Location and Allocation of Service Units on a Congested Network with Time Varying Demand Rates

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Abstract

The service system design problem arises in the design of telecommunication networks, refuse collection and disposal networks in public sector, transportation planning, and location of emergency medical facilities. The problem seeks to locate service facilities, determine their capacities and assign users to those facilities under time varying demand conditions. The objective is to minimize total costs that comprises the costs of accessing facilities by users and waiting for service at these facilities as well as the cost of setting up and operating the facilities. We consider a system where a central dispatcher assigns the user to the service facility. Under Poisson demand arrival rates and general service time distributions at the facilities, the problem is setup as a network of spatially distributed facilities, modelled as M/G/1 queues and formulated as a nonlinear integer programming model. Using simple transformation and piecewise linear approximation, we present a linear reformulation of the model with large number of constraints. We compute tight upper and lower bounds and use them in an iterative constraint generation algorithm based exact solution approach. Computational results indicate that the exact approach provides optimal solution in reasonable computational times.

Keywords: Service System Design; Time Varying Demand; Nonlinear Integer Programming, Congestion, Exact Method.

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1. Introduction

The service system design problem arises in the design of telecommunication networks, refuse collection and disposal networks in public sector, transportation networks, and location of emergency facilities.

Amiri (2001) present a model of multi-hour service system design problem, where facilities were modelled as M/M/1 queue.

The contribution of this paper is two-fold. Firstly, we present a generalized model of the service system design problem with time varying demand rates, where facilities are modelled as M/G/1 queues under Poisson demand arrival rates and general service time distributions at the facilities. Secondly, we present an exact solution approach based on constraint generation algorithm that provides optimal solution to problem.

The remainder of the paper is organized as follows. In Section 2, we describe the problem and present a nonlinear mixed integer programming (MIP) formulation of the problem. Section 3 describes the linearized model and the exact solution methodology. Illustrative example, managerial insights, and computational results are reported in Sections 5. Section 6 concludes with some directions for future research.

2. Problem Formulation

2.1. Notations

To model the problem, we define the following indices and parameters:

i	:	Index for user nodes, $i \in I$;
j	:	Index for potential facility sites, $j \in J$;
k	:	Index of potential capacity levels at the facilities, $k \in K$;
F_{jk}	:	Fixed setup cost of setting up and operating a facility at site j with capacity level k;
Q_{jk}	:	Capacity of level k for facility at site j;
C_{ij}^t	:	Cost of assigning user node i to facility j during busy-hour t;
A_i^t	:	demand arrival rate for user node i during busy-hour t;
$1/\mu$	:	average demand size;
a_i^t	:	demand requirement of user node i during busy-hour t ($a_i^t = A_i^t/\mu$);
D^t	:	Unit queueing delay cost during busy-hour t;
$cv_{s_{jk}}^2$	:	Squared coefficient of variation of service times at facility j with capacity level k;

The decision variables are defined as follows:

x_{ij}^t	:	1, if user node i is assigned to a facility at site j during busy-hour t, 0 otherwise;
y_{jk}	:	1, if facility at site j is opened with capacity level k, 0 otherwise.

2.2. Formulation

Let us assume that the demand from user node i during busy-hour t be an independent random variable that follows a Poisson process with mean a_i^t. The average size of demand is $1/\mu$. Once the demand is realized at the users' end, a central dispatcher directs the user to the facility. We assume that each facility operates as a single flexible-capacity server with an infinite buffers to accommodate user nodes request waiting for service. Users arriving at the facilities are served on a first-come first-serve (FCFS) basis. If x_{ij}^t is a decision variable that equals 1 if user node i is assigned to a facility at site j during busy-hour t, then the aggregate demand arrival rate at facility j is also a random variable that follows a Poisson process with mean $\Lambda_j^t = \sum_{i \in I} A_i^t x_{ij}^t = \sum_{i \in I} \mu a_i^t x_{ij}^t$ (due to the superposition of Poisson processes).

If the service times at each facility follows a general distribution, then each facility can be modelled as an M/G/1 queue, where the average service rate of facility j, if it is allocated capacity level k, is given by $\mu Q_j = \mu \sum_{k \in K} Q_{jk} y_{jk}$ and the variance in service times is $\sigma_j^2 = \sum_{k \in K} \sigma_{jk}^2 y_{jk}$. Thus, the service system is modelled as a network of independent M/G/1 queues in which the facilities are treated as servers with service rates proportional to their capacity levels. This service rate reflects the server capacity or essentially the number of users a facility can process in a given time period.

Under steady state conditions ($\Lambda_j^t < \mu Q_j$) and first-come first-serve (FCFS) queuing discipline, the *average sojourn time* (waiting time in queue + service time) at facility j is given by the Pollaczek-Khintchine (PK) formula: $w_j = \left(\frac{1+cv_j^2}{2}\right) \frac{\Lambda_j^t}{\mu Q_j(\mu Q_j - \Lambda_j^t)} + \frac{1}{\mu Q_j}$. The average queuing delay in the system during a busy-period can be estimated as the weighted sum of the expected delay at the facilities:

$$W^t = \frac{1}{\Lambda^t} \sum_{j \in J} \Lambda_j^t w_j = \frac{1}{\Lambda^t} \sum_{j \in J} \left\{ \left(\frac{1+cv_j^2}{2}\right) \frac{(\Lambda_j^t)^2}{\mu Q_j(\mu Q_j - \Lambda_j^t)} + \frac{\Lambda_j^t}{\mu Q_j} \right\} \tag{1}$$

where $\Lambda^t = \sum_{j \in J} \Lambda_j^t$ is the total demand arrival rate during time period t.

The expression for W^t can be written as:

$$W^t = \frac{1}{\Lambda^t} \sum_{j \in J} \left\{ \left(\frac{1 + \sum_{k \in K} cv_{s_{jk}}^2 y_{jk}}{2}\right) \frac{(\sum_{i \in I} a_i x_{ij}^t)^2}{\sum_{k \in K} Q_{jk} y_{jk} \left(\sum_{k \in K} Q_{jk} y_{jk} - \sum_{i \in I} a_i x_{ij}^t\right)} + \frac{\sum_{i \in I} a_i x_{ij}^t}{\sum_{k \in K} Q_{jk} y_{jk}} \right\}$$

For M/M/1 queueing systems, this expression reduces to:

$$W_{M/M/1}^t = \frac{1}{\Lambda^t} \sum_{j \in J} \frac{\sum_{i \in I} a_i x_{ij}^t}{\sum_{k \in K} Q_{jk} y_{jk} - \sum_{i \in I} a_i x_{ij}^t}$$

If D^t denotes the average queuing delay cost during busy-hour t, then the *total congestion cost* can be expressed as a product of D^t and total expected waiting time in the system, W^t.

Assuming that there is a fixed cost (amortized over the planning period) of setting up and operating a facility equipped with adequate service capacity and a variable cost of serving the demand of a user node, the system-wide total expected cost can be expressed as:

$$Z(\mathbf{x}, \mathbf{y}) = \sum_{j \in J} \sum_{k \in K} F_{jk} y_{jk} + \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} C_{ij}^t x_{ij}^t + \sum_{t \in T} D^t W^t(\mathbf{x}, \mathbf{y})$$

The model formulated below simultaneously determines the location and the capacity of the service facilities as well as the allocation of the user node to each facilities in order to minimize the sum of congestion cost, fixed location and capacity acquisition cost, the access costs of assigning users to facilities. Besides capacity restrictions (i.e. steady state conditions) at the facilities, and the demand requirements, there are constraints which ensure that at most one capacity level is selected at the facilities. The resulting nonlinear MIP formulation of the problem is as follows:

$$[P] : \min_{\mathbf{x}, \mathbf{y}} \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} C_{ij}^t x_{ij}^t + \sum_{t \in T} D^t W^t(\mathbf{x}, \mathbf{y}) + \sum_{j \in J} \sum_{k \in K} F_{jk} y_{jk} \tag{2}$$

$$\text{s.t.} \sum_{j \in J} x_{ij}^t = 1 \qquad \forall i \in I, t \in T \tag{3}$$

$$\sum_{i \in I} a_i^t x_{ij}^t \leq \sum_{k \in K} Q_{jk} y_{jk} \qquad \forall j \in J, t \in T \tag{4}$$

$$\sum_{k \in K} y_{jk} \leq 1 \qquad \forall j \in J \tag{5}$$

$$x_{ij} \in \{0, 1\}, \quad y_{jk} \in \{0, 1\} \qquad \forall i \in I, j \in J, k \in K \tag{6}$$

Constraints (3) ensure that the total user demand is met. Constraints (4) ensure that the total demand is less than the total capacity. Alternatively, these constraints ensure that the steady state condition at every facility ($\Lambda_j \leq Q_j$) is met. Constraints (5) state that at most one capacity level is selected at a facility. Constraints (6) are nonnegativity and binary restrictions on the variables.

The nonlinearity in the model $[P_N]$ arises due to the expression for the total expected queueing delay, W^t. In the next section, we linearize the expression and present an exact solution procedure based on the cutting plane algorithm to solve the linearized model.

3. Linear Reformulation and Solution Methodology

To linearize W_j^t, we rearrange the terms in (1) and rewrite it as follows:

$$W_j^t = \left(\frac{1+cv_j^2}{2}\right)\frac{(\Lambda_j^t)^2}{\mu Q_j(\mu Q_j - \Lambda_j^t)} + \frac{\Lambda_j^t}{\mu Q_j} \quad = \frac{1}{2}\left\{\left(1+cv_j^2\right)\frac{\Lambda_j^t}{\mu Q_j - \Lambda_j^t} + \left(1-cv_j^2\right)\frac{\Lambda_j^t}{\mu Q_j}\right\}$$

Let us define two sets of nonnegative auxiliary variables ρ_j^t and R_j^t, such that

$$\rho_j^t = \frac{\Lambda_j^t}{\mu Q_j} = \frac{\sum_{i\in I} a_i^t x_{ij}}{\sum_{k\in K} Q_{jk} y_{jk}}; \quad R_j^t = \frac{\Lambda_j^t}{\mu Q_j - \Lambda_j^t} = \frac{\sum_{i\in I} a_i^t x_{ij}}{\sum_{k\in K} Q_{jk} y_{jk} - \sum_{i\in I} a_i^t x_{ij}}; \text{ and } \quad \rho_j^t = \frac{R_j^t}{1+R_j^t}$$

This implies

$$\sum_{i\in I}\sum_{j\in J} a_i^t x_{ij} = \frac{R_j^t}{1+R_j^t}\sum_{k\in K} Q_{jk} y_{jk} = \rho_j^t \sum_{k\in K} Q_{jk} y_{jk} = \sum_{k\in K} Q_{jk} z_{jk}^t, \quad \text{where} \quad z_{jk}^t = \begin{cases} \rho_j^t, & \text{if } y_{jk} = 1 \\ 0, & \text{otherwise} \end{cases}$$

Because there exists at most one capacity level k' with $y_{jk'} = 1$ while $y_{jk} = 0$ for all other capacity levels $k \neq k'$, the expression $z_{jk}^t = \rho_j^t y_{jk}$ can be ensured by adding the following set of constraints: $z_{jk}^t \leq y_{jk}$ and $\sum_{k\in K} z_{jk}^t = \rho_j^t$.

Upon substituting the auxiliary variables, the expression for W_j^t reduces to:

$$\frac{1}{2}\left\{\left(1+\sum_{k\in K} cv_{jk}^2 y_{jk}\right) R_j^t + \left(1-\sum_{k\in K} cv_{jk}^2 y_{jk}\right)\rho_j^t\right\} = \frac{1}{2}\left\{R_j^t + \rho_j^t + \sum_{k\in K} cv_{jk}^2 (w_{jk}^t - z_{jk}^t)\right\}$$

$$\text{where } w_{jk}^t = \begin{cases} R_j^t, & \text{if } y_{jk} = 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad z_{jk}^t = \begin{cases} \rho_j^t, & \text{if } y_{jk} = 1 \\ 0, & \text{otherwise} \end{cases}$$

Because there exists at most one capacity level k' with $y_{jk'} = 1$ while $y_{jk} = 0$ for all other capacity levels $k \neq k'$, the expression $w_{jk}^t = R_j^t y_{jk}$ can be ensured by adding the following set of constraints: $w_{jk}^t \leq M y_{jk}^t$ and $\sum_{k\in K} w_{jk}^t = R_j^t$, where M is a Big-M.

Differentiating the function $f(R) = \frac{R}{1+R}$ w.r.t. R, we get the first derivative $\frac{\delta f}{\delta R} = \frac{1}{(1+R)^2} > 0$, and the second derivative $\frac{\delta^2 f}{\delta (R) 2} = \frac{-2}{(1+R)^3} < 0$. This implies that the function $f(R) = \frac{R}{1+R}$ is concave in $R_j \in [0, \infty)$. Let us define the domain H of the auxiliary variable R_j as a set of indices of points $\{R_j^h\}_{h\in H}$, at which the function $\rho_j(R_j) = R_j/(1+R_j)$ can be approximated arbitrary closely by a set of piecewise linear functions that are tangent to ρ_j. This implies that $\rho_j(R_j) = R_j/(1+R_j)$ can be expressed as the finite minimum of linearizations of ρ_j at a given set of point $\{R_j^h\}_{h\in H}$ as follows:

$$\frac{R_j^t}{1+R_j^t} = \min_{h\in H}\left\{\frac{1}{(1+R_j^{ht})^2}R_j^t + \frac{(R_j^{ht})^2}{(1+R_j^{ht})^2}\right\}$$

This is equivalent to the following set of constraints:

$$\frac{R_j^t}{1+R_j^t} \le \frac{1}{(1+R_j^{ht})^2}R_j^t + \frac{(R_j^{ht})^2}{(1+R_j^{ht})^2}, \qquad \forall j\in J, h\in H$$

The above set of constraints can be rewritten as:

$$\rho_j^t \le \frac{1}{(1+R_j^{ht})^2}R_j^t + \frac{(R_j^{ht})^2}{(1+R_j^{ht})^2}, \qquad \forall j\in J, h\in H \tag{7}$$

$$\text{or } (1+R_j^{ht})^2\rho_j^t - R_j^t \le (R_j^{ht})^2 \qquad \forall j\in J, h\in H \tag{8}$$

provided $\exists\ h\in H$ such that (8) holds with equality.

The resulting linear MIP formulation is:

$$[L(H)] : \min_{\mathbf{x},\mathbf{y}} \sum_{j\in J}\sum_{k\in K} F_{jk}y_{jk} + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} C_{ij}^t x_{ij}^t + \frac{1}{2}\sum_{j\in J}\sum_{t\in T} D^t\left\{(R_j^t + \rho_j^t + \sum_{k\in K} cv_{s_{jk}}^2(w_{jk}^t - z_{jk}^t)\right\} \tag{9}$$

s.t. (3) − (5)

$$\sum_{i\in I} a_i^t x_{ij}^t - \sum_{k\in K} Q_{jk}z_{jk}^t = 0 \qquad \forall j,t \tag{10}$$

$$z_{jk}^t - y_{jk} \le 0 \qquad \forall j\in J, t\in T, k\in K \tag{11}$$

$$\sum_{k\in K} z_{jk}^t - \rho_j^t = 0 \qquad \forall j\in J, t\in T \tag{12}$$

$$(1+R_j^{ht})^2\rho_j^t - R_j^t \le (R_j^{ht})^2 \qquad \forall j\in J, t\in T, h\in H \tag{13}$$

$$w_{jk}^t - My_{jk} \le 0 \qquad \forall j\in J, t\in T, k\in K \tag{14}$$

$$\sum_{k\in K} w_{jk}^t - R_j^t = 0 \qquad \forall j\in J, t\in T \tag{15}$$

$$x_{ij}\in\{0,1\}, \quad y_{jk}\in\{0,1\} \qquad \forall i\in I, j\in J, k\in K \tag{16}$$

$$w_{jk}^t, z_{jk}^t, R_j^t, \rho_j^t \ge 0 \qquad \forall j\in J, t\in T, k\in K \tag{17}$$

3.1. Lower and Upper Bounds

The proposed exact solution approach relies on obtaining good lower and upper bounds for the linear model $[L(H)]$. The algorithm makes successive improvements to the lower bound and the corresponding upper bound as the iterations progress. Below, we present lower and upper bounds that can be used in the proposed solution approach.

Lower bound: *For every given subset of points* $\{R_j^h\}_{h\in H^q\subset H}$*, the optimal objective function value of the problem* $[L(H^q)]$ *is a lower bound on the optimal objective of* $[L(H)]$ *or* $[P_N]$*.*

Suppose, at an iteration q, we use a subset of points $\{R_j^h\}_{h\in H^q\subset H}$, and solve the corresponding problem $[L(H^q)]$, which yields the solution $(\mathbf{x}^q, \mathbf{y}^q, \rho^q, \mathbf{w}^q, \mathbf{z}^q, \mathbf{R}^q)$ and objective function value denoted by $v(L(H^q))$. Since $[L(H^q)]$ is a relaxation of the full problem $[L(H)]$, a lower bound on the optimal objective of $[L(H)]$ or $[P_N]$ is provided by $v(L(H^q))$, where

$$LB^q = v(L(H^q)) = \sum_{j\in J}\sum_{k\in K} F_{jk}y_{jk}^q + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} C_{ij}^t x_{ij}^{tq} + \frac{1}{2}\sum_{j\in J}\sum_{t\in T} D^t \left\{(R_j^{tq} + \rho_j^{tq} + \sum_{k\in K} cv_{jk}^2(w_{jk}^{tq} - z_{jk}^{tq})\right\} \quad (18)$$

Upper bound: *For any subset of points* $(R_{ij}^h)_{H^q\subset H}$*, the objective function of* $[P]$ *computed using a part of the optimal solution* $(\mathbf{x}^q, \mathbf{y}^q)$ *of* $[L(H^q)]$ *provides an upper bound to* $[L(H)]$ *or* $[P]$*.*

Consider iteration q, where we use a subset of tangent points $(R_{ij}^h)_{H^q\subset H}$ and solve the corresponding relaxed problem $[L(H^q)]$. Because the optimal solution $(\mathbf{x}^q, \mathbf{y}^q, \mathbf{R}^q)$ of $[L(H^q)]$ is a feasible solution to $[P]$, it provides an upper bound on the optimal objective of $[P]$, given by

$$UB^q = T(\mathbf{x}^q, \mathbf{y}^q) = \sum_{j\in J}\sum_{k\in K} F_{jk}y_{jk}^q + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} C_{ij}^t x_{ij}^{tq} + \frac{D^t}{2\Lambda^t}\sum_{j\in J}\left\{\left(\frac{1+\sum_{k\in K} cv_{jk}^2 y_{jk}^q}{2}\right)\frac{(\sum_{i\in I} a_i x_{ij}^{tq})^2}{\sum_{k\in K} Q_{jk}y_{jk}^q\left(\sum_{k\in K} Q_{jk}y_{jk}^q - \sum_{i\in I} a_i x_{ij}^{tq}\right)} + \frac{\sum_{i\in} a_i x_{ij}^{tq}}{\sum_{k\in K} Q_{jk}y_{jk}^q}\right\} \quad (19)$$

3.2. Exact Solution Algorithm

The algorithm makes successive improvements to the lower and upper bounds as the iteration progresses. At every iteration, a relaxed version of the linear model $[L(H)]$ is solved to obtain an optimal solution, an upper bound and a lower bound. This solution is used to generate a set of "cuts / constraints" that eliminate the best solution found so far and improve the upper bound on the remaining solutions. The procedure terminates when the gap between the current upper bound and the best lower bound is within the tolerance limits.

The algorithm starts with an initial subset $H^q \subset H$. The resulting model $[L(H^q)]$ is solved and the upper bound (UB^q) and the lower bound (LB^q) are computed using using equations (24) and (25) respectively. If the upper bound (UB^q) equals the best known lower bound (LB^q) within accepted tolerance (ϵ) at any given iteration q, then $(\mathbf{x}^q, \mathbf{y}^q)$ is an optimal solution to $[P]$ and the algorithm is terminated. Otherwise, a new set of candidate points $R_j^{th_{new}}$ is generated using the current solution $(\mathbf{x}^q)$ as follows:

$$R_j^{th_{new}} = \frac{\sum_{i\in I} a_i^{tq} x_{ij}^t}{\sum_{k\in K} Q_{jk} y_{jk}^q - \sum_{i\in I} a_i^{tq} x_{ij}^t}$$

This new set of points is appended to $(R_j^{th})_{H^q \subset H}$ and the procedure is repeated again, until the stopping criteria is reached. The algorithm is outlined below:

Algorithm 1 Algorithm for the Exact Method

Ensure: $UB \leftarrow \infty; LB \leftarrow -\infty; q \leftarrow 0$
Require: Choose an initial set of points R^h.
1: **while** $(UB - LB)/LB \geq \epsilon$ **do**
2: Solve $P_{L(H^q)}$ to obtain $(\mathbf{x}^q, \mathbf{y}^q, \rho^q, \mathbf{w}^q, \mathbf{z}^q, \mathbf{R}^q)$.
3: Update the lower bound: $LB^q \leftarrow v(P_{L(H^q)})$.
4: Update the upper bound: $UB^q \leftarrow \min\{UB^{q-1}, Z(x^q, y^q)\}$.
5: Compute new points: $R_j^{th_{new}} = \frac{\sum_{i\in I} a_i^{tq} x_{ij}^t}{\sum_{k\in K} Q_{jk} y_{jk}^q - \sum_{i\in I} a_i^{tq} x_{ij}^t}$
6: Generate new constraints: $(1 + R_j^{th_{new}})^2 \rho_j^t - R_j^t \leq (R_j^{th_{new}})^2, \quad \forall j \in J, t \in T, h \in H$
7: Append new constraints: $H^{q+1} \leftarrow H^q \cup \{h_{new}\}$
8: $q \leftarrow q + 1$
9: **end while**

4. Computational Results

In this section, we report on the performance of the proposed exact solution approach. The algorithm was coded in OPL studio and the model $[L(H^q)]$ was solved using IBM Ilog CPLEX 12.4. The experiments were conducted on a Dell machine (Precision T5600) with Intel Xeon CPU ES-2650, 2.00 GHz CPU; 16 GB RAM, Windows 7P 64 bits OS.

4.1. Test Instances

The test instances have been generated Amiri (2001)

The co-ordinates of the user nodes and the facilities are randomly generated from a square of side 100 (using a uniform distribution $U \sim (0, 100)$).

The other parameters are generated as follows:

- *User Nodes and Facilities:*

  $I \in \{50, 100, 150, 200, 250\}$
  $J \in \{5, 10, 15, 20, 25\}$
  $cv \in \{0, 0.5, 1, 1.5\}$

Throughout the experiment, we consider 5 capacity levels, i.e., $k = 1, 2, ..., 5$ and three time periods, $t = 1, 2, 3$.

- *Demand:* The mean demand requirement of the user node i is randomly generated as $U(80, 120)$.

- *Access Cost Coefficients:* Cost of providing the service to user node, C_{ij}^t is set to the $r_{ij}^t * d_{ij}$, where r_{ij}^t is a random number generated in the range $U(0.3, 0.4)$ and d_{ij} is the Euclidean distance between user nodes i and facility site j.

- *Capacity Levels* (Q_{jk}): To determine the facility capacities, we set the facility load ratio (LR). LR is defined as the average facility utilization if all the potential facilities are assigned capacity level 3 ($k = 3$) to satisfy 125% of the total demand in the network. The third capacity level is given by, $Q_{j3} = (1.25 * \text{Total Demand}/(|J| * LR)$ where Total Demand $= \sum_{i \in I} \sum_{t \in T} \Lambda_j^t)$. The other facility capacities are set to: $Q_{j1} = 0.50 * Q_{j3}$; $Q_{j2} = 0.75 * Q_{j3}$; $Q_{j4} = 1.25 * Q_{j3}$; $Q_{j5} = 1.50 * Q_{j3}$, where $Q_{j3} = (1.25 * \text{Total Demand}/(|J| * FLR)$ and Total Demand $= \sum_{i \in I} \sum_{t \in T} \Lambda_j^t)$.

- *Fixed Costs* (f_{jk}): The fixed cost of the third capacity level is given by, $f_{j3} = FCR * E_{jj_0}$, where E_{jj_0} is the Euclidean distance between the facility site j and the center of the square within which the coordinates of user nodes are generated, $j_0 = (50, 50)$. FCR, called the Fixed Cost Ratio, is a constant set at 4 originally. The other fixed costs are generated as follows: $f_{j1} = 0.60 * f_{j3}$, $f_{j2} = 0.85 * f_{j3}$, $f_{j4} = 1.15 * f_{j3}$, $f_{j5} = 1.35 * f_{j3}$. The chosen values of fixed cost for different capacity levels exhibit both an underlying economy as well as diseconomy of scale. For example, for a 25% increase in capacity (corresponding to μ_{j4} over μ_{j3}), the capacity cost increases only 15%. However, for the a 50% increase in capacity (corresponding to μ_{j5} over μ_{j3}), the capacity cost increases 35%.

- *Unit Queueing Delay Cost* (D^t) is set originally to 10.

  $D \in \{1, 10, 25, 50, 100\}$

  $LR \in \{0.4, 0.6, 0.8\}$

  $FCR \in \{2, 4, 8, 10\}$

4.2. Results

4.3. Effect of Variability in Service Times

4.4. Effect of Delay Costs

Vidyarthi et al. (2009)

Table 1: Effect of Adding a Priori Cuts on the Performnce of the Solution Algorithm

				FCR = 2, LR = 0.4						FCR = 10, LR = 0.8					
u	f	cv	θ	Total Cost	Iter.	CPU	Iter.	CPU	Red.	Total Cost	Iter.	CPU	Iter.	CPU	Red.
100	10	0.5	1	2,317	3	7	2	5	26	4,127	4	926	2	444	52
			10	2,445	4	21	2	7	67	4,823	5	933	3	484	48
			25	2,700	4	11	2	8	23	5,563	6	1137	2	883	22
			50	2,973	4	14	3	11	17	6,190	6	1233	3	1,119	9
			100	3,422	4	29	3	17	42	7,778	5	1212	3	1,130	7
		1	1	2,285	4	10	2	7	26	4,335	4	493	3	851	-73
			10	2,485	4	14	2	8	38	5,159	6	1534	2	937	39
			25	2,766	4	15	2	10	35	5,829	6	1762	3	1,021	42
			50	3,095	4	24	2	19	21	6,621	6	1899	3	1,195	37
			100	3,553	4	30	3	25	18	8,469	6	2477	3	1,661	33
		1.5	1	2,416	4	8	2	7	19	4,265	5	1581	3	1,012	36
			10	2,562	4	14	3	8	38	5,409	6	1737	3	948	45
			25	2,828	4	24	2	13	45	6,363	6	1530	3	1,154	25
			50	3,138	4	22	3	26	-19	7,787	6	2618	3	1,454	44
			100	3,688	4	70	3	31	56	9,214	5	2233	3	1,477	34
150	15	0.5	1	2,981	4	24	2	14	44	5,724	5	2741	3	1,547	44
			10	3,247	4	57	3	35	37	6,888	8	4287	4	2,642	38
			25	3,400	4	42	3	36	15	7,667	7	3558	3	1,939	46
			50	4,042	4	72	3	51	29	9,267	6	2847	3	1,880	34
			100	4,722	4	108	3	96	11	10,985	6	2650	3	1,649	38
		1	1	2,838	4	27	2	18	34	5,875	7	4265	3	1,997	53
			10	3,345	4	43	2	35	17	6,773	7	4303	5	2,878	33
			25	3,597	4	55	3	56	-1	7,890	6	3272	3	1,705	48
			50	4,036	4	90	4	101	-13	9,873	6	3303	3	1,911	42
			100	4,780	4	190	3	95	50	12,133	6	2891	3	1,858	36
		1.5	1	2,807	4	35	2	30	15	5,971	5	2358	3	1,807	23
			10	3,270	4	57	2	32	44	7,266	7	3696	3	1,802	51
			25	3,810	4	86	3	81	5	8,779	7	3679	3	1,709	54
			50	4,172	4	157	3	128	19	10,754	7	3792	3	1,866	51
			100	5,079	5	277	3	143	48	13,513	6	3706	3	1,873	49
200	20	0.5	1	3,445	4	55	2	45	17	7,448	7	4188	5	2,892	31
			10	3,798	4	179	3	114	36	8,515	8	4608	8	4,496	2
			25	4,256	3	118	2	81	31	9,904	9	5154	6	3,414	34
			50	4,764	4	184	3	155	15	11,709	6	3836	4	2,039	47
			100	5,714	4	235	3	162	31	14,258	6	3229	4	2,518	22
		1	1	3,504	4	93	3	63	33	7,508	9	5315	5	3,240	39
			10	3,914	4	127	2	145	-15	8,959	9	5512	6	3,840	30
			25	4,420	4	179	3	156	13	10,314	7	3899	4	2,281	41
			50	4,949	4	256	3	243	5	12,563	7	3892	3	2,032	48
			100	5,817	4	388	3	368	5	15,785	6	3602	4	2,450	32
		1.5	1	3,528	4	123	3	93	25	7,567	8	4782	6	3,258	32
			10	4,048	4	222	3	109	51	9,434	8	4490	6	3,415	24
			25	4,536	4	268	3	179	33	11,208	6	3537	4	2,098	41
			50	5,171	4	350	3	348	0	13,683	6	3772	5	2,878	24
			100	6,438	4	701	3	462	34	17,906	7	4014	4	2,459	39
250	25	0.5	1	3,963	3	228	2	141	38	8,910	10	5850	8	4,542	22
			10	4,406	4	488	2	345	29	10,868	9	5394	9	5,485	-2
			25	4,983	4	420	2	241	43	13,692	11	6574	12	6,947	-6
			50	5,656	4	488	3	370	24	13,768	6	3857	4	2,381	38
			100	6,665	4	934	3	455	51	17,196	6	3330	4	2,159	35
		1	1	3,912	3	517	2	105	80	8,951	9	5613	5	3,231	42
			10	4,451	4	1,081	3	233	78	11,047	12	7006	5	3,231	54
			25	5,117	4	1,233	3	391	68	12,549	9	5572	5	2,631	53
			50	5,757	4	1,544	3	685	56	15,056	6	3862	3	2,037	47
			100	7,051	5	1,981	4	1856	6	19,588	8	4732	5	3,003	37
		1.5	1	4,044	4	918	2	402	56	9,135	11	6268	6	3,714	41
			10	4,783	4	1,099	3	776	29	12,761	11	6521	10	6,183	5
			25	5,325	5	2,074	3	1656	20	13,512	7	3890	4	2,166	44
			50	6,121	5	2,204	3	1757	20	16,918	7	4178	3	1,798	57
			100	7,425	5	2,114	4	2408	-14	21,879	7	4067	5	2,757	32
		min.			3	7	2	5	-19		4	493	2	444	-73
		avg.			4	374	3	262	28		7	3587	4	2340	34
		max.			5	2204	4	2408	80		12	7006	12	6947	57

Table 2: Computational Results for FCR = 4, LR = 0.4

$\|I\|$	$\|J\|$	θ	Cv = 0.5									cv = 1									cv = 1.5								
			TOTC	AC	FC	DC	no fac.	AVU	MAXU	Iter.	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	Iter.	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	Iter.	CPU
50	5	1	1,799	82	17	1	3.3	0.74	0.86	2	8	1,926	80	18	2	3.3	0.75	0.87	2	10	1,762	80	18	2	3.6	0.67	0.77	2	8
		10	1,854	76	18	6	3.3	0.58	0.63	2	10	1,839	73	21	7	3.6	0.52	0.61	2	6	1,877	70	23	7	3.8	0.46	0.55	3	13
		25	2,089	72	20	9	3.1	0.49	0.56	2	8	2,031	68	22	10	3.4	0.44	0.50	2	13	1,975	67	23	11	3.4	0.39	0.47	2	11
		50	2,118	69	18	13	3.1	0.40	0.48	2	12	2,230	65	21	15	3.0	0.42	0.52	2	15	2,183	63	22	15	3.4	0.35	0.41	3	18
		100	2,266	62	19	20	2.9	0.37	0.44	3	12	2,429	56	23	22	3.2	0.35	0.42	3	14	2,444	55	21	24	3.5	0.32	0.38	3	20
100	10	1	2,697	73	24	2	6.9	0.76	0.90	2	23	2,663	72	25	3	6.7	0.75	0.86	2	27	2,716	73	24	3	6.7	0.71	0.83	3	29
		10	2,868	70	23	7	5.8	0.59	0.70	2	30	2,997	68	25	8	6.0	0.55	0.64	3	42	3,011	65	26	9	6.2	0.50	0.61	2	27
		25	3,161	63	26	11	5.7	0.49	0.59	2	36	3,191	65	24	11	5.5	0.46	0.53	2	40	3,343	59	28	13	5.8	0.43	0.52	3	63
		50	3,495	60	24	15	5.2	0.44	0.55	3	51	3,597	56	28	16	5.7	0.40	0.49	3	46	3,651	53	29	18	6.1	0.37	0.47	3	80
		100	3,917	52	24	24	5.8	0.38	0.50	3	43	4,132	50	26	24	6.3	0.34	0.43	3	53	4,427	46	27	27	6.2	0.34	0.43	3	91
150	15	1	3,359	70	28	3	9.4	0.80	0.90	2	101	3,383	68	29	3	10.0	0.76	0.87	3	136	3,476	69	28	4	9.9	0.72	0.84	2	149
		10	3,871	63	28	8	9.0	0.60	0.71	2	200	3,886	61	30	9	9.0	0.56	0.67	2	224	3,979	61	30	9	8.6	0.51	0.61	3	189
		25	4,225	61	27	12	8.0	0.50	0.62	3	203	4,230	59	28	13	8.0	0.46	0.57	3	241	4,338	57	29	14	8.0	0.43	0.53	2	167
		50	4,528	55	28	18	7.5	0.45	0.56	3	168	4,809	54	27	19	7.7	0.43	0.53	3	210	4,934	50	29	21	8.3	0.39	0.48	3	249
		100	5,217	47	27	26	8.2	0.39	0.49	3	247	5,563	45	28	28	9.0	0.36	0.46	3	387	5,893	41	27	31	9.0	0.35	0.44	3	489
200	20	1	4,203	68	29	3	12.5	0.80	0.90	2	658	4,095	64	33	4	13.3	0.77	0.88	2	574	4,176	63	33	4	13.9	0.72	0.84	2	717
		10	4,793	63	29	8	10.6	0.61	0.73	3	935	4,750	60	31	9	10.8	0.56	0.68	2	638	4,867	61	30	10	10.0	0.53	0.64	3	581
		25	5,149	57	30	13	9.8	0.53	0.66	3	628	5,139	55	31	14	10.2	0.47	0.59	3	549	5,499	54	31	15	10.4	0.44	0.56	3	927
		50	5,651	52	29	19	10.1	0.45	0.57	3	557	5,904	50	29	20	9.9	0.44	0.55	3	579	6,148	49	29	22	11.1	0.39	0.49	3	1727
		100	6,597	43	28	29	10.6	0.41	0.51	3	747	6,946	42	28	31	11.0	0.39	0.48	3	968	7,512	38	29	33	12.0	0.36	0.44	4	2139
250	25	1	4,719	64	33	3	15.9	0.80	0.91	4	2114	4,957	64	33	3	16.1	0.74	0.90	6	3698	4,927	62	34	5	16.1	0.74	0.86	4	2136
		10	5,466	59	32	9	13.0	0.60	0.74	5	2929	5,787	53	37	10	15.8	0.54	0.69	6	3510	5,727	57	33	10	13.0	0.51	0.64	4	2246
		25	5,940	55	31	14	12.4	0.50	0.64	3	1257	6,190	52	34	15	13.3	0.46	0.62	4	1844	6,257	50	34	16	13.1	0.44	0.55	3	1545
		50	6,788	51	29	20	11.6	0.48	0.59	3	1598	7,233	49	30	21	12.3	0.45	0.54	3	1164	7,455	44	32	24	13.0	0.41	0.52	3	1511
		100	8,009	41	29	30	12.8	0.42	0.53	4	2089	8,343	38	30	32	14.1	0.38	0.49	3	1693	9,133	36	30	34	14.9	0.36	0.45	4	1976
		Min.	1,799	41	17	1	2.90	0.37	0.44	2	8	1,839	38	18	2	3.0	0.34	0.42	2	6	1,762	36	18	2	3.4	0.32	0.38	2	8
		Avg.	4,191	61	26	13	8.26	0.54	0.65	3	587	4,330	59	28	14	8.7	0.51	0.62	3	667	4,468	57	28	15	8.8	0.47	0.57	3	684
		Max.	8,009	82	33	30	15.90	0.80	0.91	5	2,929	8,343	80	37	32	16.1	0.77	0.90	6	3,698	9,133	80	34	34	16.1	0.74	0.86	4	2,246

Table 3: Computational Results for FCR = 4, LR = 0.6

U	F	θ	FCR	LR	Cv = 0.5									cv = 1									cv = 1.5								
					TOTC	AC	FC	DC	no fac.	AVU	MAXU	ITR	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU

Table 4: Computational Results for FCR = 4, LR = 0.8

					Cv = 0.5									cv = 1									cv = 1.5								
U	F	θ	FCR	LR	TOTC	AC	FC	DC	no fac.	AVU	MAXU	ITR	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU
50	5	1	4	0.8	1,927	72	26	2	3.7	0.83	0.89	2	13	1,973	74	23	3	3.5	0.84	0.89	2	20	1,989	72	25	3	4.0	0.78	0.85	2	13
		10	4	0.8	2,179	62	28	9	3.5	0.71	0.77	2	14	2,030	64	26	10	3.9	0.62	0.70	2	11	2,158	61	28	11	3.9	0.60	0.67	2	14
		25	4	0.8	2,326	56	28	16	3.7	0.62	0.71	2	20	2,381	54	28	18	4.0	0.57	0.66	2	21	2,722	53	28	19	3.9	0.55	0.65	2	24
		50	4	0.8	2,614	53	25	22	4.0	0.54	0.62	3	23	2,765	50	25	25	4.1	0.52	0.59	2	30	3,040	44	29	27	4.5	0.48	0.54	3	27
		100	4	0.8	3,318	43	25	32	4.3	0.50	0.57	2	36	3,530	39	26	35	4.5	0.47	0.54	2	37	3,852	34	26	39	4.7	0.45	0.51	2	57
100	10	1	4	0.8	2,995	64	33	3	7.0	0.86	0.92	2	89	3,009	63	33	4	7.6	0.82	0.91	2	153	3,052	63	32	5	7.0	0.80	0.87	2	117
		10	4	0.8	3,404	59	31	10	6.5	0.70	0.78	2	135	3,521	56	32	12	6.8	0.67	0.76	2	164	3,530	51	36	14	7.6	0.60	0.70	2	93
		25	4	0.8	3,986	51	31	18	6.7	0.65	0.73	3	162	4,083	49	31	20	7.5	0.58	0.68	2	166	4,353	45	33	22	8.0	0.54	0.62	3	346
		50	4	0.8	4,466	42	31	27	7.5	0.57	0.66	3	210	5,057	42	29	28	7.8	0.55	0.63	3	329	5,256	38	30	32	8.6	0.49	0.57	3	717
		100	4	0.8	5,782	35	28	37	8.3	0.51	0.60	3	601	6,259	33	28	39	8.9	0.48	0.55	3	1188	6,840	30	26	44	9.5	0.45	0.52	3	1708
150	15	1	4	0.8	3,902	63	34	4	10.3	0.85	0.92	3	1071	3,978	62	34	4	9.8	0.83	0.91	3	1723	4,160	57	38	5	9.8	0.82	0.88	3	1614
		10	4	0.8	4,714	54	35	12	9.9	0.70	0.80	3	1007	4,875	51	36	13	10.0	0.68	0.76	3	1126	4,970	47	37	16	10.2	0.64	0.72	3	1141
		25	4	0.8	5,329	47	33	20	10.0	0.65	0.74	3	1116	5,722	46	33	21	10.8	0.59	0.68	3	1626	5,906	41	34	25	11.7	0.55	0.64	3	1529
		50	4	0.8	6,344	38	33	29	10.8	0.59	0.68	3	1455	6,622	36	32	32	11.9	0.54	0.63	3	1542	7,543	34	31	34	12.6	0.50	0.60	3	1799
		100	4	0.8	7,864	30	30	40	12.6	0.51	0.59	3	1618	8,602	27	29	44	13.1	0.49	0.56	4	2158	9,871	24	31	45	14.3	0.44	0.51	3	1920
200	20	1	4	0.8	4,848	57	39	4	12.8	0.86	0.93	5	2879	4,953	56	40	4	12.8	0.84	0.91	7	3889	5,063	55	41	5	13.3	0.79	0.88	6	3591
		10	4	0.8	6,013	54	34	12	12.1	0.72	0.81	4	2563	5,877	45	40	14	13.1	0.67	0.76	3	1730	6,327	43	40	16	13.5	0.64	0.73	4	2096
		25	4	0.8	6,587	41	38	21	13.5	0.64	0.73	3	1663	7,044	39	37	24	14.1	0.61	0.69	4	2127	7,759	38	36	26	15.3	0.56	0.66	4	2495
		50	4	0.8	7,881	35	35	31	14.8	0.58	0.66	4	2153	8,536	32	35	33	15.9	0.53	0.61	4	2089	9,457	30	34	36	16.9	0.50	0.58	3	1982
		100	4	0.8	10,193	27	31	42	16.6	0.51	0.60	4	2145	11,426	25	32	44	17.5	0.48	0.57	3	1910	12,963	24	29	47	18.9	0.45	0.54	4	2386
250	25	1												5,857	52	44	5	17.0	0.81	0.91	7	4179	6,091	50	45	5	17.3	0.76	0.87	8	4675
		10	4	0.8	7,619	40	48	12	21.5	0.61	0.79	12	7257	7,481	43	43	14	19.2	0.63	0.79	9	5161	7,428	41	41	18	16.8	0.64	0.74	4	2497
		25	4	0.8	8,099	41	37	22	18.3	0.61	0.76	6	3572	8,691	36	41	23	19.5	0.56	0.70	5	3080	9,145	34	39	27	19.0	0.56	0.65	4	2093
		50	4	0.8	9,467	32	35	32	18.1	0.59	0.67	3	1571	10,426	31	35	34	19.5	0.54	0.63	3	1872	11,231	27	36	37	21.3	0.50	0.57	3	1747
		100	4	0.8	12,467	26	31	43	20.6	0.52	0.60	5	2693																		

Table 5: Computational Results for FCR = 8, LR = 0.4

					Cv = 0.5									cv = 1									cv = 1.5								
U	F	θ	FCR	LR	TOTC	AC	FC	DC	no fac.	AVU	MAXU	ITR	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU
50	5	1	8	0.4	2,029	71	27	2	3.0	0.83	0.89	2	14	2,066	74	24	2	2.8	0.80	0.86	2	10	1,993	73	25	2	3.2	0.74	0.82	2	10
		10	8	0.4	2,307	70	25	5	2.4	0.70	0.74	2	19	2,315	66	28	6	2.9	0.63	0.69	2	11	2,223	67	26	7	2.5	0.59	0.64	2	9
		25	8	0.4	2,366	66	25	9	2.6	0.58	0.64	2	15	2,380	62	27	10	2.7	0.54	0.60	2	12	2,545	63	27	10	2.5	0.51	0.56	2	20
		50	8	0.4	2,746	58	28	13	2.6	0.54	0.61	2	22	2,583	61	25	14	2.6	0.48	0.53	3	15	2,659	59	25	16	2.3	0.49	0.53	3	22
		100	8	0.4	2,799	53	28	19	2.6	0.45	0.51	3	23	3,108	57	22	21	2.2	0.50	0.54	2	22	3,093	50	26	24	2.6	0.43	0.48	3	20
100	10	1	8	0.4	3,216	66	32	2	5.8	0.85	0.92	2	83	3,151	65	32	3	5.3	0.83	0.90	2	52	3,230	64	33	3	5.7	0.79	0.86	2	62
		10	8	0.4	3,403	61	32	7	5.0	0.68	0.74	3	77	3,604	62	31	8	4.7	0.65	0.74	3	85	3,752	62	30	8	4.6	0.61	0.67	2	80
		25	8	0.4	3,817	59	30	10	4.6	0.59	0.66	3	101	3,861	58	31	11	4.3	0.56	0.63	3	95	4,105	57	31	12	4.5	0.52	0.59	3	113
		50	8	0.4	4,176	53	32	15	4.6	0.53	0.60	3	85	4,249	54	30	16	4.4	0.51	0.57	3	78	4,496	51	31	18	4.6	0.48	0.54	3	125
		100	8	0.4	4,711	50	28	22	4.5	0.48	0.55	3	108	4,904	47	28	24	4.8	0.45	0.51	3	112	5,192	43	31	26	5.2	0.41	0.46	3	136
150	15	1	8	0.4	4,220	60	37	3	8.7	0.86	0.92	2	515	4,442	61	36	3	7.8	0.84	0.90	2	513	4,332	58	38	4	7.9	0.81	0.88	2	487
		10	8	0.4	4,759	58	34	7	6.1	0.72	0.79	3	625	4,751	58	34	8	6.9	0.63	0.75	3	560	5,001	55	35	9	6.7	0.62	0.70	2	324
		25	8	0.4	5,141	58	31	11	5.9	0.62	0.70	3	342	5,340	52	35	12	6.3	0.58	0.66	3	304	5,511	52	34	14	6.3	0.54	0.61	3	338
		50	8	0.4	5,692	51	32	16	6.0	0.56	0.64	3	293	5,785	49	33	18	6.4	0.52	0.60	3	300	6,097	45	34	21	6.7	0.49	0.55	3	467
		100	8	0.4	6,530	45	30	25	6.5	0.50	0.59	3	396	6,843	43	31	26	7.0	0.46	0.54	3	425	7,329	39	33	28	7.8	0.41	0.47	3	512
200	20	1	8	0.4	5,083	58	39	3	10.7	0.86	0.93	3	1745	5,220	56	40	3	10.3	0.83	0.91	5	3124	5,339	58	39	4	9.5	0.81	0.88	4	1985
		10	8	0.4	5,882	56	36	8	8.1	0.70	0.78	6	3296	5,996	55	37	8	8.2	0.67	0.74	4	2039	6,064	53	38	10	8.2	0.62	0.72	4	1635
		25	8	0.4	6,393	53	34	12	7.7	0.63	0.72	3	564	6,533	53	34	13	8.2	0.56	0.68	3	1093	6,927	48	37	15	8.3	0.54	0.62	3	907
		50	8	0.4	7,054	48	34	18	7.9	0.56	0.65	3	677	7,389	47	33	20	8.0	0.55	0.62	3	1050	7,940	45	34	21	8.7	0.49	0.56	4	1689
		100	8	0.4	8,262	40	34	26	8.3	0.52	0.61	3	718	8,512	37	35	28	9.3	0.46	0.53	3	1518	9,454	35	35	30	10.2	0.42	0.50	3	1888
250	25	1	8	0.4	6,895	48	50	2	14.2	0.68	0.91	10	6069	6,278	54	42	4	12.4	0.85	0.91	6	4396	6,291	56	40	4	12.3	0.80	0.88	4	3224
		10	8	0.4	8,560	38	56	6	18.5	0.48	0.74	12	7152	7,636	48	44	8	13.3	0.61	0.75	9	6941	7,477	52	38	10	9.5	0.64	0.73	3	1427
		25	8	0.4	7,913	48	39	13	10.5	0.62	0.73	6	3341	7,831	49	37	14	9.5	0.60	0.69	3	1901	8,332	45	39	16	9.9	0.56	0.66	4	2669
		50	8	0.4	8,526	46	35	19	9.2	0.59	0.68	4	1968	9,267	40	41	20	11.5	0.51	0.62	5	4015	9,357	41	36	22	11.0	0.49	0.58	3	1852
		100	8	0.4	9,905	39	33	27	10.3	0.52	0.60	5	3508	10,447	35	35	30	11.0	0.48	0.56	3	2210	11,302	33	35	31	12.8	0.42	0.49	4	2977

Table 6: Computational Results for FCR = 8, LR = 0.6

					Cv = 0.5									cv = 1									cv = 1.5								
U	F	θ	FCR	LR	TOTC	AC	FC	DC	no fac.	AVU	MAXU	ITR	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU
50	5	1	8	0.6	2,363	62	36	2	3.2	0.87	0.92	2	23	2,283	63	34	3	3.2	0.85	0.89	2	13	2,117	70	27	3	2.7	0.83	0.88	2	13
		10	8	0.6	2,203	67	25	7	2.9	0.69	0.76	2	20	2,390	63	29	9	2.9	0.70	0.75	2	9	2,575	56	34	10	3.0	0.67	0.73	3	22
		25	8	0.6	2,592	56	31	12	2.9	0.65	0.71	2	24	2,656	56	30	14	3.0	0.61	0.68	2	19	2,831	52	34	15	3.0	0.57	0.63	2	14
		50	8	0.6	2,801	51	31	18	3.0	0.57	0.64	2	18	3,021	48	33	19	3.0	0.56	0.62	3	22	3,410	48	32	20	3.0	0.53	0.58	3	20
		100	8	0.6	3,343	43	32	25	3.0	0.53	0.58	3	20	3,654	42	31	28	3.0	0.53	0.58	3	19	3,748	39	28	33	3.3	0.49	0.53	3	38
100	10	1	8	0.6	3,516	63	34	3	5.3	0.89	0.93	2	91	3,562	62	35	3	5.5	0.86	0.91	2	170	3,522	62	34	4	5.2	0.83	0.88	2	101
		10	8	0.6	3,824	56	36	8	5.1	0.73	0.80	3	112	3,986	55	35	10	5.2	0.71	0.77	2	178	4,086	54	35	11	5.3	0.66	0.73	3	109
		25	8	0.6	4,391	51	35	14	5.0	0.67	0.74	3	108	4,508	49	35	16	5.1	0.64	0.70	3	109	5,086	46	36	18	5.3	0.62	0.68	3	127
		50	8	0.6	4,893	47	32	21	5.3	0.62	0.68	3	153	4,979	44	33	23	5.7	0.57	0.62	3	117	5,600	39	37	24	6.1	0.52	0.59	3	245
		100	8	0.6	5,845	38	33	29	5.9	0.54	0.62	3	154	6,143	34	33	33	6.1	0.52	0.58	3	219	6,781	29	36	35	6.8	0.47	0.52	3	321
150	15	1	8	0.6	4,769	59	39	3	7.5	0.88	0.94	3	944	4,752	55	42	3	7.5	0.87	0.92	3	1055	4,794	55	41	4	7.2	0.84	0.90	2	863
		10	8	0.6	5,313	53	38	9	7.1	0.75	0.82	3	1291	5,625	51	39	10	7.3	0.72	0.79	2	696	5,774	51	37	12	7.4	0.68	0.75	3	681
		25	8	0.6	6,033	48	37	15	7.2	0.68	0.75	2	410	6,293	45	38	17	7.8	0.64	0.71	3	432	6,681	42	39	20	7.9	0.61	0.67	3	1406
		50	8	0.6	6,666	41	37	22	8.0	0.61	0.69	3	602	7,211	38	37	25	8.1	0.59	0.65	3	725	7,820	34	39	27	8.9	0.54	0.60	3	1695
		100	8	0.6	8,078	32	36	32	8.6	0.56	0.62	3	755	8,881	31	34	35	9.1	0.53	0.59	3	1384	9,593	27	35	37	10.0	0.48	0.54	3	1954
200	20	1	8	0.6	5,879	54	43	3	9.3	0.90	0.94	4	2548	5,932	53	43	3	9.7	0.86	0.92	6	3420	6,058	52	44	4	10.1	0.83	0.90	8	4550
		10	8	0.6	7,024	44	46	9	10.5	0.73	0.82	8	4776	6,984	47	42	11	9.4	0.72	0.79	5	3001	7,156	45	43	13	9.9	0.67	0.75	6	3528
		25	8	0.6	7,796	41	45	14	11.0	0.64	0.74	5	3217	7,839	43	40	18	10.0	0.64	0.72	3	1488	8,367	39	41	21	10.6	0.61	0.68	4	2025
		50	8	0.6	8,447	39	38	23	10.4	0.61	0.68	3	1565	9,006	33	40	27	10.7	0.60	0.66	3	1775	9,992	32	39	29	11.6	0.55	0.61	3	1920
		100	8	0.6	10,555	31	36	33	11.6	0.55	0.63	4	2249	11,335	28	36	36	12.1	0.53	0.59	5	2755	12,475	24	37	39	13.3	0.48	0.54	3	1741
250	25	1	8	0.6	7,154	48	50	3	13.3	0.83	0.94	9	5116	7,608	45	53	3	13.9	0.79	0.91	10	5858	7,479	47	49	4	13.3	0.80	0.90	9	5557
		10	8	0.6	9,567	36	57	7	17.3	0.59	0.77	11	6377	9,029	41	50	10	15.0	0.64	0.77	11	6576	9,473	37	50	12	16.2	0.62	0.74	10	5857
		25	8	0.6	10,760	31	57	12	19.0	0.50	0.69	13	7573	9,416	38	43	19	12.5	0.64	0.72	3	1947	10,242	36	43	21	13.4	0.60	0.68	4	2274
		50	8	0.6	10,455	35	41	24	12.7	0.63	0.71	4	2118	10,948	33	40	27	13.4	0.59	0.66	4	2159	12,032	29	42	29	15.0	0.53	0.60	4	2513
		100	8	0.6	12,742	31	36	34	14.3	0.56	0.62	3	2059	13,899	26	38	36	15.4	0.52	0.59	5	3172	15,224	24	38	38	17.1	0.47	0.53	4	2634

Table 7: Computational Results for FCR = 8, LR = 0.8

					Cv = 0.5									cv = 1									cv = 1.5								
U	F	θ	FCR	LR	TOTC	AC	FC	DC	no fac.	AVU	MAXU	ITR	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU
50	5	1	8	0.8	2,302	64	34	2	3.1	0.89	0.93	2	16	2,325	63	33	3	3.2	0.88	0.92	2	12	2,339	58	38	4	3.5	0.85	0.90	2	14
		10	8	0.8	2,738	54	37	8	3.1	0.77	0.83	3	27	2,821	51	39	10	3.1	0.75	0.81	2	21	2,811	49	39	12	3.2	0.71	0.77	2	16
		25	8	0.8	2,768	53	32	15	3.0	0.71	0.76	3	25	2,965	50	32	18	3.2	0.68	0.72	2	24	3,461	43	37	20	3.3	0.66	0.70	2	23
		50	8	0.8	3,379	43	35	22	3.3	0.66	0.71	2	25	3,476	39	37	24	3.8	0.58	0.64	3	28	3,614	40	34	26	4.0	0.54	0.59	3	42
		100	8	0.8	3,909	34	35	31	3.8	0.57	0.62	2	23	4,173	34	34	32	4.0	0.53	0.58	2	40	4,783	31	32	37	4.0	0.53	0.56	3	41
100	10	1	8	0.8	3,779	55	43	3	5.8	0.90	0.94	2	180	3,860	54	43	4	5.9	0.88	0.92	2	273	4,005	53	42	5	5.5	0.87	0.91	3	206
		10	8	0.8	4,513	49	41	10	5.9	0.77	0.83	3	216	4,475	48	40	12	6.0	0.73	0.80	3	239	4,836	43	43	14	6.0	0.72	0.77	3	293
		25	8	0.8	4,943	44	39	17	6.0	0.71	0.77	3	281	5,223	39	41	20	6.4	0.68	0.73	3	378	5,763	36	43	21	6.9	0.62	0.68	3	444
		50	8	0.8	5,731	38	37	25	6.5	0.66	0.71	3	376	5,944	34	39	27	7.1	0.60	0.66	3	456	6,512	30	40	30	7.6	0.56	0.61	3	947
		100	8	0.8	7,148	30	35	35	7.0	0.60	0.65	3	594	7,535	26	37	38	7.7	0.55	0.59	3	1550	8,460	25	34	41	8.3	0.51	0.56	3	1651
150	15	1	8	0.8	5,162	52	44	3	8.2	0.90	0.94	3	1414	5,283	51	45	4	8.3	0.88	0.93	5	3020	5,413	50	45	5	8.4	0.86	0.92	4	2256
		10	8	0.8	6,137	44	44	11	8.5	0.79	0.84	4	2269	6,472	41	46	13	8.9	0.75	0.81	4	2576	6,742	40	45	15	9.0	0.72	0.77	3	1798
		25	8	0.8	6,775	39	42	18	9.0	0.71	0.77	3	1756	7,452	35	44	21	9.4	0.68	0.74	3	1630	7,786	34	42	24	10.1	0.63	0.70	3	1789
		50	8	0.8	8,074	32	42	26	9.9	0.64	0.71	3	1492	8,740	29	42	29	10.2	0.62	0.68	3	1831	9,845	27	42	31	11.1	0.57	0.63	3	2042
		100	8	0.8	10,115	25	39	36	10.8	0.59	0.65	3	1741	10,945	23	38	39	11.5	0.55	0.61	3	1681	12,710	21	37	42	12.2	0.52	0.57	4	2101
200	20	1	8	0.8	6,698	45	51	3	11.1	0.90	0.95	5	3236	6,702	46	50	4	11.0	0.88	0.93	6	3708	6,799	45	50	5	11.0	0.86	0.91	7	4075
		10	8	0.8	7,811	40	48	11	11.2	0.78	0.84	7	4258	8,124	40	48	12	12.4	0.72	0.79	7	4192	8,604	33	51	16	12.3	0.71	0.77	5	3079
		25												9,330	35	44	22	12.6	0.68	0.74	3	2063	10,253	29	46	25	13.1	0.65	0.70	3	1880
		50	8	0.8	8,758	34	47	19	12.0	0.72	0.77	4	2231	11,339	27	43	30	13.7	0.62	0.68	4	2127	12,186	22	45	33	15.0	0.57	0.62	4	2484
		100	8	0.8	12,909	24	39	37	14.6	0.58	0.64	4	2511	14,621	21	40	38	15.6	0.54	0.60	4	2485	15,664	18	39	43	16.9	0.50	0.55	4	2562
250	25	1	8	0.8	7,880	45	51	3	13.0	0.90	0.95	6	3295	7,942	43	53	4	13.8	0.87	0.94	6	3837	8,222	43	52	5	13.5	0.86	0.92	6	3411
		10	8	0.8	9,521	36	53	11	15.2	0.75	0.84	9	5211	9,953	36	51	13	15.3	0.73	0.81	6	3773	10,376	35	49	16	15.5	0.70	0.78	5	3109
		25	8	0.8	10,888	32	48	19	15.0	0.71	0.78	5	2874	11,296	33	45	22	15.7	0.68	0.74	4	2116	12,298	29	47	24	17.0	0.62	0.69	3	1971
		50	8	0.8	12,652	28	44	28	16.3	0.65	0.72	3	1891	13,802	24	45	30	17.1	0.62	0.68	4	2145	15,058	23	43	34	18.6	0.57	0.63	4	2392
		100	8	0.8	16,127	22	39	39	17.7	0.60	0.67	4	2273	17,422	19	41	40	19.7	0.54	0.60	4	2520	19,991	16	41	43	21.0	0.51	0.56	5	2859

Table 8: Computational Results for FCR = 10, LR = 0.8

					Cv = 0.5									cv = 1									cv = 1.5								
U	F	θ	FCR	LR	TOTC	AC	FC	DC	no fac.	AVU	MAXU	ITR	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU	TOTC	AC	FC	DC	no fac.	AVU	MAXU	iter.	CPU
50	5	1	10	0.8	2,671	55	42	2	3.1	0.90	0.95	2	17	2,552	51	43	3.3	3.2	0.88	0.93	2.2	19	2,582	53	40	4.6	3	0.88	0.92	2.2	25
		10	10	0.8	2,887	52	42	9	3.2	0.79	0.84	2	23	2,999	59	38	9.7	3	0.76	0.80	2.2	24	3,184	55	42	12	3.1	0.73	0.77	2.2	21
		25	10	0.8	3,248	57	39	15	3	0.74	0.77	2	20	3,204	55	37	17	3	0.71	0.74	2.6	35	3,414	56	35	21	3.2	0.67	0.71	2.4	32
		50	10	0.8	3,781	58	39	21	3.1	0.69	0.73	2.6	41	4,026	55	39	24	3.4	0.65	0.69	2.4	34	4,024	48	45	24	4	0.54	0.58	2.7	37
		100	10	0.8	4,400	54	36	31	3.5	0.63	0.67	2.4	36	4,625	51	41	30	4	0.53	0.58	2.4	35	4,989	53	38	34	4.2	0.50	0.54	2.9	70
100	10	1	10	0.8	4,127	84	43	3	5.143	0.92	0.95	2	444	4,335	82	46	3.7	5.6	0.90	0.94	2.5	851	4,265	85	42	4.5	5.4	0.88	0.91	2.8	1,012
		10	10	0.8	4,823	84	43	11	5.4	0.82	0.87	2.5	484	5,159	80	47	12	5.9	0.77	0.82	2.4	937	5,409	80	47	14	6	0.74	0.78	2.6	948
		25	10	0.8	5,563	81	45	16	6	0.73	0.79	2.4	883	5,829	76	46	19	6	0.71	0.76	2.9	1,021	6,363	81	44	21	6.5	0.66	0.71	2.8	1,154
		50	10	0.8	6,190	85	38	25	6.1	0.69	0.74	3	1,119	6,621	80	41	26	6.8	0.63	0.69	3	1,195	7,787	81	44	28	7	0.60	0.65	3.1	1,454
		100	10	0.8	7,778	78	41	32	7	0.61	0.65	2.9	1,130	8,469	79	38	37	7.2	0.59	0.64	3.3	1,661	9,214	78	39	38	8.1	0.52	0.56	2.8	1,477
150	15	1	10	0.8	5,724	106	47	3	7.5	0.92	0.95	2.7	1,547	5,875	107	47	4.1	7.5	0.91	0.94	3.3	1,997	5,971	106	48	4.8	7.9	0.88	0.92	3	1,807
		10	10	0.8	6,888	113	46	11	8	0.81	0.86	4.4	2,642	6,773	105	46	13	8.6	0.77	0.82	4.8	2,878	7,266	100	48	15	8.8	0.73	0.79	3	1,802
		25	10	0.8	7,667	104	45	18	8.7	0.74	0.79	3.3	1,939	7,890	105	45	20	9.3	0.69	0.74	2.9	1,705	8,779	103	46	23	9.7	0.66	0.71	2.9	1,709
		50	10	0.8	9,267	107	44	25	9.1	0.70	0.75	3.2	1,880	9,873	104	44	28	9.9	0.65	0.69	3.3	1,911	10,754	98	43	33	10.22	0.62	0.66	3.1	1,866
		100	10	0.8	10,985	97	41	36	10.2	0.62	0.68	2.8	1,649	12,133	93	42	37	11	0.58	0.62	3.1	1,858	13,513	92	42	40	12.13	0.52	0.57	3.1	1,873
200	20	1	10	0.8	7,448	125	52	3	10.1	0.92	0.96	4.8	2,892	7,508	122	52	4.1	10.3	0.90	0.94	5.4	3,240	7,567	125	51	5	10.3	0.88	0.92	5.6	3,258
		10	10	0.8	8,515	125	50	11	11.1	0.79	0.85	7.5	4,496	8,959	125	50	13	11.2	0.77	0.82	6.4	3,840	9,434	123	50	15	11.7	0.73	0.79	5.7	3,415
		25	10	0.8	9,904	118	49	19	11.3	0.75	0.80	5.7	3,414	10,314	121	47	21	12.2	0.70	0.76	3.8	2,281	11,208	114	49	24	12.8	0.66	0.71	3.5	2,098
		50	10	0.8	11,709	123	45	27	12.2	0.69	0.74	3.5	2,039	12,563	116	46	29	13.1	0.65	0.69	3.4	2,032	13,683	118	46	31	14.4	0.59	0.64	4.8	2,878
		100	10	0.8	14,258	117	42	36	13.8	0.62	0.68	4.3	2,518	15,785	119	42	38	14.8	0.57	0.63	4.1	2,450	17,906	108	44	40	16.2	0.52	0.57	4.3	2,459
250	25	1	10	0.8	8,910	140	55	3	12.9	0.90	0.95	7.6	4,542	8,951	131	57	4	12.8	0.90	0.94	5.4	3,231	9,135	144	53	4.9	13.2	0.87	0.92	6.4	3,714
		10	10	0.8	10,868	132	57	10	15.8	0.74	0.86	9.2	5,485	11,047	146	52	13	14	0.76	0.82	5.4	3,231	12,761	132	59	14	17.67	0.67	0.80	10.3	6,183
		25	10	0.8	13,692	131	60	14	20.67	0.58	0.78	12	6,947	12,549	133	50	21	15.2	0.70	0.76	4.6	2,631	13,512	138	48	25	16	0.66	0.72	3.6	2,166
		50	10	0.8	13,768	136	46	27	15.6	0.68	0.73	4.1	2,381	15,056	143	45	30	16.6	0.64	0.70	3.4	2,037	16,918	120	50	31	18	0.59	0.63	3.2	1,798
		100	10	0.8	17,196	130	42	38	17.1	0.62	0.68	3.6	2,159	19,588	138	44	38	18.8	0.56	0.63	5	3,003	21,879	125	45	40	20.6	0.52	0.57	4.6	2,757

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