Transmission Expansion Planning Using an AC Model: Formulations and Possible Relaxations

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***Abstract*—Transmission expansion planning (TEP) is a rather complicated process which requires extensive studies to determine *when, where* and *how many* transmission facilities are needed. A well planned power system will not only enhance the system reliability, but also tend to contribute positively to the overall system operating efficiency. Starting with two mixed-integer nonlinear programming (MINLP) models, this paper explores the possibility of applying AC-based models to the TEP problem. Two nonlinear programming (NLP) relaxation models are then proposed by relaxing the binary decision variables. A reformulation-linearization-technique (RLT) based relaxation model in which all the constraints are linearized is also presented and discussed in the paper. Garvers's 6-bus test system and the IEEE 24-bus system are used to test the performance of the proposed models and related solvers. A validation process guarantees that the resultant TEP plan is strictly AC feasible. The simulation results show that by using proper reformulations or relaxations, it is possible to apply the AC models to TEP problems and obtain a good solution.**

***Index Terms*-- Transmission expansion planning, mathematical programming, ACOPF, MINLP, reformulation, relaxation.**

I. NOMENCLATURE

b_k	Admittance of line k
b_{k0}	Shunt admittance of line k
c_k	Investment cost of the line k
c_2	Quadratic cost coefficient of generator g
c_1	Linear cost coefficient of generator g
c_0	Fixed cost coefficient of generator g
e_i	Real part of the complex bus voltage V_i
f_i	Imaginary part of the complex bus voltage V_i
g_k	Conductance of line k
g_{k0}	Shunt conductance of line k
M	Disjunctive factor, a large positive number
M$	Million dollars
Obj.	Objective function
PD_i	Total active power of the load at bus i
P_k	Active power flow on line k
PG_g	Active power output of generator g
$PG_{g.\max}$	Active power capacity of the generator g
Q_k	Reactive power flow on line k
QD_i	Total reactive power of the load at bus i
QG_g	Reactive power output of generator g
$QG_{g.\max}$	Reactive power capacity of the generator g
$\mathbf{R}_+$	Set of positive real numbers
$S_{k.\max}$	MVA capacity of line k
V_i	Voltage at bus i
$V_{\max}$	Bus voltage upper bound
$V_{\min}$	Bus voltage lower bound
x_L, y_L	Lower bound of x, y
x_U, y_U	Upper bound of x, y
z_k	Binary decision variable for line investment: 1 for build, 0 for not build (z_k is a continuous variable in the relaxed models)
θ_{ij}	Angle difference between bus i and bus j
$\theta_{\max}$	Maximum angle difference between bus i and bus j
Ω_g	Set of generators

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II. INTRODUCTION

TRANSMISSION expansion planning (TEP) is a process to determine an optimal strategy to expand the existing power system transmission network to meet the demand of the possible load growth and the proposed generators, while maintaining reliability and security performance of the power system. TEP is typically a rather complicated process that requires extensive studies to determine *where*, *when* and *how many* transmission facilities are needed. Due to the increasing complexity of modern power systems and the requirements of the deregulated market process, electric utilities have gradually realized that a well-planned power system will not only help enhance the system reliability, but will also contribute positively to the overall system operating efficiency. Traditionally, the lack of efficient computing tools has usually prevented the used of sophisticated mathematical modeling in solving the TEP problem to determine locations for placing new transmission facilities. Today, the computational performance of the computers has improved dramatically and so have optimization algorithms, which make the rigorous modeling and computing of the TEP problem possible.

The TEP problem by nature can be regarded as an optimal power flow (OPF) problem with discrete constraints. In an AC power system, it is generally known that the modeling of the ACOPF is a nonlinear programming (NLP) problem. Similarly, by adding discrete variables into the problem, the modeling of the TEP problem using an AC model (ACTEP) can be referred to as a mixed-integer nonlinear programming (MINLP) problem. Mathematically, the MINLP problems are usually considered as one of the classes of problems that are the most difficult to solve due to their intrinsic complexity. In the world of complexity, problems are generally classified as P or NP based on the effort needed to solve them. *e.g.*, a class P problem can be solved in polynomial time by a deterministic Turing machine, while a class NP problem cannot. Further, a problem is regarded as NP-hard if solving it in polynomial time would make possible to solve all the problems in class NP in polynomial time. Particularly, if a problem is NP-hard and it is also an NP problem, then it is known as an NP-complete problem [1]. The complexity of the MINLP problems is usually NP-hard or even NP-complete [2]. If P ≠ NP, then the relationship of P, NP, NP-hard and NP-complete problems can be described in Fig. 1.

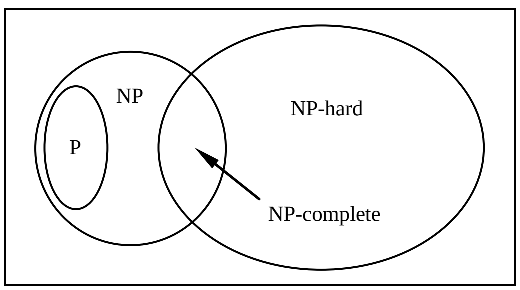

Fig. 1. Relationship of P, NP, NP-hard and NP-complete problems, P ≠ NP

The AC modeling of TEP problem is rarely seen in the literature. Reference [1] is a recent report, where an ACOPF based MINLP model is proposed and solved by using interior point method. In most of the published literature, TEP problems are modeled as a mixed-integer linear programming (MILP) problem based on the DC approximation of the network model [4]-[6]. This is the case because compared to the MINLP model, MILP is much easier to solve. One issue needs to be noted that although MINLP problems are quite hard, they are not unsolvable. In fact, quite a few solvers such as KNITRO, BONMIN, COUENNE and BARON are capable of solving MINLP-types problems and can obtain a reasonably good solution in an acceptable time frame. Among these solvers, COUENNE and BARON are designed for solving both convex and non-convex MINLP problems. KNITRO and BONMIN are designed for solving only convex MINLP problems exactly, while for con-convex MINLP problems, heuristic solutions will be given [7].

On the other hand, solving the MINLP problems is not the only choice. In fact, when the original problem is impossible or too expensive to solve, relaxation can always be considered as an option. By eliminating integer variables, the MINLP problems can be relaxed to an NLP problem, which has a potential for an easier solution. The core issue for solving global optimization problems is the use of linear or convex programming relaxations that convexify the original problem and aid the solution process. During the process of the convexification, the tightness of the relaxations plays a crucial role because it will directly affect the accuracy of the relaxed model [8]. A reformulation-linearization-technique (RLT) is usually used to generate such tight linear programming relaxations for not only constructing exact solution algorithms, but also to design powerful relaxation models for large classes of discrete combinatorial and continuous non-convex programming problems [9]-[10].

This paper explores the possibility of applying AC-based models to the TEP problems. The contributions of the paper are twofold:

1) The AC-based TEP models and their possible relaxations are proposed and discussed in detail. The paper starts with two MINLP models, and followed by two NLP models where the binary variables are relaxed. A RLT-based relaxation model is also presented in which all the constraints are linearized.
2) Many results are published on Garver's 6-bus system [3], [5], [11]. But when examined closely, few are AC feasible. This paper performs an ACOPF validation process on the expanded system, which guarantees the TEP plan obtained is strictly AC feasible.

The remainder of the paper is organized as follows: Section III reviews the ACOPF formulations. Section IV presents two MINLP-based ACTEP models. Section V presents the possible reformulations and relaxations of the MINLP models. Simulation results are provided in Section VI to compare and validate the performance of the proposed models. Finally, concluding remarks are drawn in Section VII.

III. The ACOPF Formulations

The formulation of the ACOPF is the foundation of the formulation of the TEP problem, because solving the TEP problem can essentially be regarded as solving an ACOPF with discrete constraints. In this section, the standard ACOPF formulation will be reviewed both in polar form and rectangular form, and serve as basis of the TEP formulations presented in the following sections.

Choosing to minimize the total generation cost and assuming the generators have quadratic cost functions, the standard ACOPF formulation in polar form can be written as follows,

$$\min \sum_{g\in\Omega_g} \left(c_2 PG_g^2 + c_1 PG_g + c_0 \right) \tag{1}$$

$$\sum_{g\in i} PG_g - \sum_{d\in i} PD_d + \sum_{k\in i} P_k = 0 \tag{2}$$

$$\sum_{g\in i} QG_g - \sum_{d\in i} QD_d + \sum_{k\in i} Q_k = 0 \tag{3}$$

$$PG_{g.\min} \le PG_g \le PG_{g.\max} \tag{4}$$

$$QG_{g.\min} \le QG_g \le QG_{g.\max} \tag{5}$$

$$V_{\min} \le V_i \le V_{\max} \tag{6}$$

$$-\theta_{\max} \le \theta_{ij} \le \theta_{\max} \tag{7}$$

$$0 \le P_k^2 + Q_k^2 \le S_{k.\max}^2 \tag{8}$$

where the power balance constraints for active power and reactive power at each bus are represented in (2) and (3), respectively. The generator outputs constraints for active power and reactive power are shown in (4) and (5), respectively. Notice that in reality, the $P_{\min}$ and the $Q_{\min}$ of a generator are not necessarily 0. In fact, many generators have the ability to absorb reactive power, so that $Q_{g.\min}$ usually has a negative value. Moreover, for some pumped storage generators, even $P_{g.\min}$ may be a negative number during certain generation periods. However, one fundamental assumption for ACOPF problems is that the on and off status of the generators do not change. In other words: the unit commitment problem is not considered in ACOPF. The voltage magnitude and angle constraints are shown in (6) and (7), respectively. In static power flow studies, the angle differences θ_{ij} between two buses that are directly connected are usually rather small, typically not more than $\pi/6$ in radian. In power systems, there are no separate limits for active power and reactive power on a line. The line flow constraints are enforced by limiting the apparent power flows (MVAs) as shown in (8). Assuming all the transformers have standard turns ratios, then the active power P_k and reactive power Q_k of a certain branch are given by,

$$P_k = V_i^2\left(g_k + g_{k0}\right) - V_i V_j\left(g_k \cos\theta_{ij} + b_k \sin\theta_{ij}\right) \tag{9}$$

$$Q_k = -V_i^2\left(b_k + b_{k0}\right) + V_i V_j\left(b_k \cos\theta_{ij} - g_k \sin\theta_{ij}\right) \tag{10}$$

Although the polar form of the formulations is more frequently used in ACOPF studies, the rectangular form should never be overlooked because it does offer some favorable computational properties and may be applied to TEP modeling. In order to formulate the rectangular form of power flow equations, the polar form can be rewritten as follows,

$$\dot{V}_i = V_i \cos\theta_i + jV_i \sin\theta_i = e_i + jf_i\,, \tag{11}$$

where e_i and f_i are the real and imaginary part of the complex bus voltage V_i. Substituting (11) into (9) and (10), and letting $g_{sh} = g_k + g_{k0}$, $b_{sh} = b_k + b_{k0}$, the active power and reactive line flows are given by,

$$P_k = \left(e_i^2 + f_i^2\right) g_{sh} - \left[g_k\left(e_i e_j + f_i f_j\right) + b_k\left(e_j f_i - e_i f_j\right)\right] \tag{12}$$

$$Q_k = -\left(e_i^2 + f_i^2\right) b_{sh} + \left[b_k\left(e_i e_j + f_i f_j\right) - g_k\left(e_j f_i - e_i f_j\right)\right]. \tag{13}$$

In rectangular form, (6) and (7) can be rewritten as:

$$V_{i.\min}^2 \le e_i^2 + f_i^2 \le V_{i.\max}^2 \tag{14}$$

Keeping the objective function and all other constraints intact, then (1)-(5), (8) and (12)-(13) together constitute the rectangular form of the ACOPF formulation.

IV. MINLP-Based ACTEP Models

As discussed in Section III, the TEP problem can be regarded as an extension of the OPF problem, in which the parameters to be optimized are not only limited to continuous variable but also integer (binary) variables. Notice that the constraints in the ACOPF model are highly non-linear, after introducing discrete variables, the resultant TEP model will be a MINLP problem. In this section, two MINLP-based TEP formulations are presented.

In a TEP problem, is it straightforward to model the number of lines as integer variables. For example, if the number of lines that need to be built in a transmission corridor is n_k, then n_k is an integer variable whose value can be any integer not greater than the maximum number of lines allowed $n_{k.\max}$. Instead of considering how many lines should be built in a given transmission corridor, this paper, however, formulates the problem alternatively and uses a binary decision variable z_k to determine whether a certain line should be built or not. By doing so, the set of multi-stage integer variables n_k are converted to a set of the variables which have only two stages: either 1 or 0, and therefore reduces the problem complexity. By using the binary variables approach, the first ACTEP model is shown as follows,

MIP1:

$$\min \sum_{k\in\Omega_k} c_k z_k + \sum_{g\in\Omega_g}\left(c_2 PG_g^2 + c_1 PG_g + c_0\right) \tag{15}$$

$$\sum_{g\in i} PG_g - \sum_{d\in i} PD_d + \sum_{k\in i} P_k = 0$$

$$\sum_{g\in i} QG_g - \sum_{d\in i} QD_d + \sum_{k\in i} Q_k = 0$$

$$PG_{g.\min} \le PG_g \le PG_{g.\max}$$

$$QG_{g.\min} \le QG_g \le QG_{g.\max}$$

$$V_{\min} \le V_i \le V_{\max}$$

$$-\theta_{\max} \le \theta_{ij} \le \theta_{\max}$$

$$0 \le P_k^2 + Q_k^2 \le S_{k.\max}^2$$

where the active and reactive line flows are given by,

$$P_k = z_k\left[V_i^2\left(g_k + g_{k0}\right) - V_i V_j\left(g_k \cos\theta_{ij} + b_k \sin\theta_{ij}\right)\right] \tag{16}$$

$$Q_k = z_k\left[-V_i^2\left(b_k + b_{k0}\right) + V_i V_j\left(b_k \cos\theta_{ij} - g_k \sin\theta_{ij}\right)\right]. \tag{17}$$

Comparing the TEP model *MIP1* with the ACOPF model shown in Section III, differences exist only in (15)-(17). The long term objective of the long term TEP problem is to maximize the social welfare or market surplus, which is equivalent to minimizing the sum of the investment cost and the operating cost as shown in (15) if a perfect inelastic demand curve is assumed. In order to determine whether a certain line should be built or not, the line flows are modeled as the products of the binary decision variables and the original trigonometry expressions as shown in (12) and (13). For those lines that already exist in the system, z_k are fixed to be 1. For those lines which are potentially considered as the candidate lines to be built, the algorithm can freely choose z_k to be either be 1 or 0. A new line is selected to be built if and only if the corresponding decision variable $z_k = 1$.

The *MIP1* model provides a fairly straightforward MINLP-based ACTEP models. The main difficulty in using this formulation lies in the equality constraints (16) and (17). These equality constraints are products of binary variables and non-convex non-linear expressions, which could be tricky to satisfy and extremely hard to evaluate mathematically. In fact, efforts can be made to develop a soft version of *MIP1* model by using the following disjunctive method,

MIP2:

$$\min \sum_{k\in\Omega_k} c_k z_k + \sum_{g\in\Omega_g}\left(c_2 PG_g^2 + c_1 PG_g + c_0\right)$$

$$\sum_{g\in i} PG_g - \sum_{d\in i} PD_d + \sum_{k\in i} P_k = 0$$

$$\sum_{g\in i} QG_g - \sum_{d\in i} QD_d + \sum_{k\in i} Q_k = 0$$

$$PG_{g.\min} \le PG_g \le PG_{g.\max}$$

$$QG_{g.\min} \le QG_g \le QG_{g.\max}$$

$$V_{\min} \le V_i \le V_{\max}$$

$$-\theta_{\max} \le \theta_{ij} \le \theta_{\max}$$

$$0 \le P_k^2 + Q_k^2 \le z_k S_{k.\max}^2 \ , \tag{18}$$

where the active and reactive line flows are given by,

$$P_k - V_i^2\left(g_k + g_{k0}\right) + V_i V_j\left(g_k \cos\theta_{ij} + b_k \sin\theta_{ij}\right) \le (1-z_k) M_k \tag{19}$$

$$P_k - V_i^2\left(g_k + g_{k0}\right) + V_i V_j\left(g_k \cos\theta_{ij} + b_k \sin\theta_{ij}\right) \ge (z_k-1) M_k \tag{20}$$

$$Q_k + V_i^2\left(b_k + b_{k0}\right) - V_i V_j\left(b_k \cos\theta_{ij} - g_k \sin\theta_{ij}\right) \le (1-z_k) M_k \tag{21}$$

$$Q_k + V_i^2\left(b_k + b_{k0}\right) - V_i V_j\left(b_k \cos\theta_{ij} - g_k \sin\theta_{ij}\right) \ge (z_k-1) M_k \tag{22}$$

Compared with model *MIP1*, the changes in *MIP2* are in (18)-(22). Notice that instead of using hard equality constraints, the *MIP2* model splits every equality constraint into two inequality constraints with the opposite signs. When certain conditions are satisfied, the two inequality constraints will squeeze each other and act as an equality constraint. For example: the constraint (19) and (20) indicates that if z_k is 1, *i.e.* the line exists or the line is selected to be built, then the R.H.S. of these two constraints will force the line active power flow equation to hold; otherwise, if z_k is 0, *i.e.* the line is not selected, then the disjunctive factor M_k needs to make the two constraints not binding. Similar logic applies to (21) and (22) as well. As the disjunctive factor, M_k should be sufficiently large so that the constraints are not binding when lines are not selected. But a too large M_k may also result in numerical difficulties and make the problem hard to solve. The minimum sufficient M_k can be determined by using the approach in [6]. Furthermore, when a line exists, the total power flow on the line should be kept within its MVA rating; otherwise, the power flow should be zero, as indicated in (18).

Notice that even though *MIP2* is a soft version of *MIP1*, it is still an MINLP model, and constraints like (18) could still give the solvers a hard time to solve (see case studies). In order to make the model more solvable, relaxations seem to be the only possible choice. The possible relaxations of the above models are discussed in the next section.

V. Relaxations of the ACTEP Model

Mathematically, the relaxation refers to a modeling strategy that approximates the original problem and typically has a larger feasible region, so that the solution of the approximation requires less effort than the solution to the original problem. A solution of the relaxed problem may not necessarily be the exact solution of the original problem but should be reasonably close and provides key information about the original problem. For TEP problems, the main objective is to eliminate the integer constraints to reduce the complexity of the original problem. This section proposes three possible relaxations based on the ACTEP models presented in Section IV. The first two are NLP relaxations, and the third relaxation is a RLT-based model, where the original MINLP problem is linearized and relaxed as a linear programming problem.

A. The NLP Relaxations

The two NLP relaxations are based on the *MIP2* model shown in Section IV. By slightly changing the formulation, the integer constraints in *MIP2* can be removed and this will result in an NLP model as follows,

NLP1:

$$\min \sum_{k\in\Omega_k} c_k z_k + \sum_{g\in\Omega_g}\left(c_2 PG_g^2 + c_1 PG_g + c_0\right)$$

$$\sum_{g\in i} PG_g - \sum_{d\in i} PD_d + \sum_{k\in i} P_k = 0$$

$$\sum_{g\in i} QG_g - \sum_{d\in i} QD_d + \sum_{k\in i} Q_k = 0$$

$$PG_{g.\min} \le PG_g \le PG_{g.\max}$$

$$QG_{g.\min} \le QG_g \le QG_{g.\max}$$

$$V_{\min} \le V_i \le V_{\max}$$

$$-\theta_{\max} \le \theta_{ij} \le \theta_{\max}$$

$$0 \le P_k^2 + Q_k^2 \le z_k S_{k.\max}^2$$

$$z_k\left(1-z_k\right) \le \varepsilon \tag{23}$$

$$0 \le z_k \le 1 \ . \tag{24}$$

Notice that by adding constraint (23), the z_k do not necessarily need to be a binary variable. Instead, z_k can be re-defined as a continuous variable between 0 and 1 as shown in (24). The TEP formulation is therefore reduced to a NLP model. Ideally, constraint (23) may be written as $z_k(1 - z_k) = 0$. The evaluation of an equality constraint usually results in numerical difficulties and could make the relaxed NLP problem as hard as the original MINLP problem. Therefore practically, it is recommended to use the inequality form in (23), where ε is a small positive number.

Apart from the approach in *NLP1*, another relaxation strategy is also possible. Instead of using the *zero product* constraints as shown in (23), a penalty term can be imposed on the objective function and this will result in the following relaxed NLP model,

NLP2:

$$\min \sum_{k\in\Omega_k} c_k z_k\left[A\sin\left(\pi z_k\right)+1\right] + \sum_{g\in\Omega_g}\left(c_2 PG_g^2 + c_1 PG_g + c_0\right) \tag{25}$$

$$\sum_{g\in i} PG_g - \sum_{d\in i} PD_d + \sum_{k\in i} P_k = 0$$
$$\sum_{g\in i} QG_g - \sum_{d\in i} QD_d + \sum_{k\in i} Q_k = 0$$
$$PG_{g.\min} \le PG_g \le PG_{g.\max}$$
$$QG_{g.\min} \le QG_g \le QG_{g.\max}$$
$$V_{\min} \le V_i \le V_{\max}$$
$$-\theta_{\max} \le \theta_{ij} \le \theta_{\max}$$
$$0 \le P_k^2 + Q_k^2 \le z_k S_{k.\max}^2$$
$$0 \le z_k \le 1$$

As can be observed from the *NLP2* model shown above, a sinusoidal penalty term [$A\sin(\pi z_k) + 1$] is imposed on (25). The penalty makes the first term of the resultant new objective function a product of the line investment cost and the penalty term. Notice that when z_k is equal to either 1 or 0, the value of the penalty term is 1 and has no effect on the original objective function; otherwise, the large positive coefficient A of the sine function will impose a large value on the objective function which makes it impossible to be the optimal solution. In other words, the incentive of the penalty term is to obtain the optimal solution only when z_k is equal to either 0 or 1. The sketch of the penalty function is shown in Fig. 2.

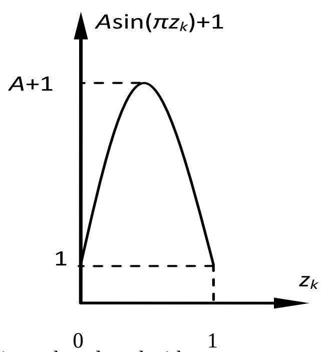

Fig. 2. Penalty function value plotted with respect to z_k

B. The RLT-based Relaxation

Before applying the RLT-based relaxation to the TEP problems, a trivial example motivates the basic concept of the RLT. Considering the following minimization problem,

$$min\ \ (x+y) \tag{26}$$

subject to,

$$x - y + xy = 1 \tag{27}$$
$$x, y \in \mathbf{R}_+$$
$$(x_L, y_L) \le (x, y) \le (x_U, y_U)$$

It can be observed that this model has a bilinear term xy and therefore it is non-convex. In order to obtain the convex relaxation, a new variable w is introduced to replace the binary term so that the constraint (27) can be rewritten as,

$$x - y + w = 1 \tag{28}$$

Notice that the upper bound and lower bound of the existing variables x and y are given respectively as (x_U, y_U) and (x_L, y_L), and there is no additional constraint on w, then a tight bound which is also known as the McCormick convex relaxation [8] can be calculated by solving the following inequalities:

$$\begin{aligned} w &\ge x_L y + y_L x - x_L y_L \\ w &\ge x_U y + y_U x - x_U y_U \\ w &\le x_U y + y_L x - x_U y_L \\ w &\le x_L y + y_U x - x_L y_U \end{aligned} \quad . \tag{29}$$

By using the above RLT-based relaxation, the original bilinear problem can be relaxed to a linear programming problem, which is apparently convex and much easier to solve. However, the main drawback of the RLT relaxation is the excessive size of the resulting LP relaxation. In fact, an RLT-based relaxation will increase the size of the problem by 2 to 4 times. Also, by using the RLT based relaxation, it is hard to control the degree of the relaxation. This means that the problem may easily become too relaxed and therefore lose some of the key information that should be maintained.

In order to apply RLT to the TEP formulation, the rectangular form of the power flow equations as shown in Section III is used. By sequentially inserting the dummy variables, all the bilinear terms can eventually be rewritten as a linear expression. The RLT-based TEP model is shown as follows,

RLT:

$$\min \sum_{k\in\Omega_k} c_k z_k + \sum_{g\in\Omega_g} \left(c_2 PG_g^2 + c_1 PG_g + c_0 \right)$$
$$\sum_{g\in i} PG_g - \sum_{d\in i} PD_d + \sum_{k\in i} P_k = 0$$
$$\sum_{g\in i} QG_g - \sum_{d\in i} QD_d + \sum_{k\in i} Q_k = 0$$
$$PG_{g.\min} \le PG_g \le PG_{g.\max}$$
$$QG_{g.\min} \le QG_g \le QG_{g.\max}$$
$$V_{\min} \le V_i \le V_{\max}$$
$$-\theta_{\max} \le \theta_{ij} \le \theta_{\max}$$
$$0 \le P_k \le S_{k.\max} \tag{30}$$
$$P_k - g_{sh}\left(X1_k + X2_k\right) + g_k\left(X3_k + X4_k\right) + b_k\left(X5_k - X6_k\right) \le (1 - z_k) M_k \tag{31}$$
$$P_k - g_{sh}\left(X1_k + X2_k\right) + g_k\left(X3_k + X4_k\right) + b_k\left(X5_k - X6_k\right) \ge (z_k - 1) M_k \tag{32}$$
$$Q_k + b_{sh}\left(X1_k + X2_k\right) - b_k\left(X3_k + X4_k\right) + g_k\left(X5_k - X6_k\right) \le (1 - z_k) M_k \tag{33}$$
$$Q_k + b_{sh}\left(X1_k + X2_k\right) - b_k\left(X3_k + X4_k\right) + g_k\left(X5_k - X6_k\right) \ge (z_k - 1) M_k \tag{34}$$

where $X1 = e_i^2$, $X2 = e_j^2$, $X3 = e_i e_j$, $X4 = f_i f_j$, $X5 = e_j f_i$ and $X6 = e_i f_j$. In steady state power system analysis, it is usually assumed that,

$$0.95 \le V_i \le 1.05$$
$$-0.5 \le \sin\theta_{ij} \le 0.5$$
$$\sqrt{3}/2 \le \cos\theta_{ij} \le 1$$

Recall that $e_i = V_i\cos\theta_i$ and $f_i = V_i\sin\theta_i$, the bounds on e_i, e_j, f_i and f_j can be therefore calculated as,

$$0.8227 \le (e_i, e_j) \le 1.05$$

$$-0.525 \le (f_i, f_j) \le 0.525$$

Thus, (29) can then be used to derive the bounds on $X1$ to $X6$. Notice that in order to obtain a fully linearized model, a further relaxation is used in (30), which is based on the assumption that at the transmission level, namely $Q_k << P_k$.

VI. Case Studies

In this section, simulation results are reported on two test systems. First, the proposed MINLP-based ACTEP models and their relaxations will be tested on Garver's 6-bus system to compare and analyze the performance of each proposed model. Later, the long-term TEP results will be shown using the IEEE 24-bus reliability test system (RTS). All the models are programmed in AMPL [12]. The computing platform used to perform all the simulations is a Linux workstation with an Intel *i*7 2600 4-core CPU at 3.40 GHz and 16 GB of RAM.

A. Garver's 6-bus System

Garver's 6-bus test system [11] has 6 buses, 6 initial lines, 5 loads and 3 generators. The generator at bus 6 is isolated from the main system. The system parameters can be found in [3]. For all cases in this section, it is assumed that the maximum number of lines allowable in a transmission corridor is 2. Since there are 6 existing lines, therefore the maximum possible number of lines can be built is $2C_6^2 - 6 = 24$. In order to compare with the published results, the operating costs are *not* included in the objective functions.

Three MINLP solvers: COUENNE 0.4.0 [13], BONMIN 1.5.1 [14] and KNITRO 8.0.0 [15] are used for solving the two MINLP models presented in Section IV. Among all three solvers, only COUENNE is designed for obtaining a global optimal solution for both convex and non-convex MINLP problem. BONMIN and KNITRO ensure optimal solutions only when the objective and all the constraints are convex, otherwise, heuristic solutions will be given.

TABLE III TEP Results for the MIP models

Model	Solver	KNITRO	BONMIN	COUENNE
MIP1	Objective	1056	1056	Time limit reached
	Lines	Build all lines	Build all lines	
MIP2	Objective	Iteration limit reached	677	Time limit reached
	Lines		Build 17 lines	

As one can observe from Table III, for the *MIP1* model, the heuristic solutions given by KNITRO and BONMIN are simply to build all the lines. This is rather a trivial feasible solution because the solvers are not really *selecting* a specific choice. For the *MIP2* model, KNITRO gives a solution of 130 M$, while BONMIN gives 667 M$. It is interesting to note that although COUENNE claims that it can solve general MINLP problems exactly, however, the solution time could be extremely long. For both MINLP-based models, COUENNE fails to return a solution within the 24-hour time limit. In a word, the performance of the solvers for the MINLP-based models is still quite limited.

The NLP models and the RLT model are solved only by KNITRO and BONMIN. Although the first two models are reduced to NLP problems, they are still non-convex and therefore are global optimization problems. For non-convex global optimization problems, obtaining a global minimum cannot be guaranteed in polynomial time. However, the solution can be significantly affected by the starting points. In order to obtain a better solution, the multi-start function in KNITRO and BONMIN is used. For both NLP models, the number of multi-starts is set to 2000. In other words, 2000 sets of different starting points are automatically initiated by the solvers. Also, the optimality tolerance and the feasibility tolerance of the solvers are set to 10^{-9}, respectively. The TEP results of the relaxation models are shown in Table IV.

TABLE IV TEP Results for the NLP and RLT Models

Model	Solver	KNITRO	BONMIN
NLP1	Objective (M$)	406	473
	Lines	Build 11 lines	Build 12 lines
NLP2	Objective (M$)	180	804
	Lines	(1-5), (2-3), (2-6)×2, (3-5), (4-6)×2	Build 21 lines
RLT	Objective (M$)	110	110
	Lines	(2-6), (3-5), (4-6)×2	(2-6), (3-5), (4-6)×2

As can be observed from Table IV that for *NLP1* model, KNITRO gives a lower objective value (406 M$) and fewer lines to build as compared to the results gives by BONMIN after 2000 restarts. For the *NLP2* model, KNITRO again gives a much lower objective value (180 M$) as compared to the results gives by BONMIN. It should be pointed out that based on the simulation experience, the selection of the penalty factor A can significantly influence the results. A penalty factor that is not sufficiently large may cause the decision variables z_k fail to converge to 0 or 1. However, a too large penalty factor could bring convergence problem and end up with a higher objective value. In this case, the value of penalty factor A used for KNITRO that gives the best solution is 10^9, while for BONMIN is 10^3. The *RLT* model, as discussed in the previous section, is a completely linearized model, and is therefore a MILP model. For the *RLT* model, both the solvers identify the same objective value and the same set of lines to be built. Among all the results in Table IV, the *RLT* model gives the lowest objective value.

During the process of the relaxations and due to the intrinsic limit of the solvers, it is likely that some key information of the original model is not strictly maintained in the relaxed models. Therefore, the "optimal" plan obtained by the TEP model may turn out to be infeasible in power flow studies. As a result, a validation process is necessary to ensure that the TEP plans are AC feasible. In this section, the four TEP plans with the lowest objective functions are validated by running an ACOPF study, the results are shown in Table V.

As observed from Table V that although the *RLT* model gives the lowest objective value, the TEP plan is, however, infeasible in ACOPF. This is probably due to too much relaxation in the model. In fact, one has little control during the relaxation process by using the RLT-base model. The following three solutions are all AC feasible. Therefore, one can conclude that the solution given by *NLP2* (marked **bold** in Table V) is the *best known* ACTEP solution for Garver's test system. The expanded network is shown in Fig. 4.

TABLE V TEP RESULTS COMPARISON FOR THE GARVER'S SYSTEM

Model	Solver	Objective (M$)	ACOPF Results
RLT	KNITRO / BONMIN	110	No convergence
NLP2	**KNITRO**	**180**	**Converged**
NLP1	KNITRO	406	Converged
NLP1	BONMIN	473	Converged

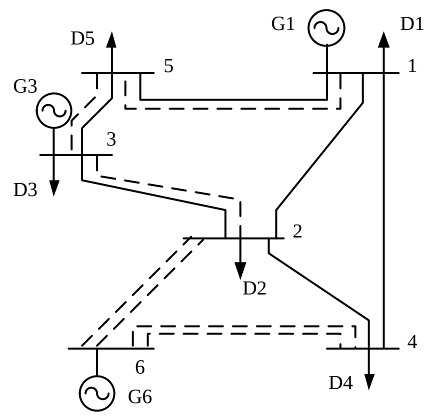

Fig. 4. Garver's 6-bus test system after TEP

Fig. 5 shows the effect of multi-starts on the ACTEP solutions using *NLP2* model. It can be observed that the quality of the solution can be significantly improved with the increase number of multi-starts. Meanwhile, one should be aware that this is at a cost of the computing time.

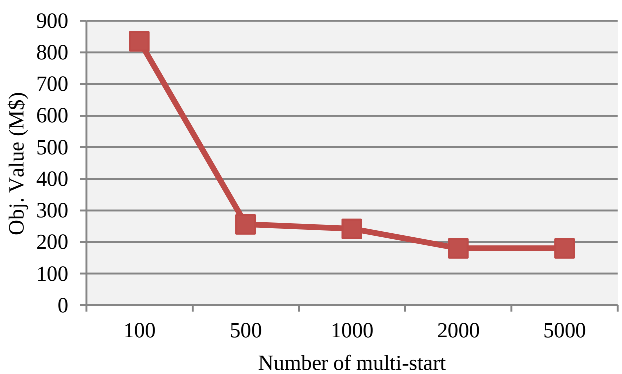

Fig. 5. Effect of multi-starts on the ACTEP solutions

Table VI shows the comparison of Garver's 6-bus system TEP results obtained in this paper and some results available in the published literature. One can observe that the AC model gives a higher objective value and requires more lines to be built. Notice that since the DC lossless model and the DC lossy model are based on the MILP model, the global optimality of the solution is guaranteed. However, since the AC model is a non-convex NLP model and it therefore is NP-hard. The only conclusion can be drawn is that this solution is the *best known* ACTEP solution for Garver's 6-bus system.

Although the solution process of Garver's system looks slightly complicated, the results are not completely desperate, In fact, it is worth noting that the results given by the *NLP2* model are quite promising. Therefore, the authors decided to test the models on a larger system: IEEE 24-bus RTS system.

TABLE VI COMPARISON OF THE TEP RESULTS

Model	DC Lossless [5]	DC Lossy [5]	AC
Objective (M$)	110	140	180
Lines	(3-5), (4-6)×3	(2-6)×2, (3-5), (4-6)×2	(1-5), (2-3), (2-6)×2, (3-5), (4-6)×2
AC Feasible?	No	No	Yes

B. *IEEE 24-bus System*

The IEEE 24-bus system used in this paper is the same as the one used in [5]. It has 33 generators connected at 10 buses, and 21 loads. The line investment cost data and the system parameters can be found in [5] and [16], respectively. The total load is 2850 MW. The objective function in this case is to minimize the sum of line investment cost and the operating cost for 20 years. The AC model and solver used in this study are *NLP2* and KNITRO, respectively. Similar to the previous case, the penalty factor used in this case is 10^9, the number of multi-starts is set to 2000, and the optimality tolerance and the feasibility tolerance of the solvers are set to 10^{-9}.

In order to perform the ACTEP studies, five lines that are related to bus 1, bus 2 and bus 7 are removed (1 – 2, 1 – 4, 1 – 5, 2 – 4 and 7 – 8), which means bus 1, bus 2 and bus 7 are isolated from the rest of the system. Instead, a new candidate line set is listed in Table VII.

TABLE VII PARAMETERS FOR CANDIDATE LINES

Number	Corridor	Cost (M$)	Rating (MW)
1	1 – 2	7.04	175
2	1 – 4	106.92	175
3	1 – 5	42.78	175
4	2 – 4	64.14	175
5	2 – 6	97.2	175
6	7 – 2	7.04	175
7	7 – 4	106.92	175
8	7 – 5	64.14	175
9	7 – 8	31.08	175

Two cases are studied for the IEEE 24-bus system. In Case 1, the TEP model is run without any additional constraints. In Case 2, an additional constraint is added in consideration of the steady state security requirement, which is the number of lines connect to one bus should be greater or equal to 2. The ACTEP results are shown in Table VIII.

As observed from Table VIII, for Case 1, the AC model requires building 3 lines, thus bus 1, bus 2, bus 4, bus 5 and bus 7 in the system will be radially connected, while in Case 2 where the security requirement is added, two more lines 2 – 4 and 1 – 2 are required, and therefore eliminates the radial line. Both the DC lossless model and the DC lossy model give the same result of building 3 lines when security requirement is not considered. It should be pointed out that if considering security requirement, then the DC-based models give the same results as Case 2 of the AC model (not shown in Table VIII).

TABLE VIII COMPARISON OF TEP RESULTS USING DIFFERENT MODELS

Corridor	**DC lossless model [4]**	**DC lossy model [4]**	**AC model**	
			Case 1	**Case 2**
1 – 2	1	1		1
1 – 5			1	1
2 – 4				1
7 – 2	1	1	1	1
7 – 8	1	1	1	1
Investment cost (M$)	45.16	45.16	87.94	152.08
Losses[1] (MW)	58.77	58.77	55.72	54.06
Annual operating cost (M$)	560.3	560.3	558.7	558.0
CPU time	0.08 s	0.16 s	2.3 h (2000 restarts)	2.5 h (2000 restarts)

[1]The losses and the annual operating cost are obtained from ACOPF

VII. SUMMARY AND CONCLUSIONS

This paper explored the possibility of applying AC-based models to the TEP problem. Five TEP models include two MINLP-based models, two NLP relaxed models and a RLT-based linear model are presented in this paper. The formulation of each model is shown and discussed in detail. A validation process guarantees the resultant TEP plan is *strictly AC feasible*. The conclusions of this paper are:

- The AC model can be applied to model TEP problems. The solution of MINLP-based ACTEP models is still challenging.
- By reformulation and relaxation, it is possible to solve the NLP-based ACTEP problem and obtain a local optimal solution.
- In order to obtain a high quality solution, the optimality and feasibility tolerances of the solver should be set as small as possible, and the use of the multi-start option is a necessity.
- For the *NLP2* model, proper penalty factors should be selected. The *RLT* model may not work well for the TEP problem due to the lack of control of the relaxation.

Comparing results, the ACTEP solutions typically require building more lines and therefore give a higher objective cost than the results given by the DC-based MILP models. This occurs because of the voltage and reactive power issues. However, due to the non-convex nature of the ACTEP models, it is hard to judge the global optimality of the results. The multi-start option may be a good choice, but this option will also result in the slowness in computation of the ACTEP model. It should be pointed out that the potential of the MINLP/NLP model for solving large scale TEP problems appears to be an open problem which requires more research.

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IX. REFERENCES

[1] A. Wigderson, "P, NP and mathematics – a computational complexity perspective", *in Proc. Int. Congress of Mathematicians*, 2006.

[2] A. N. Letchford, "A tutorial on mixed-integer non-linear programming," [Online]. Available: http://www.lancs.ac.uk/staff/letchfoa/talks/MINLP.pdf

[3] M. J. Rider, A.V. Garcia and R. Romero, "Power system transmission network expansion planning using AC model," *IET Gener. Transm. Distrib*., vol. 1, no. 5, pp. 731–742, Sept. 2007.

[4] H. Zhang, V. Vittal, G. T. Heydt and J. Quintero, "A mixed-integer linear programming approach for multi-stage security-constrained transmission expansion planning," *To appear in IEEE Trans. Power Syst*., 2012.

[5] N. Alguacil, A. L. Motto and A. J. Conejo, "Transmission expansion planning: a mixed-integer LP approach," *IEEE Trans. Power Syst.*, vol. 18, no. 3, pp. 1070 - 1077, Aug. 2003.

[6] L. Bahiense, G. C. Oliveira, M. Pereira and S. Granville, "A mixed integer disjunctive model for transmission network expansion," *IEEE Trans. Power Syst.*, vol. 16, no. 3, pp. 560–565, Aug. 2001.

[7] M. R. Bussieck and S. Vigerske, "MINLP solver software," [Online]. Available: http://www.math.hu-berlin.de/~stefan/minlpsoft.pdf

[8] H. D. Sherali and W. P. Adams, *A reformulation-linearization technique for solving discrete and continuous non-convex problems*. Kluwer Academic Publishers, 1999.

[9] L. Liberti and C. Pantelides, "An exact reformulation algorithm for large nonconvex NLPs involving bilinear terms," *Journal of Global Optimization*, no. 36, pp. 161–189, April 2006.

[10] J. A. Taylor and F. S. Hover, "Linear relaxations for transmission system planning," *IEEE Trans. Power Syst.*, vol. 26, no. 4, pp. 2533-2538, Nov. 2011.

[11] L. L. Garver, "Transmission net estimation using linear programming," *IEEE Trans. Power Apparat. Syst.*, vol. PAS-89, pp. 1688–1697, Sept./Oct. 1970.

[12] R. Fourer, D. M. Gay and B. W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*, 2nd edition, Florence, KY: Duxbury Press, 2002.

[13] P. Belotti, Couenne: a user's manual. Technical report, Lehigh University, 2009. [Online]. Available: http://www.coin-or.org/Couenne/

[14] P. Bonami and J. Lee, Bonmin user's manual. Technical report, IBM Corporation, June 2007. [Online]. Available: https://projects.coin-or.org/Bonmin

[15] R. A. Waltz and T. D. Plantenga, Knitro user's manual, version 7.0, Sept. 2010, [Online]. Available: http://seldon.it.northwestern.edu/sscc/knitro/Knitro70_UserManual.pdf

[16] R. D. Zimmerman and C. Murillo-Sánchez, MATPOWER user's manual. [Online]. Available: http://www.pserc.cornell.edu/mat-power

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