Solution Methods for the Periodic
Petrol Station Replenishment Problem

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Abstract

In this paper we introduce the Periodic Petrol Station Replenishment Problem (PPSRP) over a T-day planning horizon and we describe four heuristic methods for its solution. Even though all the proposed heuristics belong to the common partitioning-then-routing paradigm, they differ in the way of assigning the stations to each day of the horizon. The resulting daily routing problems are then solved exactly till optimality. Moreover, an improvement procedure is also developed with the aim of ensuring a better solution quality. Our heuristics are tested and compared on two instances of a real-life test problem and our computational results show encouraging improvements with respect to a human planning solution.

Keywords: petrol delivery; periodic constraints; vehicle routing problem.

1. Introduction

The problem of planning petrol delivery to the distribution stations is well recognized in the Operations Research literature under the name of Petrol Station Replenishment Problem (PSRP). It consists in making simultaneously several decisions such as determining the minimum number of trucks required, assigning the stations to the available trucks, defining a feasible route for each truck, settling whether drivers' work overtime is required, etc. The objective to be achieved is usually defined as the minimization of the traveled distance by the trucks to serve all the stations.

Most of the works presented in the literature solve this problem over a time period of one single day ignoring the fact that solving the PSRP over a T-day planning horizon may yield delivery savings for the company (see for example Cornillier et al., 2012). However, most of the petrol replenishment problems are characterized by the fact that the stations should not be served at each day of the T-day planning horizon but rather a specified number of times. This will introduce a further characterization of the PSRP, namely the periodic nature of the application. Specifically, each station i must be served r_i days within the time horizon, and these service days are assigned to i by selecting one of the feasible combinations of r_i service days with the objective of minimizing the total distance traveled by the trucks.

In this paper, we define a new variant of the above described problem that we will denote as the Periodic Petrol Station Replenishment Problem (PPSRP) and we propose novel three-phase heuristic methods for its solution. Our approaches consist of

assigning first the service combinations by solving different integer programming models, defining then the routing for each day and finally using a local improvement technique to further reduce the delivery distance. A real-life application has been chosen for testing and assessing the performance of our solution methods. Satisfactory results have been obtained even in terms, for some of the proposed heuristics, of savings with respect to the solution currently adopted by the company.

The paper is organized as follows. Next section will be devoted to surveying the related works proposed in the literature and highlighting the original contribution of this paper. Section 3 will formally define the PPSRP and describe the periodicity constraints to be implemented. Section 4 will be dedicated to the development of our heuristic approaches to solve the PPSRP. The computational performance of the heuristics will be discussed in section 5 and finally some concluding remarks and future developments will be drawn in section 6.

2. Literature Review

The problem introduced in this paper finds its root in two different classes of distribution problems that are broadly studied in the literature, namely the PSRP and the periodic Vehicle Routing Problem (VRP). From one side, the PPSRP is a natural extension of the PSRP by prolonging the planning horizon to cover a T-day period. The 1-day variant of the problem has been subject to an extensive research activity and several mathematical models and numerical methods have been proposed for its solution. A not exhaustive list of these works include besides the collection of papers by Cornillier et al. (2008a, 2008b, 2009, and 2012), the contributions of Brown and Graves (1981), Brown et al. (1987), Ben Abdelaziz et al. (2002), Rizzoli et al. (2003), Ng et al. (2008), Surjandari et al. (2011) and Boctor et al. (2012). Only few papers have considered the multi-period aspect in the context of PSRP but none of them included explicitly the periodic nature of the problem. Specifically, Taqa Allah et al. (2000) have extended the planning horizon of the PSRP to the Multi-period case and have proposed several heuristic methods for the solution of the so-called MPSRP. They considered in their paper a single depot and an unlimited homogeneous truck fleet. Malepart et al. (2003) have tackled also the MPSRP in a real-life context and included the case in which the distribution company can decide for some stations the quantity to be delivered at each visit service. The last and more recent work that considered a multi-period horizon is due to Cornillier et al. (2008b) who proposed a heuristic that maximizes the profit calculated as the difference between the delivery revenue and the sum of regular and overtime drivers' costs.

Thus, to the best of our knowledge this paper represents a first tentative to tackle a PSRP which is characterized by periodic service requirements of the petrol stations.

From the other side, the PPSRP finds also its root in the literature of the periodic VRP that attracted the interest of many researchers over the few two decades (see for example Gaudioso et al. (1992), Chao et al. (1995) and Cordeau et al. (1997)). The two

problems have many similarities and the main difference between the two problems consists in the fact that the PPSRP should include additional constraints related, for example, to the drivers' shifts length.

3. Problem Description

The PPSRP is formally defined in terms of graph theory on a undirected connected network $G=(V, A)$. The set of vertices $V=\{1,\ldots,i,\ldots,n\}$ includes all the stations to be served whereas vertex 0 denotes the petrol depot. With $A=\{(i,j): \forall i \text{ and } j \in V \cup \{0\}\}$ we denote the set of all undirected edges that connect all the vertices of the network. To each edge $(i, j) \in A$ is associated a traversal cost c_{ij} that generally represents its length. An example of a region to be served and its resulting graph are represented in Figure 1.
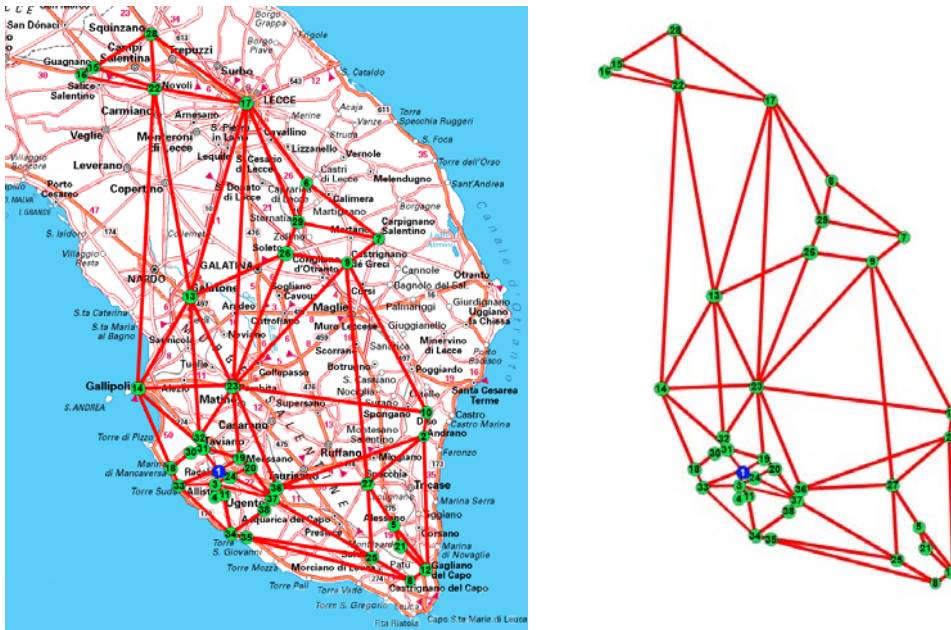

Figure 1 – An example of a region to be served (left) and its corresponding connected network (right). The stations correspond to the nodes of the graph and the roads are the undirected edges

The objective of the PPSRP consists in finding a minimum distance routing to serve all the stations considering an extended planning horizon of T days.

Within this T-day horizon, each station i must be visited r_i times, with at most one visit per day. These visits are assigned to i by selecting one out of a given set of feasible combinations of r_i visit days. Several mathematical representations of the periodicity aspect within the routing problems have been discussed for example in Paletta and Triki (2004). In this paper we will suppose that the set C_i for each city $i \in V$ is defined on the basis of the periodic sequence of combinations. Consequently, each station specifies the number of service visits r_i during the T-day planning horizon, and it must

be visited every $\frac{T}{r_i}$ days. An example of feasible combinations for T=6 is reported in Table 1.

Frequency of service r_i	Feasible sequence of combinations C_i
1	$C_i = \{(1),(2),(3),(4),(5),(6)\}$
2	$C_i = \{(1,4), (2,5), (3,6)\}$
3	$C_i = \{(1,3,5), (2,4,6)\}$
6	$C_i = \{(1,2,3,4,5,6)\}$

Table 1 – Periodic sequence combinations for $T = 6$

The resulting network model can be expressed in terms of combinatorial optimization having two sets of decision variables. The first set defines the routing solution and the corresponding truck assignment, i.e. it will give clear insight on which truck will serve which stations in which day in order to satisfy all the demand. The second set of variables will choose for each station the best periodicity combination that will minimize the overall delivery distance. The integer model with all its constraints is cumbersome and represents only slight differences with respect to the mathematical model reported, for example, in Cornillier et al. (2008b). For this reason and for brevity we avoid to report the whole model in this paper and we focus in the sequel on the innovative heuristic methods developed for its solution.

4. Heuristic Algorithms

Since PPSRP instances are generally characterized by high number of variables and constraints due mainly to the extended time horizon and also to the big number of stations to be served, we have chosen to solve the problem heuristically. We propose here four different heuristics all based on the idea of splitting the task of assigning the periodicity combinations to the stations from that of performing the routing rather than dealing with both the tasks simultaneously (see Figure 1). The routing task is carried out then by solving exactly the resulting VRPs, one for each day of the planning horizon. Finally all the heuristics are concluded by running an improvement procedure that is common to all the four approaches.
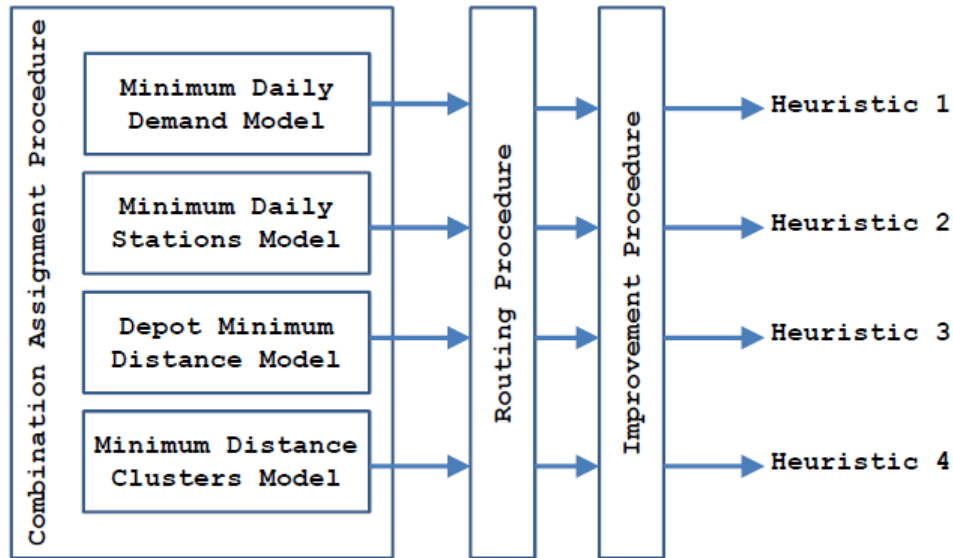
Figure 1 – The four heuristic solution approaches

4.1. Combination Assignment Procedure

The procedure of assigning feasible combinations to the stations is carried out by solving exactly (by using any general purpose software) one of the integer programming models proposed below. The notation to be used through this section is the following:

T days of the planning horizon

n number of stations

d_{it} demand of petrol of station $i \in V$ for day t (i.e. weekly demand of i divided by r_i)

C_i feasible periodicity combinations for station $i \in V$

a_{rt} input constant that equals 1 if day t belongs to combination r, and 0 otherwise

NV number of available vehicles

Q_v capacity of vehicle $v=1,...,NV$

c_{ij} distance from station i and station j with $i, j \in V \cup \{0\}$

4.1.1. Heuristic 1: `Minimum_Daily_Demand`

The idea here is to minimize the total demand to be served each single day of the planning horizon. In this way, the workload will be as much balanced as possible and consequently the number of vehicles will be minimized at a daily basis. While this procedure has the advantage of being easy to understand and to implement, it has the disadvantage of not taking into account the geographical position of the stations. The proposed integer program has y_{ir} as decision variables that define the periodicity combination to be assigned to each station, as follows:

y_{ir} is 1 if combination r is assigned to station i, 0 otherwise.

Moreover, by denoting by D_t the total quantity of petrol to be served on day t, the mathematical model can be expressed as:

$$\text{Min} \max_{t=1,...,T} D_t \tag{1}$$

Subject to:

$$\sum_{r \in C_i} y_{ir} = 1 \qquad\qquad i = 1,...,n \tag{2}$$

$$\sum_{i=1}^{n} \sum_{r \in C_i} a_{rt} d_{it} y_{ir} \leq D_t \qquad\qquad t = 1,...,T \tag{3}$$

$$y_{ir} \in \{0,1\} \qquad\qquad r \in C_i, i = 1,...,n \tag{4}$$

The objective function (1) minimizes the highest quantity of petrol to be served over all the days of the planning horizon. It is expressed here as a min-max objective function but it can be easily transformed to a standard minimization problem by using conventional operations research techniques. Constraints (2) force the model to choose only one periodic combination among all the feasible combinations. Constraints (3) restrict the daily quantity D_t to be delivered to match at least the total demand to be served on day t. Finally, constraints (4) impose the domain of the decision variables.

4.1.2. Heuristic 2: `Minimum_Daily_Stations`

The idea of this heuristic is to minimize the per day total number of stations to be served. Even in this case the attempt aims at balancing the workload expressed in term of number of stations and consequently at minimizing the number of vehicles to be used on a daily basis. This heuristics share the same advantage of its predecessor of being easy to understand and to implement, and moreover ensure a balanced workload for the trucks over the T days of the horizon. However, both the heuristics share also the same disadvantage of ignoring the geographical position of the stations.

By using the same binary variables y_{ir} as defined previously and, moreover, by introducing a new variable W_{nt} defining the number of stations to be visited during day t, the mathematical model is expressed as:

$$\text{Min} \max_{t=1,...,T} W_{nt} \tag{5}$$

Subject to:

$$\sum_{r \in C_i}^{n} y_{ir} = 1 \qquad\qquad i = 1,...,n \tag{6}$$

$$\sum_{i=1}^{n} \sum_{r \in C_i} a_{rt} y_{ir} = W_{nt} \qquad\qquad t = 1,...,T \tag{7}$$

$$y_{ir} \in \{0,1\} \qquad\qquad r \in C_i, i = 1,...,n \tag{8}$$

$$W_{nt} \quad Integer \qquad\qquad t = 1,...,T \tag{9}$$

The objective function (5) makes use of the new variable W_{nt} in order to minimize the highest number of stations to be served over all the days of the planning horizon. Constraints (6) assigns as before only one periodic combination to each station. Constraints (7) impose logic relations between the new variable W_{nt} and the assignment variables y_{ir} on a daily basis, and the domain of the decision variables is defined through constraints (8) and (9).

4.1.3. Heuristic 3: `Depot_Minimum_Distance`

The aim of this heuristic is to solve a relaxation of a PVRP-like in order to minimize the total distance between the depot (node 0) and all the stations to be served. Even though the resulting model is a simplified version of the PVRP, it will maintain some of the complexity but on the other hand it has the advantage of making use of more information since it takes into consideration the geographical position of the stations with respect to the depot.

The integer model has two sets of variables. Besides, the binary variables y_{ir} as previously used we need to introduce also the following binary variables:

x_{0it} is 1 if station i is assigned to the depot (node 0) on day t, and 0 otherwise

The optimization model can be summarized as follows:

$$\text{Min} \quad \sum_{i=1}^{n} \sum_{t=1}^{T} c_{0i} x_{0it} \tag{10}$$

Subject to:

$$\sum_{r \in C_i} y_{ir} = 1 \qquad\qquad i = 1,...,n \tag{11}$$

$$x_{0it} = \sum_{r \in C_i} a_{rt} y_{ir} \qquad\qquad i = 1,...,n,\ t = 1,...,T \tag{12}$$

$$\sum_{i=1}^{n} d_{it} x_{0it} \leq \sum_{v=1}^{NV} Q_v \qquad\qquad t = 1,...,T \tag{13}$$

$$x_{0it} \in \{0,1\} \qquad\qquad i = 1,...,n,\ t = 1,...,T \tag{14}$$

$$y_{ir} \in \{0,1\} \qquad\qquad i = 1,...,n,\ r \in C_i \tag{15}$$

The objective function (10) minimizes the total distance of all the stations with respect to the depot over the days of the planning horizon. Constraints (11) have the same role as in the previous models, whereas Constraints (12) impose logic relations between the two sets of variables x_{0it} and y_{ir}. Inequalities (13) are capacity constraints that ensure, for each day t, the respect of the trucks' capacity and, finally, the integrity of the decision variables are defined by constraints (14) and (15). It should be noted that this

model is similar to problem (1)—(4) and the main difference between the two problems consists in the objective function (1) that becomes here (10).

4.1.4. Heuristic 4: `Minimum_Distance_Clusters`

This heuristic is an extension of the previous one allowing the minimization of the total distance of the stations with respect to virtual centers, one for each day of the horizon, rather than considering only the depot. This clustering strategy allows to exploit better the information about the distances and, hopefully, generates more compact sets of stations from the geographical viewpoint. The obvious disadvantage is the complexity of the resulting model involving a big number of binary variables.

For simplicity, the virtual centers are chosen here among a set of specific positions that coincide with some of the stations, so that the topology of the network remain unaffected. The integer model makes use of the variables y_{ir} and, moreover, introduces the following two sets of binary variables:

x_{ijt} is 1 if station i is assigned to the virtual centre j in day t, and 0 otherwise

z_{jt} is 1 if virtual centre j is assigned to day t, and 0 otherwise

The integer model corresponding to heuristic 4 is as follows:

$$\text{Min} \quad \sum_{i=1}^{n}\sum_{j=1}^{T}\sum_{t=1}^{T} c_{ij} x_{ijt} \tag{16}$$

Subject to:

$$\sum_{r \in C_i} y_{ir} = 1 \qquad\qquad i = 1,...,n \tag{17}$$

$$\sum_{j=1}^{T} x_{ijt} = \sum_{r \in C_i} a_{rt} y_{ir} \qquad\qquad i = 1,...,n,\ t = 1,...,T \tag{18}$$

$$\sum_{j=1}^{T} z_{jt} = 1 \qquad\qquad t = 1,...,T \tag{19}$$

$$\sum_{i=1}^{n} d_{it} x_{ijt} \leq \left(\sum_{v=1}^{NV} Q_v\right) z_{jt} \qquad\qquad t = 1,...,T,\ j = 1,...,T \tag{20}$$

$$x_{ijt} \in \{0,1\} \qquad\qquad i = 1,...,n,\ t = 1,...,T,\ j = 1,...,T \tag{21}$$

$$y_{ir} \in \{0,1\} \qquad\qquad i = 1,...,n,\ r \in C_i \tag{22}$$

$$z_{jt} \in \{0,1\} \qquad\qquad t = 1,...,T,\ j = 1,...,T \tag{23}$$

Similarly to the previous heuristic, the objective function (16) minimizes the total distance of all the stations with respect to the T virtual centres (one for each day of the planning horizon). All the other Constraints have the same meaning and role as in the

previous heuristic except for some slight adjustments due to presence of T virtual centres and the introduction of the new set of variables z_{jt}. We should just mention that the notation may be further simplified in the above model by merging the indexes j and t (since they represent the same set of horizon days and have a one-to-one relationship) but we preferred, for the sake of clarity, to distinguish between the temporal index t and the spatial index j.

Finally, we should note that the last two heuristics seem to be more challenging computationally since they include the geographical aspect of the problem and, thus, involve additional decision variables with respect to the former two methods. Possible higher quality solutions to be yielded by the latter heuristics may be paid by longer computational time when solving large-scale problems. This time-quality trade-off maybe very useful when solving several instances with different sizes.

4.2. Solving the VRPs: `Routing_Procedure`

Once the hard task of assigning the stations to the days of the planning horizon, while respecting the periodicity constraints on each station, has been solved we need to define the routes for each day. The `Routing_Procedure` takes as input the sets of stations corresponding to each day (i.e. the output of any of the models of subsection 4.1) and generates a set of routes for the available trucks. This is nothing else but solving independently T standard VRPs, one for each day of the horizon. Since each resulting VRP is relatively small with respect to the original PPSRP we decided to solve these T problems exactly by using a standard VRP model and a general purpose optimization software packages.

4.3. Getting a Better Solution: `Improvement_Procedure`

The final step of our solution approaches consists in trying to improve the quality of the solution (minimize further the overall distance) by applying a local search technique. Theoretically, a room for the improvement should be available since the assignment of the stations to the T days has been done heuristically without any guarantee to get the best feasible solution. Starting from a set of routes assigned to the available trucks for every day of the planning horizon, the technique employed here is based on switching any two stations assigned to two different service days and check if any improvement is taking place. While checking, at each switching operation, the improvement in the total distance, we should check also the feasibility of the solution expressed in terms of service periodicity of each station and capacity of the corresponding trucks. This procedure is easy to understand but requires a big effort for its implementation, since no standard software are available to achieve this task.

5. Computational Results

This section is dedicated to the application of our heuristic methods to solve a real-life test problem and to analyze and compare their performance. All the models introduced

above have been implemented by using the LINGO commercial package 8.0 as modeling language that has been interfaced with the state-of-the-art solver CPLEX 12.1 for the solution of the resulting integer programs (Lindo, 2006 and Ibm Ilog, 2011 respectively). On the other hand, the improvement procedure required specific programming skills and, thus, the language JAVA has been chosen for its implementation. A computer with an Intel Core 2 Duo Processor 2.53GHz and 4GB of RAM has been used to run all the resulting codes.

5.1. Real-life Test Problem

To our knowledge, there are no standard test problems available as benchmark for the PPSRP, since we believe it is the first time this problem is being tackled. For this reason, we will compare the quality of our heuristics' solutions from one side with respect to the solution currently adopted by the case company, and from the other with respect to the best feasible solution that can be reached by the exact solver.

The case study consists in a regional company located in the south of Italy having a central petrol depot from which a heterogeneous fleet of trucks starts daily trips to serve $n = 38$ petrol stations dislocated in different positions of the regional territory. The planning horizon for this problem is a week having 6 working days (i.e. $T = 6$). The weekly demand of petrol varies from one station to another requiring in some cases only one delivery visit but most of the cases a multi-visit service fractioned into 2 or 3 deliveries per week. The available trucks have different capacities allowing the model to choose for each route the one that fits better the model's constraints.

Concerning the drivers, generally speaking the company has enough drivers available during all the weekdays except for the sixth working day (i.e. the first weekend day) in which the number of drivers is limited because of the unavailability of most of them to work. This limitation has been considered in our models by dropping, whenever is possible, the sequence combinations that involve day 6. For example, when $r_i = 2$ we have considered only the set $C_i = \{(1,4), (2,5)\}$ as feasible combinations and we disregarded the combination (3,6) in order to take into account the drivers' unavailability. We kept, however, both the sequences $C_i = \{(1,3,5), (2,4,6)\}$ for the stations requiring 3 visits per week.

The integrated PPSRP model corresponding to our application resulted in a large-scale problem having 84740 constraints and 84701 variables. The state-of-the-art Cplex software package was not able to yield an exactly solution for the problem but has generated at least a feasible upper bound solution of 13975 km/week. Indeed, after more than 30 CPU hours and a huge number of iterations the software has given a fault memory error due to the big number of subproblems generated during the Branch-and-Bound method.

For completeness, we also collected data for a different instance of the test problem in order to ascertain about the performance of our heuristic procedures. The new instance

refers to another working week with different demand of petrol to be served and also different frequencies of periodicity.

5.2. Results and Discussion

We start by assessing the solution quality of each approach (including the exact method) before and after applying the improvement procedure and the corresponding CPU time. The results collected in Table 2, show, from one side, the superiority of Heuristic 4, both as solution quality and CPU time, with respect to the other heuristics and also with respect to the feasible upper bound obtained by Cplex. From the other side, the positive effect of the improvement_procedure on all the method, except Heuristic 4, is clear. This means that the solution generated by Heuristic 4 is probably close enough to the optimal solution so that no margin was left for improvement.

Approach	Initial Solution (Km) Before Improvement	Final Solution (Km) After Improvement	Total CPU Time (sec.)
Exact Method	13975	13360	109628
Heuristic 1	14382	14281	1021
Heuristic 2	13744	13036	3264
Heuristic 3	13587	13340	2126
Heuristic 4	12193	12193	591

**Table 2 – Solution costs (in terms of distance) and CPU times**

The second set of experiments deals with the assessment of our approaches with respect to the solution currently adopted by the company. Such a solution has not been obtained through the use of any decision support tool but just as the result of a long experience of the company's human operators in managing not only the loading/unloading operations but also the transportation phase. For the data corresponding to the week we took into consideration, the actual distance run by the trucks for serving all the 38 stations resulted to be 14820 kms. It is evident from Table 3, thus, that all our approaches improve the human operator solution and, particularly, Heuristic 4 that reaches, after improvement, a relative distance saving of 17.7% with respect to the company's decisions. These same results are also depicted in Figures 2 and 3.

Approach	Distance (Km)	Savings (km)	Savings (%)
Human Operator	14820	---------	---------
Exact Method	13360	1469	9.9
Heuristic 1	14281	539	3.6
Heuristic 2	13036	1784	12.0
Heuristic 3	13340	1480	10.0
Heuristic 4	12193	2627	17.7

Table 3 – Comparison between our solutions and the company's solution
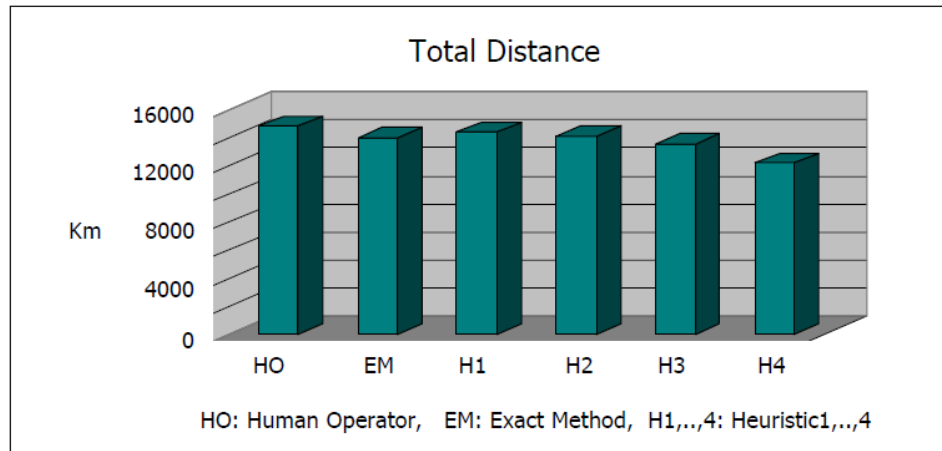
Figure 2 – Comparison between our solutions and the company's solution
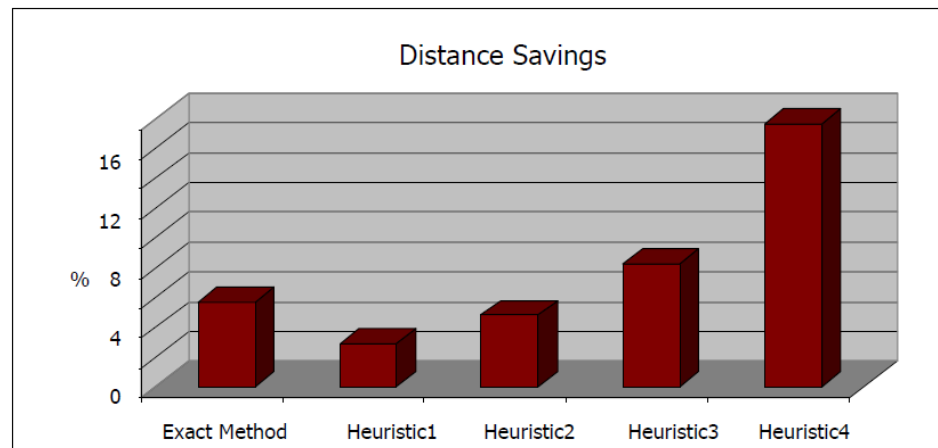
Figure 3 – Savings obtained with respect to company's solution

Finally, for the sake of completeness, we tested our heuristic procedures on a different set of data for the same distribution network. This kind of sensitivity analysis experiment have the aim of checking the robustness of the obtained solutions, and specially verifying the conclusions related to Heuristic 4. This new set of data has been purposely collected for another week belonging to a different season and different demand pattern with respect to the previous data. The two sets differ only by the amount of petrol to be served to each station and, consequently, by their frequency of service.

Approach	Initial Solution (Km) Before Improvement	Final solution (Km) After Improvement	Savings (%): Final vs. Company's solutions
Human Operator	14770	--------	--------
Heuristic 1	15053	14814	--------
Heuristic 2	14197	13987	5.3
Heuristic 3	12848	12828	13.1
Heuristic 4	12819	12819	13.2

Table 4 – Comparison of the solutions for different test data

The results reported in Table 4 show that these new results substantially confirm the trend observed in the previous test (except for Heuristic 1). More specifically, Heuristic 4 shows once again to outperform all the other approaches. Moreover, even in this case the `Improvement_Procedure` has no effect on the initial solution obtained, confirming our intuitive conclusion that Heuristic 4 generates already a nearly optimal solution. From another point of view, the results reported in the final column of Table 4 confirm the improvement that three of our heuristics can achieve with respect to the solution adopted by the company for that week.

6. Concluding Remarks

In this paper, we have presented and discussed the Petrol station Replenishment Problem with Periodicity constraints. We have proposed four heuristic approaches based on three-phases each: the assignment of a periodicity combinations to each station, the construction of trucks tours, and finally the tours improvement phase. The performance of the proposed heuristics has been tested on two different instances of a real-life periodic problem. Both the instances have shown the superiority of Heuristic 4 with respect to the others reaching an average improvement of more than 15% (in terms of traveled distance) with respect to the solutions actually adopted by the company while keeping the execution time within reasonable limits. Future developments in this direction consists in solving another more challenge real-life problem having a National scale for a petrol distribution company operating in Oman. Besides the difficulties that will be introduced by the problem's size, the new application will be also characterized by further complexities such as uncertainty related to the drivers availability and regulatory restrictions on the citizenship of the drivers to accomplish some routes.

References

Ben Abdelaziz F, Roucairol C, Bacha C (2002), Deliveries of liquid fuels to SNDP gas stations using vehicles with multiple compartments. In: Systems Man and Cybernetics IEEE International Conference. Hammamet, Tunisia.

Boctor F, Renaud J, Cornillier F (2011), Trip packing in petrol stations replenishment. Omega 39:86–98.

Brown G G, Ellis C J, Graves G W, Ronen D (1987), Realtime wide area dispatch of Mobil tank trucks. Interfaces 17(1):107–120.

Brown G G, Graves G W (1981), Realtime dispatch of petroleum tank trucks. Management Science 27:19–32.

Chao I-M, Golden B L, Wasil E (1995), An improved heuristic for the period vehicle routing problem. Networks 26:25-44.

Christofides N, Beasley J E (1984), The period routing problem. *Networks* 14:237-256.

Cordeau J-F, Gendreau M, Laporte G (1997), A tabu search heuristic for periodic and multi-depot vehicle routing problems. Networks 30:105-119.

Cornillier F, Boctor F, Laporte G, Renaud J (2008a), An Exact Algorithm for the Petrol Station Replenishment Problem, J. of the Operational Research Society 59: 607–615.

Cornillier F, Boctor F, Laporte G, Renaud J (2008b), A heuristic for the multi-period petrol station replenishment problem. European J. of Operational Research 191:295–305.

Cornillier F, Boctor F, Renaud J (2012), heuristics for the multi-depot petrol station replenishment problem with time windows. European J. of Operational Research 220:361–369.

Cornillier F, Laporte G, Boctor F, Renaud J (2009), The petrol station replenishment problem with time windows. Computers and Operations Research 36:919–935.

Gaudioso M, Paletta G (1992). A heuristic for the periodic vehicle routing problem. Transportation Science 26:86-92.

Ibm Ilog Cplex Optimizer (2011), http://www-01.ibm.com/software/integration/optimization/cplex-optimizer/.

Lindo Systems Inc. (2006), Optimization Modeling with Lingo, www.lindo.com.

Malépart V, Boctor F, Renaud J, Labilois S (2003), Nouvelles approches pour l'approvisionnement des stations d'essence. Revue Française de Gestion Industrielle 22:15–31.

Ng W L, Leung S H, Lam J P, Pan S W (2008), Petrol delivery tanker assignment and routing: a case study in Hong Kong. J. of the Operational Research Society 59:1191–1200.

Paletta G, Triki C (2004), Solving the Asymmetric Traveling Salesman Problem with Periodic Constraints. Networks 44:31-37.

Rizzoli A, Casagrande N, Donati A, Gambardella L, Lepori D, Montemanni R, Pina P, Zaffalon M (2003), Planning and optimisation of vehicle routes for fuel oil

distribution. In: MODSIM International Congress on Modelling and Simulation. Townsville, Australia.

Surjandari I, Rachman A, Dianawati F, Wibowo R P (2011), Petrol delivery assignment with multi-product, multi-depot, split deliveries and time windows. International J. of Modeling and Optimization 1(5):375–379.

Taqa allah D, Renaud J, Boctor FF (2000), Le problème d'approvisionnement des stations d'essence. APII-JESA J. Européen des Systèmes Automatisés 34:11–33.