A First Course in Linear Optimization

— a dynamic book —

by
Jon Lee

Third Edition (Version 3.00)

REEX PRESS

Jon Lee

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Go Forward

This is a book on linear optimization, written in LaTeX. I started it, aiming it at the course IOE 510, a masters-level course at the University of Michigan. Use it as is, or adapt it to your course! It is an ongoing project. It is alive! It can be used, modified (the LaTeX source is available) and redistributed as anyone pleases, subject to the terms of the Creative Commons Attribution 3.0 Unported License (CC BY 3.0)  . Please take special note that you can *share* (copy and redistribute in any medium or format) and *adapt* (remix, transform, and build upon for any purpose, even commercially) this material, but you must give appropriate credit, provide a link to the license, and indicate if changes were made. You may do so in any reasonable manner, but not in any way that suggests that I endorse you or your use. If you are interested in endorsements, speak to my agent.

I started this material, but I don't control so much what you do with it. Control is sometimes overrated — and I am a control freak, so I should know!

I hope that you find this material useful. If not, I am happy to refund what you paid to me.

Jon Lee
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Ann Arbor, Michigan
started March 2013

Preface

This book is a treatment of linear optimization meant for students who are reasonably comfortable with matrix algebra (or willing to get comfortable rapidly). It is *not* a goal of mine to teach anyone how to solve small problems by hand. My goals are to introduce: (i) the mathematics and algorithmics of the subject at a beginning mathematical level, (ii) algorithmically-aware modeling techniques, and (iii) high-level computational tools for studying and developing optimization algorithms (in particular, `MATLAB` and `AMPL`).

Proofs are given when they are important in understanding the algorithmics. I make free use of the inverse of a matrix. But it should be understood, for example, that $B^{-1}b$ is meant as a mathematical expression for the solution of the square linear system of equations $Bx = b$. I am not in any way suggesting that an efficient way to calculate the solution of a large (often sparse) linear system is to calculate an inverse! Also, I avoid the dual simplex algorithm (e.g., even in describing branch-and-bound and cutting-plane algorithms), preferring to just think about the ordinary simplex algorithm applied to the dual problem. Again, my goal is not to describe the most efficient way to do matrix algebra!

Illustrations are woefully few. Though if Lagrange could not be bothered[1], who am I to aim higher? Still, I am gradually improving this aspect, and many of the algorithms are *illustrated* in the modern way, with computer code.

The material that I present was mostly well known by the 1960's. As a student at Cornell in the late 70's and early 80's, I learned and got excited about linear optimization from Bob Bland, Les Trotter and Lou Billera, using [1] and [5]. The present book is a treatment of some of that material, with additional material on integer-linear optimization, mostly which I originally learned from George Nemhauser and Les. But there is new material too; in particular, a "deconstructed post-modern" version of Gomory pure and mixed-integer cuts. There is nothing here on interior-point algorithms and the ellipsoid algorithm; don't tell Mike Todd!

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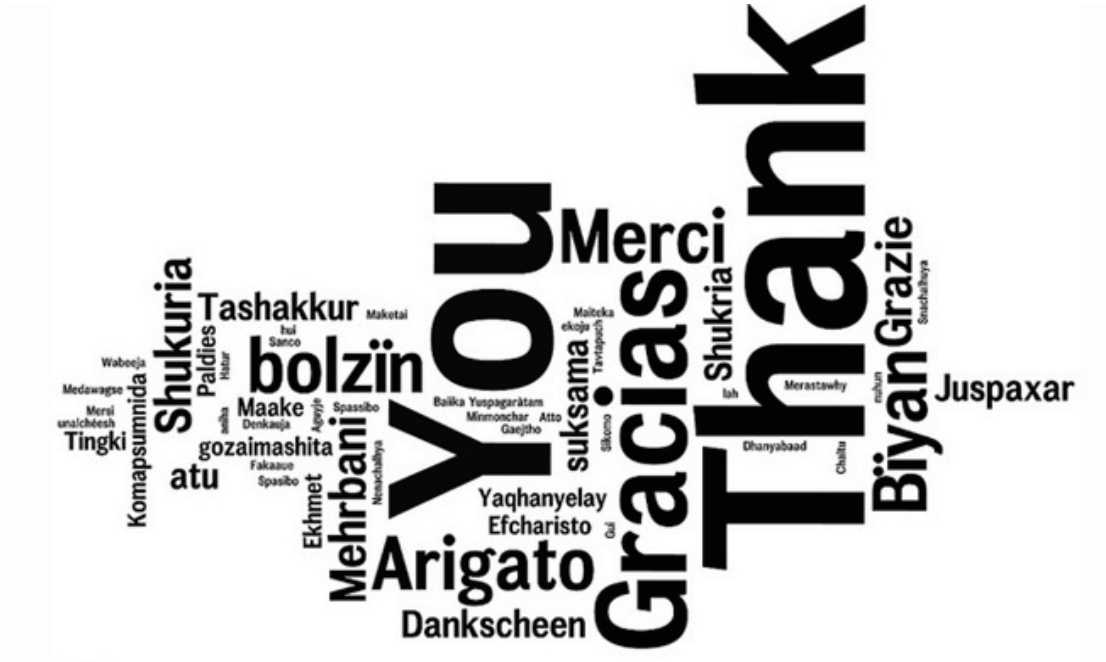
started March 2013
(or maybe really in Ithaca, NY in 1979)

Serious Acknowledgments

Throw me some serious funding for this project, and I will acknowledge you — seriously!

Many of the pictures in this book were found floating around on the web. I am making "*fair use*" of them as they float through this document. Of course, I gratefully acknowledge those who own them.

Hearty thanks to many students and to Prof. Siqian Shen for pointing out typos in an earlier version.



Dedication

For students *(even Ohio students)*. Not for publishers — not this time. Maybe next time.

The Nitty Gritty

You can always get all released editions of this book (in `.pdf` format) from my web page and the materials to produce them (LaTeX source, etc.) from me.

If you decide that you need to recompile the LaTeX to change something, then you should be aware of two things. First, there is no user's manual nor help desk. Second, I use a lot of include files, for incorporating code listings and output produced by software like `Matlab` and `AMPL`. Make sure that you understand where everything is pulled from if you recompile the LaTeX. In particular, if you change *anything* in the directory that the LaTeX file is in, something can well change in the `.pdf` output. For example, `Matlab` and `AMPL` scripts are pulled into the book, and likewise for the output of those scripts. If you want to play, and do not want to change those parts of the book, play in another directory.

I make significant use of software. Everything seems to work with:

```
MATLAB R2016b
AMPL 20170711
CPLEX 12.7.1.0
Mathematica 11.1.0.0
WinEdt 10.1
MiKTeX 2.9
```

Use of older versions is inexcusable. Newer versions will surely break things. Nonetheless, if you can report success or failure on *newer* versions, please let me know.

I use lots of LaTeX packages (which, as you may know, makes things rather fragile). I could not possibly gather the version numbers of those — I do have a day job! (but I do endeavor to keep my packages up to date).

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