Principle of optimal accuracy classification for metrological purposes

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Abstract

The conceptions of optimizing a quantitative classification hierarchy and the criterion of informational optimality have been used for modeling universal accuracy classification scale. Proposed model is based on test uncertainty ratios determined in conformity with optimal levels of confidence in evaluating measurement uncertainty, as well as on the optimal classification structure. The author believes that this paper may be of interest and practical value for professionals engaged in improving metrological estimations.

Keywords: Information optimality; Accuracy classification; Measurement uncertainty

1. Introduction

The way of optimizing a quantitative hierarchy [1] and developed on this base Universal Classification Scale (UCS) consists of the set of dimensionless base-line numerical values, forming system's structure: elements, subclasses, classes, and system groups. While many of existing classification systems, based on informational optimality, might be identified in frame of this system, in application to metrological accuracy classifications UCS can be adapted when relative accuracy characteristics are expressed in the form of variances of measurement uncertainties which, in turn, are being evaluated according to standard methods [2]. Besides, further research has resulted in the need to specify optimization criteria more precise which, as established, are located within the admissible informational limits [3], as well as to determine and substantiate optimal numerical characteristics of above constituents of the system.

Taking all this into consideration, the suitable for metrology Universal Accuracy Classification Scale (UACS) in application to expanded measurement uncertainties has been achieved by the transformation of UCS data in permissible limits with substantiated informatively best values for tabulated data and structural constituents. Outcomes of such transforming are presented in this paper.

2. Principle and scheme of accuracy classification

The UACS creating is based on the following optimal measurement traceability characteristics:

(a) the informatively sufficient interval for test uncertainty ratio (TUR) between $TUR_{min} = \sqrt{(2\pi)}$, related to 92% level of confidence for measurement uncertainty evaluation, and $TUR_{max} = \sqrt{(4\pi)}$, related to 96% level of confidence. In terms of information theory TUR_{min} and TUR_{max} bear on the boundary of information redundancy absence and the maximum permissible redundancy respectively [4]. There is 95% confidence proven [4] as the optimal level within above boundary values that is related to $TUR_{med} = 0.5(TUR_{max} + TUR_{min})$ and is bearing on the optimal information redundancy;

(b) the optimality of integers 6 and 7, representing key structural base of UACS, which on each hierarchical level consists of seven relative classification elements forming six subclasses. For being convinced in this we shall deal with the problem of optimal classification integer τ_o as an informatively sufficient number τ of levels and their classification periodicity in such hierarchies. Over the averaged out estimation the periodicity is achievable to certain number (τ) of multiplied by itself ratios TUR_{med}/TUR_{min} and TUR_{max}/TUR_{med}. Since a traceability chain is characterized by the number τ of links between (τ + 1) of elements, equations $TUR_{med}^{\ \tau} = TUR_{min}^{\ \tau+1}$, and $TUR_{max}^{\ \tau} = TUR_{med}^{\ \tau+1}$ are true within the range of minimum and maximum test uncertainty ratios. There are two optimums for the estimation error $\delta_{TUR}(\tau)$ intrinsic to these equations:

$$\tau_{o1} = \arg\min |\delta_{TUR1}(\tau)| = \arg\min |TUR_{med}^{\ 1/\tau} - TUR_{max}/TUR_{med}| \quad (1)$$
$$\tau_{o2} = \arg\min |\delta_{TUR2}(\tau)| = \arg\min |TUR_{med}^{\ 1/\tau} - TUR_{med}/TUR_{min}| \quad (2)$$

Graphically the results of calculations with (1) and (2) are presented in Fig. 1, which show in fact the availability of two integers of optimum: τ_{o1} = 6 and τ_{o2} = 7.

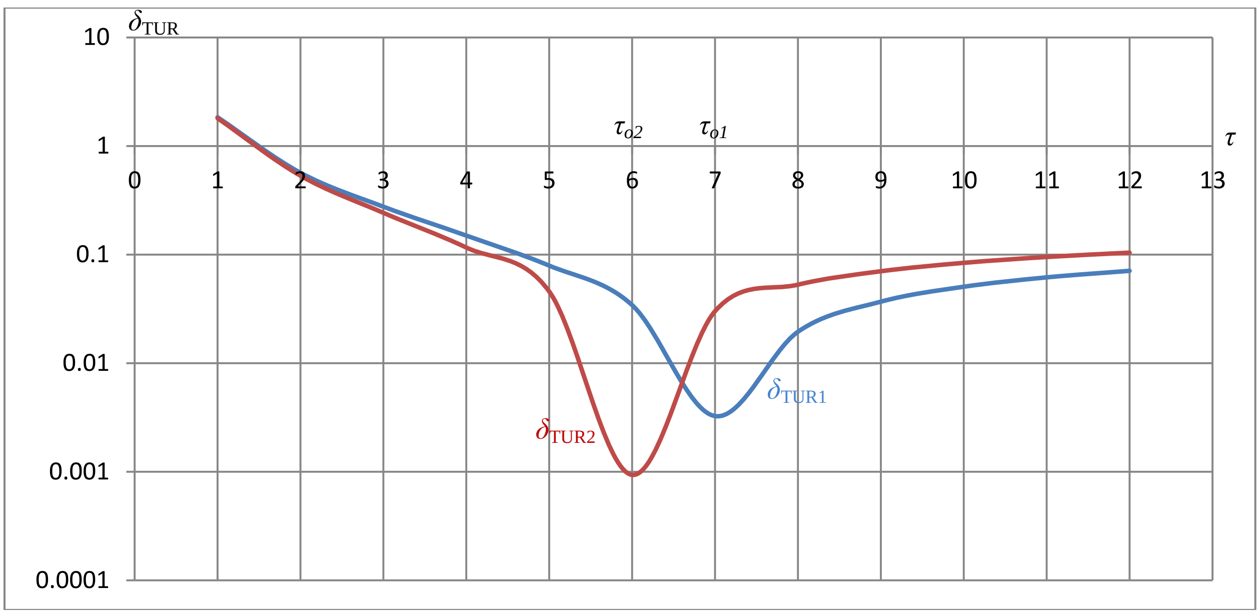

Fig. 1: Informational optimality of 6 and 7 as traceability chain integers

From these considerations we proceed to forming UACS, and Fig. 2 illustrates three ways of achieving this goal for limited number of accuracy classes.

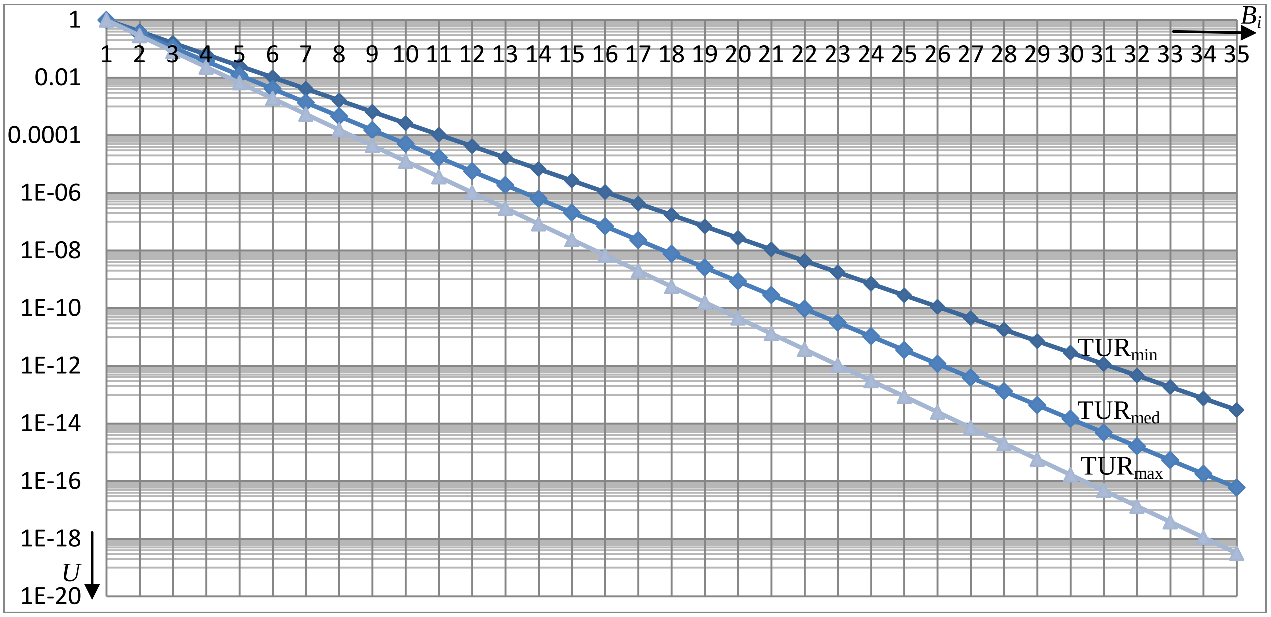

Fig.2: Relations between boundaries in numbering of accuracy classification (B_i) and measurement uncertainties (U) for key levels of optimal TUR

Since TUR_{min} meets the requirement of necessity and sufficiency, the further consideration is bearing upon this level of optimality.

Each class of accuracy (C_i) in UACS can be determined as belonging to the interval within the uncertainties corresponding to $C_i = (B_{i+1} - B_i)$. Illustratively the fragment of the classification consisting of first six levels of accuracy one can find in Fig. 3.

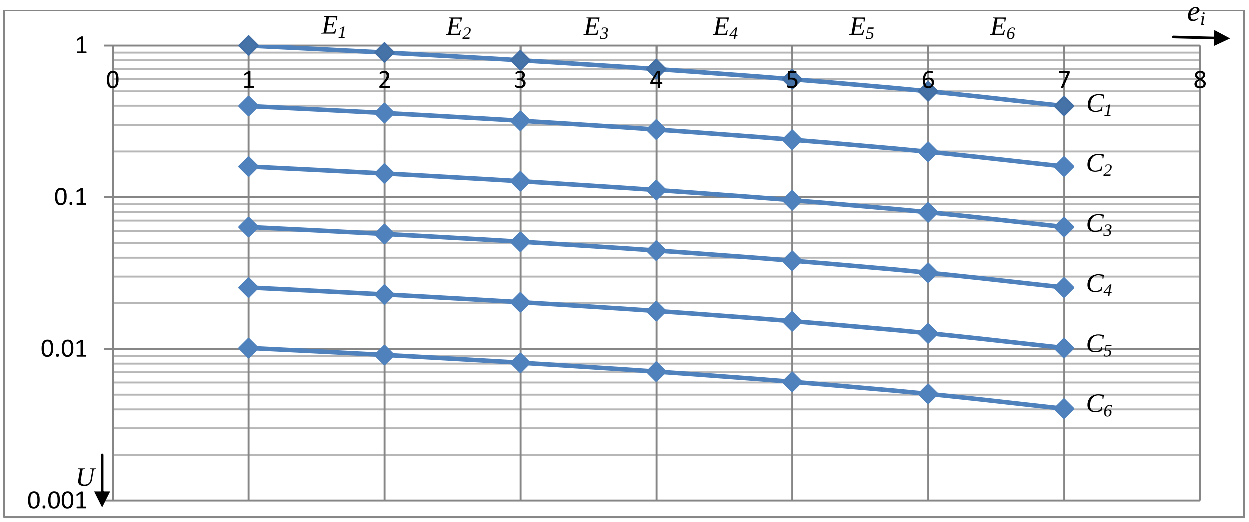

Fig. 3: Fragment of the classification consisting of first six levels of accuracy

An accuracy class (C_i) involves six subclasses (E_j) between seven classification elements (e_j) as measurement uncertainties. Thus, identification to the full of measurement methods, standards and instruments in the system of optimal accuracy classification requires specifying both symbols as classification index C_iE_j. This can be carried out by using the Table 1.

Table 1: Universal Accuracy Classification Scale (UACS)

Classes	Relative measurement uncertainties as classification elements (e_j), and subclasses (E_j)						
	E_1 (e_1–e_2)	E_2 (e_2–e_3)	E_3 (e_3–e_4)	E_4 (e_4–e_5)	E_5 (e_5–e_6)	E_6 (e_6–e_7)	
	e_1	**e_2**	**e_3**	**e_4**	**e_5**	**e_6**	**e_7**
C_1	1	0.9	0.8	0.7	0.6	0.5	0.4
C_2	0.40	0.36	0.32	0.28	0.24	0.20	0.16
C_3	0.16	0.14	0.13	0.11	0.10	0.08	0.06
C_4	0.060	0.057	0.051	0.044	0.038	0.032	0.025
C_5	0.025	0.023	0.020	0.018	0.015	0.013	0.010
C_6	1.0E-02	9.0E-3	8.0E-3	7.0E-3	6.0E-3	5.0E-3	4.0E-3
C_7	4.0E-3	3.6E-3	3.2E-3	2.8E-3	2.4E-3	2.0E-3	1.6E-3
C_8	1.6E-3	1.45E-3	1.29E-3	1.13E-3	9.7E-4	8.0E-4	6.4E-4
C_9	6.4E-4	5.8E-4	5.1E-4	4.5E-4	3.9E-4	3.2E-4	2.6E-4
C_{10}	2.6E-4	2.3E-4	2.1E-4	1.8E-4	1.5E-4	1.3E-4	1.0E-4
C_{11}	1.0E-4	9.2E-05	8.2E-05	7.2E-05	6.1E-05	5.1E-05	4.1E-05
C_{12}	4.1E-05	3.7E-05	3.3E-05	2.9E-05	2.5E-05	2.0E-05	1.6E-05
C_{13}	1.6E-05	1.5E-05	1.3E-05	1.1E-05	9.8E-06	8.1E-06	6.5E-06
C_{14}	6.5E-06	5.9E-06	5.2E-06	4.6E-06	3.9E-06	3.3E-06	2.6E-06
C_{15}	2.6E-06	2.3E-06	2.1E-06	1.8E-06	1.6E-06	1.3E-06	1.0E-06
C_{16}	1.0E-06	9.3E-07	8.3E-07	7.3E-07	6.2E-07	5.2E-07	4.1E-07
C_{17}	4.1E-07	3.7E-07	3.3E-07	2.9E-07	2.5E-07	2.1E-07	1.7E-07
C_{18}	1.7E-07	1.5E-07	1.3E-07	1.2E-07	9.9E-08	8.2E-08	6.6E-08
C_{19}	6.6E-08	5.9E-08	5.3E-08	4.6E-08	4.0E-08	3.3E-08	2.6E-08
C_{20}	2.6E-08	2.4E-08	2.1E-08	1.8E-08	1.6E-08	1.3E-08	1.0E-08
C_{21}	1.0E-08	9.4E-09	8.4E-09	7.3E-09	6.3E-09	5.2E-09	4.2E-09
C_{22}	4.2E-09	3.8E-09	3.3E-09	2.9E-09	2.5E-09	2.1E-09	1.7E-09
C_{23}	1.7E-09	1.5E-09	1.3E-09	1.2E-09	1.0E-09	8.3E-10	6.7E-10
C_{24}	6.7E-10	6.0E-10	5.3E-10	4.7E-10	4.0E-10	3.3E-10	2.7E-10
C_{25}	2.7E-10	2.4E-10	2.1E-10	1.9E-10	1.6E-10	1.3E-10	1.1E-10
C_{26}	1.1E-10	9.5E-11	8.5E-11	7.4E-11	6.4E-11	5.3E-11	4.2E-11
C_{27}	4.2E-11	3.8E-11	3.4E-11	3.0E-11	2.5E-11	2.1E-11	1.7E-11
C_{28}	1.7E-11	1.5E-11	1.4E-11	1.2E-11	1.0E-11	8.4E-12	6.7E-12
C_{29}	6.7E-12	6.1E-12	5.4E-12	4.7E-12	4.0E-12	3.4E-12	2.7E-12
C_{30}	2.7E-12	2.4E-12	2.2E-12	1.9E-12	1.6E-12	1.3E-12	1.1E-12
C_{31}	1.1E-12	9.7E-13	8.6E-13	7.5E-13	6.4E-13	5.4E-13	4.3E-13
C_{32}	4.3E-13	3.9E-13	3.4E-13	3.0E-13	2.6E-13	2.1E-13	1.7E-13
C_{33}	1.7E-13	1.5E-13	1.4E-13	1.2E-13	1.0E-13	8.5E-14	6.8E-14
C_{34}	6.8E-14	6.1E-14	5.5E-14	4.8E-14	4.1E-14	3.4E-14	2.7E-14
C_{35}	2.7E-14	2.5E-14	2.2E-14	1.9E-14	1.6E-14	1.4E-14	1.1E-14
C_{36}	1.1E-14	9.8E-15	8.7E-15	7.6E-15	6.5E-15	5.4E-15	4.3E-15
C_{37}	4.3E-15	3.9E-15	3.5E-15	3.0E-15	2.6E-15	2.2E-15	1.7E-15
C_{38}	1.7E-15	1.6E-15	1.4E-15	1.2E-15	1.0E-15	8.6E-16	6.9E-16
C_{39}	6.9E-16	6.2E-16	5.5E-16	4.8E-16	4.1E-16	3.4E-16	2.8E-16
C_{40}	2.8E-16	2.5E-16	2.2E-16	1.9E-16	1.7E-16	1.4E-16	1.1E-16

For example, if $U = 0.5*10^{-7}$ is the minimum relative uncertainty in calibrating gauge blocks by the interferometer, then this value in terms of correct classification of accuracy corresponds to $C_{19}E_3$ of UACS.

Table 1 comprises 40 hierarchy levels, while formally the UACS might be represented as the infinite number of levels. However, applied metrology imposes restrictions. Besides, there is fundamental physical restriction to measurement as *Heisenberg uncertainty* that confines the levels of such a scale in principle.

3. Conclusion

Proposed optimization technique enables determining of real classification level of accuracy, expressed by relative measurement uncertainty for each calibration/testing laboratory (in the form of CMC – calibration and measurement capability [5]) in any field of measurement, as well as of accuracy level of any measurement method, standard, instrument, and measurement system. As the first practical step, on the basis of this work the guiding document [6] has been prepared for the introduction in The National Physical Laboratory of Israel.

4. References

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