing the function with respect to the feasible $(\boldsymbol{\alpha}, \boldsymbol{\beta}, z)$ is equivalent to minimizing

$$h(z) := \min_{(\boldsymbol{\alpha},\boldsymbol{\beta}) \in \mathcal{S}} \Phi(\boldsymbol{\alpha}, \boldsymbol{\beta}, z)$$

with respect to $z \in [\underline{z}, \overline{z}]$. Accordingly, $\Phi(\boldsymbol{\alpha}, \boldsymbol{\beta}, z)$ is linear in z for fixed $(\boldsymbol{\alpha}, \boldsymbol{\beta}) \in \mathcal{S}$, and therefore, the objective function $h(z)$ is piecewise-linear concave in z (see Fig 3). Hence, to derive the optimal value, it is enough to solve $\min_{(\boldsymbol{\alpha},\boldsymbol{\beta}) \in \mathcal{S}} \Phi(\boldsymbol{\alpha}, \boldsymbol{\beta}, z)$ with a fixed $z = \underline{z}$ and $z = \overline{z}$, and choose the optimal solution z^* with the smaller objective function value between $h(\underline{z})$ and $h(\overline{z})$.

The problem with fixed $z = \underline{z}$ or $z = \overline{z}$ is still difficult to compute because of $\phi_i(\boldsymbol{\beta}_i) := \max_{\boldsymbol{E}_i \in \mathcal{U}_i} (-K_i \boldsymbol{p}_i^S + \boldsymbol{\beta}_i)^\top \boldsymbol{E}_i$ in the objective function. If $\mathcal{U}_i$ is described as a polyhedral set defined by

$$\mathcal{U}_i = \Big\{ \boldsymbol{E}_i : \boldsymbol{A}_i \boldsymbol{E}_i \leq \boldsymbol{b}_i \Big\}, \tag{13}$$

we can obtain the dual problem of $\phi_i(\boldsymbol{\beta}_i) = \max_{\boldsymbol{E}_i \in \mathcal{U}_i} (-K_i \boldsymbol{p}_i^S + \boldsymbol{\beta}_i)^\top \boldsymbol{E}_i$ as

$$\begin{array}{ll} \underset{\boldsymbol{\tau}_i}{\text{minimize}} & \boldsymbol{b}_i^\top \boldsymbol{\tau}_i \\ \text{subject to} & \boldsymbol{A}_i^\top \boldsymbol{\tau}_i = -K_i \boldsymbol{p}_i^S + \boldsymbol{\beta}_i, \quad \boldsymbol{\tau}_i \geq \mathbf{0}, \end{array}$$

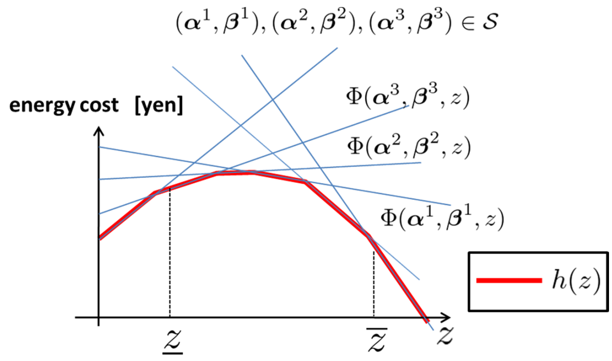

Fig 3: Optimal value function $h(z)$ in z.

and therefore, the problem of $h(\underline{z}) = \min_{(\boldsymbol{\alpha},\boldsymbol{\beta})\in\mathcal{S}} \Phi(\boldsymbol{\alpha},\boldsymbol{\beta},\underline{z})$ now becomes an LP:

$$\begin{array}{ll}
\underset{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau}}{\text{minimize}} & C\underline{z} + \sum_{i=1}^{T}\left\{\underline{z}\boldsymbol{b}_i^\top\boldsymbol{\tau}_i + \boldsymbol{D}_i^\top\boldsymbol{\alpha}_i + K_i{\boldsymbol{p}_i^P}^\top\boldsymbol{D}_i\right\} \\
\text{subject to} & \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i = K_i\boldsymbol{p}_i^R, \quad \boldsymbol{A}_i^\top\boldsymbol{\tau}_i = -K_i\boldsymbol{p}_i^S + \boldsymbol{\beta}_i, \quad (i=1,\cdots,T) \\
 & \boldsymbol{\alpha}_i \geq \boldsymbol{0}, \quad \boldsymbol{\beta}_i \geq \boldsymbol{0}, \quad \boldsymbol{\tau}_i \geq \boldsymbol{0}, \quad (i=1,\cdots,T).
\end{array}$$

The problem of $h(\overline{z})$ similarly reduces to an LP.

If we assume ellipsoidal uncertainty set [1, 2] for $\mathcal{U}_i$ using matrix $\boldsymbol{A}_i$ as

$$\mathcal{U}_i = \left\{\boldsymbol{E}_i = \boldsymbol{a}_i^0 + \boldsymbol{A}_i\boldsymbol{u}_i \ : \ ||\boldsymbol{u}_i|| \leq 1\right\}, \tag{14}$$

the coefficient $\phi_i(\boldsymbol{\beta}_i)$ reduces to

$$\phi_i(\boldsymbol{\beta}_i) := \max_{\boldsymbol{E}_i\in\mathcal{U}_i}(-K_i\boldsymbol{p}_i^S + \boldsymbol{\beta}_i)^\top\boldsymbol{E}_i = {\boldsymbol{a}_i^0}^\top(-K_i\boldsymbol{p}_i^S + \boldsymbol{\beta}_i) + ||\boldsymbol{A}_i^\top(-K_i\boldsymbol{p}_i^S + \boldsymbol{\beta}_i)||.$$

Therefore, $\min_{(\boldsymbol{\alpha},\boldsymbol{\beta})\in\mathcal{S}} \Phi(\boldsymbol{\alpha},\boldsymbol{\beta},z)$ with fixed z is a second order cone programming (SOCP) problem. We can solve such an SOCP by using SeDuMi [18]. If $\mathcal{U}_i$ is described by a particular set such as box, polyhedral or ellipsoidal set, $\min_{(\boldsymbol{\alpha},\boldsymbol{\beta})\in\mathcal{S}} \Phi(\boldsymbol{\alpha},\boldsymbol{\beta},z)$ with fixed z reduces to a tractable convex programming problem. In such cases, we can easily obtain $h(\underline{z})$ and $h(\overline{z})$.

4.2 Robust Optimization for a Gross Tariff System

The optimization model (3) for a gross tariff system is a special case of model (7) for a net tariff system, as Remark 3.3 shows. We can regard (12) with $\boldsymbol{p}_i^R = (\boldsymbol{p}_i^S - \boldsymbol{p}_i^P - \boldsymbol{p}_i^G) = \boldsymbol{0}$ as a robust variant of (3).

Indeed, the robust variant of (3) can be written as the following simplified problem:

$$\min_{\underline{z}\leq z\leq\overline{z}} Cz + \sum_{i=1}^{T}\left\{\max_{\boldsymbol{E}_i\in\mathcal{U}_i} -K_i{\boldsymbol{p}_i^S}^\top\boldsymbol{E}_i z + K_i{\boldsymbol{p}_i^P}^\top\boldsymbol{D}_i\right\}. \tag{15}$$

This is the same as the robust model (12) with $\boldsymbol{p}_i^R = \boldsymbol{0}$, because the optimal $(\boldsymbol{\alpha}_i,\boldsymbol{\beta}_i)$ of (12) must be zero vectors, due to $\boldsymbol{p}_i^R = \boldsymbol{0}$. We can solve the robust problem in exactly the same way as solving (12). If the uncertainty set is described as (14), we obtain

$$\max_{\boldsymbol{E}_i\in\mathcal{U}_i} -K_i{\boldsymbol{p}_i^S}^\top\boldsymbol{E}_i = -K_i{\boldsymbol{a}_i^0}^\top\boldsymbol{p}_i^S + K_i||\boldsymbol{A}_i^\top\boldsymbol{p}_i^S||.$$

Thus, to obtain the optimal solution of (15), we only have to solve an SOCP with $z = \underline{z}$ and one with $z = \overline{z}$.

Table 3: Nominal data in our model (7)

p^S_{ij}	48 yen/kWh†	$i = 1, \cdots, T,\ j = 1, \cdots, 24$
p^P_{ij}	8.41 yen/kWh	$i = 1, \cdots, T,\ j = 1, \cdots, 6, 23, 24$
	26.75 yen/kWh	$i = 1, \cdots, T,\ j = 7, \cdots, 22$
p^G_{ij}	5 yen/kWh	$i = 1, \cdots, T,\ j = 1, \cdots, 24$
C	31,500 yen/kW	
$\overline{z}$	15000 kW	
K_i	1 day	$i = 1, \cdots, T$
T	365 periods	
H_i	24 hour	$i = 1, \cdots, T$
F^P_{ij}	0.391 kg-CO_2/kWh	$i = 1, \cdots, T,\ j = 1, \cdots, 6, 23, 24$
	0.514 kg-CO_2/kWh	$i = 1, \cdots, T,\ j = 7, \cdots, 22$
F^M	70 kg-CO_2/kW-year	
G	0.25 $\times CO_2$ of 1990's electricity production	

† The new Japanese tariff system sets p^S_{ij} to 48, whereas the old system sets it equal to p^P_{ij}.

5 Numerical Experiments

We present a computational study to evaluate our model (7) for the new net metering policy of Japan and its robust variant (12). Our experiments used data on $5,000$ households in the Tohoku region of Japan. Electricity in this region is provided by Tohoku Electric Power Co., Inc. Electricity purchase prices for residential buildings are determined by the company. Table 3 shows the data necessary for (7) and (12). F^P_{ij} (CO2 emission factor of purchased electricity) and F^M (CO2 emissions from production of silicon PV modules) are taken from [12, 15]. The upper bound on the size of the PV system $\overline{z}$ was $15,000$ kW. This is a reasonable upper bound, because it is achievable if all 5,000 households install PV systems with a maximum of 3 kW each. To estimate the PV-generated electricity E_{ij}, we used global solar irradiation data for Sendai, a city in the Tohoku region, from [11].

We also used the average hourly electricity demand per household, shown in [20], to which we multiplied $5,000$ as the electricity demand D_{ij} of 5,000 households. Throughout the numerical experiments, we used the increase in energy costs as the "energy cost". The increase is defined as the change resulting from introducing the PV system, i.e.,

increased energy cost [yen] = optimal value of (7) - total cost of not installing PV system.

A negative value of the increase in energy cost means that the total energy cost is reduced as a result of introducing the PV system.

5.1 Evaluation of Optimal Solutions for Model (7)

First, we compared our model (7) with the existing LP model (2) given the net feed-in tariff system. In the case of $p^S_{ij} = p^P_{ij}$, both models provide the same optimal solution $(z, \boldsymbol{x}^S, \boldsymbol{x}^P, \boldsymbol{x}^C)$ and the same energy cost. Fig 4 shows the optimal increase in energy costs for (2) and (7) with a fixed PV size z. We find that energy costs of both models are the same for every z and that the optimal solution of both models exists at $\underline{z}$. Fig 5 depicts the identical optimal solution $(x^S_{ij}, x^P_{ij}, x^C_{ij})$, $j = 1, \ldots, 24$, of both models in the ith day (concretely, May 1st, 2009). In the optimal solution, electricity was not bought and sold at the same time. On the other hand,

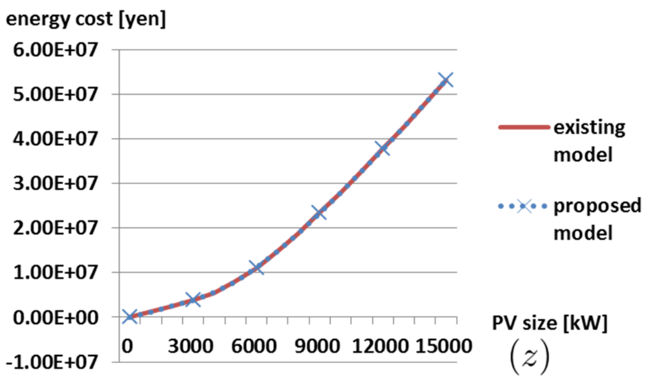

Fig 4: Annual increase in energy costs with $p^S_{ij} = p^P_{ij}$.

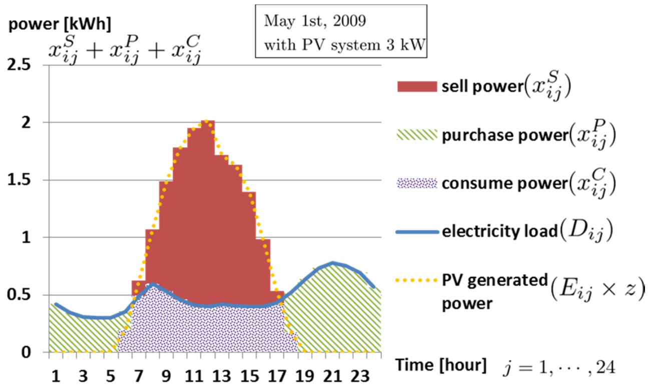

Fig 5: Optimal energy trading in a day (1st May) with existing model (2) under $p^S_{ij} = p^P_{ij}$. Our model (7) gives the same results with $p^S_{ij} = p^P_{ij}$ (and also with $p^S_{ij} = 48$).

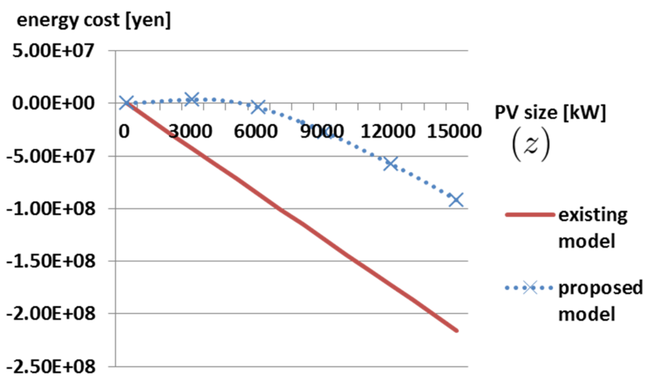

Fig 6: Annual increase in energy costs with $p^S_{ij} = 48$.

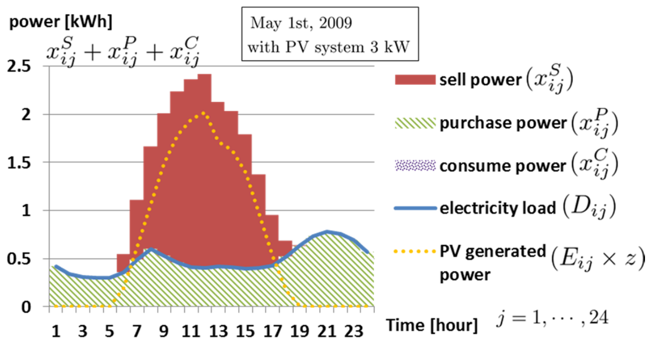

Fig 7: Optimal energy trading in a day (1st May) with existing model (2) under $p^S_{ij} = 48$.

in the case of $p_{ij}^S = 48$ yen/kWh, the models gave different optimal solutions (see Fig 6). The difference was caused by an unrealizable solution using the existing model (2), where $x_{ij}^P > 0$ and $x_{ij}^S > 0$. It happened during the 6 am-18 pm time period, as shown in Fig 7. Therefore, we ought to use our model (7) with the current Japanese net tariff system when $p_{ij}^S > p_{ij}^P$. In this case, the upper bound $\overline{z}$ of the PV size z corresponds to the optimal solution of our model (7).

Second, we investigated the sensitivity of the optimal solution of our model (7) for each z by changing the input parameters of the model. Fig 8 shows the annual increase in energy costs and total electricity for each PV size z. We can see that the optimal value is concave in z, and the optimal PV size is attained at $\overline{z}$. We can also see that purchased energy becomes smaller as the PV size z becomes larger.

Fig 9 shows the total CO2 reduction to be had by installing a z kW PV system. The CO2 emissions during manufacture of the PV system $F^M \times z$ increase in proportion to the PV size z. However, the increases in sold electricity and self-consumed electricity decrease the amount of CO2, since they reduce the amount of electricity generated by electric power companies. In total, CO2 emissions decrease as z grows. Fig 10 shows the lower bound of the PV size, $\underline{z}$, to reach the CO2 reduction goal. The lower bound $\underline{z}$ becomes large when we set the CO2 reduction goal to a high value.

Fig 11 shows the relation between annual increase in energy costs and lifetime of the PV system for the three cases of $z = 5,000$, $z = 10,000$ and $z = 15,000$. If the PV lifetime is less than 17 years, increases in energy costs are all positive for the three cases of z. This implies that installing a PV system will cost much more than not installing one if the PV lifetime is less than 17 years. Furthermore, at 15 years where the three curves cross, the optimal PV size changes; a large-size PV system is preferable if PV lifetime is more than 15 years, and a small-size PV system is preferable, otherwise.

Fig 12 shows the impact of changing the electricity sales price p_{ij}^S on the annual increase in energy costs. The electricity sales price directly affects the optimal energy cost of (7). Given a $15,000$ kW PV system, $p^S = 34$ yen/kWh is the threshold electricity price at which we can get a profit. Furthermore, price fluctuations have a larger effect on a large PV system. We can see that if $p^S < 34$ yen/kWh, $\underline{z}$ is the optimal PV size, whereas $\overline{z}$ is optimal if $p^S > 34$.

5.2 Evaluation of the Robust Model (12)

We examined the robust variant (12) that incorporates uncertain weather conditions in the deterministic model (7). In Section 5.1, we estimated the amount of PV-generated electricity using historical solar irradiation data in 2009 for our model (7). The optimal PV size z^* and increase in energy costs change depending on the solar irradiation data (see Fig 13). This figure shows the minimum increase in energy costs for each size of the PV system in the following four scenarios for solar irradiation data (based on historical daily (24 hours) solar irradiation data during 2000-2009):

sum_day: solar irradiation data for 365 days. The minimum daily irradiation (i.e., daily sum of irradiations) for each calender date was chosen from data recorded during 2000-2009.

sum_month: solar irradiation data for 12 months. The minimum monthly irradiation (i.e., monthly sum of the irradiation) for each calender month was chosen from data recorded during 2000-2009.

sum_year: solar irradiation data in the selected year. The minimum yearly irradiation (i.e., yearly sum of the irradiation) was chosen from data recorded during 2000-2009.

2009year: solar irradiation data in 2009.

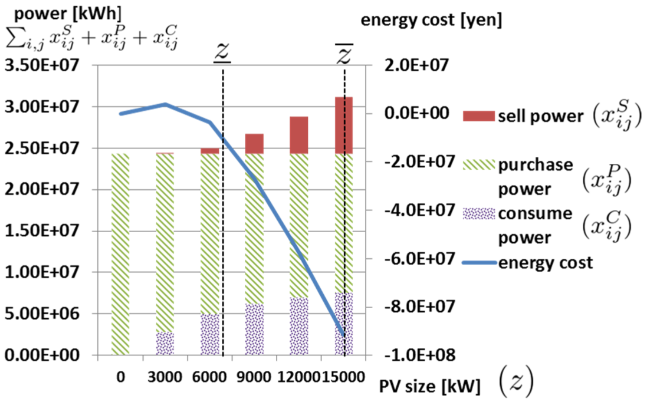

Fig 8: Annual increase in energy costs and total electricity.

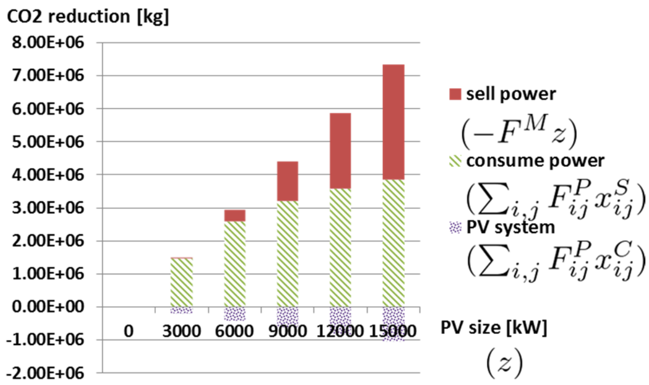

Fig 9: Amount of CO2 reduction.

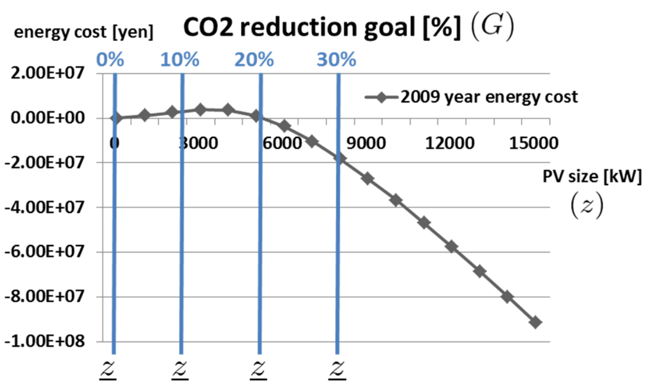

Fig 10: Minimum requirement of PV size z to achieve a CO2 reduction goal.

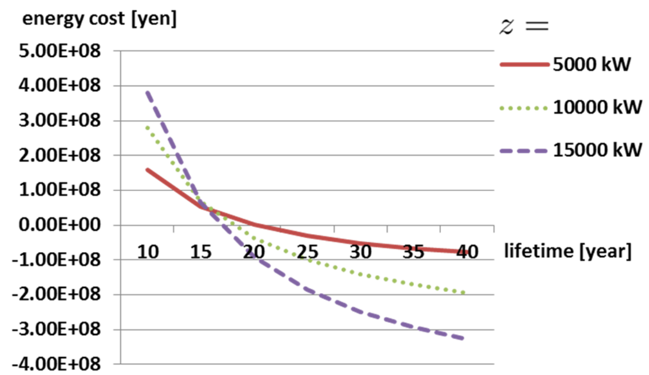

Fig 11: Relationship between annual increase in energy costs and PV system's lifetime.

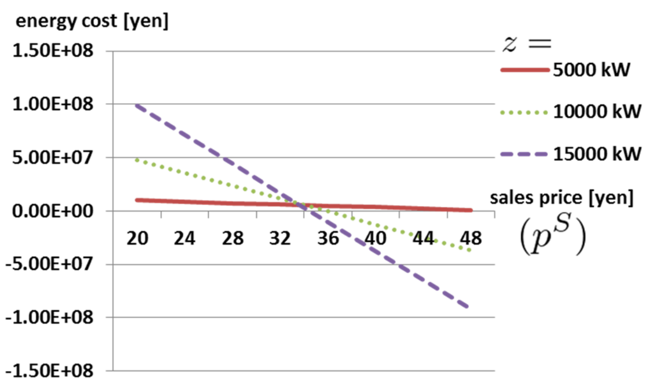

Fig 12: Relationship between annual increase in energy costs and electricity sales price p_{ij}^S.

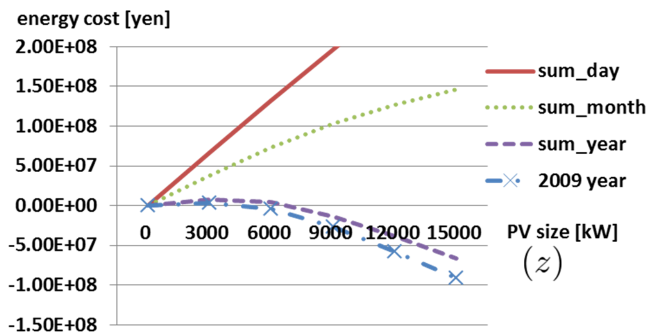

Fig 13: Increase in energy costs in four scenarios of solar irradiation.

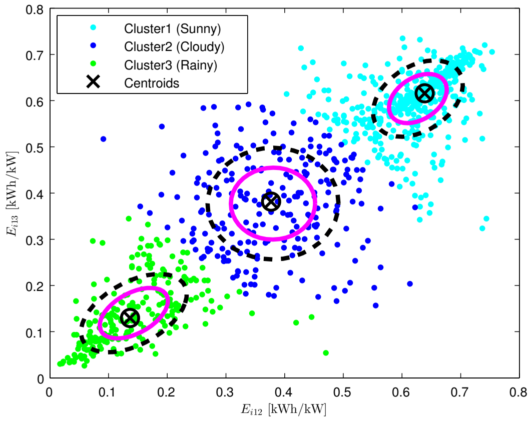

Fig 14: Two-dimensional projections (12-13PM) of confidence ellipsoids for three weather conditions (three clusters) in Spring. Each plot shows PV-generated electricity E_{ij} during 2000-2009 Spring and it is assigned to one of three clusters. Ellipsoids contain $100(1-\alpha)\%$ of normally distributed data of each cluster. Centroids: α=1, Pink ellipsoids: α=0.75, Black ellipsoids: α=0.5.

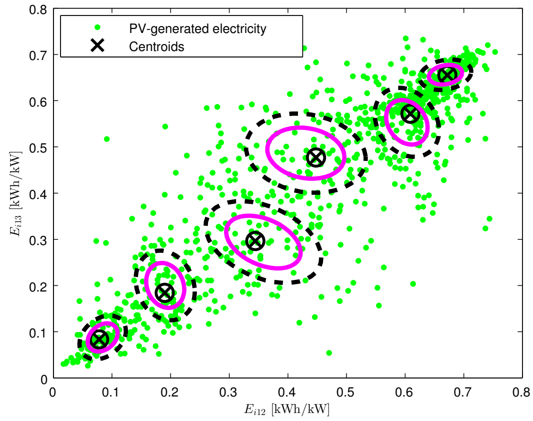

Fig 15: Two-dimensional projections (12-13PM) of confidence ellipsoids for six weather conditions (six clusters) in Spring. Each plot shows PV-generated electricity E_{ij} during 2000-2009 Spring. Centroids: α=1, Pink: α=0.75, Black: α=0.5.

sum_day is the most "pessimistic" irradiation data for PV electricity E_{ij}. The 2009 data would be reasonable if similar weather conditions occurred again. However, to meet a CO2 reduction constraint in the future, the optimal solution obtained with only the 2009 irradiation data is unreliable.

The objective function is concave in z, and therefore, the smaller of $\underline{z}$ and $\overline{z}$ is the optimal value. The optimal solution is $z^* = \underline{z}$ kW in the cases of "sum_day" and "sum_month", whereas the optimal solution is $z^* = \overline{z}$ kW in the remaining cases. We can see that the optimal solution z^* and the optimal energy cost depend on the assumed solar irradiation. Therefore, we need to apply a robust optimization approach to solve the PV size problem for uncertain irradiation data. To do so, we need to define the uncertainty set, i.e., the set of all potentially realizable values of the uncertain parameter, for PV-generated electricity $\boldsymbol{E}_i$ by using historical irradiation data. The way of constructing the uncertainty set $\mathcal{U}_i$ is a very important consideration in practice. Below, we try two kinds of uncertainty set: ellipsoidal uncertainty and polyhedral uncertainty.

Ellipsoidal uncertainty: The setting of the ellipsoidal uncertainty (14) is as follows: To start with, collect 10-year hourly irradiation data running the course of 2000-2009, compute a set of 24-dimensional vectors $\widehat{\boldsymbol{E}}$ by (1) and classify it into several groups (for example, sunny/cloudy/rainy days) by using the K-means method. Then, for each group i, estimate the normal distribution of PV-generated electricity $\boldsymbol{E}_i$ by using the mean and covariance matrix from data $\widehat{\boldsymbol{E}}$ of the group. Finally, construct an ellipse-shaped joint $100(1-\alpha)\%$ confidence region $\mathcal{U}_i$ for $\boldsymbol{E}_i$ in each group, $i = 1, \ldots, T$. The smaller the confidence level α is, the larger the confidence ellipse will be. We regard the ellipsoid as the uncertainty set for each group.

The amount of light which comes from the sky varies widely according to the season of year. We set the four seasons as follows: Spring (March 1 - May 31), Summer (Jun 1 - August 31), Autumn (September 1 - November 30) and Winter (December 1 - February 28). It is appropriate that the uncertainty sets take these seasons into account in addition to the above-mentioned groups of weather conditions.

For example, we assumed three weather conditions (sunny/cloudy/rainy) for each of the four seasons. Fig 14 shows three clusters of PV-generated electricity $\boldsymbol{E}_i$ that were constructed by applying the K-means method to Spring empirical PV electricity data 2000-2009. We call the case *three clusters*. We defined 12 uncertainty sets in total for a one-year PV electricity data set by assuming three weather conditions for all seasons. In Fig 15, we similarly assumed six weather conditions, i.e., *six clusters*, for $\boldsymbol{E}_i$ of Spring irradiation data. In total, we defined 24 uncertainty sets through a year (i.e., six uncertainty sets for each season).

We constructed robust optimization model (12) using the uncertainty set $\mathcal{U}_i \subset \mathcal{R}^{H_i}$ for $\boldsymbol{E}_i$ and the number of days K_i in each group, $i = 1, \ldots, T$. We experimented with different numbers of weather conditions; three clusters for each season or six clusters for each season. We set (4 $\times$ (the number of clusters),24) to (T, H_i) and set the number of days, assigned by the K-means method for the ith cluster, $i = 1, \ldots, T$, to K_i.

We briefly show how to construct the ellipsoidal uncertainty set of (14) for PV-generated electricity $\boldsymbol{E}_i$ in the ith group. We assume that $\boldsymbol{E}_i \sim N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$, where the mean $\boldsymbol{\mu}_i$ and covariance matrix $\boldsymbol{\Sigma}_i$ are computed from observations in the ith group. The confidence ellipsoid with confidence level of α is expressed as $(\boldsymbol{E}_i - \boldsymbol{\mu}_i)^\top \boldsymbol{\Sigma}_i^{-1}(\boldsymbol{E}_i - \boldsymbol{\mu}_i) \leq \chi^2(\alpha)$, where $\chi^2(\alpha)$ follows chi-square distribution. Thus, we can transform the ellipsoid into the follow form:

$$||\frac{\boldsymbol{\Sigma}_i^{-\frac{1}{2}}}{\sqrt{\chi^2(\alpha)}}(\boldsymbol{E}_i - \boldsymbol{\mu}_i)|| \leq 1, \tag{16}$$

which implies correspondence with (14) through $\boldsymbol{a}_i^0 = \boldsymbol{\mu}_i$ and matrix $\boldsymbol{A}_i = \sqrt{\chi^2(\alpha)}\boldsymbol{\Sigma}_i^{\frac{1}{2}}$. As shown in the previous section, the robust formulation with ellipsoidal uncertainty sets results in an SOCP. Figs 14 and 15 draw ellipsoids of different sizes; centroids for $\alpha = 1$, pink ellipsoids for $\alpha = 0.75$ and black ones for $\alpha = 0.5$.

Fig 16 and Fig 17 show the annual increase in energy costs for each PV size z on the assumption of three clusters and six clusters for each season, respectively. We can see that increase in energy costs is large when the range of uncertainty is wide (i.e., α is small) and the number of clusters is small. We can confirm that robust optimal solutions are in $\underline{z}$ or $\overline{z}$ and the solutions of Fig 17 are less conservative than those of Fig 16. Indeed, for the case of $\alpha = 0.75$, the costs of PV systems are more favorably evaluated in Fig 17 than in Fig 16. This is caused by mitigated worst-case scenarios in six clusters. For example, in the extreme case with $\alpha \approx 0$ and only one cluster, we have to assume the extremely worst-case scenario such as a day with no sunshine at all through a year.

Table 4 evaluates robust optimal solutions in terms of achievement rate of the CO2 reduction goal using new data set; artificially generated dataset and unused historical data of 2010. To compute achievement rates, we used randomly generated 100 sets of test samples from $N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$, $i = 1, \ldots, T$. We see that the parameter $\alpha = 0.5$ gives us a rather robust solution that satisfies the CO2 reduction goal with high probability in the case of three clusters. For unused historical data of 2010, all robust optimal solutions satisfied the CO2 reduction goal. We also can see that the minimum requirement $\underline{z}$ for achieving the goal increases when α becomes small or the number of clusters becomes small.

Polyhedral uncertainty: Here, we do not assume that $\boldsymbol{E}_i$ follows a probability distribution when setting the uncertainty sets $\mathcal{U}_i$. Instead, we use a polyhedral uncertainty set (13) for $\mathcal{U}_i$ by taking the convex hull of each year's PV generation $\boldsymbol{E}_i \in \mathcal{R}^{H_i}$ on 1st May during 2000-2009 (see Fig 18). We considered two different polyhedral uncertainty sets: a one-year uncertainty set and four seasonal uncertainty sets. Regarding the one-year uncertainty set, we assumed $(T, K_i, H_i) = (1, 1, 365 \times 24)$ and constructed a one-year uncertainty set $\mathcal{U}_1$ for $\boldsymbol{E}_1 \in \mathcal{R}^{H_1}$. Regarding the seasonal uncertainty sets, we assumed $(T, K_i, H_i) = (4, 1, \text{the number of days in the } i\text{th period} \times$

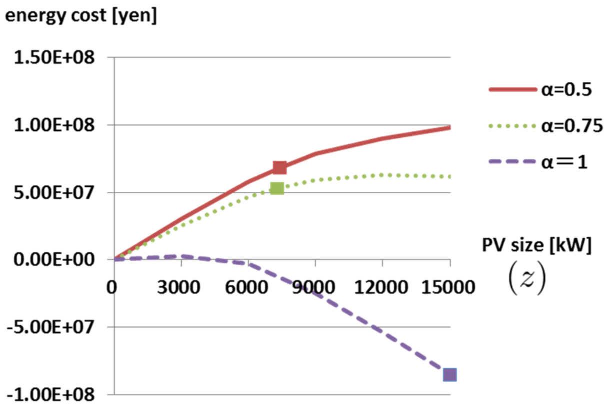

Fig 16: Three clusters: comparison of increased energy costs among robust optimization models (12) with three confidence ellipsoids. Ellipsoids contain $100(1-\alpha)\%$ of normally distributed data. The squares show the robust optimal solutions which can achieve the CO2 reduction goal.

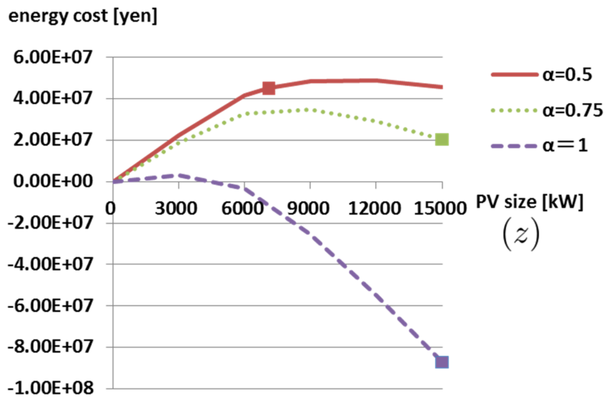

Fig 17: Six clusters: comparison of increased energy costs among robust optimization models (12) with six confidence ellipsoids. The squares show the robust optimal solutions which can achieve the CO2 reduction goal.

Table 4: Achievement rates of the CO2 reduction goal of 25 % by 2020

Three clusters	$\alpha=0.5$	$\alpha=0.75$	$\alpha=1$
$\underline{z}$	7397	7286	6864
Rate for normal distributed data	90%	87%	60%
Evaluation for new data of 2010	$\surd$	$\surd$	$\surd$
robust optimal solution z^*	7397	7286	15000
Six clusters	$\alpha=0.5$	$\alpha=0.75$	$\alpha=1$
$\underline{z}$	7114	7064	6864
Rate for normal distributed data	84%	83%	49%
Evaluation for new data of 2010	$\surd$	$\surd$	$\surd$
robust optimal solution z^*	7114	15000	15000

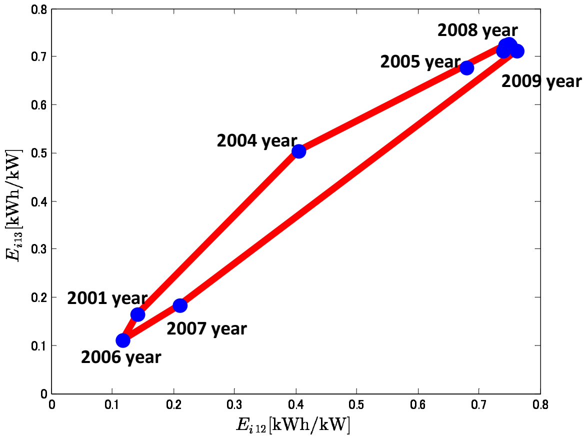

Fig 18: Two-dimensional projections (12-13PM) of polyhedral uncertainty set for $\boldsymbol{E}_1 \in \mathcal{R}^{H_1}$ on 1st May during 2000-2009.

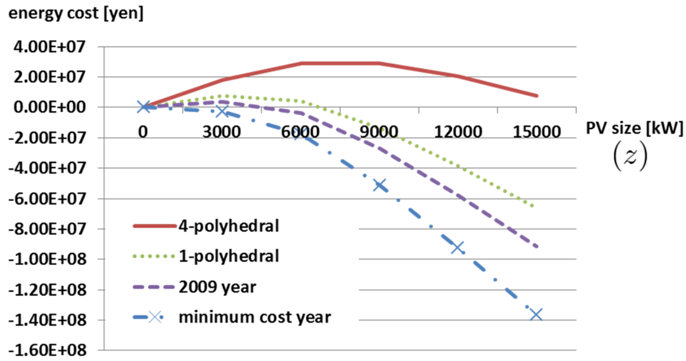

Fig 19: Comparison of increased energy costs among robust optimization models: (12) with seasonal uncertainty sets (4-polyhedral), (12) with one-year uncertainty set (1-polyhedral), deterministic model (7) with 2009 data, and model (7) with 2004 data (2004 is the year with the largest annual solar irradiation during 2000-2009).

24) and constructed seasonal uncertainty sets $\mathcal{U}_i$ for $\boldsymbol{E}_i \in \mathcal{R}^{H_i}$, $i = 1, \ldots, T$. The robust model with polyhedral uncertainty results in an LP model given a fixed z. Fig 19 shows the minimum increase in energy costs with a fixed z for four models: robust optimization model (12) with the four seasonal uncertainty sets, the model (12) with the one-year uncertainty set, the deterministic model (7) with 2009 solar irradiation data, and the model (7) with 2004 irradiation data. Here, 2004 is the year with the largest annual solar irradiation during 2000-2009.

The robust model (12) with seasonal uncertainty sets provides the most pessimistic solution among these four models, since the model assumes the worst case in $\mathcal{U}_i$ for PV electricity $\boldsymbol{E}_i$ in each season. On the other hand, the robust model (12) with the one-year uncertainty set provides a solution that is equal to the worst-case solution among the optimal ones of the deterministic models (7) which use each of the 2000-2009 data sets.

With the exception of the robust model (12) with four seasonal uncertainty sets, the results shown in Fig 19 imply the positive economic impacts of PV systems. The results may be less positive if the uncertainty sets had a much more pessimistic setting. The robust decision for the PV system size depends on the definition of the uncertainty sets, and this means that it is very important to construct appropriate uncertainty sets in practice. We need to adjust uncertainty sets to the required level of robustness in order to ensure that the resulting solution meets a certain CO2 reduction target with a minimum cost even in the worst case.

6 Conclusion

We described a new approach to determining the optimal size of a residential grid-connected photovoltaic (PV) system to meet a certain CO2 reduction target at a minimum cost under the net feed-in tariff system with a premium rate for electricity from residential PV systems. We extended our approach by using a robust optimization technique to cope with uncertainty in photovoltaic power generation caused by weather variability. This study yielded the following findings:

- The given target of CO2 emission reduction determines a lower bound for the size of the PV system.

- The amount of CO_2 emission reduction increases with z even if the entire life-cycle is taken into account.
- In both the deterministic model (7) and its robust variant (12), the optimal size of the PV system is at the lower or upper bound in the current Japanese tariff system.
- The optimal PV size depends on the parameter. For example, the electricity price $p^S = 34$ and 15-year PV lifetime are thresholds for changing the optimal PV size.
- The robust optimization model provides a PV size prediction that is robust to uncertain solar irradiation data and makes it possible to certainly meet a future CO_2 reduction target.
- We obtain a very robust solution with large uncertainty sets, leading to the optimal PV size of $\underline{z}$.

The robust decision for the PV system size depends on the definition of the uncertainty sets, and therefore, it is very important to construct appropriate uncertainty sets in practice. We may need to conduct further investigations on how to construct uncertainty sets for PV-generated electricity E_{ij}.

Our robust optimization model deals with the uncertainty in PV electricity E_{ij}. It would be interesting to modify the model by taking into account uncertainties in future electricity demand D_{ij} and electricity prices, in addition to E_{ij}.

Our deterministic and robust models determine the best array size for PV systems. It may be possible to extend these models so that they optimize not only the array size but also the array surface tilt angle and configuration, as is done in [6].

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