Tight and Compact MIP Formulation of Configuration-Based Combined-Cycle Units

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***Abstract*—Private investors, flexibility, efficiency and environmental requirements from deregulated markets have led the existence and building of a significant number of combined-cycle gas turbines (CCGTs) in many power systems. These plants represent a complicated optimization problem for the short-term planning unit commitment (UC) carried out by independent system operators due to their multiple operating configurations. Accordingly, this paper proposes a mixed-integer linear programming (MIP) formulation of the configuration-based model of CCGTs, which is commonly utilized for bid/offering market processes. This formulation is simultaneously tighter and more compact than analogous MIP-based models, hence it presents a lower computational burden. The computational efficiency of the proposed formulation is supported by solving network-constrained UC case studies, of different size and complexity, using three of the leading commercial MIP solvers: CPLEX, GUROBI and XPRESS.**

***Index Terms*—Mixed-integer linear programming MIP, combined cycle unit, unit commitment, tight MIP formulation.**

NOMENCLATURE

Uppercase letters are used for denoting parameters and sets. Lowercase letters denote variables and indexes.

A. Indexes and Sets

$g \in \mathcal{G}$	Generating units, running from 1 to G.
$x, y \in \mathcal{M}_g$	Modes, running from 0 to M_g.
$x', y' \in \mathcal{M}_g$	All modes in $\mathcal{M}_g$ different than $x = 0$.
$\mathcal{M}_g^{\mathrm{F},x}$	Feasible transitions between modes x and y, where $x \neq y$.
$t \in \mathcal{T}$	Hourly periods, running from 1 to T hours.

B. Parameters

$C_g^{\mathrm{LV},x}$	Linear variable cost of each mode [\$/MWh].
$C_g^{\mathrm{NL},x}$	No-load cost of each mode [\$/h].
$C_g^{\mathrm{T},xy}$	Transition cost between modes [\$].
$\overline{P}_g^x$	Maximum power output of each mode [MW].
$\underline{P}_g^x$	Minimum power output of each mode [MW].
RD_g^x	Ramp-down rate of each mode [MW/h].
RU_g^x	Ramp-up rate of each mode [MW/h].
RD_g^{xy}	Ramp-down rate between two modes [MW/h].
RU_g^{xy}	Ramp-up rate between two modes [MW/h].
TD_g^x	Minimum downtime of each mode [h].
TU_g^x	Minimum uptime of each mode [h].

C. Variables

p_{gt}^x	Power output of each mode above its minimum output [MWh].
r_{gt}^x	Spinning reserve provided by each mode [MW].
u_{gt}^x	Commitment status of each mode. Binary variable which is equal to 1 if mode x is on and 0 otherwise.
v_{gt}^{xy}	Transition between modes. Binary variable which is equal to 1 if there is a transition from mode x to y and 0 otherwise

Acronyms

TC	Proposed tight and compact formulation.
Ref	Reference formulation (see Appendix).
MX	Mixed formulation between TC and Ref.

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I. INTRODUCTION

A. Motivation

THE number of installed combined-cycles gas turbines (CCGTs) has been increasing worldwide due to the movement toward deregulation and competition, the entering of private investors to electricity markets, low cost of capital investment, operation flexibility, fast response, high efficiency and environmental concerns [1], [2]. The quantity of CCGTs and their multiple operating configurations considerably increase the computational burden of the unit commitment (UC) problem executed by independent system operators (ISOs). The UC is a large-scale, mixed-integer and non-convex problem [1], [3], [4] whose size is continuously increasing due to policy changes (e.g. emission control [5]), penetration of renewable sources (e.g. wind generation [6]) and fuels evolution (e.g. natural gas integration [7]). It is then imperative to develop computationally efficient formulations for CCGTS to not add a significant burden to the already complex UC problems.

Mixed-integer programming (MIP) has become a very popular framework to deal with the complexity of UC and CCGT models, because 1) it has been recognized as the most computationally efficient framework to solve large-scale problems involving integer variables, and 2) it provides high flexibility in developing comprehensive models [3], [8].

CCGT models have been already proposed under the MIP framework in [8]–[11]. These references contributed practical formulations that allow ISOs to represent the operation of CCGTs in UCs. However, their main drawback is the creation of large and weak (not tight) models which greatly increase the computational burden of UC problems. Therefore, tighter and more compact MIP-based CCGT models are needed to create more computationally efficient UC formulations.

B. Tight and Compact MIP Formulations

The computational performance of an MIP formulation is mainly influenced by its tightness and compactness [12], [13]. The tightness of an MIP formulation refers to the distance between the integer solution and the associated linear programming (LP) relaxation, and it defines the search space that the solver requires to explore in order to compute the (optimal integer) solution. The compactness of an MIP formulation refers to its size and defines the searching speed that the solver takes to find the optimal solution.

Research on improving MIP formulations is usually focused on tightening rather than on compacting. An MIP model is typically tightened by adding a huge number of constraints, which increases the problem size [13]. Although this tightening strategy reduces the search space, solvers may demand more time to explore it during the branch-and-cut process because they are now required to repeatedly solve larger LPs. On the other hand, compact formulations commonly provide weak lower bounds. In short, creating tight and compact computationally efficient models is a non-trivial task. Obvious formulations are very weak (not tight) or very large, and trying to improve the tightness (compactness) normally means harming the compactness (tightness).

In the case of UC problems, there have been efforts affecting single sets of constraints for single-cycle units [14]–[20]. The work in [14] and [15] describes the convex hull of the minimum up/downtime constraints for the 1-binary and 3-binary format, respectively. Although both formulations are ideal in terms of tightness, the formulation in [15] is considerably more compact which results in a much lower computational burden. Refs. [16]–[18] propose tight linear approximations for quadratic generation costs. The work in [19] and [20] presents large (exponential) classes of facet-defining inequalities for ramping constraints. Although these inequalities further tighten the UC problem, they should not be added directly to the formulation, because the resulting model will take much more time to solve, i.e., the solver will be required to repetitively solve considerably larger LPs. To obtain a computational advantage, these inequalities must be appropriately introduced as cuts within a branch-and-cut algorithm [19], [20].

There have also been efforts in obtaining simultaneously tight and compact UC formulations for multiple sets of constraints, again only for single-cycle units [21]–[24]. The work in [23] and [24] provide the convex hull description for basic operating constraints of a single generating unit in power-based and the traditional energy-based UC problems, respectively. These convex hulls contain constraints for 1) generation limits, 2) minimum up/down times, 3) startup/shutdown capabilities, and 4) startup/shutdown power trajectories. Using these convex hulls as the core of UC models lead to simultaneously tight and compact MIP formulations. However, to the best of our knowledge, there are no works regarding tightening and/or compacting configuration-based CCGT formulations. All cited references aim at tightening and/or compacting constraints for single-cycle units.

C. Combined Cycle Modeling

Basically, a CCGT consists of combustion turbines (CTs), each with a heat recovery steam generator (HRSG) that drives steam turbines (STs). Steam and combustion turbines have an electrical generator capable of producing power. Mainly, three different representations of CCGTs in the UC problem can be identified in the literature to model the multiple operating configurations between CTs and STs [25]–[27]. The simplest representation is the *Aggregate modeling* in which the CCGT corresponds to an aggregated pseudo unit treated as a regular thermal generator, ignoring all different operating configurations. Due to its simplicity and low computational burden, this approximation is implemented by several ISOs such as ISO NE [28], NYISO [27], PJM [26], IESO [29] and XM[1] [30].

A second representation is the *Component (or Physical unit) modeling*. In this approach each of the physical components of the CCGT is modeled with its technical characteristics. This technique is more suitable for security analysis than for the scheduling of units [9], [26].

Finally, CCGTs are also represented by *Configuration-based (or Mode) modeling*. This approach represents the CCGT operational characteristics by using multiple and mutually exclusive configurations, modes or combinations. These modes 1) depend on the different combinations that can be composed by CTs, HRSG and STs; 2) have their own technical characteristics (e.g. minimum up/downtimes, ramps, etc.); and 3) follow predefined transition paths. ISOs, such as CAISO [31], ERCOT [26] and MISO [32], are migrating their models to this representation because it is recognized as more suitable for bid/offer processing and scheduling [26], [33]. However, considering individual mode representations and transition constraints result in more (binary) variables and constraints added to the UC problem. This size increase has been tackled by simplifying the problem, e.g., [11] and [34]. The former reference ignores some modes according with their input/output function costs and in the latter, an ISO directly limits the number of transitions allowed for each mode. Nevertheless, since one of the first MIP configuration-based model was proposed by [8], no efforts have been found in the literature trying to improve the quality of the MIP formulation and hence its computational performance.

D. Contributions and Paper Organization

This paper presents an alternative MIP reformulation of the configuration-based modeling of CCGTs in the UC problem.

[1]XM, Compañía de Expertos en Mercados. Colombian system operator.

Becuase this formulation describes the same integer problem as in [6], [8]–[10], [35], it obtains the same optimal results.

The main contributions of this paper are:

1) A tight MIP-based formulation for the configuration-based modeling of CCGTs in the UC problem. This tightness is mainly achieved by reformulating the within- and between-modes ramping constraints, and by adapting the single-cycle units constraints of [15] to a multiple-unit configuration in such a way that the resulting minimum up/down constraints implicitly model the mode-transition costs and the feasible transition between modes. As a result, the proposed model reduces the computational burden of analogous CCGT formulations.
2) The proposed formulation is simultaneously tighter and more compact than analogous models in the literature. This overcomes the main drawback of common tightening strategies which create larger models as a direct consequence of adding a huge number of inequalities. Then, the proposed formulation boosts MIP-solving convergence, because it reduces the search space explored by the solver (due to the tightness) and simultaneously increases the solver's searching speed (due to the compactness).
3) This tight and compact MIP formulation for CCGTs can help ISOs to solve their comprehensive UC problems more efficiently.

The reminder of this paper is organized as follows. Section II presents the mathematical formulation of the configuration-based model of CCGTs. This section also conceptually evaluates the tightness and compactness characteristics of the proposed model. Section III provides illustrative case studies that validate the performance of the formulation. Finally, Section IV draws the main conclusions and suggests future works.

II. Configuration-Based Modeling of CCGTs

A. Mathematical model

The configuration-based modeling of CCGTs represents all feasible combinations between CTs and STs by modes or configurations. Transitions between modes follow a predefined state diagram, as shown in Fig. 1. The "Mode 0" is introduced to represent the state when the CCGT unit is offline. For the sake of brevity, this section only addresses the technical constraints for the configuration-based modeling of CCGTs. However, including these equations in a complete UC formulation is straightforward. The reader is referred to [21] and [36] for more complete UC formulations.

1) Objective Function: The aim of short-term scheduling problems is to minimize the total operating costs. For a configuration-based model of CCGT, these costs are defined as (i) production cost and (ii) mode transition costs

$$\min \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}}\sum_{x'\in\mathcal{M}_g}\Bigg[\underbrace{C_g^{\mathrm{NL},x'}u_{gt}^{x'}+C_g^{\mathrm{LV},x'}\Big(\underline{P}_g^{x'}u_{gt}^{x'}+p_{gt}^{x'}\Big)}_{i} + \sum_{y\in\mathcal{M}_g^{\mathrm{F},x'}}\underbrace{C_g^{\mathrm{T},x'y}v_{gt}^{x'y}}_{ii}\Bigg]. \quad (1)$$

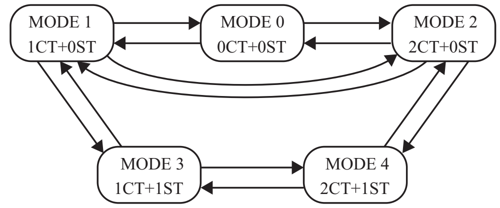
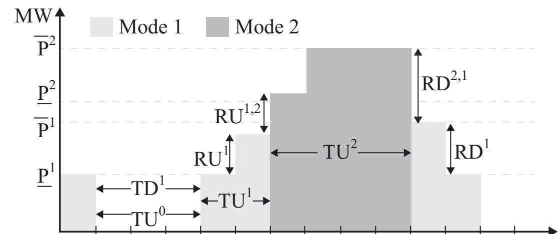


Figure 1: State transition diagram for a CCGT with 2 CTs and 1 ST in two consecutive periods.



u^0	0	1	1	1	0	0	0	0	0	0	0	0	1
u^1	1	0	0	0	1	1	0	0	0	0	1	1	0
$v^{0,1}$	0	0	0	0	1	0	0	0	0	0	0	0	0
$v^{1,0}$	0	1	0	0	0	0	0	0	0	0	0	0	1

Figure 2: Behavior of binary variables.

This paper focuses on CCGTs technical constraints, hence, for the sake of simplicity, linear production costs $C_g^{\mathrm{LV},x}$ are employed.

2) Transition, State Coupling and Minimum up/down Constraints: Fig. 2 depicts the behavior of the commitment u_{gt}^x and transition v_{gt}^{yx} binary variables included in the formulation. Notice that when any mode $x \neq 0$ is on, then $u^x = 1$, $u^0 = 0$, and the constraints ruled by the parameters TU^x and TD^0 are active. Similarly, when all modes $x \neq 0$ are off then $u^x = 0$, $u^0 = 1$, and the constraints ruled by the parameters TD^x TU^0 are active. Additionally, when there is a transition from mode x to y, $v^{xy} = 1$ and 0 otherwise.

Initially, the following constraint guarantees that CCGT modes are mutually exclusive, as also modeled in [8], [9]

$$\sum_{x\in\mathcal{M}_g} u_{gt}^x = 1 \quad \forall g,t. \quad (2)$$

Then, we define v_{gt}^{yx} as a binary variable representing transition between modes. Given that this variable can be read as the startup of mode x and the shutdown of mode y, we are able to adapt the constraints used to schedule single-cycle units proposed in [15] to represents modes and transitions of a CCGT unit. As a result, transitions between modes are ruled by

$$u_{gt}^{x'} - u_{g,t-1}^{x'} = \sum_{y\in\mathcal{M}_g^{\mathrm{F},x'}} v_{gt}^{yx'} - \sum_{y\in\mathcal{M}_g^{\mathrm{F},x'}} v_{gt}^{x'y} \quad \forall g,x',t \quad (3)$$

and minimum up and downtime constraints for each mode are expressed as:

$$\sum_{i=t-TU_g^x+1}^{t} \sum_{y\in\mathcal{M}_g^{\mathrm{F},x}} v_{gi}^{yx} \le u_{gt}^x \quad \forall g,x,t\in\left[TU_g^x,T\right] \tag{4}$$

$$\sum_{i=t-TD_g^x+1}^{t} \sum_{x\in\mathcal{M}_g^{\mathrm{F},y}} v_{gi}^{xy} \le 1-u_{gt}^x \quad \forall g,x,t\in\left[TD_g^x,T\right]. \tag{5}$$

Transition variables are controlled by (2)-(5) guaranteeing $v_{gt}^{xy}=1$ when there is a transition from mode x to y and $v_{gt}^{xy}=0$ otherwise.

It is important to highlight that the minimum up/down constraints of a single-cycle unit can be adapted in different ways to take into account multiple modes of CCGTs (e.g., see (A.5)-(A.12) in the Appendix). However, by defining v_{gt}^{yx} as the variable representing transition between modes, we are able to use constraints (3)-(5) to control the minimum up/down limits as well as: 1) the mode-transition costs, which can be directly accounted in the objective function (1) as $C_g^{\mathrm{T},xy}v_{gt}^{xy}$; and 2) the feasible transitions between modes, which are ruled inside the startup/shutdown logic constraint (3). In short, (3)-(5) help to obtain a compact formulation, as further discussed in Section II-B2.

3) Generation Limits: The total mode production is modeled in two blocks: the minimum power output $\underline{P}_g^x$ that is generated just by being committed and the generation over that minimum p_{gt}^x. This allows to formulate tighter and more compact models, especially when modeling ramp-rate constraints [21], [22], [36]. The generation limit over the power output and spinning reserve is set as

$$p_{gt}^{x'}+r_{gt}^{x'} \le \left(\overline{P}_g^{x'}-\underline{P}_g^{x'}\right)u_{gt}^{x'} \qquad \forall g,x',t. \tag{6}$$

The total production and reserve of a unit can be easily obtained as the sum of all modes production $\sum_{x'\in\mathcal{M}_g}\left(\underline{P}_g^{x'}u_{gt}^{x'}+p_{gt}^{x'}\right)$ and all modes reserve $\sum_{x'\in\mathcal{M}_g} r_{gt}^{x'}$, respectively.

4) Ramping Constraints: The ramping constraints within a mode and between modes are enforced by:

$$\begin{aligned} p_{g,t-1}^{x'}-p_{gt}^{x'}-\sum_{y'\in\mathcal{M}_g^{\mathrm{F},x}} p_{gt}^{y'} &\le RD_g^{x'}u_{g,t-1}^{x'} \\ -\sum_{y\in\mathcal{M}_g^{\mathrm{F},x'}}&\left(RD_g^{x'}+\underline{P}_g^{x'}-\underline{P}_g^{y'}-RD_g^{x'y}\right)v_{gt}^{x'y} \quad \forall g,x',t \end{aligned} \tag{7}$$

$$\begin{aligned} \left(p_{gt}^{x'}+r_{gt}^{x'}\right)-p_{g,t-1}^{x'}-\sum_{y'\in\mathcal{M}_g^{\mathrm{F},x}} p_{g,t-1}^{y'} &\le RU_g^{x'}u_{gt}^{x'} \\ -\sum_{y\in\mathcal{M}_g^{\mathrm{F},x'}}&\left(RU_g^{x'}+\underline{P}_g^{x'}-\underline{P}_g^{y'}-RU_g^{yx'}\right)v_{gt}^{yx'} \quad \forall g,x',t. \end{aligned} \tag{8}$$

Fig. 2 illustrates the use of ramp-rate limits for transitions within and between modes that result from (7) and (8). These constraints work as follows:

Within-mode ramps: when a given mode x is on for two consecutive periods, $v_{gt}^{xy}=0$ for all x,y because of (3). Then, (7) becomes $p_{g,t-1}^x-p_{gt}^x\le RD_g^x$ and (8) becomes $\left(p_{gt}^x+r_{gt}^x\right)-p_{g,t-1}^x\le RU_g^x$, which are the ramp limit constraints within a mode.

Table I: Comparison of all possible vertices between *TC* and *Ref* for $T=3$

	TC		*Ref*	
TU,TD	Integer	Fractional	Integer	Fractional
1,1	17	0	58	5836
1,2	13	12	46	9077
2,1	11	0	40	16108
2,2	11	0	40	10490

Between-modes ramps: when there is a transition from mode x' to y', $v_{gt}^{x'y'}=1$ and $v_{gt}^{xy}=0$ for all remaining modes different than x',y'. Consequently, (7) becomes $\left(\underline{P}_g^{x'}+p_{g,t-1}^{x'}\right)-\left(\underline{P}_g^{y'}+p_{gt}^{y'}\right)\le RD_g^{x'y'}$ and (8) becomes $\left(\underline{P}_g^{y'}+p_{gt}^{y'}+r_{gt}^{y'}\right)-\left(\underline{P}_g^{x'}+p_{g,t-1}^{x'}\right)\le RU_g^{x'y'}$, which are the ramp limit constraints between modes.

B. Tightness and Compactness Evaluation

Through this document, the tight and compact formulation presented in this paper for the configuration-based modeling of CCGTs, labeled as *TC*, is compared with the formulation of CCGTs presented in [8], [9], labeled as *Ref* and presented in (A.1)-(A.18) in the Appendix. This model is chosen as reference because 1) the authors in [8], [9] claim that it was built following the guidelines proposed in [37] and [12] to obtain a good MIP formulation; 2) it has been widely used in the literature; and 3) other CCGT formulations are actually small modifications of it, e.g. [6], [10], [35]. Bear in mind that the constraints of *TC* and *Ref* are exactly equivalent in an MIP sense but different in an LP sense.

The tightness and compactness of *TC* are conceptually compared with *Ref*, in the following subsections.

1) Tightness: Ideally, an MIP problem can be formulated so that the feasible region of the corresponding LP relaxation becomes the convex hull of the MIP problem. This is the smallest convex region containing all feasible integer points of the MIP problem. When a convex hull is formulated for an MIP problem, we could solve it as an LP since each vertex is a point satisfying integrality constraints [37]. Unfortunately, in many practical problems there is an enormous number of inequalities needed to describe the convex hull and the effort required to obtain it outweighs the computation demanded to solve the original MIP formulation [12], [37]. However, for the CCGT optimization case, it is possible to tighten the feasible region of the LP problem close to the convex hull.

In order to compare the tightness between *TC* and *Ref*, we first describe their feasible regions in terms of number of vertices and then we discuss theoretically the main reasons that make *TC* tighter. With the help of PORTA [38], we compute all vertices of the polytope described by constraints that only involve integer variables. Accordingly, the *TC* polytope is defined by (2)-(5) together with $c_{gt}^{\mathrm{T}}=\sum_{x'}\sum_{y\in\mathcal{M}_g^{\mathrm{F},x}} C_g^{\mathrm{T},xy}v_{gt}^{xy}$, which counts transition costs between modes; and the *Ref* polytope is defined by (A.2)-(A.12). Vertices are computed for a CCGT unit containing 1 CT and 1 ST for different time spans (up to $T=8$) and different combinations of TU and TD. For the sake of simplicity, we assume that $TU=TU^x$ and $TD=TD^x$ for all modes x.

Table I shows the number of integer and fractional vertices for the case of $T=3$. The complete set of experiments can

Table II: Comparison of number of constraints per CCGT unit

	Costs	States+Transitions +startup/shutdown	Up/down times	Gen. Limits	Ramp rates	Total
TC	0	$T\cdot M$	$M\,(2T-TU-TD+2)$	$T\,(M-1)$	$2T\,(M-1)$	$T\,(7M-4)+M\,(2-TU-TD)$
Ref	$T\cdot M^{\mathrm{F}}$	$T\,(3M+1)$	$M\,(2T-TU-TD+2)$	$2T\cdot M$	$2T\left(M^{\mathrm{F}}+M-1\right)$	$T\left(3M^{\mathrm{F}}+9M-1\right)+M\,(2-TU-TD)$
Ref-TC†	$T\cdot M^{\mathrm{F}}$	$T\,(2M+1)$	0	$T\,(M+1)$	$2T\cdot M^{\mathrm{F}}$	$T\left(3M^{\mathrm{F}}+3M+2\right)$

† difference in number of constraints between *Ref* and *TC*

be found in www.iit.upcomillas.es/aramos/PORTA-CCGT.rar. When $TU\geq TD$, the polytope described by *TC* is the tightest possible because it only contains integer vertices, contrary to *Ref*. For the case where $TU<TD$, *TC* presents considerably fewer number of fractional vertices than *Ref*. Note that *Ref* always presents more integer vertices, mainly because *Ref* uses extra inequalities to define transition cost variables c^{T}_{gt} (see (A.2)), and this enlarges the feasible region, i.e., solutions where $c^{\mathrm{T}}_{gt}\neq 0$ are always feasible. Contrariwise, the feasible region of TC is tighter because it forces $c^{\mathrm{T}}_{gt}=0$ if there is not a transition between modes ($v^{xy}_{gt}=0$).

It is important to highlight that although (3)-(5) are adapted from the convex hull of the minimum up/down of a single-cycle unit [15], they are not a convex hull for multi-mode units (fractional vertices appear for the cases where $TD>TU$, see Table I). This is because any change made to a convex hull may not lead to a convex hull, and even combining two convex hulls does not necessarily lead to a convex hull [24].

Now, we detail two other reasons supporting why *TC* is tighter than *Ref*. First, *TC* models (2)-(5) in such a way that all transition variables v^{xy}_{gt} are forced to take binary values when variables u^{x}_{gt} are defined as binary, even if v^{xy}_{gt} are declared as continuous. As similarly shown in [15], [21] for the case of single-cycle units, the case for multi-mode CCGT units is explained as follows:

1) When mode i is off for two consecutive periods: $u^{i}_{gt}, u^{i}_{g,t-1}=0$, then (4) forces $v^{yi}_{gt}=0$ and (3) ensures that $v^{iy}_{gt}=0$ for all y.
2) When mode i is on for two consecutive periods: $u^{i}_{gt}, u^{i}_{g,t-1}=1$, then (5) forces $v^{iy}_{gt}=0$ and (3) ensures that $v^{yi}_{gt}=0$ for all y. Moreover, (2) imposes $u^{y}_{gt}, u^{y}_{g,t-1}=0$ for all $y\neq i$ and from case 1) $v^{xy}_{gt}=0$ for all remaining possible transitions.
3) When there is a transition from mode i to j: $u^{i}_{g,t-1}, u^{j}_{gt}=1$ and (2) ensures $u^{x}_{g,t-1}, u^{y}_{gt}=0$ for all $x\neq i, y\neq j$. From case 1) $v^{xy}_{gt}=0$ for all $x\neq i,j$ and $y\neq i,j$. As a result, the only transition variables that can be 1 are v^{ij}_{gt} and v^{ji}_{gt}. Finally, (3) guarantees $v^{ji}_{gt}=0$ and $v^{ij}_{gt}=1$.

Although v^{xy}_{gt} can be declared as continuous, it is recommended to define them as binary. This strategy does not increase the complexity of the MIP solving process, and this actually allows the solver to look for opportunities that can exploit their integrality characteristic, as discussed in [19], [21].

The second reason that makes *TC* tighter than *Ref* is that constraints (2)-(8) are free of *big-M* parameters. These parameters are used by [8], [9] to relax transition-costs (A.2), state coupling (A.4) and ramp-rate constraints (A.15)-(A.18). *Big-M* inequalities considerably harm the tightness of MIP formulations, so they must be avoided when possible [12].

2) Compactness: Although the number of constraints is considered to be the best simple predictor of the LP models' difficulty [12], the number of nonzeros also has a significant impact on solving times, see [21] and references therein. This section conceptually compares the number of constraints and variables between *TC* and *Ref* in function of the number of modes M, feasible transitions M^{F} and periods T. The number of nonzeros is numerically evaluated in Section III-B1.

Table II shows the number of constraints per CCGT unit, excluding variable bounds. Notice that, in total, *Ref* formulates polynomially more constraints than *TC*. Major savings are associated with calculating transition costs, with ensuring the feasible transition between modes and with modeling ramp-rate constraints. Initially, the reduction in transition costs (column costs) results from the definition of binary variables v^{xy}_{gt} representing feasible mode transitions. Consequently, all costs in *TC* are directly computed in the objective function, while *Ref* formulates them through $T\cdot M^{\mathrm{F}}$ (*big-M*) constraints per CCGT (see (A.2)). Also, the mode-transition variables v^{xy}_{gt} allow *TC* to model the feasible transition between modes inside the startup/shutdown logic equation (3), thus avoiding $T\cdot M$ (*big-M*) constraints (A.4) apart from the traditional startup/shutdown logic constraints (A.5) (see States+Transitions+startup/shutdown column). In short, modeling the minimum up/down constraints using variables v^{xy}_{gt}, instead of the traditional startup/shutdown variables per mode, allows *TC* to formulate costs, transitions and startup/shutdown constraints using $T(2M+M^{\mathrm{F}}+1)$ fewer equations than *Ref*, where $T\left(M+M^{\mathrm{F}}\right)$of those are *big-M* constraints.

The size reduction in ramp-rate constraints is because the set of the four inequalities in *Ref* (A.15)-(A.18), resulting in $2T\left(M^{\mathrm{F}}+M-1\right)$ *big-M* constraints, is compacted into a set of two inequalities in *TC* (7)-(8), thus saving $T\cdot M^{\mathrm{F}}$ constraints.

It is relevant to highlight that *TC* only defines (4) and (5) for all modes x, including Mode 0 ($x=0$). All the remaining constraints are set for x' (for all $x\neq 0$), i.e., those constraints are not imposed when the CCGT unit is offline. This enables *TC* to reduce the number of constraints required to ensure generation limits, mode transitions and ramp rates.

Finally, Table III compares the number of variables per CCGTs between *TC* and *Ref*. *TC* requires $3T$ fewer continuous variables than *Ref*, mainly because *TC* computes transition costs directly in the objective function by using v^{xy}_{gt}, and *TC* defines production and reserve variables only for modes different than Mode 0. However, *TC* commonly formulates more binary variables than *Ref*. This is due to the definition of transition variables v^{xy}_{gt}. Bear in mind, however, that v^{xy}_{gt} take binary values even if declared as continuous, as detailed in Section II-B1.

Table III: Comparison of number of variables.

	Continuous	Integer	Total
TC	$2T(M-1)$	$T\left(M+M^{\mathrm{F}}\right)$	$T(3M+M^{\mathrm{F}}-2)$
Ref	$T(2M+1)$	$3TM$	$T(5M+1)$
Ref-TC†	$3T$	$T(2M-M^{\mathrm{F}})$	$T(2M-M^{\mathrm{F}}+3)$

† difference in number of variables between *Ref* and *TC*

Table IV: Types of CCGTs

	Type1	Type2	Type3
Turbines	2CT+1ST	2CT+2ST	4CT+2ST
# modes	5	6	11
# feasible transitions	12	16	42

Table V: Set of experiments

	1 Day (24 h)	2 Days (48 h)	4 Days (96 h)
4 Units		4 of Type1	
8 Units		8 of Type1	
10 Units		8 of Type1 + 2 of Type2	
12 Units		8 of Type1 + 2 of Type2 + 2 of Type3	
16 Units		8 of Type1 + 2 of Type2 + 6 of Type3	

III. Numerical Results

This section is divided into two parts. The first part describes the case studies and formulations that were implemented to illustrate the performance of the model presented in Section II. The second part compares optimization models in terms of problem size and computational performance in order to evaluate the tightness and compactness properties of the formulations.

A. Formulations and Case Studies

1) Formulations: Three configuration-based formulations for CCGTs are implemented:

1) *TC*: The tight and compact formulation (1)-(8) presented in this paper.
2) *Ref:* The formulation (A.1)-(A.18) presented in the Appendix, which is chosen as reference because of the reasons exposed in Section II-B1.
3) *MX*: A formulation that is a mix between *TC* and *Ref*. *MX* is composed by (1) and (3)-(5) from *TC*, and all remaining constraints from *Ref,* i.e., (A.3) and (A.13)-(A.18). This model is implemented in order to illustrate the effect of adopting only the proposed constraints that relate binary variables (3)-(5). Bear in mind that these constraints not only replace the minimum up/down constraints of *Ref* (A.5)-(A.12), but also replace the mode-transition costs (A.2) and the feasible transitions between modes (A.4), as discussed in Section II-A2.

It is relevant to highlight that all models describe the same MIP problem, so they provide the same optimal results e.g., operating costs and commitments. The difference between these models is how constraints are formulated.

2) Case Studies: The IEEE 118-bus system [9] is implemented to evaluate the performance of the configuration-based modeling of CCGTs in a network-constrained UC. The system is composed of 118 buses, 186 transmission lines, 91 demand sides and 54 thermal units. All 54 thermal units, in *TC*, *Ref* and *MX*, are modeled using the tight and compact UC formulation presented in [21], and using shift factors for the DC power flow [4]. Three different types of CCGTs are included into the system, as indicated in Table IV. The number of turbines determines the quantity of modes and feasible transitions of each unit. In addition, we create different case studies by adding from 4 to 16 CCGTs to the power system, as shown in Table V. Furthermore, five different CCGT combinations are carried out for three planing horizons of 1, 2 and 4 days with hourly time steps. This results in a total of 15 different case studies. All system data are available online at www.iit.upcomillas.es/aramos/IEEE118_CCGT.xls.

In order to illustrate the performance of the formulations, all experiments are solved using three of the leading commercial MIP solvers, CPLEX 12.6.0 [39], GUROBI 5.6 [40] and XPRESS 25.01.05 [41]. All instances were solved for three different optimality tolerances (OptTol), 5E-3, 1E-3 and 1E-4. The problems are solved until they hit the time limit of 5 hours or until they reach the given OptTol. Solvers' defaults were used for all the experiments, which were run on an Intel-i7 2.4-GHz personal computer with 16 GB of RAM memory.

In short, the set of 15 case studies is executed nine times because of the three different solvers and three OptTols. This results in a total of 135 experiments.

B. Comparing Different Formulations

1) Problem Size: Table VI shows the problem size increment per CCGT unit for models *Ref* and *MX* with respect to *TC* for the case of 1-day and all combinations of CCGTs. The problem size for 2- and 4-day cases is approximately 2 and 4 times as big as the 1-day case, respectively. For clarity, Table VI only shows the increment of constraints, nonzero elements, integer and continuous variables that different CCGT models add to the network-constrained UC formulation. That is, a hypothetical case of 0 CCGTs added to the network-constrained UC would imply a row of zeros in Table VI.

Although, on average, *TC* formulated 20% more binary variables than *Ref* for all time spans and all combinations of units, it reduced the number of constraints and nonzero elements by more than 3 and 1.6 times, respectively. By comparing *MX* with *TC*, we can observe that the proposed ramp rate model (7)-(8) contributed with more than half of the reduction in the number of constraints and around 40% in the number of nonzero elements. As discussed in Section II-B2, the number of constraints and nonzero elements evaluates the compactness of a formulation. Consequently, *TC* is considerably more compact than *Ref*.

2) Computational Performance: The goal of this section is not to compare the performance of the solvers, but rather to observe the general performance of the formulations despite the solver used. To this end, Figs. 3 and 4 depict solving times upwardly organized for each formulation, for all experiments and for the three solvers. Horizontal axes of Figs. 3 and 4 show the 45 experiments for each optimality tolerance and each planing horizon, respectively. For example, for a given OptTol in Fig. 3, there are 5 different combinations of CCGTs for each of the 3 different planing horizons solved using 3 solvers (CPLEX, GUROBI and XPRESS).

Table VI: Increment in p.u. with respect to TC of problem size for different formulations - 1 Day case

Case	# of constraints			# of integer variables			# of continuous variables			# of nonzero elements		
(# units)	TC	*Ref*	MX	TC	*Ref*	*MX*	*TC*	*Ref*	*MX*	TC	*Ref*	MX
4 U	2665	3.14	2.04	1632	0.88	1.00	864	1.44	1.11	18609	1.64	1.44
8 U	5289	3.16	2.05	3264	0.88	1.00	1728	1.44	1.11	38001	1.62	1.44
10 U	6873	3.20	2.08	4318	0.87	1.00	2258	1.43	1.11	50985	1.60	1.43
12 U	9877	3.38	2.21	6864	0.78	1.00	3264	1.35	1.09	81369	1.56	1.42
16 U	15881	3.53	2.33	11949	0.71	1.00	5283	1.29	1.07	142473	1.53	1.41

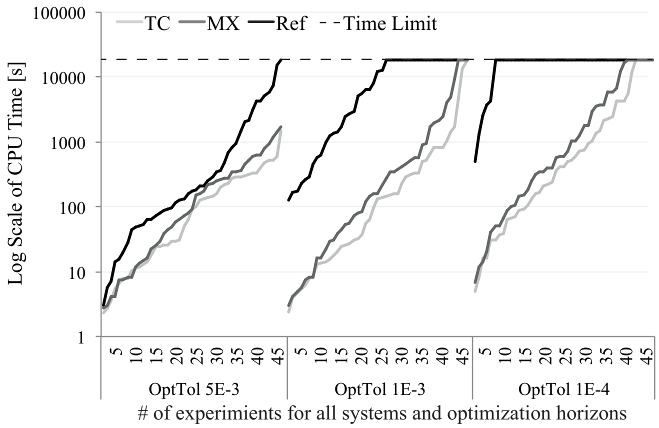


Figure 3: Solving times for all formulations grouped by OptTol

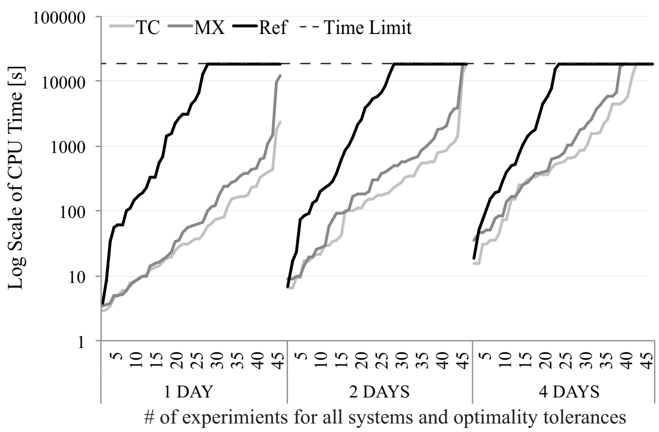


Figure 4: Solving times for all formulations grouped by Days

Clearly *TC* and *MX* solved much faster than *Ref* despite the solver used. This difference was more relevant for stricter OptTols as shown in Fig. 3. For an OptTol of 1E-4 the fastest *Ref* was 100 times slower than the fastest *TC*. Due to the time limit of 5 hours, for an OptTol of 1E-3, *TC* solved 98% of the experiments while *Ref* solved 58%; and for an OptTol of 1E-4, *TC* solved the problem in 90% of the experiments while *Ref* only managed 11%. A different point of view is given in Fig. 4 where results are grouped according to the planning horizon. This representation aims to depict how solving times are affected by the problem size. Again, *TC* was demonstrated to be the fastest formulation.

Although Figs. 3 and 4 provide a general overview of the performance of each formulation despite the solver used, we complement these results using solving-times ratios through box-and-whisker plots, see Fig. 5. These ratios are obtained using *TC* as reference, for example, when comparing solving times between *Ref* and *TC*, for each instance, two solving times are obtained, one for *Ref* and one for *TC*. Ratios are then computed by dividing the solving time of *Ref* by the corresponding solving time of *TC*. Each box in Fig. 5 contains the instances that were solved without reaching the time limit; that is, those instances that hit the time limit were excluded from these graphs. As a consequence, the quantity of results available to construct the boxes decreases for lower optimality tolerances, where *Ref* was not able to solve most of the instances (see Fig. 3).

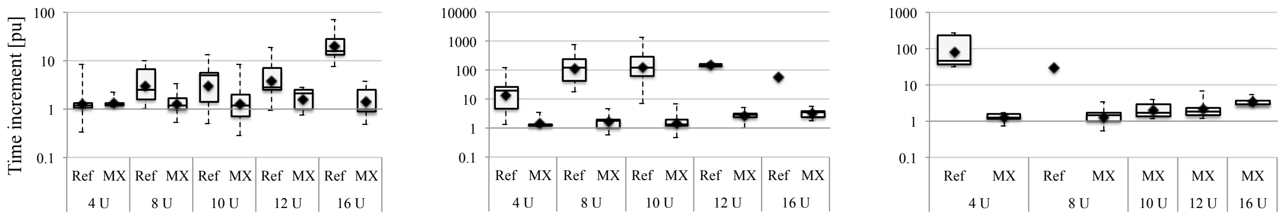
In addition, the geometric mean for each case is marked by a black rhombus in Fig. 5 to illustrate the general performance of *Ref* and *MX* with respect to *TC*. In general, *TC* was the fastest formulation because on average it was up to 3.4 and 148 times faster than *MX* and *Ref*, respectively.

Table VII presents a summary of the computational performance of different formulations for 1 day with an OptTol of 1E-3 under CPLEX. Similar results were obtained for the other optimality gaps and solvers, hence similar conclusions can be drawn. It can be observed that *TC* could find the optimal solutions faster than *MX* and *Ref*. For cases with 12 and 16 CCGT units *Ref* did not reach the required OptTol within the 5 hours time limit, while *TC* achieved it in 11.8 and 124.9 seconds, respectively.

The tightness of the formulations can be measured by the integrality gap. This parameter is defined as $(Z_{MIP} - Z_{LP})/Z_{MIP}$, where Z_{LP} is the optimal value of the (initial) relaxed LP problem and Z_{MIP} is the (best) integer solution found when the MIP problem is solved. The integrality gaps obtained for *TC* were about one third of those for *Ref*, which indicate that *TC* is considerably tighter. For the cases shown in Table VII, *TC* and *MX* were able to solve the network-constrained UC problems exploring far fewer nodes than *Ref*, and even before starting the branch-and-bound process (0 Nodes).

In summary, it is relevant to remark that:

- In general, for all 135 experiments, *TC* was able to solve 96% of the instances while *MX* managed 93% and *Ref* succeeded in only 56% of the instances.
- Considering only the successful cases, *TC* was in average 69.8 times faster than *Ref* and 2.4 times faster than *MX*.
- The *MX* formulation was faster than *Ref* due to the tightness and compactness of the modeling of feasible transition between modes, mode-transition costs and minimum up/down times.
- Further improvements of *TC* over *MX* are due to the proposed ramp-rate constraints in the formulation.
- Gains in solving times increased notably for smaller



(a) OptTol 5E-3 (b) OptTol 1E-3 (c) OptTol 1E-4

Figure 5: Solving times with respect to *TC*

Table VII: Computational performance of different formulations. CPLEX - 1 Day - OptTol 1E-3

Case	CPU Time [s]			IntGap			OptTol ($\times 10^{-3}$)			Nodes ($\times 10^{3}$)			Iterations ($\times 10^{3}$)		
(# units)	*Ref*	*MX*	*TC*	*Ref*	*MX*	*TC*	*Ref*	*MX*	*TC*	*Ref*	*MX*	*TC*	*Ref*	*MX*	*TC*
4 U	291.6	2.5	2.4	0.4	0.2	0.1	1.00	0.67	0.92	8.0	0	0	464.3	8.2	10.2
8 U	2993.3	4.1	3.9	0.8	0.2	0.2	1.00	0.58	0.56	43.8	0	0	4306.9	9.0	10.7
10 U	6421.3	5.7	4.8	0.8	0.3	0.2	1.00	0.84	0.94	71.0	0	0	5554.7	10.7	11.7
12 U	18000	18.6	11.8	1.1	0.5	0.4	1.12	0.86	0.80	103.9	0.019	0.005	11202.2	20.9	20.4
16 U	18000	266.8	124.9	4.3	2.3	1.3	1.25	0.94	0.91	46.0	0.601	0.992	6782.6	117.8	105.3

optimality tolerances. This outcome clarifies the effect of tighter and more compact formulations in the branching process.

- *Ref* was faster than *TC* for three specific cases by using one specific solver, GUROBI. These cases resulted for the largest OptTol (5E-3) where the solvers' initial heuristics played an important role for convergence. One of these three cases, 4 days 4 CCGTs, was solved without exploring any node while the other two instances, 2 days 12 CCGTs and 4 days 10 CCGTs, required some branching.

IV. Conclusions and Future Work

This paper presented a tight and compact MIP formulation for the configuration-based model of CCGTs in the UC problem. This formulation was conceptually and numerically demonstrated to be tighter and more compact than analogous models available in the literature. Consequently, the computational burden of CCGTs in UC problems is reduced, optimal solutions can be obtained significantly faster. This computational efficiency was supported by solving network-constrained UC case studies, of different size and complexity, using three of the leading commercial MIP solvers, CPLEX, GUROBI and XPRESS. The proposed tight and compact MIP-based CCGT model can support ISOs to migrate from their aggregate models to configuration-based representations of CCGTs, taking the advantages of bid/offer processing and scheduling offered by the configuration-based representation of CCGTs.

Although the proposed formulation presents significant computational advantages other models available in the literature, it could be further improved by: 1) Obtaining the convex hull for the minimum up/down constraints for CCGTs, including the mode-transition costs and the feasible transitions between modes. Notice that although (3)-(5) are adapted from the convex hull of the minimum up/down of a single-cycle unit [15], they are not a convex hull for multi-mode units (fractional vertices appear for the cases where $TD>TU$, see Table I). 2) Obtaining facet-defining inequalities for the intra- and inter-mode ramping constraints. Although there are facet-defining inequalities for ramping constraints for single-cycle units available in the literature [19], [20], these inequalities cannot be directly used for the intra- and the inter-mode ramping constraints for CCGTs. New facet-defining inequalities should then be obtained taking into account multi-mode units. 3) Combining tight and compact UC formulations with decentralized optimization may also speed up the solving process, as shown in [42].

Appendix

This section details the CCGT model presented in [8], [9], which is used as the reference (*Ref*) model in this document. In is important to highlight that this *Ref* formulation describes the same integer problem as the formulation presented in Section II, where (1) is equivalent to (A.1); (2) to (A.3); (3)-(5) to (A.2) and (A.4)-(A.12); (6) to (A.13)-(A.14); and (7)-(8) to (A.15)-(A.18).

We use the same nomenclature as in Section II. Newer nomenclature is defined as it appears in the text.

1) Objective function:

$$\min \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}} \left[\sum_{x\in\mathcal{M}_g} \left(C_g^{\mathrm{NL},x} u_{gt}^x + C_g^{\mathrm{LV},x} \widehat{p}_{gt}^x \right) + c_{gt}^{\mathrm{T}} \right] \quad \text{(A.1)}$$

where c_{gt}^{T} is a continuous variable representing the mode transition cost [\$], which is obtained from

$$c_{gt}^{\mathrm{T}} \geq C_g^{\mathrm{T},xy} - M\left(2 - u_{g,t-1}^x - u_{gt}^y\right) \quad \forall g, x, y \in \mathcal{M}_g^{\mathrm{F}',x}, t \quad \text{(A.2)}$$

where the set $\mathcal{M}_g^{\mathrm{F}',x}$ contains the feasible transitions between modes x and y, including $x=y$. The *big-M* parameter is set to $C_g^{\mathrm{T},xy}$. As suggested in [9], [12], all the *big-M* parameters for *Ref* are set to the smallest possible value without imposing a spurious restriction, thus making the least damage to the tightness of the MIP formulation.

2) Transitions, state coupling and startup/shutdown constraints: All modes are defined as mutually exclusive

$$\sum_{x\in\mathcal{M}_g} u^x_{gt} = 1 \quad \forall g,t \tag{A.3}$$

and the transitions between modes are ruled by

$$1-\sum_{y\in\mathcal{M}_g^{\mathrm{F}',x}} u^y_{g,t+1} + \sum_{y\notin\mathcal{M}_g^{\mathrm{F}',x}} u^y_{g,t+1} = M\big(1-u^x_{gt}\big) \quad \forall g,x,t \tag{A.4}$$

where the *big-M* parameter is set to 2.

The startup and shutdown logic of a mode is given by

$$u^x_{gt} - u^x_{g,t-1} = v^x_{gt} - w^x_{gt} \quad \forall g,x,t \tag{A.5}$$

$$v^x_{gt} + w^x_{gt} \le 1 \quad \forall g,x,t. \tag{A.6}$$

where variables v^x_{gt} and w^x_{gt} stand for the startup and shutdown of mode x, respectively.

3) Minimum up/down Constraints:

$$\sum_{t=1}^{TU^x_{g0}} \big(1-u^x_{gt}\big) = 0 \quad \forall g,x \tag{A.7}$$

$$\sum_{i=t}^{t+TU^x_g-1} u^x_{gi} \ge TU^x_g v^x_{gt} \quad \forall g,x,t \in \big[TU^x_{g0}+1, T-TU^x_g+1\big] \tag{A.8}$$

$$\sum_{i=t}^{T} \big(u^x_{gi} - v^x_{gt}\big) \ge 0 \quad \forall g,x,t \in [T-TU_g+2, T] \tag{A.9}$$

$$\sum_{t=1}^{TD^x_{g0}} u^x_{gt} = 0 \quad \forall g,x \tag{A.10}$$

$$\sum_{i=t}^{t+TD^x_g-1} \big(1-u^x_{gi}\big) \ge TD^x_g w^x_{gt} \quad \forall g,x,t \in \big[TD^x_{g0}+1, T-TD^x_g+1\big] \tag{A.11}$$

$$\sum_{i=t}^{T} \big(1-u^x_{gi}-w^x_{gt}\big) \ge 0 \quad \forall g,x,t \in \big[T-TD^x_g+2, T\big] \tag{A.12}$$

where TU^x_{g0}, TD^x_{g0} are the number of hours a mode must be initially up or down, respectively.

4) Generation Limits:

$$\widehat{p}^x_{gt} \ge \underline{P}^x_g u^x_{gt} \qquad \forall g,x,t \tag{A.13}$$

$$\widehat{p}^x_{gt} + r^x_{gt} \le \overline{P}^x_g u^x_{gt} \qquad \forall g,x,t \tag{A.14}$$

where $\widehat{p}^x_{gt}$ is the total power output of each mode [MW].

The total production and reserve of a unit are computed as the sum of all modes production $\sum_{x\in\mathcal{M}_g}\widehat{p}^x_{gt}$ and all modes reserve $\sum_{x\in\mathcal{M}_g} r^x_{gt}$, respectively.

5) Ramping Constraints: The ramping constraints within the same mode are represented as

$$\left(\widehat{p}^{x'}_{gt} + r^{x'}_{gt}\right) - \widehat{p}^{x'}_{g,t-1} \le RU^{x'}_g + M\left(2-u^{x'}_{g,t-1} - u^{x'}_{gt}\right) \quad \forall g,x',t \tag{A.15}$$

$$\widehat{p}^{x'}_{g,t-1} - \widehat{p}^{x'}_{gt} \le RD^{x'}_g + M\left(2-u^{x'}_{g,t-1} - u^{x'}_{gt}\right) \quad \forall g,x',t \tag{A.16}$$

where the *big-M* parameter is set to $\overline{P}^x_g$.

The ramping constraints between modes are represented as

$$\left(\widehat{p}^{y}_{gt} + r^{y}_{gt}\right) - \widehat{p}^{x}_{g,t-1} \le RU^{xy}_g + M\left(2-u^{x}_{g,t-1} - u^{y}_{gt}\right) \quad \forall g,x,y\in\mathcal{M}_g^{\mathrm{F},x},t \tag{A.17}$$

$$\widehat{p}^{x}_{g,t-1} - \widehat{p}^{y}_{gt} \le RD^{xy}_g + M\left(2-u^{x}_{g,t-1} - u^{y}_{gt}\right) \quad \forall g,x,y\in\mathcal{M}_g^{\mathrm{F},x},t \tag{A.18}$$

where the *big-M* parameter is set to the maximum generation capacity of the CCGT unit $\overline{P}_g$.

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