A several new mixed integer linear programming formulations for exploration of online social networks

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Abstract

The goal of this paper is to identify the most promising sets of closest assignment constraints from the literature, in order to improve mixed integer linear programming formulations for exploration of information flow within a social network. The direct comparison between proposed formulations is performed on standard single source capacitated facility location problem instances. Therefore, in contrast to previous work, these results could be verified, validated and used in direct comparison by some possible future method.

Key words: Facility Location, Combinatorial Optimization, Social Networks

1. Introduction

Social networks are usually defined in the literature as structures made of actors (generally humans) linked by relations (Wasserman & Faust, 1994). For example, these relations may be phone calls or kinship. Consequently, the social networks are mostly represented as graphs, therefore social network optimization is closely related to graph optimization techniques. Therefore, some concepts from these techniques may be used for research of online networks.

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In this work, two out of three problems from (Stanimirović & Mišković, 2013) for efficient analysis of an online social network is considered. Developed mixed integer linear programming models provide new information on linking behaviors and flow between nodes. The intention is to design an efficient search strategy to identify nodes that exchange information containing some keywords, which further can indicate that identified nodes belongs to the same interest group. More precisely, the intention of these problems is to determine exactly p locations for establishing control devices, such that all user nodes from given social network are analyzed in the most efficient way with the opened control devices.

Stanimirović & Mišković (2013) proposed mixed integer linear programming (MILP) formulations for three problems (they wrongly named them as models) for efficient analysis of an online social network. In order to solve these problems of larger dimensions, they also proposed three evolutionary metaheuristic approaches. Unfortunately, their paper is full of errors and deceptions. For example, all three evolutionary approaches were used crossover probabilities which were *larger than 1* (Stanimirović & Mišković, 2013, p. 241). Since the authors performed experiments on the problem instances that were generated randomly by themselves, and are unavailable to other researchers, the only valuable scientific content of that paper seems to be MILP formulations.

2. Existing mathematical formulations

In this section, three problems that consider exploration of online social networks are presented, as they are defined in (Stanimirović & Mišković, 2013). In all three problems (in that paper problems are wrongly named as models), the same input data and data structures, which are related to online social networks, are considered. Let $I = \{1, 2, ..., m\}$ be the set of user nodes in the network, and $J = \{1, 2, ..., n\}$ a set of potential locations for establishing control devices. The non-negative matrix $c_{ij} | i \in I, j \in J$ represents the cost of searching through the data from a user node i by a control device j. These costs may depend on the amount of time needed to explore one unit of data originating from i by device j, the distance and speed of Internet connection between them, etc. In all three problems, exactly p potential locations are established, and each user is assigned to one or more open control devices in order to minimize the overall search process of online social network.

2.1. Problem 1

This problem consists of establishing (exactly p) control devices and assign each user to exactly one, most favorable open device, such that the maximal load of an established control device is minimized. The load of an established control device is equal to the sum of searching costs through this control device. It is required that each user is assigned to the most favorable open device, i.e. established control device with minimal searching cost from it.

The Problem 1 can be mathematically formulated in the following way. Let $S \subset J$ denote the set of established control devices, $j_i \in S$ is the most favorable established device for user i, and $I_j = \{j \in J | j = j_i \; for \; some \; i \in I\}$ is set of users assigned to j. Then, objective function value $obj_{P1}(S) = \max\limits_{j \in S} \sum\limits_{i \in I_j} c_{ij}$, and optimal objective value of Problem 1 can be defined as $opt_{P1} = \min\limits_{S \subset J, |S| = p} \mathrm{obj}_{P1}(\mathrm{S})$.

The mixed integer linear programming formulation introduced in (Stanimirović & Mišković, 2013) have decision variables $x_{ij} = 1$, if user i is assigned to established control device j, and $x_{ij} = 0$ otherwise. Next, variables $y_j = \begin{cases} 1, & j \in S \\ 0, & j \notin S \end{cases}$ and real variable $z = \max\limits_{j \in S} \sum\limits_{i \in I_j} c_{ij} = \mathrm{obj}_{P1}(\mathrm{S})$.

Using the previous notation, MILP of Problem 1 was defined as follows:

$$\min z \tag{1}$$

such that:

$$\sum_{j \in J} y_j = p \tag{2}$$

$$x_{ij} \leq y_j, \quad for \; all \; i \in I, j \in J \tag{3}$$

$$\sum_{j \in J} x_{ij} = 1, \quad for \; all \; i \in I \tag{4}$$

$$\sum_{k \in J} c_{ik} \cdot x_{ik} \leq c_{ij} + M \cdot (1 - y_j), \quad for \; all \; i \in I, j \in J \tag{5}$$

$$\sum_{i \in I} c_{ij} \cdot x_{ij} \leq z, \quad for \; all \; j \in J \tag{6}$$

$$x_{ij}, y_j \in \{0,1\}\,,\ z \geq 0 \quad for\ all\ i \in I, j \in J \tag{7}$$

The objective function (1) minimizes the maximal time needed to search the data coming from all user nodes assigned to an established control device. Constraint (2) indicates that exactly p resource nodes are established. By constraints (3) and (4) it is ensured that each user node is assigned to exactly one established resource node. By constraints (5), each user is assigned to its most favorable open device. In this constraint, large positive constant M ($M > \max\limits_{i \in I, j \in J} c_{ij}$) is used, in order to dispose bounds on searching costs of non-established control devices. Constraints (6) impose lower bounds on the value of objective value z, while (7) reflect the nature of decision variables x_{ij}, y_j and z.

Note that constraints (5) were almost the same as closest assignment constraints (8) proposed by Berman et al. (2009), in the context of equitable location problem. The only slight difference is in the case $y_j = 0$, when right hand side of (5) was $c_{ij} + M$ while right hand side of (8) was M. Since $M > \max\limits_{i \in I, j \in J} c_{ij}$ it is theoretically the same, which also can be seen from computational results in Section 4.

$$\sum_{k \in J} c_{ik} \cdot x_{ik} + (M - c_{ij}) \cdot y_j \leq M, \quad for\ all\ i \in I, j \in J \tag{8}$$

2.2. Problem 2

Similarly to Problem 1, this problem also consists of establishing (exactly p) control devices and assign each user to exactly one, most favorable open device. Instead of optimizing the maximal load in the Problem 1, now the goal is to minimize the load balance among control devices. The load balance of an established control device is equal to the difference between maximal and minimal load among established control devices.

The Problem 2 can be mathematically formulated as
$opt_{P2} = \min\limits_{S \subset J, |S| = p} \text{obj}_{P2}(\text{S})$, where $obj_{P2}(S) = \max\limits_{j \in S} \sum\limits_{i \in I_j} c_{ij} - \min\limits_{j \in S} \sum\limits_{i \in I_j} c_{ij}$.

The mixed integer linear programming (MILP) formulation introduced in (Stanimirović & Mišković, 2013) have the same decision variables as for Problem 1, with one additional real variable $z_{min} = \min\limits_{j \in S} \sum\limits_{i \in I_j} c_{ij}$

Using the previous notation, MILP of Problem 2 was defined as follows:

$$\min z - z_{min} \tag{9}$$

subject to conditions (2)-(7) with additional requirements such that:

$$\sum_{i \in I} c_{ij} \cdot x_{ij} + K \cdot (1 - y_j) \geq z_{min}, \quad for\ all\ j \in J \tag{10}$$

$$z_{min} \geq 0 \tag{11}$$

The objective function (9) minimizes the load balance among control devices. Note that, in (Stanimirović & Mišković, 2013, p. 230) sentence "By objective function (10) we want to provide a load balance among established control devices, as much as possible." is wrong, since load balance was minimized, not maximized!

Large positive constant K in conditions (10) must holds $K > \sum_{i \in I} c_{ij}$ in order to dispose zero value load of non-established control devices. Constraint (11) impose upper bounds on the real decision variable z_{min}.

2.3. Problem 3

In the Problem 3, user can be assigned to more than one established control device and such device may not be most favorable one. In this scheme, a user node may split its out-coming data flow to several established control devices, i.e. user requirements can be (partially) served by some different established devices but out-coming flow of the considered user node must be equal to sum of these flow parts. The objective of the Problem 3 is to establish (exactly p) control devices and to divide user requirements through open control devices in order to minimize the load balance of control devices.

The mixed integer linear programming (MILP) formulation of Problem 3 (Stanimirović & Mišković, 2013) have the binary decision variables $y_j, j = 1, ..., n$, real variables x_{ij} and real variable z. Using the previous notation, MILP was formulated as follows:

$$\min z \tag{12}$$

subject to conditions (2)-(4) with additional requirements:

$$c_{ij} \cdot x_{ij} \leq z, \quad for\ all\ i \in I, j \in J \tag{13}$$

$$x_{ij} \in [0,1], y_j \in \{0,1\}\,,\; z \geq 0 \quad for\ all\ i \in I, j \in J \tag{14}$$

The objective function (12) minimizes the load among control devices. Constraints (13) impose lower bounds on the value of objective value z, while (14) reflect the nature of decision variables x_{ij}, y_j and z.

2.4. Illustrative example

Example 1 (Stanimirović & Mišković, 2013) Let us consider a small network with $m = 10$ user nodes and $n = 6$ potential locations for establishing control devices. Suppose that exactly $p = 3$ control devices are to be established with cost matrix

$$C = \begin{bmatrix} 10163 & 14 & 73 & 489 & 14588 & 125 \\ 113 & 234 & 29 & 12365 & 12657 & 265 \\ 12050 & 12955 & 132 & 73 & 368 & 0 \\ 12114 & 12765 & 114 & 221 & 42 & 143 \\ 192 & 14245 & 122 & 13123 & 169 & 33 \\ 10533 & 12446 & 195 & 294 & 325 & 133 \\ 25 & 171 & 393 & 385 & 11333 & 10765 \\ 370 & 14645 & 116 & 292 & 14748 & 449 \\ 286 & 13273 & 245 & 14095 & 497 & 82 \\ 476 & 11263 & 187 & 124 & 14359 & 275 \end{bmatrix}$$

Optimal objective function values of Problems 1-3 on this example are:

- Problem 1: Optimal solution value is 419, with established control devices 2, 4 and 6;
- Problem 2: Optimal solution value is 28, with established control devices 2, 4 and 6;
- Problem 3: Optimal solution value is 90.213645, with established control devices 1, 3 and 6. Note that, in (Stanimirović & Mišković, 2013) is erroneously presented optimal solution for this problem as 90.124 with correct set of established control devices.

□

3. New mixed integer linear programming formulations

In this section it will be presented new mixed integer programming formulations for Problems 1 and 2. General idea of improvement is to replace constraints (5), by some other ones. Espejo et al. (2012) give the survey of closest assignment constraints (CAC) from the literature, proposed one such constraint and it gives a theoretical analysis of all of them. Except $BDTW$ constraints proposed by Berman et al. (2009), which was already presented in previous section in (8), all other approaches would be described in detail.

Due to (Espejo et al., 2012), the earliest work in which CAC were considered in an integer programming modeling was in (Rojeski & ReVelle, 1970). It was used for solving the budget constrained median problem where the proposed constraints RR are:

$$x_{ij} + \sum_{k \in J: c_{ik} < c_{ij}} y_k \geq y_j, \quad for\ all\ i \in I, j \in J \tag{15}$$

Wagner & Falkson (1975) introduced WF set of constraints for ensuring closest center behavior in an integer-linear location-allocation model, as follows:

$$\sum_{k \in J: c_{ik} > c_{ij}} x_{ik} + y_j \leq 1, \quad for\ all\ i \in I, j \in J \tag{16}$$

Next linear set of constraints CC designed to force closest assignment was proposed by Church & Cohon (1976) for siting energy facilities and afterward by Hanjoul & Peeters (1987).

$$\sum_{k \in J: c_{ik} \leq c_{ij}} x_{ik} \geq y_j, \quad for\ all\ i \in I, j \in J \tag{17}$$

Dobson & Karmarkar (1987) proposed a DK constraints for solving the competitive location problem on a network:

$$x_{ij} + y_k \leq 1 \quad for\ all\ i \in I, j, k \in J\ such\ that\ c_{ik} < c_{ij} \tag{18}$$

Several years ago, Cánovas et al. (2007) proposed $CGLM$ for SImple Plant Location Problem with Order:

$$y_j + \sum_{k \in J: c_{ik} > c_{ij}} x_{ik} + \sum_{k \in J: c_{ik} \leq c_{ij}, c_{lk} > c_{lj}} x_{lk} \leq 1, \quad for\ all\ i, l \in I, j \in J \tag{19}$$

Belotti et al. (2007) proposed $BLMN$ constraints for solving the obnoxious p-median problem:

$$p \cdot \sum_{k \in J: c_{ik} > c_{ij}} x_{ik} \leq \sum_{k \in J: c_{ik} > c_{ij}} y_k, \quad for\ all\ i \in I, j \in J \tag{20}$$

Finally, Marín (2011) and Espejo et al. (2012) have proposed newest CACs known in the literature as *M* and *EMR*, respectively:

$$q_{ij} \cdot \sum_{k \in J: c_{ik} \geq c_{ij}} x_{ik} + \sum_{k \in J: c_{ik} < c_{ij}} y_k \leq q_{ij}, \quad for\ all\ i \in I, j \in J \tag{21}$$

$$\begin{aligned} \sum_{k \in J: c_{ik} < c_{ij}, |\theta_{ij}| - |\theta_{ik}| \leq p} (|\theta_{ik}| + (p - |\theta_{ij}|)^-) \cdot x_{ik} + q_{ij} \cdot \sum_{k \in J: c_{ik} \geq c_{ij}} x_{ik} + \\ \sum_{k \in J: c_{ik} < c_{ij}} y_k \leq q_{ij}, \quad for\ all\ i \in I, j \in J \end{aligned} \tag{22}$$

where $q_{ij} = min\{p, |\theta_{ij}|\}$, $\theta_{ij} = \{k | c_{ik} < c_{ij}\}$ and $z^- = min\{z, 0\}$.

The detailed analysis of all these constraints will be omitted since it is already done in (Espejo et al., 2012). It would be mentioned that some of them could not work in the case of ties between costs, i.e. cost from one user node to two or more potential control devices can be equal. Since this situation can occurs with Problems 1 and 2, these ones must be omitted from the consideration.

4. Computational results

All computational experiments were carried out on an Intel Core i7-3770 on 3.4 GHz with 4 GB RAM under Windows XP. In order to compare efficiency of proposed MILP formulations, CPLEX 12.5 solver is used on Single Source Capacitated Plant Location Problem (SSCPLP) instances `http://www-eio.upc.es/~elena/sscplp/index.html` from $m = 20, n = 10$ up to $m = 90, n = 30$ with $p = 5$ in all cases.

Since *RR* could not work in the case of ties between costs and *CGLM* (*BLMN*) running times is 1-2 orders of magnitude larger than others, they are omitted from consideration.

The results for Problem 1 are presented in Table 1 in the following format:

- Instance name;

- Optimal solution (obtained using all formulations);

- Running time of CPLEX solver based on MILP with *SM* constraints (5) from (Stanimirović & Mišković, 2013);

- Running time of CPLEX solver based on MILP with *WF* constraints (16) from (Wagner & Falkson, 1975);

- Running time of CPLEX solver based on MILP with *CC* constraints (17) from (Church & Cohon, 1976);

- Running time of CPLEX solver based on MILP with *DK* constraints (18) from (Dobson & Karmarkar, 1987);

- Running time of CPLEX solver based on MILP with *BDTW* constraints (8) from (Berman et al., 2009);

- Running time of CPLEX solver based on MILP with *M* constraints (21) from (Marín, 2011);

- Running time of CPLEX solver based on MILP with *EMR* constraints (22) from (Espejo et al., 2012);

- Minimal running time among all formulations.

At the end of Table 1 is presented sum of all running times (rows) and number of cases when present formulation has strictly smaller running time compared to all other formulations. Computational results on Problem 2 is presented on the same way in Table 2.

Row *Sum* in both Table 1 and 2 shows that the CPLEX solver with MILP formulation based on *WF* closest assignment constraints (16) proposed by Wagner & Falkson (1975) has minimal sum of running times for both problems. However, from last row in both tables, we can see that each formulation has, at least once, strictly better running time than all others. As it can be expected earlier, performance differences of *SM* and *BDTW* is minor, since *SM* closest assignment constraints proposed in (Stanimirović & Mišković, 2013) are based on *BDTW* ones from (Berman et al., 2009).

It can be seen that Problem 2 is much harder to solve than Problem 1, since running times is 6-14 times larger. On the other side, dispersion of running times is much higher for Problem 1, since closest assignment constraints obviously have much more impact on it. From last column, it can be seen that for Problem 1, sum of minimal running times is more than

Table 1: Running times for Problem 1

Inst.	*Opt*	*SM*	*WF*	*CC*	*DK*	*BDTW*	*M*	*EMR*	t_{min}
p1	61	0.171	0.078	0.328	0.078	0.171	0.093	0.140	0.078
p2	54	0.093	0.093	0.093	0.109	0.093	0.062	0.062	0.062
p3	55	0.093	0.078	0.078	0.078	0.078	0.093	0.109	0.078
p4	50	0.046	0.046	0.046	0.046	0.046	0.062	0.062	0.046
p5	86	0.078	0.078	0.359	0.078	0.078	0.078	0.093	0.078
p6	70	0.187	0.187	0.218	0.140	0.187	0.218	0.218	0.140
p7	96	0.593	0.593	0.484	0.656	0.593	0.796	0.781	0.484
p8	76	0.281	0.281	0.656	0.296	0.296	0.859	0.578	0.281
p9	95	0.328	0.312	0.687	0.343	0.343	0.625	0.640	0.312
p10	68	0.296	0.281	0.296	0.281	0.296	0.421	0.437	0.281
p11	89	0.687	0.640	0.781	0.718	0.656	0.437	0.437	0.437
p12	75	0.296	0.328	0.281	0.296	0.312	0.359	0.406	0.281
p13	75	0.296	0.281	0.281	0.296	0.296	0.375	0.406	0.281
p14	86	0.281	0.281	0.484	0.281	0.296	0.765	0.750	0.281
p15	54	0.281	0.281	0.406	0.281	0.296	0.515	0.546	0.281
p16	54	0.296	0.281	0.406	0.296	0.296	0.515	0.531	0.281
p17	72	0.281	0.265	0.312	0.296	0.281	0.734	0.453	0.265
p18	105	2.171	2.203	3.109	2.093	2.390	3.093	4.062	2.093
p19	91	1.797	1.750	3.781	2.031	1.828	3.156	3.140	1.750
p20	99	1.812	1.875	3.656	1.828	1.765	2.921	3.500	1.765
p21	103	1.843	1.890	2.468	1.750	1.812	2.562	2.812	1.750
p22	103	1.406	1.250	2.437	1.265	1.265	2.640	3.093	1.250
p23	103	2.171	2.141	3.890	1.812	2.125	2.859	2.578	1.812
p24	102	2.484	2.437	4.703	2.593	2.640	3.062	3.610	2.437
p25	95	1.296	1.156	2.843	1.281	1.234	3.218	3.375	1.156
p26	130	3.125	3.015	5.265	3.485	3.250	3.421	4.046	3.015
p27	121	2.468	2.390	4.625	2.515	2.515	3.406	3.921	2.390
p28	111	2.828	2.875	3.937	2.984	3.015	3.765	3.828	2.828
p29	131	3.250	3.234	3.625	3.203	3.250	4.640	4.281	3.203
p30	130	3.328	3.406	5.656	3.781	3.485	4.375	5.062	3.328
p31	109	2.718	2.640	3.312	2.484	2.703	3.859	4.468	2.484
p32	112	2.312	2.281	4.796	2.938	2.359	3.796	4.343	2.281
p33	131	2.671	2.500	4.453	2.688	2.765	3.703	3.687	2.500
p34	150	19.218	20.110	31.828	17.343	19.250	170.672	41.937	17.343
p35	145	19.546	24.031	42.047	26.265	21.703	151.687	221.375	19.546
p36	152	20.625	20.750	44.546	25.093	21.843	282.437	35.484	20.625
p37	143	16.750	18.796	30.406	16.781	15.906	42.671	35.235	15.906
p38	145	20.578	19.046	28.438	16.093	18.125	36.219	45.468	16.093
p39	151	21.843	21.468	34.250	25.031	20.188	43.468	35.375	20.188
p40	136	15.281	16.390	36.484	25.468	15.281	31.937	45.703	15.281
p41	149	19.671	19.171	99.718	105.343	18.921	188.406	208.531	18.921
p42	188	23.156	25.859	35.343	31.953	21.844	41.812	30.734	21.844
p43	197	212.484	192.375	150.062	118.875	212.718	236.266	193.546	118.875
p44	164	34.890	34.875	35.828	27.718	34.109	37.843	35.390	27.718
p45	151	21.953	20.953	49.859	23.265	21.375	20.593	43.000	20.593
p46	198	35.484	31.781	161.687	33.671	33.937	273.390	49.796	31.781
p47	186	31.750	31.953	37.046	29.500	31.875	295.328	37.781	29.500
p48	183	23.531	25.250	38.812	23.281	23.343	34.016	32.781	23.281
p49	180	29.921	31.766	44.656	26.500	27.734	329.437	59.015	26.500
p50	219	34.734	37.078	156.453	184.234	33.609	425.093	54.062	33.609
p51	219	44.953	45.843	65.281	33.000	47.718	59.593	64.234	33.000
p52	212	30.625	30.235	55.375	149.688	30.250	68.640	51.890	30.235
p53	233	51.515	51.234	54.156	181.937	50.765	49.016	257.953	49.016
p54	221	177.125	182.109	217.422	247.125	181.234	461.937	351.328	177.125
p55	215	297.531	280.187	46.281	33.843	299.359	57.890	55.093	33.843
p56	229	190.453	190.562	58.953	226.218	179.984	306.625	299.781	58.953
p57	238	42.265	45.890	56.921	44.859	45.125	64.641	68.765	42.265
Sum		1478.146	1459.139	1680.604	1716.384	1469.211	3771.100	2420.712	942.030
N.Best		6	13	3	15	6	2	0	

Table 2: Running times for Problem 2

Inst.	*Opt*	*SM*	*WF*	*CC*	*DK*	*BDTW*	*M*	*EMR*	t_{min}
p1	19	0.203	0.203	0.234	0.203	0.218	0.218	0.250	0.203
p2	16	0.203	0.187	0.218	0.187	0.578	0.156	0.218	0.156
p3	31	0.218	0.218	0.218	0.203	0.234	0.234	0.234	0.203
p4	13	0.218	0.218	0.218	0.250	0.203	0.203	0.296	0.203
p5	23	0.218	0.218	0.234	0.203	0.234	0.203	0.281	0.203
p6	30	0.234	0.234	0.250	0.250	0.250	0.187	0.250	0.187
p7	14	0.921	0.937	1.328	0.968	0.921	0.843	1.078	0.843
p8	11	0.906	0.968	1.093	0.968	0.859	1.187	1.078	0.859
p9	12	0.968	0.953	1.453	0.984	1.031	1.141	1.453	0.953
p10	12	0.828	0.828	0.890	1.078	0.859	0.984	1.500	0.828
p11	15	1.031	0.968	1.187	1.109	1.015	1.062	0.812	0.812
p12	17	1.046	1.031	1.000	1.078	0.968	1.031	1.234	0.968
p13	17	1.000	0.968	1.046	1.078	1.062	1.109	1.218	0.968
p14	7	0.937	1.156	1.187	1.218	0.859	1.359	1.078	0.859
p15	14	0.859	0.859	1.390	1.109	0.906	1.031	1.468	0.859
p16	14	0.875	0.890	1.343	1.140	0.859	1.062	1.546	0.859
p17	13	1.093	1.125	1.109	1.093	1.125	1.140	1.296	1.093
p18	6	5.531	6.015	13.937	6.828	5.625	21.031	6.703	5.531
p19	6	8.328	8.765	16.343	6.578	5.843	12.546	28.687	5.843
p20	5	6.031	6.671	16.281	6.031	8.953	25.812	10.890	6.031
p21	6	5.718	5.796	16.828	9.484	5.593	23.187	34.218	5.593
p22	11	5.250	7.375	13.796	9.515	5.312	11.890	9.281	5.250
p23	13	5.718	8.625	28.828	9.656	5.765	31.937	27.593	5.718
p24	3	7.125	9.562	20.234	7.046	8.484	31.984	26.125	7.046
p25	12	7.750	8.046	18.906	6.171	8.703	25.422	7.671	6.171
p26	10	16.671	16.093	32.312	13.343	13.609	39.640	39.750	13.343
p27	9	14.640	14.671	48.328	11.843	10.812	36.640	36.015	10.812
p28	11	9.766	7.171	21.015	11.609	10.188	37.296	11.187	7.171
p29	9	12.078	13.703	22.593	8.718	9.859	16.812	44.890	8.718
p30	2	14.250	9.734	31.843	10.235	14.343	42.265	39.484	9.734
p31	9	11.015	11.328	43.828	12.093	11.796	33.687	46.781	11.015
p32	8	13.250	12.046	22.265	12.234	13.312	34.687	37.046	12.046
p33	11	14.140	11.296	41.843	10.438	12.812	30.937	42.500	10.438
p34	6	235.781	236.688	376.890	270.171	233.593	638.125	647.796	233.593
p35	5	283.187	282.859	454.922	247.187	282.953	733.343	635.609	247.187
p36	8	242.187	241.000	306.546	253.703	239.484	639.625	537.453	239.484
p37	1	248.109	242.203	303.203	238.297	247.750	477.312	631.421	238.297
p38	6	254.843	256.359	377.421	235.765	252.062	645.765	742.953	235.765
p39	3	222.843	224.828	341.328	257.156	223.468	669.578	738.921	222.843
p40	7	268.875	267.593	353.984	267.296	267.671	696.953	671.921	267.296
p41	5	299.828	299.953	375.578	275.109	299.937	638.203	765.578	275.109
p42	6	391.735	392.546	594.250	427.828	396.015	1127.235	1296.625	391.735
p43	5	499.984	499.812	547.188	552.187	500.437	1161.359	1145.234	499.812
p44	3	428.515	427.250	484.156	387.531	429.687	877.687	1030.265	387.531
p45	6	415.125	413.296	450.094	428.593	414.328	823.125	852.015	413.296
p46	5	415.953	416.187	991.875	459.109	415.703	991.047	1033.203	415.703
p47	7	495.484	487.437	634.796	409.781	495.968	1085.515	1123.781	409.781
p48	7	461.984	452.343	697.000	394.609	463.500	1084.000	1074.609	394.609
p49	6	410.391	412.656	543.312	408.265	410.390	844.796	847.843	408.265
p50	7	573.234	575.000	2006.250	583.515	576.172	1733.687	1909.406	573.234
p51	4	637.797	617.937	1659.453	525.875	628.859	1375.187	1710.656	525.875
p52	2	694.031	682.563	2017.766	644.390	692.718	1561.593	1648.625	644.390
p53	6	665.250	678.985	1966.671	686.546	670.672	1795.750	1390.313	665.250
p54	4	698.000	692.421	1189.984	742.546	694.265	1592.828	1884.922	692.421
p55	5	672.718	675.390	1948.031	858.359	677.015	1789.109	2102.703	672.718
p56	5	659.234	657.531	1995.593	720.250	656.110	2157.437	1590.109	656.110
p57	3	668.093	654.094	1884.781	572.312	669.593	1443.593	1477.875	572.312
Sum		11012.200	10955.789	22924.650	11011.321	11001.540	27051.775	27953.947	10424.132
N.Best		9	8	0	18	11	3	1	

35% smaller than *WF* sum (942.030 compared to 1459.139 seconds) while for Problem 2 it is 4.85% smaller (10424.132 compared to 10955.789 seconds). Therefore, if we can find effective way of cooperation (or distinction) between proposed constraints we would obtain significant savings.

5. Conclusions

The presented paper considers the problem of exploration of online social networks. Several mixed integer linear programming formulations based on closest assignment constraints from the literature are proposed. Experimental results contain direct comparison between proposed formulations and they clearly indicate that the standard solvers based on proposed formulations are efficient in optimally solving the problems of exploration of online social networks. The future work can be directed in investigation of effective way of cooperation or distinction between proposed constraints and construction of some exact method based on these formulations.

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