Robust Unit Commitment with Dispatchable Wind: An LP Reformulation of the Second Stage

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***Abstract*—The increasing penetration of uncertain generation such as wind and solar in power systems imposes new challenges to the Unit Commitment (UC) problem, one of the most critical tasks in power systems operations. The two most common approaches to address these challenges — stochastic and robust optimization — have drawbacks that prevent or restrict their application to real-world systems. This paper demonstrates that an adaptive robust UC in which, by considering wind dispatch flexibility, the second-stage problem, usually being non-convex, can be represented with an equivalent linear program (LP). Consequently, the full two-stage robust UC formulation, which is typically a bi-level problem, can be translated into an equivalent single-level mixed-integer program. Experiments on the IEEE 118-bus test system show that the computation time, and the number of scenarios and violations can be significantly reduced in the unified stochastic and robust approach compared to a pure stochastic approach. In this paper, the formulation is evaluated considering dispatchable wind (i.e., allowing wind curtailment), but it can be applied to any uncertain source with the possibility of being curtailed.**

***Index Terms*—Stochastic optimization, robust optimization, dispatchable wind, unit commitment.**

I. Introduction

To be prepared for future demand, network operators decide about start-up and shutdown schedules of power generating units some time (typically a day) up front. The objective of this so-called unit commitment problem (UC) is to minimize generation costs while meeting power system constraints. A higher penetration of variable and uncertain generation (e.g., wind and solar power) significantly increases the uncertainty in the net forecasted future demand, and thereby also makes the UC optimization problem more complex [1].

The two main approaches for dealing with the uncertainty in demand and generation are stochastic optimization and robust optimization. Stochastic optimization (SO) is a generalization of standard optimization, but typically the objective is to minimize the expected costs over a set of possible future scenarios. However, SO is considered impractical by most system operators, mainly because of its computational burden [1]. Additionally, the most important goal for these operators is to ensure safe operation of the network without black-outs, and SO does not give sufficient guarantees on meeting the related network constraints in all realizations. Moreover, SO requires a large number of scenarios and their associated probability distribution, which is hard to obtain.

In robust optimization (RO) [2], [3] the costs are minimized considering the feasibility of *all* possible realizations of the modeled uncertainty. Typically the resulting schedules are over-conservative: although the probability of the realization of the worst case is virtually nil, the chosen schedule is robust for this realization, and therefore much more costly than what is actually required. Recent work on a budget of uncertainty aims to solve this over-conservatism by modelling a smaller uncertainty set in a flexible way [2]. However, the robust UC typically requires solving a bilevel optimization problem, where the outer level is a mixed-integer linear programming (MIP) minimization problem, and the inner level is usually a maximization *bilinear program*, which is a nonconcave problem. Such a problem is typically solved using an iterative process and linear approximations, but this can only guarantee a locally optimal solution [2]. Although there are different alternatives to solve the bilinear problem [4], finding optimal robust solutions in time is still a major challenge.

The principal contributions of this paper are as follows:

1) By allowing wind curtailment for a basic (box) uncertainty set for wind, we prove that the second stage of a fully adaptive robust UC problem which is typically nonlinear and NP-hard [2] has an equivalent LP formulation, which can be solved in polynomial time.
2) Consequently, the full two-stage robust formulation can be translated into an equivalent single-level MIP problem. This allows solving realistically-sized problem instances very close to optimality, instead of relying on approximations using iterative algorithms for the bi-level robust UC. [2], [3], [5].
3) This robust formulation can also be used in the unified stochastic and robust approach proposed in [3], thus overcoming the over-conservatism of the pure robust approach.
4) Experiments confirm the relatively short computation times for (very close to) optimal solutions for the robust and unified stochastic-robust approaches with this efficient formulation.

The remainder of this paper is organized as follows. Section II details the proposed single-level robust UC reformulation with dispatchable wind, and shows how to complement stochastic UC by incorporating the robust part. Section III provides and discusses results from several experiments, where a comparison between robust, stochastic and unified UC formulations is made. Finally, relevant conclusions and future works are drawn in Section IV.

II. Robust, Stochastic, and Unified UC Formulations with Dispatchable Wind

This section first defines the robust UC problem, then provides the proof of the second-stage LP reformulation. After that, the stochastic UC is presented, followed by a description of the unified stochastic and robust approach.

For the sake of brevity, the formulations in this section are presented in a compact matrix way. The details of the complete UC formulation are given in Appendix A.

A. Robust UC

The robust optimization approach to UC (sometimes called *adaptive* robust optimization [2]) typically consists of two stages, where the costs for (first-stage) commitment $\boldsymbol{x}$ decisions and the costs of the (second-stage) dispatch in the worst-case wind scenario are minimized over a planning horizon $\mathcal{T}$ of length $T = |\mathcal{T}|$ [2], [3], [6]. This worst-case scenario $\boldsymbol{\xi}$ is taken from a continuous interval of minimum and maximum wind nodal injection for each time interval and each bus, called the *uncertainty set*.

Definition 1. Given a vector with the lowest ($\underline{\mathbf{w}}$) and one with highest ($\overline{\mathbf{w}}$) possible wind power injection for each time interval $t \in \mathcal{T}$ and each node $b \in \mathcal{B}^{\mathrm{W}}$, the *uncertainty set* Ξ is defined by a continuous range between these extremes, i.e.,

$$\Xi\left(\underline{\mathbf{w}}, \overline{\mathbf{w}}\right) = \left\{\boldsymbol{\xi} \in \mathbb{R}^{BT} : \underline{\mathrm{w}}_{bt} \leq \xi_{bt} \leq \overline{\mathrm{w}}_{bt},\ \forall b \in \mathcal{B}^{\mathrm{W}}, t \in \mathcal{T}\right\}$$

where $\mathcal{B}^{\mathrm{W}}$ denotes the set of wind nodes, and B^{W} the number of such nodes ($B^{\mathrm{W}} = \left|\mathcal{B}^{\mathrm{W}}\right|$).

This uncertainty set can be taken very large for very robust solutions, or relatively tight to get less conservative solutions, allowing some control over conservatism [7].

Given such minimum and maximum possible wind power injection, the objective is to minimize the sum of the costs $\mathbf{b}^\top \boldsymbol{x}$ for the units' commitment related decisions for each time interval (e.g., on/off and startup/shutdown) and the worst-case dispatch cost $\max_{\boldsymbol{\xi}\in\Xi} \min_{\boldsymbol{p}(\cdot),\boldsymbol{w}(\cdot)} \left(\mathbf{c}^\top \boldsymbol{p}\left(\boldsymbol{\xi}\right) + \mathbf{d}^\top \boldsymbol{w}\left(\boldsymbol{\xi}\right)\right)$. The continuous variable $\boldsymbol{p}$ is a vector of units' dispatch decisions for each time interval. The continuous variable $\boldsymbol{w}$ is a vector of each wind production dispatch decision for each bus with wind power injections, and for each time interval.

Through this paper wind is considered to be dispatchable; that is, wind curtailment is allowed. Therefore, the uncertain parameter is the *maximum* possible dispatchable wind ($\boldsymbol{\xi}$) that can be produced (i.e., available wind capacity) rather than the wind dispatch ($\boldsymbol{w}$) itself, which is a decision variable.

Definition 2. The *two-stage robust UC formulation* S given minimum ($\underline{\mathbf{w}}$) and maximum ($\overline{\mathbf{w}}$) possible wind power injection is the following $S\left(\underline{\mathbf{w}}, \overline{\mathbf{w}}\right) =$

$$\begin{aligned}
\min_{\boldsymbol{x}} & \left(\mathbf{b}^\top \boldsymbol{x} + \max_{\boldsymbol{\xi}\in\Xi(\underline{\mathbf{w}},\overline{\mathbf{w}})} \min_{\boldsymbol{p}(\cdot),\boldsymbol{w}(\cdot)} \left(\mathbf{c}^\top \boldsymbol{p}\left(\boldsymbol{\xi}\right) + \mathbf{d}^\top \boldsymbol{w}\left(\boldsymbol{\xi}\right)\right)\right) \\
\text{s.t. } & \mathbf{F}\boldsymbol{x} \leq \mathbf{f},\ \boldsymbol{x} \text{ is binary} && (1)\\
& \mathbf{H}\boldsymbol{p}\left(\boldsymbol{\xi}\right) + \mathbf{J}\boldsymbol{w}\left(\boldsymbol{\xi}\right) \leq \mathbf{h} && (2)\\
& \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{p}\left(\boldsymbol{\xi}\right) + \mathbf{C}\boldsymbol{w}\left(\boldsymbol{\xi}\right) \leq \mathbf{g} && (3)\\
& \boldsymbol{w}\left(\boldsymbol{\xi}\right) \leq \boldsymbol{\xi} && (4)\\
& \boldsymbol{p}, \boldsymbol{w} \geq 0 && (5)
\end{aligned}$$

where $\boldsymbol{x}, \boldsymbol{p}$ and $\boldsymbol{w}$ are variables as discussed above.

Constraint (1) involves only commitment-related constraints, e.g., minimum up/down times. Constraint (2) contains dispatch-related constraints, e.g., energy balance, transmission limit constraints, and ramping constraints. Constraint (3) couples the commitment and dispatch decisions, e.g., minimum and maximum generation capacity constraints. Finally, (4) guarantees that the wind dispatch cannot exceed the available wind production.

Note that only the right hand side of (4) has an explicit dependence on the uncertain parameter $\boldsymbol{\xi}$, while the vectors $\mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{f}, \mathbf{h}$, and $\mathbf{g}$ together with matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{F}, \mathbf{H}$ and $\mathbf{J}$ are taken to be deterministic and exactly known. The second-stage variables $\boldsymbol{p}\left(\boldsymbol{\xi}\right)$ and $\boldsymbol{w}\left(\boldsymbol{\xi}\right)$ are a function of the uncertain parameter $\boldsymbol{\xi}$, and hence fully adaptive to any realization of the wind uncertainty.

Wind dispatch cost is usually considered to be zero. However, the parameter $\mathbf{d}$ is included to consider the possibility where this cost is different from zero. In some power systems, this cost is negative [8], [9], and it can be set to a large negative value to simulate situations where curtailment is not desired [8]. Defining negative values for parameter $\mathbf{d}$ is equivalent to penalizing wind curtailment in the objective function.

Even solving just the second stage (max-min part) of this fully adaptive problem S typically requires solving a bilinear program, which is in general NP-hard [2]. However, we show below that this second stage can, in fact, be formulated as a linear program (LP), which can be solved in polynomial time. This LP can then be used in an MIP formulation (without the bilinear term) of the whole two-stage unit commitment problem, and we show that this is equivalent to S.

Definition 3. The *two-stage robust UC formulation* $RO\left(\underline{\mathbf{w}}\right)$ given minimum possible wind power injection ($\underline{\mathbf{w}}$) is the following MIP:

$$\begin{aligned}
RO\left(\underline{\mathbf{w}}\right) = \min_{\boldsymbol{x},\boldsymbol{p},\boldsymbol{w}} \ & \mathbf{b}^\top \boldsymbol{x} + \mathbf{c}^\top \boldsymbol{p} + \mathbf{d}^\top \boldsymbol{w} \\
\text{s.t. } & \mathbf{F}\boldsymbol{x} \leq \mathbf{f},\ \boldsymbol{x} \text{ is binary} && (6)\\
& \mathbf{H}\boldsymbol{p} + \mathbf{J}\boldsymbol{w} \leq \mathbf{h} && (7)\\
& \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{p} + \mathbf{C}\boldsymbol{w} \leq \mathbf{g} && (8)\\
& \boldsymbol{w} \leq \underline{\mathbf{w}} && (9)\\
& \boldsymbol{p}, \boldsymbol{w} \geq 0. && (10)
\end{aligned}$$

Theorem 4. *For each minimum ($\underline{\mathbf{w}}$) and maximum ($\overline{\mathbf{w}}$) possible wind power injection, the second stage problem ($\max_{\boldsymbol{\xi}\in\Xi} \min_{\boldsymbol{p},\boldsymbol{w}} \mathbf{c}^\top \boldsymbol{p} + \mathbf{d}^\top \boldsymbol{w}$) can be formulated as an LP, and thus $S\left(\underline{\mathbf{w}}, \overline{\mathbf{w}}\right) = RO\left(\underline{\mathbf{w}}\right)$.*

Proof: We start by following a similar procedure to that described in [2], where the second-stage optimization problem of (1)-(4) is reformulated as a single-stage optimization problem with full adaptability. This corresponds to the single-stage problem once the first-stage variables $\boldsymbol{x}$ have been fixed. The completely adaptive formulation of this problem is then

$$\begin{aligned}
\max_{\boldsymbol{\xi}\in\Xi} \min_{\boldsymbol{p}(\cdot),\boldsymbol{w}(\cdot)} \ & \mathbf{c}^\top \boldsymbol{p}\left(\boldsymbol{\xi}\right) + \mathbf{d}^\top \boldsymbol{w}\left(\boldsymbol{\xi}\right) \\
\text{s.t. } & \mathbf{H}\boldsymbol{p}\left(\boldsymbol{\xi}\right) + \mathbf{J}\boldsymbol{w}\left(\boldsymbol{\xi}\right) \leq \mathbf{h} && (11)\\
& \mathbf{B}\boldsymbol{p}\left(\boldsymbol{\xi}\right) + \mathbf{C}\boldsymbol{w}\left(\boldsymbol{\xi}\right) \leq \tilde{\mathbf{g}} && (12)\\
& \boldsymbol{w}\left(\boldsymbol{\xi}\right) \leq \boldsymbol{\xi} && (13)\\
& \boldsymbol{p}, \boldsymbol{w} \geq 0 && (14)
\end{aligned}$$

where $\tilde{\mathbf{g}} = \mathbf{g} - \mathbf{A}\boldsymbol{x}$.

To get a max-max formulation, we obtain the dual of the dispatch problem $\min_{\boldsymbol{p},\boldsymbol{w}} \mathbf{c}^\top \boldsymbol{p}(\boldsymbol{\xi}) + \mathbf{d}^\top \boldsymbol{w}(\boldsymbol{\xi})$:

$$\begin{aligned} \max_{\boldsymbol{\lambda},\boldsymbol{\varphi},\boldsymbol{\eta}} \ & \mathbf{h}^\top \boldsymbol{\lambda} + \tilde{\mathbf{g}}^\top \boldsymbol{\varphi} + \boldsymbol{\xi}^\top \boldsymbol{\eta} \\ \text{s.t.} \ & \mathbf{H}^\top \boldsymbol{\lambda} + \mathbf{B}^\top \boldsymbol{\varphi} \geq \mathbf{c} && (15) \\ & \mathbf{J}^\top \boldsymbol{\lambda} + \mathbf{C}^\top \boldsymbol{\varphi} + \boldsymbol{\eta} \geq \mathbf{d} && (16) \\ & \boldsymbol{\lambda}, \boldsymbol{\varphi}, \boldsymbol{\eta} \geq 0 && (17) \end{aligned}$$

where $\boldsymbol{\lambda}, \boldsymbol{\varphi}, \boldsymbol{\eta}$ are the dual variables of constraints (11)-(13).

Now, the second-stage problem $\max_{\boldsymbol{\xi}\in\boldsymbol{\Xi}} \min_{\boldsymbol{p},\boldsymbol{w}} \mathbf{c}^\top \boldsymbol{p} + \mathbf{d}^\top \boldsymbol{w}$ becomes

$$\begin{aligned} \max_{\boldsymbol{\lambda},\boldsymbol{\varphi},\boldsymbol{\eta},\boldsymbol{\xi}} \ & \mathbf{h}^\top \boldsymbol{\lambda} + \tilde{\mathbf{g}}^\top \boldsymbol{\varphi} + \boldsymbol{\xi}^\top \boldsymbol{\eta} \\ \text{s.t.} \ & \mathbf{H}^\top \boldsymbol{\lambda} + \mathbf{B}^\top \boldsymbol{\varphi} \geq \mathbf{c} && (18) \\ & \mathbf{J}^\top \boldsymbol{\lambda} + \mathbf{C}^\top \boldsymbol{\varphi} + \boldsymbol{\eta} \geq \mathbf{d} && (19) \\ & \boldsymbol{\xi} \leq \overline{\mathbf{w}} && (20) \\ & \boldsymbol{\xi} \geq \underline{\mathbf{w}} && (21) \\ & \boldsymbol{\lambda}, \boldsymbol{\varphi}, \boldsymbol{\eta}, \boldsymbol{\xi} \geq 0. && (22) \end{aligned}$$

This objective function contains the bilinear term $\boldsymbol{\xi}^\top \boldsymbol{\eta} = \sum_{b\in\mathcal{B}^{\mathrm{w}},t\in\mathcal{T}} \xi_{bt}\eta_{bt}$ which, in general, is non-convex and NP-hard to solve [2]. However, under the condition that variables $\boldsymbol{\xi}$ are independent from the rest of the system, a bilinear program has an equivalent linear formulation. Dantzig [10] shows that substituting one variable for the bilinear term can lead to a convex optimization problem. In this case, we substitute $\omega_{bt} = \xi_{bt}\eta_{bt}$ and multiply (20) and (21) by η_{bt} and obtain the following formulation where variables ξ_{bt} no longer appear [11, Theorem 10.2]:

$$\begin{aligned} \max_{\boldsymbol{\omega},\boldsymbol{\lambda},\boldsymbol{\varphi},\boldsymbol{\eta}} \ & \mathbf{h}^\top \boldsymbol{\lambda} + \tilde{\mathbf{g}}^\top \boldsymbol{\varphi} + \sum_{b\in\mathcal{B}^{\mathrm{w}},t\in\mathcal{T}} \omega_{bt} \\ \text{s.t.} \ & \mathbf{H}^\top \boldsymbol{\lambda} + \mathbf{B}^\top \boldsymbol{\varphi} \geq \mathbf{c} && (23) \\ & \mathbf{J}^\top \boldsymbol{\lambda} + \mathbf{C}^\top \boldsymbol{\varphi} + \boldsymbol{\eta} \geq \mathbf{d} && (24) \\ & \overline{\mathbf{W}}\boldsymbol{\eta} - \boldsymbol{\omega} \geq 0 && (25) \\ & \boldsymbol{\omega} - \underline{\boldsymbol{W}}\boldsymbol{\eta} \geq 0 && (26) \\ & \boldsymbol{\lambda}, \boldsymbol{\varphi}, \boldsymbol{\eta}, \boldsymbol{\omega} \geq 0 && (27) \end{aligned}$$

where $\overline{\mathbf{W}}$ and $\underline{\boldsymbol{W}}$ are diagonal matrices containing the vectors $\overline{\mathbf{w}}$ and $\underline{\mathbf{w}}$ in the diagonal, respectively.

As mentioned in [11, Theorem 10.2], this linear problem (23)-(27) is only equivalent to the bilinear problem (18)-(22) when $\omega_{bt} \neq 0$ is not possible if $\eta_{bt} = 0$. Notice that the linear problem satisfies this condition since (25) guarantees that $\omega_{bt} = 0$ if $\eta_{bt} = 0$.

Next, we obtain the dual of the linear problem (23)-(27):

$$\begin{aligned} \min_{\boldsymbol{p},\boldsymbol{w},\boldsymbol{v}^+,\boldsymbol{v}^-} \ & \mathbf{c}^\top \boldsymbol{p} + \mathbf{d}^\top \boldsymbol{w} \\ \text{s.t.} \ & \mathbf{H}\boldsymbol{p} + \mathbf{J}\boldsymbol{w} \leq \mathbf{h} && (28) \\ & \mathbf{B}\boldsymbol{p} + \mathbf{C}\boldsymbol{w} \leq \tilde{\mathbf{g}} && (29) \\ & \boldsymbol{w} + \overline{\mathbf{W}}^\top \boldsymbol{v}^+ - \underline{\boldsymbol{W}}^\top \boldsymbol{v}^- \leq 0 && (30) \\ & \boldsymbol{v}^- - \boldsymbol{v}^+ \leq \mathbf{1} && (31) \\ & \boldsymbol{p}, \boldsymbol{w}, \boldsymbol{v}^+, \boldsymbol{v}^- \geq 0 \end{aligned}$$

where $\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{v}^+, \boldsymbol{v}^-$ are the dual variables of constraints (23)-(26), respectively.

By applying the Fourier Motzkin elimination, we obtain the following equivalent formulation where variables $\boldsymbol{v}^+$ and $\boldsymbol{v}^-$ no longer appear:

$$\begin{aligned} \min_{\boldsymbol{p},\boldsymbol{w}} \ & \mathbf{c}^\top \boldsymbol{p} + \mathbf{d}^\top \boldsymbol{w} \\ \text{s.t.} \ & \mathbf{H}\boldsymbol{p} + \mathbf{J}\boldsymbol{w} \leq \mathbf{h} && (32) \\ & \mathbf{B}\boldsymbol{p} + \mathbf{C}\boldsymbol{w} \leq \tilde{\mathbf{g}} && (33) \\ & \boldsymbol{w} \leq \underline{\mathbf{w}} && (34) \\ & \boldsymbol{p}, \boldsymbol{w} \geq 0. && (35) \end{aligned}$$

In short, the second-stage max-min problem (11)-(14), which is non-convex [2], [4], is equivalent to the LP formulation (32)-(35). Consequently, the complete two-stage UC formulation with wind dispatch $S(\underline{\mathbf{w}}, \overline{\mathbf{w}})$ is equivalent to $RO(\underline{\mathbf{w}})$. (The step-by-step Fourier Motzkin procedure can be found in our technical report [12].) ■

The main element of this proof is a generalization of a similar result on adaptive robust optimization for LP problems [11], [13]. Since the uncertainty affecting every one of the constraints (13) is independent of each other, and the uncertainty set is defined as a continuous interval, the fully adaptive solution of the second-stage problem is equivalent to the static (or non-adaptive) one, as proven in [13] and further discussed in [14]. That is, we can obtain the solution of the second-stage ARO model (11)-(14) by solving its static robust reformulation, which result is (32)-(35).

It is important to highlight that this *RO* problem is a deterministic formulation for only one scenario: the worst-case wind scenario, i.e., the lowest expected wind $\underline{\mathbf{w}}$ within the uncertainty set Ξ. If this formulation has an optimal solution $\boldsymbol{w}^*$ then it guarantees that all other possible wind realizations within the uncertainty set are feasible. That is, all scenarios can become $\boldsymbol{w}^*$ by curtailment. However, this solution may be too conservative because it does not guarantee that wind scenarios above $\underline{\mathbf{w}}$ can be dispatched.

B. Stochastic UC

To validate our formulation and to illustrate its computational properties, we compare *RO* to a stochastic optimization (*SO*) formulation. In *SO*, the first stage determines the day-ahead units' on/off decisions, and the second-stage dispatch decisions are taken by minimizing the expected costs over a set of scenarios $\mathcal{S}$.

Definition 5. For a fixed commitment decision $\boldsymbol{x}$ and wind expected wind availability $\mathbf{w}$, the dispatch variables of thermal $\boldsymbol{p}$ and wind $\boldsymbol{w}$ units are constrained by a set of feasible solutions:

$$\Omega(\boldsymbol{x}, \mathbf{w}) = \{\boldsymbol{p}, \boldsymbol{w} : \mathbf{H}\boldsymbol{p} + \mathbf{J}\boldsymbol{w} \leq \mathbf{h}, \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{p} + \mathbf{C}\boldsymbol{w} \leq \mathbf{g}, \ 0 \leq \boldsymbol{w} \leq \mathbf{w}, \boldsymbol{p} \geq 0\}. \quad (36)$$

Definition 6. The *SO* formulation is the following MIP:

$$\begin{aligned} SO(\mathcal{S}) = \min_{\boldsymbol{x}} \mathbf{b}^\top \boldsymbol{x} + & \min_{\boldsymbol{p_s},\boldsymbol{w_s}\in\Omega(\boldsymbol{x},\mathbf{w_s})\forall s\in\mathcal{S}} \underset{}{E}\left(\mathbf{c}^\top \boldsymbol{p_s} + \mathbf{d}^\top \boldsymbol{w_s}\right) \\ \text{s.t. } & \mathbf{F}\boldsymbol{x} \leq \mathbf{f}, \ \boldsymbol{x} \text{ is binary} && (37) \end{aligned}$$

where $E(\cdot)$ is the expected cost function. The index s represents scenarios that belong to a set $\mathcal{S}$. The positive variables $\boldsymbol{p_s}$ and $\boldsymbol{w_s}$ are the dispatch decisions of thermal and wind units.

C. Unified Stochastic and Robust Optimization Approach

Disadvantages of both the stochastic and the robust optimization approaches can be addressed by the unified approach (*SR*) [3]. This formulation reduces the over-conservatism of pure *RO* because an expected value is now optimized over a set of scenarios, and it does not require the large quantity of scenarios to guarantee feasibility as pure *SO* does.

In the unified *SR* UC, the first stage again determines the day-ahead unit commitment decisions. The second stage contains the economic dispatch decisions for the generating units under each scenario for the stochastic optimization part *and* the worst-case scenario. As proposed in [3], the parameter $\alpha \in [0, 1]$ represents the weight of the expected total generation cost and, accordingly, $1 - \alpha$ represents the weight of the generation cost of the worst case.

Definition 7. The *SR* UC formulation is the following MIP:

$$SR(\underline{\mathbf{w}}) = \min_{\boldsymbol{x}} \mathbf{b}^\top \boldsymbol{x} + (1-\alpha) \min_{\underline{\boldsymbol{p}}, \underline{\boldsymbol{w}} \in \Omega(\boldsymbol{x}, \underline{\mathbf{w}})} \left(\mathbf{c}^\top \underline{\boldsymbol{p}} + \mathbf{d}^\top \underline{\boldsymbol{w}}\right)$$

$$+ \alpha \min_{\boldsymbol{p_s}, \boldsymbol{w_s} \in \Omega(\boldsymbol{x}, \mathbf{w_s}) \forall s \in \mathcal{S}} E\left(\mathbf{c}^\top \boldsymbol{p_s} + \mathbf{d}^\top \boldsymbol{w_s}\right) \quad (38)$$

$$\text{s.t. } \mathbf{F}\boldsymbol{x} \leq \mathbf{f}, \ \boldsymbol{x} \text{ is binary} \quad (39)$$

$$\boldsymbol{w_s} \geq \underline{\boldsymbol{w}}, \ \forall s \in \mathcal{S} \quad (40)$$

where $\underline{\boldsymbol{p}}$ and $\underline{\boldsymbol{w}}$ are the variables representing the generating dispatch decisions for the robust optimization part; likewise, the variables $\boldsymbol{p_s}$ and $\boldsymbol{w_s}$ are the generating dispatch decisions for the stochastic optimization part. The set $\Omega(\cdot)$ is the set of feasible dispatch solutions defined in Definition 5.

Constraint (40) guarantees that all the wind dispatch scenarios for the stochastic part are greater than or equal to the worst-case wind dispatch of the robust part, thus guaranteeing that the stochastic solution is indeed protected by the robust solution. Therefore, any uncertain wind realization above $\underline{\boldsymbol{w}}$ is protected since, in the worst-case, it can be curtailed to $\underline{\boldsymbol{w}}$.

Finally, the worst-case wind scenario for the robust part can be defined without using new information, by using the wind scenarios from the stochastic part as follows:

$$\underline{\mathrm{w}}_{bt} = \inf_{s \in \mathcal{S}} \left(\mathrm{w}_{sbt}\right) \quad \forall b, t \quad (41)$$

where $\inf(\cdot)$ is the infimum function, parameters w_{sbt} are the wind scenarios used in the stochastic part, and parameters $\underline{\mathrm{w}}_{bt}$ define the worst-case wind scenario to be used for the robust part. Even though no additional information is used, adding the worst-case scenario can significantly improve the robustness of the pure stochastic solution, as shown in Section III-B2.

III. Numerical Results

To validate our MIP formulation for robust optimization (*RO*), presented in Section II-A, we compare outcomes, their costs, and the required run time to a standard stochastic optimization (*SO*) approach (Section II-B) with different numbers of scenarios, as well as to unified stochastic and robust to stochastic optimization (*SR*), detailed in Section II-C. For the *SR*, we first establish the weight α for the stochastic part in the objective function. Also we analyse the effect of changing the penalty for wind curtailment.

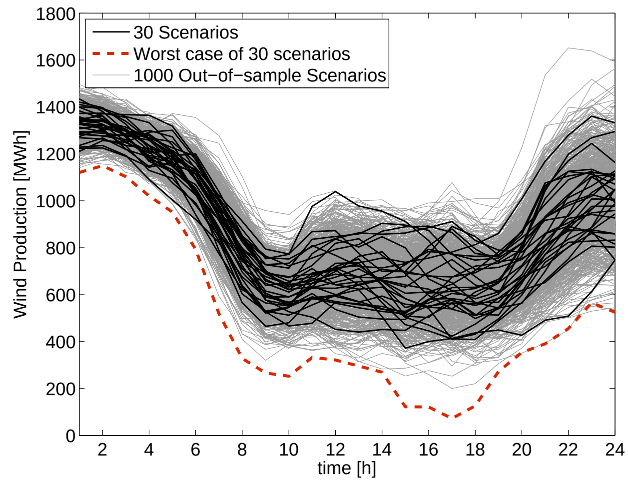

Fig. 1. Wind production can develop in significantly different ways over 24 hours. The 30 selected scenarios (and their worst case) used for UC are shown on a background the 1000 out-of-sample scenarios.

A. Experimental Setup

As a case study, we use the IEEE 118-bus test system containing three different buses, with wind production, and it was adapted to consider startup and shutdown power trajectories [15]. This IEEE-118 bus system has 186 transmission lines, 54 thermal generator units, 91 loads, and three buses with wind production. The penalty costs for demand-balance and transmission-limits violations are set to 10000 $/MWh and 5000 $/MWh [16], respectively. In addition, wind curtailment is penalized as 300 $/MWh (about ten times the average wind bid in some US markets [8], [9]) to simulate cases where wind curtailment is not desired.

All experiments are solved using CPLEX 12.6.1 using default parameters [17]. All instances are solved until they reach an optimality tolerance of $5 \cdot 10^{-4}$. All experiments were run on an Intel-Xeon 3.7-GHz personal computer with 16 GB of RAM memory.

To compare the performance of the different UC models, we make a clear difference between the scheduling and (out-of-sample) evaluation stages. The scheduling stage solves the different UC models and obtains their commitment policy using a small representative number of wind scenarios, up to 30 scenarios. The evaluation stage, for each fixed commitment policy, solves a network-constrained economic dispatch problem repetitively for a set of 1000 out-of-sample wind scenarios (see Fig. 1), thus obtaining an accurate estimate of the expected performance of each UC policy.

To generate scenarios for the uncertain wind power production, we use Latin Hypercube Sampling (LHS). We assume that the wind production follows a multivariate normal distribution with predicted nominal value and volatility matrix. The

Table I
PROBLEM SIZE COMPARISON OF UC FORMULATIONS

UC	SC #	Constraints	Continuous variables	Binary variables	Nonzero Elements
SO	5	56367	35640	6390	1413190
	30	309567	213833	6390	8332689
RO	1	18447	9713	6390	311070
SR^{30}	5	66855	42761	6390	1689430
	30	321848	220954	6390	8614565

idea in applying LHS is to optimally distribute the samples to explore the whole area in the experimental region, avoiding the creation of scenarios that are too similar (clusters) [18].

The worst-case wind scenario for the *RO* and SR^{30} models (from 5 to 30 scenarios) is obtained by taking the lowest wind expected production for each time point and for each bus from the set of 30 (ins-sample) scenarios. Then, from (41), this worst-case wind scenario is $\underline{\mathrm{w}}_{bt}^{30} = \inf_s \left(\mathrm{w}_{sbt}^{30}\right) \forall b, t$. Fig. 1 shows the 30 scenarios and the worst-case scenario for the aggregated (from all buses) wind production.

We then assess the performance of the different UC policies on six aspects. For the scheduling stage: 1) the fixed production costs (FxdCost [k$]), which includes non-load, startup and shutdown costs, and 2) the time required to solve the UC problem (CPU Time [s]). For the evaluation stage we record 3) the average of the total production costs including the wind curtailment penalization (AvgTCW [k$]) and 4) without the penalization (AvgTC [k$]), which separates the effect of wind curtailment penalization from the production costs; 5) the volatility of these costs represented by their standard deviation (StdTC [k$]); 6) the total accumulated number of violations in both demand-balance and transmission-limits constraints (# Viol); and 7) the average percentage of wind that was curtailed (% WCurt).

B. Results

Table I shows an overview of the problem sizes for the smallest and the largest set of scenarios for different UC formulations. Despite the number of modeled scenarios, all the UCs have the same number of binary variables, because they all obtain the same quantity of binary (first-stage) decisions. The number of constraints differs significantly. Note that *SR* is slightly larger than *SO* because *SR* adds an extra scenario representing the worst-case scenario.

1) Different Objective Weights for the SR *Approach:* First, we aim to establish the optimal balance between the costs of the worst-case component and the other scenarios in the objective function in the *SR* formulation. Table II presents the results for 5 scenarios with an objective weight α ranging from 0 (all weight to worst-case scenario) to 1 (all weight to stochastic scenarios) in steps of 0.1.

It is interesting to notice that *SR* took much longer to solve with $\alpha = 0$ than with other values of α. This can be explained by the fact that all of the other dispatches (of the 5 scenarios of the stochastic part) contain values in the objective function that help the solver to find the optimal solution more quickly. Additionally, the stochastic part of *SR* helps to accommodate high values of wind (lower WCurt) and to lower the AvgTCW, but when $\alpha = 0$, this effect disappears completely. As a consequence, the *SR* solution tends to become the same as

Table II
DIFFERENT LEVELS OF α FOR SR^{30} WITH 5 SCENARIOS

	Scheduling		Out-of-sample Evaluation				
α #	FxdCost [k$]	CPU Time [s]	AvgTCW [k$]	AvgTC [k$]	StdTC [k$]	Viol #	WCurt %
0	58.76	346.7	855.10	750.25	15.4	0	1.66
0.1	66.92	43.6	788.02	753.85	15.7	0	0.53
0.2	67.09	54.1	784.94	754.39	15.8	0	0.48
0.3	66.85	48.8	775.95	755.46	15.9	0	0.32
0.4	68.51	34.3	774.34	756.04	15.8	0	0.28
0.5	66.16	35.4	773.47	755.8	15.9	0	0.27
0.6	66.16	47.5	773.47	755.8	15.9	0	0.27
0.7	67.92	57.4	771.89	756.87	16.0	0	0.23
0.8	64.90	62.0	767.32	754.89	16.4	0	0.19
0.9	62.56	63.1	766.39	752.57	16.5	0	0.21
1	67.01	39.5	765.87	751.08	16.2	0	0.23

the *RO* solution (see Table III), but the problem takes much longer to be solved because the *SR* formulation is significantly larger (see Table I).

From the results we observe that as α increases, AvgTCW decreases. This is because the problem becomes less conservative when the robust part of *SR* has a smaller weight. The same behaviour was previously observed in [3]. Even though this is not exactly true for AvgTC, for high values of α (above 0.7) AvgTC also tends to decrease. Furthermore, it should be noted that there is also a tendency of wind curtailment to decrease as α increases, but where the lowest value was obtained when $\alpha = 0.8$. Finally, we observe that results are not very sensitive to α: in the range $[0.3, 1]$ we see a difference in total costs of at most 2%.

Summarizing, as the weight of the worst-case cost component $(1 - \alpha)$ decreases, AvgTCW decreases and the volatility of the solution (StdTC) increases, which is not surprising as the problem then becomes less conservative. However, when ignoring the worst-case ($\alpha = 1$), the fixed cost and curtailment are a bit higher, so based on these results, we set $\alpha = 0.9$ for the remaining experiments, noting that the results are rather robust against the selected value. In addition, similar results were found for the case of *SR* when 30 scenarios where used for the stochastic part.

2) Robust, Stochastic, and Unified: Table III presents the results for all three UC formulations. Regarding *SO* and *RO*, we can clearly observe the differences widely discussed in the literature [2], [3]. First, as expected, the higher the number of scenarios, the better the *SO* performance. Second, on the one hand, the stochastic formulation *SO* using 30 scenarios achieves the lowest AvgTC and guarantees robustness (Viol=0). On the other hand, *RO* guarantees robustness by only optimizing for the worst-case scenario, but scheduled too few reserves (lower FxdCost) to ensure that higher wind production levels could be dispatched (around 10x larger WCurt than *SO*). However, compared to *SO* with 30 scenarios, the *RO* formulation proposed in this paper could be solved more than an order of magnitude faster (above 16x).

Including the *RO* formulation into a *SO* formulation gives us a so-called *SR* formulation. Even when the worst case is based on only the 5 scenarios used for *SO* (SR^5), see Fig. 2, the quality of the of the UC improves significantly: SR^5 shows no violations (instead of 171), a cost reduction of more than 4%, and a volatility (StdTC) reduction of 70%. The most important part, however, is that, in contrast to adding scenarios into the

Table III
COMPARISON BETWEEN DIFFERENT UC FORMULATIONS

UC	SC	Scheduling		Out-of-sample Evaluation				
	#	FxdCost [k$]	CPU Time [s]	AvgTCW [k$]	AvgTC [k$]	StdTC [k$]	Viol #	WCurt %
SO	5	65.09	38.0	805.63	792.15	220.1	171	0.22
	10	63.20	184.8	782.10	767.33	110.1	86	0.23
	15	62.89	277.0	782.16	765.97	110.1	86	0.25
	20	66.33	266.8	780.13	770.03	110.1	86	0.16
	25	65.04	460.8	772.68	760.38	65.9	50	0.20
	30	64.52	980.6	763.82	753.73	16.3	0	0.16
RO	1	59.14	58.2	855.33	750.50	15.4	0	1.66
SR^5	5	63.19	60.4	774.05	760.29	66.0	0	0.21
SR^{30}	5	62.56	63.1	766.39	752.57	16.5	0	0.21
	10	62.56	164.6	765.73	751.83	16.5	0	0.22
	15	66.74	324.0	765.02	754.70	16.4	0	0.16
	20	66.19	251.6	764.53	755.53	16.2	0	0.17
	25	66.97	649.9	764.42	753.61	16.3	0	0.15
	30	68.59	520.3	764.38	754.88	16.4	0	0.14

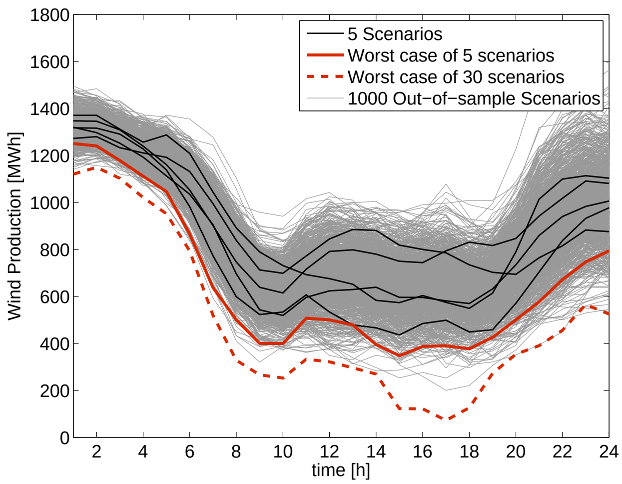

Fig. 2. When only five scenarios are selected to be used for solving the UC problem, the worst case is typically more optimistic than the worst case of 30 scenarios.

SO, basing the worst-case scenario on more scenarios comes at almost no extra costs in terms of run time. We therefore continue with a comparison where the worst case is based on 30 scenarios, SR^{30}.

Compared with *SO* with 25 scenarios, which had 50 violations, SR^{30} with 5 scenarios presents even lower costs (AvgTCW) and a further reduction of volatility, still completely avoiding violations. Moreover, it solves 7.3x faster than a comparable solution of *SO*. In general, the performance of a stochastic formulation with few scenarios is dramatically improved by adding the worst-case scenario to the formulation. This is because the stochastic part lowers the expected costs and the robust part avoids violations, as also concluded in [3]. Furthermore, the computational burden of *SR* is not significantly affected, compared to *SO* (it was even lower in some cases).

3) Wind Penalization: The approach we put forward in this paper is enabled by wind curtailment. In some power systems, wind curtailment may be undesirable; however, violations of the demand balance and of transmission-capacity limits are even worse. As explained earlier all these types of violations are penalized in the UC formulation (demand balance with 10000 $/MWh and transmission limits with 5000 $/MWh). The objective of this final experiment is to study the effect of different penalties on wind curtailment.

Table IV shows the performance of *SO* with 5 and 30 scenarios, *SR* with 5 scenarios, and *RO* for different negative values of wind bids (WB), which is equivalent to applying wind curtailment penalization. In general for a given UC formulation, as the wind curtailment penalization increases, the FxdCost increases. This results from the need to schedule more resources (reserves) to better accommodate different wind realizations. This, in turn, also increases the average total costs (AvgTCW and AvgTC).

When wind curtailment is not penalized (Wind Bid (WB)=0) the UC approaches carry the least quantity of reserves (reflected by the lowest FxdCost). Surprisingly, *SO30* reported violations for the case in which the penalization of wind curtailment was set to zero. Therefore, the least quantity of reserves might be insufficient to avoid violations in the out-of-sample evaluation stage, as observed in the case of *SO30*. It should be noted that *SR05* scheduled the least quantity of resources (lowest FxdCost) and presented the lowest average total costs. Surprisingly, even though *RO* was expected to be over-conservative for the case in which the wind bid is equal to zero, its curtailment and average costs were similar to *SR05*.

For non-zero wind bids, *RO* presents the lowest volatility (reflected by the lowest StdTC), but also the highest curtailment. This is a result of its conservative policy, which avoids infeasible solutions but cannot guarantee that a high production of wind will be dispatched. This also leads *RO* to have the highest AvgTCW when compared to *SO30* and *SR05*.

In general, in these cases, higher penalties lead to higher fixed costs. In addition, for the highest penalty, *SR05* presented a slightly higher AvgTCW and sometimes slightly lower AvgTC than *SO30*, being thus very similar to *SO30*, while solving the problem more than an order of magnitude faster. Furthermore, *SR05* could avoid violations for all the cases.

IV. CONCLUSIONS AND FUTURE WORK

By allowing wind curtailment (with a box uncertainty set for wind), the second-stage bilinear formulation of a fully adaptive robust UC problem has an equivalent LP reformulation. This reformulation shows that the worst-case wind (available capacity) scenario can be found before solving the robust UC. The resulting complete bi-level robust UC formulation thus is equivalent to a single-level MIP problem. As expected from the differences between a bilinear and an LP formulation, the computational burden is consequentially significantly reduced.

The efficient formulation can also be used as part of the unified stochastic and robust approach [3]. This unified formulation reduces the over-conservatism of pure robust UC because an expected value is now optimized on a set of scenarios, and it does not require the large quantity of scenarios to guarantee feasibility as pure stochastic UC does. More importantly, the computational burden of the unified approach remains low, since the proposed robust UC solution just adds a single extra scenario to the stochastic UC.

Table IV
COMPARISON BETWEEN DIFFERENT UC FORMULATIONS FOR DIFFERENT PENALIZATIONS OF WIND CURTAILMENT

		Scheduling		Out-of-sample Evaluation				
WB	UC	FxdCost [k$]	CPU Time [s]	AvgTCW [k$]	AvgTC [k$]	StdTC [k$]	Viol #	WCurt %
0	*SO05*	33.13	22.1	818.55	818.55	295.2	326	3.9
	SO30	37.00	361.5	743.25	743.25	91.7	45	3.63
	SR05	31.83	30.7	733.65	733.65	16.0	0	3.38
	RO	31.91	3.4	734.12	734.12	16.0	0	3.17
25	*SO05*	41.94	21.2	856.51	849.35	397.9	333	1.39
	SO30	40.85	535.6	743.49	737.44	15.9	0	1.17
	SR05	40.42	33.3	744.18	738.65	15.8	0	1.07
	RO	49.01	4.3	754.19	744.21	15.4	0	1.91
50	*SO05*	43.58	33.3	861.47	852.66	397.9	333	0.85
	SO30	42.31	608.3	748.65	739.62	15.9	0	0.87
	SR05	40.64	40.8	749.11	740.08	15.8	0	0.87
	RO	50.18	3.8	769.13	747.53	15.4	0	2.06
75	*SO05*	51.94	75.1	866.28	858.17	397.9	341	0.53
	SO30	56.46	4094	752.3	746.65	16.0	0	0.36
	SR05	42.07	66.9	753.21	742.15	16.1	0	0.70
	RO	54.72	30.5	780.35	748.66	15.4	0	2.02
100	*SO05*	52.19	60.6	868.07	860	397.9	340	0.39
	SO30	57.65	1181.4	754.08	747.48	16.0	0	0.32
	SR05	59.51	89.9	754.29	747.9	16.2	6	0.31
	RO	57.97	20	784.2	749.56	15.5	0	1.65
200	*SO05*	54.31	35.8	873.63	861.4	397.9	333	0.29
	SO30	59.71	794.2	760.02	750	16.2	0	0.24
	SR05	62.36	48.5	760.52	750.79	16.3	0	0.23
	RO	59.14	79	820.4	750.49	15.4	0	1.66
300	*SO05*	65.09	38	805.63	792.15	110.1	171	0.22
	SO30	64.52	980.6	763.82	753.73	16.3	0	0.16
	SR05	62.56	63.1	766.39	752.57	16.3	0	0.21
	RO	59.14	58.2	855.33	750.5	15.4	0	1.66
400	*SO05*	69.06	31.4	785.13	772.88	110.2	86	0.14
	SO30	66.73	821.7	767.01	755.26	16.3	0	0.14
	SR05	67.66	85.7	769.93	755.42	16.4	0	0.17
	RO	59.14	72.8	890.29	750.55	15.4	0	1.66
500	*SO05*	75.87	31.4	791.63	778.52	129	87	0.12
	SO30	68.72	717.2	769.85	756.9	16.3	0	0.12
	SR05	71.6	68.6	771.87	759.3	16.0	0	0.12
	RO	58.77	63.5	925.03	750.44	15.3	0	1.66

The formulation has been evaluated only for wind curtailment, but it would be useful for any uncertain source that can be curtailed, in particular generation from solar panels. Another straightforward application of the results in this paper would be to incorporate the worst-case solution in a deterministic UC formulation based on reserves, thereby greatly improving its robustness without significantly affecting its computational burden. We also believe that our formulation could help in the design of new heuristics for the two-stage robust UC problem including dynamic sets of uncertainty [4].

APPENDIX

A. Nomenclature

Indexes and Sets:

$g \in \mathcal{G}$	Generating units, running from 1 to G.
$b \in \mathcal{B}$	Buses, running from 1 to B.
$l \in \mathcal{L}$	Transmission lines, running from 1 to L.
$t \in \mathcal{T}$	Hourly periods, running from 1 to T hours.

System Parameters:

D_{bt}	Demand on bus b for hour t [MWh].
$\overline{F}_l$	Power flow limit on transmission line l [MW].
Γ_{lb}	Shift factor for line l associated with bus b [p.u.].
Γ^{G}_{lg}	Shift factor for line l associated with unit g [p.u.].
W_{sbt}	Wind scenarios for the stochastic part [MWh].
$\underline{\mathrm{W}}_{bt}$	Worst-case wind scenario for the robust part [MWh].

Unit's Parameters:

C^{V}_g	Variable production cost of unit g [$/MWh].
C^{VW}_b	Variable production cost of wind [$/MWh].
C^{NL}_g	No-load cost [$/h].
C^{SD}_g	Shutdown cost [$].
C^{SU}_g	Startup cost [$].
$\overline{P}_g$	Maximum power output [MW].
$\underline{P}_g$	Minimum power output [MW].
RD_g	Ramp-down capability [MW/h].
RU_g	Ramp-up capability [MW/h].
SD_g	Shutdown ramping capability [MW/h].
SU_g	Startup ramping capability [MW/h].
TD_g	Minimum down time [h].
TU_g	Minimum up time [h].

Decision Variables:

w_{bt}	Wind production for hour t at bus b [MWh].
p_{gt}	Energy output above minimum output of unit g for hour t [MWh].
$\widehat{p}_{gt}$	Total energy output of unit g for hour t.
u_{gt}	Binary variable which is equal to 1 if the unit is producing above minimum output and 0 otherwise.
v_{gt}	Binary variable which takes the value of 1 if the unit starts up and 0 otherwise.
z_{gt}	Binary variable which takes the value of 1 if the unit shuts down and 0 otherwise.

B. UC Formulation

Here, we present the set of constraints for quick-start units (which can startup within one hour) and single-startup costs. The formulations also take into account slow-start units and variable startup costs, which depend on how long the unit have been offline. The reader is referred to [6], [15], [19], [20] for further details.

The UC seeks to minimize all production costs:

$$\begin{aligned}\min_{\boldsymbol{x}} & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\left[C^{\mathrm{NL}}_g u_{gt} + C^{\mathrm{SU}}_g v_{gt} + C^{\mathrm{SD}}_g z_{gt}\right] \\ & + (1-\alpha) \min_{\underline{\widehat{p}}_{gt}, \underline{w}_{gt} \in \Omega\left(u_{gt}, v_{gt}, z_{gt}, \underline{\mathrm{W}}_{\mathrm{bt}}\right)} f\left(\underline{\widehat{p}}_{gt}, \underline{w}_{bt}\right) \\ & + \alpha \min_{\widehat{p}_{sgt}, w_{sgt} \in \Omega\left(u_{gt}, v_{gt}, z_{gt}, \mathrm{W}_{\mathrm{sbt}}\right) \forall s\in\mathcal{S}} E\left(f\left(\widehat{p}_{sgt}, w_{sbt}\right)\right)\end{aligned} \tag{A1}$$

$$\text{s.t. } u_{gt} - u_{g,t-1} = v_{gt} - z_{gt} \quad \forall g,t \tag{A2}$$

$$\sum_{i=t-TU_g+1}^{t} v_{gi} \le u_{gt} \quad \forall g, t\in[TU_g, T] \tag{A3}$$

$$\sum_{i=t-TD_g+1}^{t} z_{gi} \le 1 - u_{gt} \quad \forall g, t\in[TD_g, T] \tag{A4}$$

$$w_{sbt} \ge \underline{w}_{bt}, \; \forall s\in\mathcal{S}, b\in\mathcal{B}^{\mathrm{W}}, t \tag{A5}$$

where $E(\cdot)$ is the expected cost function, and $f(\cdot)$ is the (second-stage) dispatch cost function defined as

$$f\left(\widehat{p}_{sgt}, w_{sbt}\right) = \sum_{t\in\mathcal{T}} \left[\sum_{g\in\mathcal{G}} C_g^{\mathrm{V}} \widehat{p}_{sgt} + \sum_{b\in\mathcal{B}^{\mathrm{W}}} C_b^{\mathrm{VW}} w_{sbt} \right].$$

Constraints (A2)-(A4) are the commitment-related constraints: the startup/shutdown logic, minimum uptime and minimum downtime constraints, respectively.

The dispatch solutions for a given $\widehat{p}_{gt}, w_{bt}$ are constrained by a set of feasible solutions $\Omega\left(u_{gt}, v_{gt}, z_{gt}, \mathrm{W}_{\mathrm{bt}}\right)$ defined by (A6)-(A15), for a fixed commitment u_{gt}, startup v_{gt} and shutdown z_{gt} decisions, and an expected wind available capacity W_{bt}.

1) System-wide Constraints: The energy demand balance is guaranteed as follows:

$$\sum_{g\in\mathcal{G}} \widehat{p}_{gt} = \sum_{b\in\mathcal{B}} D_{bt} - \sum_{b\in\mathcal{B}^{\mathrm{W}}} w_{bt} \quad \forall t \tag{A6}$$

and power-flow transmission limits are ensured with

$$-\overline{F}_l \leq \sum_{g\in\mathcal{G}} \Gamma_{lg}^{\mathrm{G}} \widehat{p}_{gt} + \sum_{b\in\mathcal{B}^{\mathrm{W}}} \Gamma_{lb} w_{bt} - \sum_{b\in\mathcal{B}} \Gamma_{lb} D_{bt}^{\mathrm{E}} \leq \overline{F}_l \quad \forall l, t \tag{A7}$$

2) Constraints of Units' Energy Production: The energy production of thermal units must be within their capacity limits:

$$p_{gt} \leq \left(\overline{P}_g - \underline{P}_g\right) u_{gt} - \left(\overline{P}_g - SD_g\right) z_{g,t+1} - \max\left(SD_g - SU_g, 0\right) v_{g,t} \quad \forall g \in \mathcal{G}^1, t \tag{A8}$$

$$p_{gt} \leq \left(\overline{P}_g - \underline{P}_g\right) u_{gt} - \left(\overline{P}_g - SU_g\right) v_{gt} - \max\left(SU_g - SD_g, 0\right) z_{g,t+1} \quad \forall g \in \mathcal{G}^1, t \tag{A9}$$

$$p_{gt} \leq \left(\overline{P}_g - \underline{P}_g\right) u_{gt} - \left(\overline{P}_g - SU_g\right) v_{gt} - \left(\overline{P}_g - SD_g\right) z_{g,t+1} \quad \forall g \notin \mathcal{G}^1, t \tag{A10}$$

where $\mathcal{G}^1$ is defined as the units in $\mathcal{G}$ with $TU_g = 1$.

Ramping-capability limits are ensured with:

$$-RD_g \leq p_{gt} - p_{g,t-1} \leq RU_g \quad \forall g, t. \tag{A11}$$

The total energy production for thermal and wind units are obtained as follows:

$$\widehat{p}_{gt} = \underline{P}_g u_{gt} + e_{gt} \quad \forall g, t \tag{A12}$$

$$w_{bt} \leq \mathrm{W}_{bt} \quad \forall b \in \mathcal{B}^{\mathrm{W}}, t \tag{A13}$$

Finally, non-negative constraints for decision variables:

$$p_{gt} \geq 0 \quad \forall g, t \tag{A14}$$

$$w_{bt} \geq 0 \quad \forall b \in \mathcal{B}^{\mathrm{W}}, t \tag{A15}$$

It is important to highlight that the set of constraints (A2)-(A4) together with (A8)-(A10) and (A12) is the tightest possible representation (convex hull) for an unit operation [21].

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