Power-Capacity and Ramp-Capability Reserves for Wind Integration in Power-Based UC

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***Abstract*—This paper proposes a power-based network-constrained unit commitment (UC) model as an alternative to the traditional deterministic UCs to deal with wind generation uncertainty. The formulation draws a clear distinction between power-capacity and ramp-capability reserves to deal with wind production uncertainty. These power and ramp requirements can be obtained from wind forecast information. The model is formulated as a power-based UC, which schedules power-trajectories instead of the traditional energy-blocks and takes into account the inherent startup and shutdown power trajectories of thermal units. These characteristics allow a correct representation of unit's ramp schedule which define their ramp availability for reserves. The proposed formulation significantly decreases operation costs if compared to traditional deterministic and stochastic UC formulations while simultaneously lowering the computational burden. The operation cost comparison is made through 5-min economic dispatch simulation under hundreds of out-of-sample wind generation scenarios.**

***Index Terms*—Mixed-integer programming, operating reserves, power-capacity reserves, ramp-capability reserves, unit commitment.**

NOMENCLATURE

A. Indexes and Sets

$g \in \mathcal{G}$	Generating units, running from 1 to G.
$b \in \mathcal{B}$	Buses, running from 1 to B.
$l \in \mathcal{L}$	Transmission lines, running from 1 to L.
$t \in \mathcal{T}$	Hourly periods, running from 1 to T hours.

B. Parameters

D_{bt}	Power demand on bus b at the end of hour t [MW].
Γ_{lb}	Shift factor for line l associated with bus b [p.u.].
Γ^{P}_{lg}	Shift factor for line l associated with unit g [p.u.].
$\overline{F}_l$	Flow limit on transmission line l [MW].
$\overline{P}_g$	Maximum power output [MW].
$\underline{P}_g$	Minimum power output [MW].
RD_g	Ramp-down capability [MW/h].
RU_g	Ramp-up capability [MW/h].
SD_g	Startup ramping capability [MW/h].
SU_g	Shutdown ramping capability [MW/h].
W_{bt}	Nominal forecasted wind power at end of hour t [MW].
$\overline{W}_{bt}$	Upper bound of the forecasted wind power at the end of hour t [MW].
$\underline{W}_{bt}$	Lower bound of the forecasted wind power at the end of hour t [MW].
$W^{\mathrm{R}-}_{bt}$	Ramp-down forecasted wind requirement for the whole hour t [MW/h].
$W^{\mathrm{R}+}_{bt}$	Ramp-up forecasted wind requirement for the whole hour t [MW/h].

C. First-stage Variables

r^{-}_{gt}	Down power-capacity reserve scheduled [MW].
r^{+}_{gt}	Up power-capacity reserve scheduled [MW].
$r^{\mathrm{R}-}_{gt}$	Down ramp-capability reserve scheduled [MW/h].
$r^{\mathrm{R}+}_{gt}$	Up ramp-capability reserve scheduled [MW/h].
u_{gt}	Binary variable which is equal to 1 if the unit is producing above $\underline{P}_g$ and 0 otherwise.
v_{gt}	Binary variable which takes the value of 1 if the unit starts up and 0 otherwise.
z_{gt}	Binary variable which takes the value of 1 if the unit shuts down and 0 otherwise.

D. Second-stage Variables

p_{gt}	Power output above minimum output at the end of hour t [MW].
$\widehat{p}_{gt}$	Total power output at the end of hour t, including startup and shutdown trajectories [MW].
$\overline{r}_{gt}$	Reserve deployment to provide the upper-wind dispatch $\overline{w}_{bt}$ [MW].
$\underline{r}_{gt}$	Reserve deployment to provide the lower-wind dispatch $\underline{w}_{bt}$ [MW].
w_{bt}	Wind dispatch for the nominal wind case W_{bt} [MW].
$\overline{w}_{bt}$	Wind dispatch for the upper bound wind $\overline{W}_{bt}$ [MW].
$\underline{w}_{bt}$	Wind dispatch for the lower bound wind $\underline{W}_{bt}$ [MW].

E. Functions

$c^{\mathrm{F}}_{gt}(\cdot)$	Fixed production cost [\$].
$c^{\mathrm{V}}_{gt}(\cdot)$	Variable production cost [\$].

I. INTRODUCTION

INrecent years, high penetration of variable generating sources, such as wind power, has challenged independent system operators (ISO) in keeping a reliable power system operation. The deviation between expected and real wind production must be absorbed by the power system resources (reserves), which must be available and ready to be deployed in real time. To guarantee this availability, the system resources must be committed in advance, usually day-ahead, by solving the so-called unit commitment (UC) problem.

A. Literature Review

1) Dealing with Uncertainty in UC: Stochastic and robust optimization have gained substantial popularity for UC optimization under parameter uncertainty. In the stochastic optimization approach, the stochasticity can be represented through an explicit description of scenarios and their associated probability [1], [2]. This approach presents however some practical limitations: 1) it may be difficult to obtain an accurate probability distribution of the uncertainty; and 2) a large number of scenario samples is required to obtain robust solutions, which results in a computationally intensive problem (often intractable).

The robust optimization approach partly overcomes these disadvantages 1) by requiring moderate information about the underlying uncertainty, such as the mean and the range of the uncertain data; and 2) by immunizing the solution against all realizations of the data within the uncertainty range. However, it may be too conservative, since the objective function is to minimize the worst-case cost scenario, which may never be realized in practice. To deal with overconservatism, 1) a parameter commonly called budget-of-uncertainty is introduced in the optimization problem to control the conservatism of the robust solution [3], [4]; and 2) more recently, [4] proposes an unified stochastic and robust UC model that takes advantage of both stochastic and robust optimization approaches, where the objective is to achieve a low expected total cost while ensuring the system robustness.

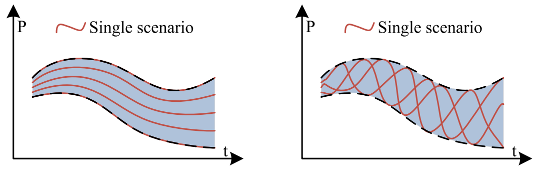
Although the computational burden of adaptive robust UC does not depend on the number of scenarios, it requires solving a mixed integer programming (MIP) problem together with a bilinear program to obtain the worst-case scenario. This problem is considerably more complex to solve than a pure MIP, requires ad-hoc solving strategies [3], [4], and it can also considerably increase the computational burden of UC problems.

In short, although stochastic and robust UCs are powerful tools to deal with uncertainty, they are computationally intensive. This is the reason why traditional deterministic formulations remain valid and widely used by ISOs worldwide. Therefore, it is needed to develop improved deterministic formulations that better exploit the flexibility of the power system and better face wind uncertainty.

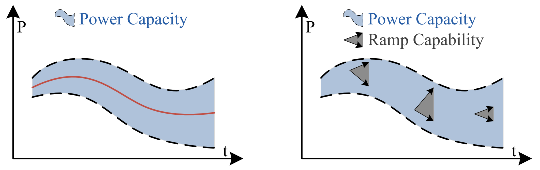
2) Power-Capacity and Ramp-Capability Reserves: In order to solve the day-ahead UC it is necessary to take into account that wind generation is subject to uncertainty. As the wind power forecasting error can be significant 24 hours in advance, the range of possible values of wind power for each hour of the following day can be very broad. As a consequence, ISOs need to schedule some power-capacity reserve to guarantee that committed system resources will be able to cope with any value of wind generation that can be realised within that range.

When getting closer to the real time, for instance one hour in advance, the range of possible values for the next hour is smaller. However, even within such short time interval, wind generation can increase or decrease its value at a rate that will require that conventional generators adapt their output to follow that ramp to keep the demand-supply balance. Therefore, apart from the day-ahead power-capacity reserve, it will be necessary to ensure that for any hour, the committed system resources will be able to cope with the expected maximum ramp of variation of the wind generation. Thus, a ramp-capability reserve is also needed.

To illustrate the need of a clear differentiation between power-capacity and ramp-capability reserves, consider the following example. Figs. 1a and 1b show two different set of wind scenarios which present the same power-capacity uncertainty ranges, but completely different ramp uncertainty ranges. Dealing with the scenarios in Fig. 1b requires higher ramp-capability, although both set of scenarios demand the same power-capacity requirements. In fact, some power systems have experienced short-term scarcity events caused by resources with sufficient power capacity but insufficient ramp capability [5]. In response, ISOs are developing market-based ramping products, thus making a clear difference between power-capacity and ramp-capability requirements [5], [6].



(a) Stochastic (low ramp) (b) Stochastic (high ramp)



(c) Traditional Deterministic (d) Proposed Deterministic

Fig. 1: Different approaches to deal with wind uncertainty

A stochastic UC implicitly captures both reserve requirements through scenarios, e.g., see Figs. 1a and 1b. However, to correctly represent these reserve requirements, a large number of scenarios is needed, resulting in a high computational cost. On the other hand, the traditional deterministic UCs can only ensure a given power-capacity reserve, see Fig. 1d, but it cannot guarantee different ramping requirements to deal with either of the scenarios in Figs. 1a and 1b. Although deterministic UCs remains being the ISOs' dominant practice nowadays due to the low computational burden, it does not efficiently exploit the system flexibility to deal with the specific requirements imposed by wind generation uncertainty.

3) Power-based UC: Conventional day-ahead UC formulations fail to deal with ramp capabilities appropriately. Inefficient ramp management arises from applying ramp-constraints to energy levels or (hourly) averaged generation levels; consequently, energy schedules may not be feasible [7]. In addition, traditional UC models assume that units start/end their production at their minimum output. That is, the intrinsic startup and shutdown power trajectories of units are ignored. As a consequence, there may be a high amount of energy that is not allocated by UC but it is inherently present in real time, thus affecting the total load balance and causing a negative economic impact [8]. For further details of the drawbacks of conventional UC scheduling approaches, the reader is referred to [9], [10] and references therein.

To overcome these drawbacks, [10] proposes the power-based UC (or ramping scheduling) approach. This approach uses piece-wise linear power trajectories for both generating units and demand instead of the commonly established staircase profile for energy blocks. The use of an instantaneous power profile allows the model to efficiently schedule reserves and ramping resources. In comparison with conventional UC models, the power-based UC approach guarantees that, first, energy schedules can be delivered and, second, that operating reserves can be deployed respecting the ramping and capacity limits of generating units. In addition, the model takes into account the normally neglected power trajectories that occur during the startup and shutdown processes, thus optimally scheduling them to provide energy and ramp, which help to satisfy the power demand.

B. Power-Capacity and Ramp-Capability Reserves in Power-Based UC: An Overview

This paper proposes a power-based network-constrained UC model as an alternative to the traditional deterministic UCs to deal with wind generation uncertainty. The proposed UC gives flexibility to the power system to face wind uncertainty. This flexibility is provided by drawing a clear distinction between power-capacity and ramp-capability reserve requirements (Fig. 1d), and by optimally dispatching wind generating units. Allowing a different value for ramp-capability reserve requirements results in a more realistic setting, as discussed above. Wind dispatch flexibility is modelled by considering curtailment in the UC formulation. Curtailment may appear due to either economic reasons or technical reasons, e.g., insufficient network capacity. This flexibility helps to reduce the reserve requirements since part of the uncertainty can be faced by curtailment, as practiced in ERCOT and MISO. Introducing other renewable energy sources to the formulation is straightforward if they can be curtailed.

The model is formulated as a power-based UC, which schedules power-trajectories instead of the traditional energy-blocks, and it takes into account the inherent startup and shutdown power trajectories of thermal units. These characteristics allow a correct representation of unit's ramp schedule [7], [8] which define their ramp availability for reserves [10].

The formulation is represented as a mixed integer programming (MIP) problem, which has become the leading approach in the electricity sector due to significant improvements on MIP solvers. The core of the proposed MIP formulation is built upon the convex-hull and the tight-and-compact formulations presented in [11] and [8], respectively, thus taking advantage of their mathematical properties. These formulations reinforce the convergence speed by reducing the search space (tightness) and at the same time by increasing the searching speed with which solvers explore that reduced space (compactness).

We present an extensive numerical study on the IEEE 118-bus test system, where we compare the proposed formulation with the stochastic and with the deterministic approaches. To perform comparisons and to obtain an accurate estimate of the performance of each UC policy, the hourly commitment obtained from each UC approach is evaluated through a 5-min economic dispatch for 200 out-of-sample scenarios.

C. Contributions and Paper Organization

The principal contributions of this paper are as follows:

1) The proposed formulation explicitly includes a pre-specified nodal power-capacity and ramp-capability reserve requirements, which can be obtained from wind forecast information. The formulation explicitly models the interdependency between the power-capacity and ramp-capability reserves; i.e., providing ramp-capability means providing power-capacity, but providing power-capacity does not necessarily means providing a given level of ramp-capability.
2) Although the proposed UC formulation optimizes over a nominal wind scenario, it also includes the worst-case wind scenario proposed in [12], then the UC solution guarantees that the system has enough flexibility to adapt to any wind uncertain realization. The level of conservatism of the solution is controlled by the reserve parameters and wind curtailment flexibility. That is, once the reserve requirements are fixed, the proposed UC reshape these requirements by considering curtailment.
3) We develop a practical compact mixed-integer programming (MIP) formulation to deal with wind uncertainty. The complete formulation remains compact since it only needs two reserve requirements, unlike the stochastic approach where problem size depends on the number of considered scenarios.
4) The proposed deterministic UC can be used by ISOs to ensure that enough power-capacity and ramp-capability resources are available to deal with wind uncertainty in real-time operation. ISOs can also adjust the level of conservatism of the solution by adjusting the reserve requirements, based on their preferences and on their available information of wind uncertainty.

The remainder of this paper is organized as follows. Section II details the mathematical formulation of the different operating reserves and their links with the ramp schedules. Section III presents some numerical examples as well as a comparison with the deterministic and stochastic UC approaches. Finally, concluding remarks are made in Section IV.

II. MATHEMATICAL FORMULATION

This section presents the proposed mathematical formulation of the power-based UC. This section first discusses the relationship between the wind uncertainty range and the power system reserve requirements. The next part is devoted to modelling the reserve constraints for generating units and the network constraints. Finally, the objective function is defined.

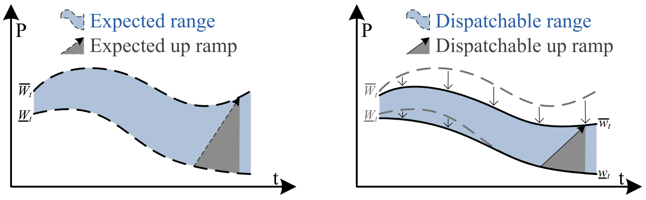
A. Wind Uncertainty Range and Power System Requirements

The first step to define the level of reserves. In this paper, the uncertain parameters are the power-capacity and ramp-capability ranges of wind production, see Fig. 1d. The wind power-capacity uncertainty range of node b at time t is defined by the upper and lower bounds $\left[\overline{W}_{bt}, \underline{W}_{bt}\right]$. The wind ramp uncertainty range of node b at time t is defined by $\left[W_{bt}^{\mathrm{R}-}, W_{bt}^{\mathrm{R}+}\right]$. The nominal value of wind ramp is defined by the trajectory of the nominal wind power production W_{bt}.

In this paper, we consider the nominal value of wind production W_{bt} as the middle value of the uncertainty range, i.e., $\left(\overline{W}_{bt} + \underline{W}_{bt}\right)/2$. The proposed formulation is general, so ISOs could define any other nominal wind value, e.g., the most expected wind production. The only limitation is that the nominal value of wind production must be defined within the wind uncertainty range.

The flexibility that brings the fact that wind generation can be curtailed is taken into account. Thus, the possible dispatched wind range that results from the UC may (shrink) be different than the forecasted range, as shown in Fig. 2.

To allow curtailment in the formulation, the wind-dispatch variables are bounded by their associated wind forecast



(a) Forecasted range (b) Dispatchable range

Fig. 2: Forecasted and dispatchable wind uncertainty ranges

bounds:

$$0 \le \underline{w}_{bt} \le \underline{W}_{bt},\ 0 \le w_{bt} \le W_{bt},\ 0 \le \overline{w}_{bt} \le \overline{W}_{bt} \quad \forall b,t \tag{1}$$

and we define the variables $w^{\mathrm{R+}}_{bt}$ and $w^{\mathrm{R-}}_{bt}$ as the maximum ramp up and down range, exceeding wind nominal values, that can fit within the dispatchable uncertainty range, respectively:

$$w^{\mathrm{R+}}_{bt} = (\overline{w}_{bt} - w_{bt}) + \left(w_{b,t-1} - \underline{w}_{b,t-1}\right) \quad \forall b,t \tag{2}$$
$$w^{\mathrm{R-}}_{bt} = (\overline{w}_{b,t-1} - w_{b,t-1}) + (w_{bt} - \underline{w}_{bt}) \quad \forall b,t. \tag{3}$$

Once the wind uncertainty ranges are defined, the power system must supply demand and reserves for these ranges:

$$\sum_{g\in\mathcal{G}} \widehat{p}_{gt} = \sum_{b\in\mathcal{B}} (D_{bt} - w_{bt}) \quad \forall t \tag{4}$$
$$\sum_{g\in\mathcal{G}} r^{+}_{gt} \ge \sum_{b\in\mathcal{B}} (w_{bt} - \underline{w}_{bt}) \quad \forall t \tag{5}$$
$$\sum_{g\in\mathcal{G}} r^{-}_{gt} \ge \sum_{b\in\mathcal{B}} (\overline{w}_{bt} - w_{bt}) \quad \forall t \tag{6}$$
$$\sum_{g\in\mathcal{G}} r^{\mathrm{R+}}_{gt} \ge \sum_{b\in\mathcal{B}} \inf\left(\tilde{W}^{\mathrm{R-}}_{bt}, w^{\mathrm{R-}}_{bt}\right) \quad \forall t \tag{7}$$
$$\sum_{g\in\mathcal{G}} r^{\mathrm{R-}}_{gt} \ge \sum_{b\in\mathcal{B}} \inf\left(\tilde{W}^{\mathrm{R+}}_{bt}, w^{\mathrm{R+}}_{bt}\right) \quad \forall t. \tag{8}$$

where (4) is a power balance at the end of hour t. Be aware that the energy balance for the whole hour is automatically achieved by satisfying the power demand at the beginning and end of each hour, and by considering a piecewise-linear power profile for demand and generation [10].

Equality (4) ensures that the system provides the power and ramp requirements for the wind nominal case. Constraints (5)-(6) and (7)-(8) guarantee that the system can provide the maximum power and ramp deviations from the nominal case, respectively. Parameters $\tilde{W}^{\mathrm{R+}}_{bt}$ and $\tilde{W}^{R-}_{bt}$ are the maximum up and down ramp deviations from the nominal ramp, respectively, and are obtained as follows:

$$\tilde{W}^{\mathrm{R+}}_{bt} = W^{\mathrm{R+}}_{bt} - (W_{bt} - W_{b,t-1}) \quad \forall b,t \tag{9}$$
$$\tilde{W}^{\mathrm{R-}}_{bt} = W^{\mathrm{R-}}_{bt} - (W_{b,t-1} - W_{bt}) \quad \forall b,t \tag{10}$$

The infimum functions in (7) and (8) guarantee that the ramp requirement do not exceed the scheduled wind range by choosing the minimum value between the forecasted ramp requirement and the maximum possible ramp within the scheduled wind range. An MIP equivalent formulation for the infimum function in (7) and (8) is provided in [13].

B. Individual Unit's Constraints

This section presents a set of constraints that guarantee that a unit can provide any power trajectory within its scheduled

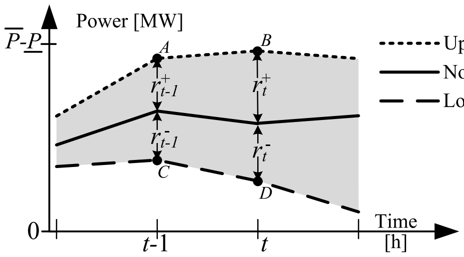


Fig. 3: Unit's operating range

ramp-capability $r^{\mathrm{R+}}_{gt}, r^{\mathrm{R-}}_{gt}$ and power-capacity r^{+}_{gt}, r^{-}_{gt} reserve ranges. Fig. 3 shows how the nominal case and the power-capacity reserves define upper and lower envelopes for units' operation.

1) Commitment Logic: The relation between the commitment, startup and shutdown variables is given by:

$$u_{gt} - u_{g,t-1} = v_{gt} - z_{gt} \quad \forall g,t. \tag{11}$$

Constraints imposing the minimum up/down times and different startup types are also included, see [10].

2) Total Power Output for The Nominal Production: The proposed formulation considers slow- and quick-start units. For the sake of brevity, we present the set of constraints for quick-start units, which can startup within one hour:

$$\widehat{p}_{gt} = \underline{P}_g \left(u_{gt} + v_{g,t+1}\right) + p_{gt} \quad \forall g,t. \tag{12}$$

The slow-start units are included into the formulation by only modifying (12), thus including shutdown and different-startup power trajectories that take longer than one hour. The reader is referred to [8], [10], [11] for further details.

3) Power-Capacity Reserves: The upper and lower envelopes must be within the unit's capacity limits, see Fig. 3:

$$p_{gt} + r^{+}_{gt} \le \left(\overline{P}_g - \underline{P}_g\right) u_{gt} - \left(\overline{P}_g - SD_g\right) z_{g,t+1} + \left(SU_g - \underline{P}_g\right) v_{g,t+1} \quad \forall g,t \tag{13}$$
$$p_{gt} - r^{-}_{gt} \ge 0 \quad \forall g,t \tag{14}$$

4) Ramp-Capability Reserves: The unit's nominal production defines the ramp-capability that is available in every period:

$$p_{gt} - p_{g,t-1} + r^{\mathrm{R+}}_{gt} \le RU_g u_{gt} + \left(SU_g - \underline{P}_g\right) v_{g,t+1} \quad \forall g,t \tag{15}$$
$$-p_{gt} + p_{g,t-1} + r^{\mathrm{R-}}_{gt} \le RD_g u_{gt} + \left(SD_g - \underline{P}_g\right) z_{gt} \quad \forall g,t \tag{16}$$

5) Relationship Between Power-Capacity and Ramp-Capability Reserves: The following constraints ensure that the unit operate within the ramp limits on either the upper or lower envelopes, respectively:

$$-r^{\mathrm{R-}}_{gt} \le r^{+}_{gt} - r^{+}_{g,t-1} \le r^{\mathrm{R+}}_{gt} \quad \forall g,t \tag{17}$$
$$-r^{\mathrm{R-}}_{gt} \le r^{-}_{gt} - r^{-}_{g,t-1} \le r^{\mathrm{R+}}_{gt} \quad \forall g,t \tag{18}$$

where (17) and (18) can be obtained from Fig. 3, see Appendix A for further details.

The available up (down) ramp-capability $r^{\mathrm{R+}}_{gt}$ $(r^{\mathrm{R-}}_{gt})$ is bounded by the maximum upwards (downwards) power change that is possible within power-capacity operating range, $C \to B$

($A \rightarrow D$) in Fig. 3:

$$r^{\mathrm{R}+}_{gt} \leq r^{-}_{g,t-1} + r^{+}_{gt} \quad \forall g,t \tag{19}$$

$$r^{\mathrm{R}-}_{gt} \leq r^{+}_{g,t-1} + r^{-}_{gt} \quad \forall g,t. \tag{20}$$

Constraints (19) and (20) guarantee that once the unit is scheduled to provide ramp-capability reserve, there is a scheduled power-capacity range that can allow this ramp-capability deployment.

Finally, all these reserve variables are defined as positive:

$$r^{+}_{gt}, r^{-}_{gt}, r^{\mathrm{R}+}_{gt}, r^{\mathrm{R}-}_{gt} \geq 0 \quad \forall g,t. \tag{21}$$

In summary, constraints (13)-(21) guarantee that the unit can provide any power trajectory within its scheduled ramp-capability and power-capacity reserve ranges.

C. Network Constraints

[12] shows that by finding a feasible dispatch for the lowest expected wind bound $\underline{w}_{bt}$, all other possible wind realizations within the uncertainty range are feasible. That is, all scenarios can become $\underline{w}_{bt}$ by curtailment. Consequently, all scenarios can be dispatched and, in the worst case, the maximum quantity of wind that can be dispatched for any scenario would be $\underline{w}_{bt}$. Now, by ensuring a feasible dispatch for the upper expected wind bound $\overline{w}_{bt}$, we guarantee that wind scenarios up to $\overline{w}_{bt}$ can also be dispatched.

Now, we need to find the units' reserve deployments $\overline{r}_{gt}$ and $\underline{r}_{gt}$ for the upper and lower expected wind bounds, respectively. These reserve deployments must be within the scheduled power capacity limits:

$$-r^{-}_{gt} \leq \overline{r}_{gt}, \underline{r}_{gt} \leq r^{+}_{gt} \quad \forall g,t \tag{22}$$

and they must also satisfy ramp limit constraints:

$$-r^{\mathrm{R}-}_{gt} \leq \overline{r}_{gt} - \overline{r}_{g,t-1} \leq r^{\mathrm{R}+}_{gt} \quad \forall g,t \tag{23}$$

$$-r^{\mathrm{R}-}_{gt} \leq \underline{r}_{gt} - \underline{r}_{g,t-1} \leq r^{\mathrm{R}+}_{gt} \quad \forall g,t. \tag{24}$$

Finally the transmission capacity constraints are enforced for both the upper and lower expected wind bounds:

$$-\overline{F}_l \leq \sum_{g \in \mathcal{G}} \Gamma^{\mathrm{P}}_{lg} \left(\widehat{p}_{gt} + \overline{r}_{gt}\right) + \sum_{b \in \mathcal{B}} \Gamma_{lb} \left(\overline{w}_{bt} - D_{bt}\right) \leq \overline{F}_l \quad \forall l,t \tag{25}$$

$$-\overline{F}_l \leq \sum_{g \in \mathcal{G}} \Gamma^{\mathrm{P}}_{lg} \left(\widehat{p}_{gt} + \underline{r}_{gt}\right) + \sum_{b \in \mathcal{B}} \Gamma_{lb} \left(\underline{w}_{bt} - D_{bt}\right) \leq \overline{F}_l \quad \forall l,t \tag{26}$$

The demand balances for these scenarios are guaranteed by (4) together with:

$$\sum_{g \in \mathcal{G}} \overline{r}_{gt} = \sum_{b \in \mathcal{B}} \left(w_{bt} - \overline{w}_{bt}\right) \quad \forall t \tag{27}$$

$$\sum_{g \in \mathcal{G}} \underline{r}_{gt} = \sum_{b \in \mathcal{B}} \left(w_{bt} - \underline{w}_{bt}\right) \quad \forall t \tag{28}$$

and the nominal wind production must be within its upper and lower wind dispatches:

$$\underline{w}_{bt} \leq w_{bt} \leq \overline{w}_{bt} \quad \forall b,t. \tag{29}$$

Notice that total reserve deployment for the upper wind dispatch (27) is negative, this means that the power system must decrease its overall generation when wind production is above the nominal value. Notice in (27) and (28) that the power-capacity reserve requirements are provided by $\overline{r}_{gt}, \underline{r}_{gt}$ then these variables provide the limits on r^{-}_{gt}, r^{+}_{gt}. In other words, variables $\overline{r}_{gt}, \underline{r}_{gt}$ will be equal to either r^{-}_{gt} or r^{+}_{gt}. Therefore, (23) and (24) are more constrained and dominate (17) and (18), that is, (17) and (18) are then redundant.

Although (5) and (6) ensure that the units can provide the required power-capacity reserves, they do not guarantee that there is transmission capacity available to deploy them. However, constraints (25)-(29) guarantee that these power-capacity reserves can be deployed.

D. Objective Function

The proposed UC formulation optimizes over a nominal value of wind production and some weight can be given to the upper and lower wind bounds:

$$\min \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} \Bigg[\underbrace{c^{\mathrm{F}}_{gt}\left(u_{gt}, v_{gt}, z_{gt}\right)}_{\text{First stage}} + \underbrace{\alpha_1 c^{\mathrm{V}}_{gt}\left(\widehat{p}_{gt}\right) + \alpha_2 c^{\mathrm{V}}_{gt}\left(\widehat{p}_{gt} + \overline{r}_{gt}\right) + \alpha_3 c^{\mathrm{V}}_{gt}\left(\widehat{p}_{gt} + \underline{r}_{gt}\right)}_{\text{Second stage}} \Bigg] \tag{30}$$

where $\alpha_1 + \alpha_2 + \alpha_3 = 1$. Henceforth, we set $\alpha_2 = \alpha_3 = \frac{\alpha}{2}$, hence $\alpha_1 = (1 - \alpha)$. The weight α gives the flexibility to ISOs to give importance to the limits of the uncertainty range.

Similarly to the robust and stochastic approaches, the first-stage counts the fixed production cost $c^{\mathrm{F}}_{gt}(\cdot)$ which is composed by the no-load, shutdown and different startup costs, depending on how long the unit has been offline [10]. The second stage counts the variable production cost $c^{\mathrm{V}}_{gt}(\cdot)$ that is calculated based on the units' energy production, which can be easily obtained from $\widehat{p}_{gt}$ [10].

III. NUMERICAL RESULTS

The performance of our proposed approach is evaluated using the modified IEEE 118-bus test system, available online at www.iit.upcomillas.es/aramos/IEEE118_SUSD-Ramps.xls, for a time span of 24 hours. The system has 118 buses, 186 transmission lines, 91 loads, 54 thermal units and three wind units. The power system data are based on that in [2] and it was adapted to consider startup and shutdown power trajectories. All tests were carried out using CPLEX 12.6 [14] on an Intel-i7 3.4-GHz personal computer with 16 GB of RAM memory. The problems are solved until they hit a time limit of 7200 seconds or until they reach an optimality tolerance of 0.05%.

In this section, we first show the procedure used to evaluate the performance of the UC solutions. Then, we perform sensitivity analysis of the proposed formulation in terms of the objective weight and uncertainty range. Finally, we compare the performance of the proposed approach with the traditional deterministic and stochastic approaches.

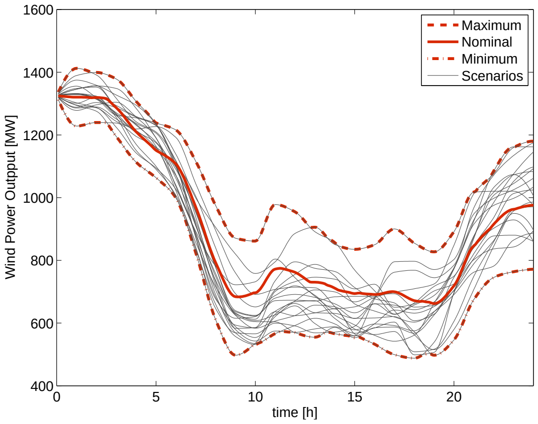


Fig. 4: Representation of wind uncertainty over time, scenarios and bounds

A. *Evaluating Approach*

The uncertainty model: we use latin hypercube sampling (LHS) to generate scenarios for the uncertain wind production. We assume that the wind production follows a multivariate normal distribution with predicted value W and volatility matrix Σ. The idea in applying LHS is to optimally distribute the samples to explore the whole area in the experimental region, avoiding the creation of scenarios that are too similar (clusters) [15].

To compare the performance of the different UC approaches, we make a clear difference between the scheduling stage and the validation stage. The computational experiments proceed as follows.

1) Scheduling stage: solve the different UC models and obtain the hourly commitment solutions, using 20 wind scenarios for each of the three wind units. Fig. 4 shows the aggregated wind production of these wind scenarios.
2) Out-of-sample validation stage: for each fixed UC solution, solve a 5-min economic dispatch problem repetitively for a set of 200 new wind scenarios. Notice that around the 20% of these out-of-sample scenarios fall outside the uncertainty bounds shown in Fig. 4.

In the 5-min economic dispatch, we introduce penalty costs for the violation of some constraints to mimic the high costs due to corrective actions in real time operations. The penalty costs are set to 10000 and 5000 $/MWh for demand-balance and transmission-limits violations, respectively, as suggested in [16] (similarly to [3], [4]). These penalty costs represent the expensive real-time corrective actions that an ISO needs to take in the event that the actual system condition significantly deviates from the expected condition, such as dispatching fast-start units, voltage reduction or load shedding.

We show the performance of the UC strategies in eight aspects, two related with the scheduling stage and six with the validation stage. These aspects, presented in Tables I to III, are described ass follows. Scheduling stage: 1) the fixed production costs described in Section II-D (UC [k$]), and 2) the number of startups (# SU). These two aspects indicate the commitment decisions that were needed by each approach to prepare the system to deal with the given wind uncertainty. Validation stage: 3) the average dispatch costs (Average), indicates the economic efficiency of the UC decision; 4) the volatility of these costs (Std), represented by the standard deviation of dispatch costs, which indicates the reliability of the real-time dispatch operation under the UC decision; 5) the dispatch cost of the worst-case scenario (Worst), indicates how robust the UC decision is against the worst-case scenario (from the 200 out-of-sample scenarios); 6) number of scenarios where there were violations in either demand-balance or transmission-limits constraints (# Sc); 7) total number of these violations (# Tot); and 8) total accumulated energy that could not be accommodated, demand-balance violations (MWh). The last three aspects also indicate how robust the UC decision is against different wind scenarios.

Table I: SENSITIVITY OF OBJECTIVE WEIGHT α

α	Scheduling Hourly		Validation: 5-min Economic Dispatch					
			Dispatch Costs [k$]			Violations		
	UC [k$]	# SU	Average	Std	Worst	# Sc	# Tot	MWh
0	52.026	14	771.115	14.351	814.471	2	2	0.038
0.1	51.986	14	770.823	14.365	814.223	2	2	0.038
0.2	51.949	14	770.970	14.348	814.087	2	2	0.048
0.3	51.986	14	770.806	14.364	814.206	2	2	0.038
0.4	51.961	14	770.928	14.392	814.201	2	2	0.038
0.5	51.351	13	771.642	14.361	814.667	2	2	0.038
0.6	51.259	13	771.822	14.408	815.037	0	0	0.000
0.7	50.446	14	772.659	14.325	815.602	1	1	0.004
0.8	50.623	14	772.657	14.378	816.045	5	5	0.108
0.9	50.435	14	772.725	14.327	815.951	5	5	0.108
1.0	49.824	13	773.503	14.355	816.718	5	5	0.108

B. *Sensitivity Analysis*

1) Changes of Objective Weight α: We test the performance of the proposed approach under different α and the results are shown in Table I. Notice that the performance does not change considerably. The maximum values of the Average, Std and Worst-case dispatch cost are 0.6% above the minimum values. These small changes are because the model guarantee feasibility through a set of hard constraints; however, the results may change considerably if we relax some constraints and introduce penalty-cost violations. Henceforth, we set $\alpha = 0.1$.

2) Changes of Uncertainty Range: Table II shows the results in the scheduling and validation stage for different values of the uncertainty range, from 0 to 100%. The 100% uncertainty range is defined by the bounds shown in Fig. 4, and the 0% is equivalent to a deterministic UC using only the nominal wind case. These ranges were equally changed to the power-capacity and ramp-capability ranges. It can be clearly observed that the larger the considered uncertainty range, the UC costs and number of startups increase because the UC solutions become more conservative. Consequently, the dispatch costs and violations decreases.

Through different uncertainty ranges, there is a significant reduction in the Average and Std dispatch costs. This significant reduction is closely related to the violations reduction and its associated costs, which represent the expensive emergency actions that the ISO has to take to maintain system reliability.

Notice that the uncertainty range of 85% presents the lowest average dispatch costs. This indicates that the uncertainty range can be slightly reduced without sacrificing the efficiency and robustness of the UC solution. We can observe in the ranges (85% and above) presenting few violations that considering

Table II: SENSITIVITY OF UNCERTAINTY RANGE

%	Scheduling Hourly		Validation: 5-min Economic Dispatch					
			Dispatch Costs [k$]			Violations		
	UC [k$]	# SU	Average	Std	Worst	# Sc	# Tot	MWh
0	46.705	10	1067.017	575.205	5479.411	103	1744	5884.536
10	46.906	10	1018.959	505.127	5017.905	101	1492	4928.511
20	46.725	10	966.994	461.833	4797.448	87	1259	3883.190
30	47.443	11	877.291	337.356	3905.236	53	759	2102.645
40	47.941	12	825.176	228.394	3130.061	31	308	1052.421
50	47.973	12	795.862	134.644	2317.292	16	145	460.961
60	48.691	13	780.770	67.952	1617.704	11	77	165.247
70	51.583	13	772.493	26.906	1039.311	6	39	43.814
80	51.442	13	770.863	14.830	831.475	4	12	3.647
85	51.930	14	770.535	14.522	814.291	3	6	2.008
90	51.911	14	770.562	14.384	814.089	2	2	0.038
95	51.934	14	770.740	14.382	814.246	2	2	0.038
100	51.986	14	770.823	14.365	814.223	2	2	0.038

Table III: BETWEEN DIFFERENT UC POLICIES UNDER THE 200 OUT-OF-SAMPLE WIND SCENARIOS

	Scheduling Hourly		Validation: 5-min Economic Dispatch					
			Dispatch Costs [k$]			Violations		
	UC [k$]	# SU	Average	Std	Worst	# Sc	# Tot	MWh
ResRPC	51.986	14	770.823	14.365	814.223	2	2	0.038
StchOpt	54.765	12	808.971	200.096	2903.841	28	259	611.473
DetRes	55.492	16	857.199	279.813	3254.877	55	611	1793.881

lower uncertainty levels leads to better economic benefit, but worse risk performance, which is represented by the standard deviation of the dispatch cost. Using this information, a proper tradeoff can be made by decision makers.

Henceforth, we set the uncertainty range to 100%.

C. Comparing the Proposed Approach with the Deterministic and Stochastic Approaches

The proposed UC formulation (ResRPC), which includes ramp-capability and power-capacity reserves, is compared with the traditional deterministic-reserve modelling (DetRes) and the stochastic (StchOpt) UC approaches. All three models are based on the power-based UC proposed in [10].

To obtain the commitment strategies of all UC approaches, we use the 20 wind scenarios shown in Fig. 4, as described in the scheduled stage in Section III-A. We assume these scenarios to be the only information available for the scheduling stage. Therefore, we use these data to describe the different wind uncertainty representation required by the different UC approaches. The proposed approach ResRPC uses the nominal wind production together with minimum and maximum bounds of power-capacity and ramp-capability, which are obtained from this set of scenarios. The stochastic approach StchOpt uses all 20 scenarios. Finally, the deterministic approach DetRes uses the nominal wind production and two hourly reserves, upwards and downwards which are defined as $\sum_b \left(W_{bt} - \underline{W}_{bt}\right)$ and $\sum_b \left(\overline{W}_{bt} - W_{bt}\right)$, respectively.

1) Reliability of Dispatch Operation: Table III compares the performance of the different UC approaches. From the scheduling stage, we can observe that DetRes commits the largest quantity of resources, because this is the only approach that cannot readjust (optimize) the given level of reserves by considering wind curtailment. That is, the reserve requirements for the deterministic approach results in a larger quantity of committed resources. On the other extreme, StchOpt commits the smallest quantity of resources (reserves), because it (endogenously) optimize the quantity of reserves required based on the 20 scheduling scenarios. Somewhere in between, ResRPC can readjust the level of reserves using the wind curtailment flexibility, leading to lower FxdCost than DetRes; however, ResRPC tends to be overconservative, because it takes into account the worst-case wind scenario, hence scheduling more resources (higher FxdCost) than StchOpt.

From the validation stage in Table III, we can observe the following:

1) The Average and Std dispatch costs of StchOpt are around 6% and 40% lower than DetRes, respectively. This clearly shows the advantages of the stochastic strategy over the deterministic one, as expected.
2) Although DetRes committed the largest quantity of resources, it is the least robust. This is mainly because the deterministic approach only models the network constraints for the nominal case and it cannot guarantee that the committed reserves can be deployed. This is in contrast to ResRPC and StchOpt, where generating units are committed taking into account that power must be delivered to specific places in the network where the uncertainty appears.
3) The Average dispatch cost of StchOpt is around 5% higher than ResRPC, and the Std for StchOpt is more than an order of magnitude higher (13.9 times). Similarly, the total quantity of violations and the energy unbalance of StchOpt is more than two (130 times) and four (16k times) orders of magnitude higher than ResRPC, respectively.

In short, the proposed approach ResRPC presents a better economic-benefit and risk performance than the deterministic and stochastic approaches for this study case. Consequently, ResRPC offers more robust commitment decisions which lead to a better system reliability.

Although we use LHS to represent the space of scenarios adequately, the performance of StchOpt may be improved by introducing a larger quantity of scenarios in the scheduling stage or by a better scenario sampling. To observe the performance of ResRPC and DetRes compared with a "perfect" stochastic approach, we carried out the economic dispatch validation using the same scenarios used by StchOpt in the scheduling stage. Table III shows the performance of the different UC approaches under the 20 scheduling scenarios. For this case, StchOpt presented the lowest Average dispatch cost, around 0.3% lower than ResRPC, but the Std and the Worst-case are higher than ResRPC. Notice that StchOpt presented constraint violations in two scenarios even though these scenarios were used in the scheduling stage. This is because the scheduling stage considers a simplified hourly piece-wise linear approximation of the 5-min smooth power profile of the set of scenarios shown in Fig. 4.

2) Computational Performance: Table V shows a comparison of problem size and computational burden between the different approaches. Notice that all three formulations have almost the same quantity of binary variables, but ResRPC has around 2.2% more than the others. This is due to the modelling of the infimum function that ResRPC requires, see

Table IV: Between Different UC Policies Under the 20 Scheduling Wind Scenarios

	5-min Economic Dispatch Simulation					
	Dispatch Costs [k$]			Violations		
	Average	Std	Worst	# Sc	# Tot	MWh
ResRPC	770.863	12.360	795.588	1	1	0.002
StchOpt	768.793	21.888	848.723	2	12	5.729
DetRes	803.457	119.146	1263.678	3	36	71.670

Table V: Problem Size and Computational Burden of The Different Approaches

	Problem Size [#]				Computational Burden	
	Constraints	Nonzero elements	Continuous variables	Binary variables	CPU Time [s]	Nodes explored
ResRPC	36141	1074712	21096	6520	90.45	250
StchOpt	225141	5600307	169776	6376	867.88	819
DetRes	18093	315424	11016	6376	8.75	29

Section II-A.

When comparing the number of constraints, nonzero elements and continuous variables, ResRPC is around twice the size of DetRes, and StchOpt is more than 12 and 6 times larger than DetRes and ResRPC, respectively. On the other hand, the CPU time of ResRPC is around an order of magnitude higher than that of DetRes, and one lower than that of StchOpt. Finally, unlike DetRes and ResRPC, the problem size and computational burden of StchOpt highly depends on the quantity of scenarios that it considers.

IV. CONCLUSIONS

In this paper, we proposed a power-based network-constrained UC formulation as an alternative to the traditional deterministic UC under wind generation uncertainty. The formulation draws a clear distinction between power-capacity and ramp-capability reserves to deal with wind production uncertainty. The model is formulated as a power-based UC, which schedules power-trajectories instead of the traditional energy-blocks and takes into account the inherent startup and shutdown power trajectories of thermal units. The formulation is compact since only needs two reserve requirements then keeping the advantages of deterministic UCs, unlike the stochastic approach which problem size depends on the quantity of scenarios. Study cases showed that the proposed formulation significantly decreases operation costs compared to traditional deterministic and stochastic UC formulations while simultaneously lowering the computational burden. The operation cost comparison was made through 5-min economic dispatch simulation under hundreds of out-of-sample wind generation scenarios. As future studies, the performance of the proposed formulation must be compared with the traditional stepwise energy-block formulations under both stochastic and robust approaches for different power systems.

APPENDIX

The ramp-up and -down constraints on the upper envelope, $A \rightarrow B$ in Fig. 3, are ensured with

$$\left(p_{gt} + r^{+}_{gt}\right) - \left(p_{g,t-1} + r^{+}_{g,t-1}\right) \le RU_{gt} \quad \forall g,t \qquad (31)$$

$$-\left(p_{gt} + r^{+}_{gt}\right) + \left(p_{g,t-1} + r^{+}_{g,t-1}\right) \le RD_{gt} \quad \forall g,t. \qquad (32)$$

On the other hand, (33) is obtained by reorganizing the ramp-up constraints given in (15) and (17), and (34) by reorganizing (16) and (17):

$$r^{+}_{gt} - r^{+}_{g,t-1} \le r^{\mathrm{R}+}_{gt} \le RU_g - p_{gt} + p_{g,t-1} \quad \forall g,t \quad (33)$$

$$-r^{+}_{gt} + r^{+}_{g,t-1} \le r^{\mathrm{R}-}_{gt} \le RD_g + p_{gt} - p_{g,t-1} \quad \forall g,t \quad (34)$$

where these two constraints are equivalent to (31) and (32), respectively. Similarly, (15) and (16) together with (18) guarantee the ramp-up and -down constraints on the lower envelope scenario, $C \rightarrow D$ in Fig. 3.

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