On Truck dock assignment problem with operational time constraint within cross docks

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Abstract

An integer programming model for the truck dock assignment problem with operational time constraint within cross docks has been proposed in (*Miao, Z.,Lim, A.,Ma, H., 2009. Truck dock assignment problem with operational time constraint within crossdocks. European Journal of Operational Research 192 (1), 105–115*). We address the following issues in this formulation: 1) from among those constraints in the model, there are n^4 constraints, which eliminate a subset of feasible solutions of the problem that has been described. Consequently, the model does not necessarily offer an optimal solution for the described problem, 2) another set of n^4 constraints, although do not cut off feasible solutions, but their upper bounds are incorrectly specified such that it leads to redundancy of a majority of them instead of contributing in numerical efficiency by tightening the polytope of LP relaxation, 3) there is a flaw in calculation of cost in the objective function such that the positive transportation costs are calculated even for *not* transporting anything. This article contributes in the following way: 1) describes the issue using an example and subsequently proposes a rectified model, 2) calculates the dimension of polytope in the revised model, 3) introduces several classes of valid inequalities resulting in an improved formulation with tighter LP-relaxation polytope facilitating resolution of larger instances and, 4) proves that some of the aforementioned valid inequalities are facets of polytope.

1. Mathematical Model (Miao et al., 2009)

According to (Miao et al., 2009; Lim et al., 2006): *an over-constrained truck dock assignment problem with time window, operational time, and capacity constraint in a transshipment network through cross docks where the number of trucks exceeds the number of docks available and the capacity of the cross dock is limited, and where the objective is to minimize the operational cost of the cargo shipment and the number of unfullled shipments is studied*.
The objective function accounts for the total cost of 1) dock operations, 2) penalties for the unfullled shipments.

The following parameters and variables are introduced therein:

Parameters:

N: set of trucks arriving at and/or departing from the cross dock
M: set of docks available in the cross dock
n: total number of trucks, that is $|N|$ ($|N|$ denotes cardinality of set N)
m: total number of docks, that is $|M|$
a_i: arrival time of truck i ($1 \leq i \leq n$)
di: departure time of truck i ($1 \leq i \leq n$)

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t_{kl}: operational time for pallets from dock k to dock l $(1 \le k, l \le m)$
f_{ij}: number of pallets transferring from truck i to truck j $(1 \le i, j \le n)$
c_{kl}: operational cost per unit time from dock k to dock l $(1 \le k, l \le m)$
p_{ij}: penalty cost per unit cargo from truck i to truck j $(1 \le i, j \le n)$
C: capacity of cross dock, i.e. the maximum number of cargos the cross dock can hold at a time
$\hat{x}_{ij}$: 1 iff truck i departs no later than truck j arrives; 0 otherwise.

Note 1 (Miao et al. (2009)). *It has been also assumed that:*

- $f_{ij} \ge 0$ *iff* $d_j \ge a_i, \forall i, j \ne i$, *otherwise* $f_{ij} = 0$ *meaning that truck i will transfer some cargo to truck j iff truck j departs no earlier than truck i arrives;*
- $a_i < d_i (1 \le i \le n)$ *which means for each truck, the arrival time should strictly smaller than its departure time;*
- $n > m$ *which satisfies the over-constrained condition;*
- *sort all the* a_i *and* d_i *in an increasing order, and let* t_r $(r = 1, 2, ..., 2n)$ *correspond to these* $2n$ *numbers such that* $t_1 \le t2 \le \cdots \le t_{2n}$*. Using this notation, we can easily formulate the set of capacity constraints later.*

Variables:

y_{ik}: 1 if truck i is assigned to dock k; 0, otherwise.
z_{ijkl}: 1 if truck i is assigned to dock k and truck j is assigned to dock l; 0 otherwise.

1.1. Mathematical model

We call this problem CROSS-DOCK:

[CROSS-DOCK]

$$min \sum_{k=1}^{m} \sum_{l=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{kl} t_{kl} z_{ijkl} + \sum_{i=1}^{n} \left(\sum_{j=1}^{n} p_{ij} f_{ij} \left(1 - \sum_{k=1}^{m} \sum_{l=1}^{m} z_{ijkl} \right) \right) \tag{1}$$

$s.t.$

$$\sum_{k=1}^{m} y_{ik} \le 1, \qquad \forall i \tag{2}$$

$$z_{ijkl} \le y_{ik}, \qquad \forall i, j, k, l : j \ne i \tag{3}$$

$$z_{ijkl} \le y_{jl}, \qquad \forall i, j, k, l : j \ne i \tag{4}$$

$$y_{ik} + y_{jl} - 1 \le z_{ijkl} \qquad \forall i, j, k, l : j \ne i \tag{5}$$

$$\hat{x}_{ij} + \hat{x}_{ji} \ge z_{ijkk} \qquad \forall i, j, k, l : j \ne i \tag{6}$$

$$\sum_{k=1}^{m} \sum_{l=1}^{m} \sum_{j}^{n} \sum_{i \in \{i : a_i \le t_r\}} f_{ij} z_{ijkl} - \sum_{k=1}^{m} \sum_{l=1}^{m} \sum_{i}^{n} \sum_{j \in \{j : d_j \le t_r\}} f_{ij} z_{ijkl} \le C \qquad \forall r \in \{1, 2, \ldots, 2n\} \tag{7}$$

$$f_{ij} z_{ijkl} (d_j - a_i - t_{kl}) \ge 0 \qquad \forall i, j, k, l : j \ne i \tag{8}$$

$$y_{ik} \in \{0, 1\}, z_{ijkl} \in \{0, 1\} \tag{9}$$

The first part in (1) accounts for the cost of transport, and the second part calculates the penalty for the unfulfilled transports. (2) ensures that a truck can be assigned to no more than one dock.

Consider constraints (8) before the others. (8) indicates that, if truck i uses dock k and truck j uses dock l then the variable z_{ijkl} *may* take 1, only if the arrival of i plus the transfer time from k to l is not later than the departure of j.

That means, there is a sense of *direction of flow* associated to each variable z_{ijkl} which is not covered in the definition of this variable in (Miao et al., 2009). Therefore, to this end, we associate such flow direction to the variable z_{ijkl}.

(3)-(4) indicate that for a pallet transfer between truck i and truck j, i can use dock k and j can use dock l, if i is allocated to k and j is allocated to l.

Constraints (5) imply that if the truck i is assigned to dock k and truck j to dock l, a bidirectional transfer between the two trucks —from i to j represented by z_{ijkl} and from j to i represented by z_{jilk}— must take place. If the transfer does not take place, then *not* both trucks can be docked. Moreover, if transfer from i to j does not take place, from j to i must not take place either. I.e. $z_{ijkl} = 1$ iff $z_{jilk} = 1$ and $z_{ijkl} = 0$ iff $z_{jilk} = 0$.

Constraints (6) ensure that truck i and truck j can use the same dock for realizing the transfer of pallets from i to j, only if their time windows do not intersect —i.e., i leaves no later than j arrives.

Constraints (7) guarantee that at every event time (arrival and/or departure of a truck), the capacity of cross dock is respected.
For a given $i, j, k, l : j \neq i$, the corresponding constraint in (8) exists only if f_{ij} and $(d_j - a_i - t_{kl})$ are nonzero, otherwise f_{ij} or $(d_j - a_i - t_{kl})$ would void the constraint. As a consequence, this would be z_{jilk} which influences the value that z_{ijkl} must take —as the left-hand side of (5) is the same for z_{ijkl} and z_{jilk}.

2. Issues in the model

The model possess several issues some of which we could identify are listed in the sequel:

2.1. *Constraints (5) eliminate correct solutions of the problem*

Case 1: Given $i, j \neq i$ such that $f_{ij} = 0$, according to Note 1, if j would have already left before i arrives then $f_{ij} = 0$ and i would not be able to deliver to j. But, if $f_{ji} \neq 0$ and a sufficient capacity on the cross dock would have been available, truck j might have already dropped off its cargo on the cross dock buffer —before it departs. This cargo can be later on transferred to i, once i arrives. Having $f_{ij} = 0$, voids the corresponding constraint in (8) and this constraint does not make any decision on z_{ijkl}. If for some k, l, $z_{jilk} = 1$, as it is possible, then constraint (5) forces z_{ijkl} to take 1, too, for a zero-size load of pallets from i to j.

When $f_{ij} = 0$ by Note 1 and $z_{ijkl} = 1$ (due to equality with z_{jilk}), we have $p_{ij} f_{ij} \left(1 - \sum_{k=1}^{m} \sum_{l=1}^{m} z_{ijkl}\right)$ equal to zero because no penalty is going to be paid for a flow that does not exist, i.e. $f_{ij} = 0$.

However, still the first term in the objective function (i.e. for the same i, j, $\sum_{k=1}^{m} \sum_{l=1}^{m} c_{kl} t_{kl} z_{ijkl}$) is contributing in the objective value by forcing to pay transport cost for pallets, which do not exist. That means, for $f_{ij} = 0$ we still have to pay for the transfer cost.

This is a direct consequence of tying up the destiny of z_{ijkl} and z_{jilk} to each other using (5).

Case 2: Given $i, j, k, l : j \neq i$, suppose that $(d_j - a_i) \geq 0$ and $(d_j - a_i - t_{kl}) \leq 0$ such that j does not leave before i arrives, but there is no sufficient time to make the transfer between dock k and dock l. However, if j has left something on the buffer before it leaves, i which arrives later can still take it and z_{jilk} can take 1. This does not fall into the category of Note 1 and f_{ij} is not necessarily 0 (might be strictly positive). Therefore, corresponding constraint in (8) exists and forces $z_{ijkl} = 0$. On the other hand, (5) forces z_{jilk} to take 0 and avoids it to take 1.

In such a case, having $z_{jilk} = 1$ in a feasible solution of the real-life problem is possible (assuming a sufficient capacity), but it is eliminated by this model.

Therefore, forcing z_{jilk} and z_{ijkl} to take the same values (i.e. having (5) in the model) is *incorrect* and those constraints imposing such conditions must be removed from CROSS-DOCK.

2.2. *Constraints* (6)

Because a variable z_{ijkl} implies a sense of direction of flow, we make the following re-definition:

z_{ijkl}: 1 if pallets of truck i which is assigned to dock k are transferred to the truck j which is assigned to dock l; 0 otherwise.

There are two different cases allowing two distinct trucks i and j use the same dock k: 1) truck i drops its pallets and leaves dock k before j arrives at dock k, 2) truck j drops its pallets and leaves dock k before truck i arrives at dock k.

Therefore, z_{ijkk} only depends on whether i leaves before j arrives or not. Whether j departs before i arrives or not is *not* directly related to z_{ijkk}. Because if j depart before i arrives (i.e. $\hat{x}_{ij} = 0$ and $\hat{x}_{ji} = 1$), still there is no possibility of transferring from i to j, but the constraint turns to $z_{ijkk} \leq \hat{x}_{ij} + \hat{x}_{ji} = 1$ which is of no effect unless it causes numerical deficiencies. While $z_{ijkk} \leq \hat{x}_{ij} = 0$, clearly sets the variable to the correct value.

Consequently, constraints (6) should be replaced by:

$$\hat{x}_{ij} \geq z_{ijkk} \qquad \forall i, j \neq i, k, l \tag{10}$$

3. Example

Let $m = 2$ be the number of docks and $n = 4$ be the number of trucks. Moreover, let: $c = t = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $p = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$, $f = \begin{pmatrix} 0 & 2 & 2 & 2 \\ 2 & 0 & 2 & 2 \\ 2 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 \end{pmatrix}$, $a = \begin{pmatrix} 15.42 \\ 15.50 \\ 17.00 \\ 16.52 \end{pmatrix}$ and $d = \begin{pmatrix} 16.41 \\ 16.41 \\ 18.00 \\ 17.57 \end{pmatrix}$.

The optimal solution, s^*, of this problem using the model CROSS-DOCK in Miao et al. (2009) has an objective value 16 and all the variables take 0. We have $\hat{x}_{13} = 1\hat{x}_{14} = 1, \hat{x}_{23} = 1, \hat{x}_{24} = 1$.

Here, the objective only accounts for the penalties because no pallet transfer takes place.

The *claim* is that this model does not correctly represent the problem. In other words, there are other feasible solutions with better objective values to the problem which are eliminated by this model (in particular, constraints (5)).

We remove constraint (5) and rectify (6) and call the model R-CROSS-DOCK:

[R-CROSS-DOCK]

$$min \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{kl}t_{kl}z_{ijk} + \sum_{i=1}^{n}\left(\sum_{j=1}^{n} p_{ij}f_{ij}\left(1 - \sum_{k=1}^{m}\sum_{l=1}^{m} z_{ijkl}\right)\right) \tag{11}$$

$s.t.$

$$\sum_{k=1}^{m} y_{ik} \leq 1, \qquad \forall i \tag{12}$$

$$z_{ijkl} \leq y_{ik}, \qquad \forall i, j, k, l \tag{13}$$

$$z_{ijkl} \leq y_{jl}, \qquad \forall i, j, k, l \tag{14}$$

$$y_{ik} + y_{jk} \leq 1 + \hat{x}_{ij} + \hat{x}_{ji} \qquad \forall i, j, k \tag{15}$$

$$z_{ijkk} \leq \hat{x}_{ij} \qquad \forall i, j \neq i, k, l \tag{16}$$

$$\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{j}^{n}\sum_{i\in\{i:a_i\leq t_r\}} f_{ij}z_{ijkl} + \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i}^{n}\sum_{j\in\{j:d_j\leq t_r\}} f_{ij}z_{ijkl} \leq C \qquad \forall r \in \{1, 2, \ldots, 2n\} \tag{17}$$

$$z_{ijkl} = 0 \qquad \forall i, j, k, l : j \neq i, (d_j - a_i - t_{kl}) \leq 0 \tag{18}$$

$$y_{ik} \in \{0, 1\}, z_{ijkl} \in \{0, 1\} \tag{19}$$

The new constraints (15) ensure that if the arrival/departure time windows of two trucks i and j overlap ($\hat{x}_{ij} = \hat{x}_{ji} = 0$), either of them can be docked at a dock k (not both).

The optimal solution, s'^{*}, of this problem using the model CROSS-DOCK has an optimal objective value of 11 and $z_{1311} = 1$, $z_{1412} = 1$, $z_{2321} = 1$, $z_{2422} = 1$, $z_{4321} = 1$, $y_{11} = 1$, $y_{22} = 1$, $y_{31} = 1$, $y_{42} = 1$ while $\hat{x}_{13} = 1$, $\hat{x}_{14} = 1$, $\hat{x}_{23} = 1$ and $\hat{x}_{24} = 1$

The relative gap between the two solution is 45.45 %.

3.1. *Why s'^{*} is not feasible in s^{*}*

A diagnosis of infeasibility has reported that an *Irreducibly Inconsistent Set (IIS)* of constraints is consisted of $y_{11} + y_{22} <= 1 + z_{1212}$ as the cause of infeasibility.

In s'^{*}, we have $y_{11} = 1$, $y_{22} = 1$. By substituting in constraint (6), we have $1 + 1 = y_{11} + y_{22} <= 1 + z_{1212}$ ($1 + 1 = y_{11} + y_{22} <= 1 + z_{2121}$) which forces $z_{2121} = 1$.

However, constraint (8) has already suggested $z_{1212} = 0$ (and $z_{2121} = 0$) as $d_j - a_i - t_{kl} < 0$ (and $d_i - a_j - t_{lk} < 0$) which is a contradiction.

4. Valid inequalities

Fischetti and Lodi (2007) have shown that rank-1 Chvàtal-Gomory inequalities are often very effective in closing the integrality gaps to an utmost degree. Moreover, while optimizing over the first Chvàtal-Gomory closure, one might succeed in identifying new classes of valid inequalities for the (M)IP models at hand, some of which might be facet-defining inequalities.

Optimizing R-CROSS-DOCK over the first Chvàtal-Gomory closure has led to identify *four* classes of valid inequalities for the model.

i) *Valid inequalities I:* If truck i is not assigned to dock k, no transfer from i to any $j \neq i$, assuming that i is allocated to k, can take place.

$$\sum_{l}^{n} z_{ijkl} \leq y_{ik}, \qquad \forall i, j \neq i, \ \ k \tag{20}$$

Similarly, for truck j and dock l,

$$\sum_{k}^{n} z_{ijkl} \leq y_{jl}, \qquad \forall i, j \neq i, \ \ l. \tag{21}$$

We refer to R-CROSS-DOCK together with (20) and (21) by R-CROSS-DOCK-1.

ii) *Valid inequalities (II) – Triangle inequalities:* if truck i is assigned to dock m and truck k to n then at most two variables among z_{ijmn}, z_{ikmn} and z_{jkmn} can take 1.

$$z_{ijmn} + z_{ikmn} + z_{jkmn} \leq y_{im} + y_{kn} \qquad \forall i, j \neq i \;, k \notin \{i, j\}, \;\; m, n \tag{22}$$

We refer to R-CROSS-DOCK together with (22) by R-CROSS-DOCK-2.

iii) *Valid inequalities (III) – the aggregation of* (3) *(and* (4)*):* By aggregating (3) (and (4)) and summing up over the dock indexes yields:

$$\sum_k z_{ijkl} \leq \sum_k y_{ik}, \qquad \forall i, j \neq i, \;\; l \tag{23}$$

$$\sum_l z_{ijkl} \leq \sum_l y_{jl}, \qquad \forall i, j \neq i, \;\; k \tag{24}$$

We refer to R-CROSS-DOCK together with (23) and (24) by R-CROSS-DOCK-3.

iv) *Valid inequalities (IV):* Clearly, for every i and $j \neq i$, we always have $\sum_{k,l} z_{ijkl} \leq 1$.

$$\sum_{k,l} z_{ijkl} \leq \sum_k y_{i\,k} \qquad \forall i, j \neq i \tag{25}$$

$$\sum_{k,l} z_{ijkl} \leq \sum_l y_{j\,l} \qquad \forall i, j \neq i \tag{26}$$

The left hand side can be zero if $\sum_k y_{i\,k} = 0$ or $\sum_k y_{j\,k} = 0$. Therefore, the following constraints are also valid.

We refer to R-CROSS-DOCK together with (25) and (26) by R-CROSS-DOCK-4.

5. Polyhedral Properties

We show that under certain conditions, R-CROSS-DOCK is a full-dimensional polytope and some of the proposed valid inequalities are indeed facet-defining inequalities (Balas, 1997).

Note 2. *Every constraint in* (16) *and* (18) *either fixes a variable to zero or is a redundant constraint.*

5.1. dimension of R-CROSS-DOCK

There are two cases: 1) all the constraints in (16) and (18) are redundant in which case we identify $|N|(|N|-1)|M||M| + |N||M|$ linearly independent points in R-CROSS-DOCK polytope or, 2) some of the constraints in (16) and (18) are implying equality (fixing some variables to 0). In this case, the dimension of polytope would be $|N|(|N|-1)|M||M| + |N||M| - |Eq|$ where $|Eq|$ is the cardinality of set of equality-implying constraint in (16) and (18).

Theorem 1. *Given $C \geq f_{ij}, \forall i, j \neq i$, if all constraints in* (16) *and* (18) *are redundant, R-CROSS-DOCK polytope is a full-dimensional polytope.*

Proof of Theorem 1. Let $\mathbf{y}^T = [y_{11}, y_{12}, \ldots, y_{1\,m}, \ldots, y_{n\,m-1}, y_{n\,m}]$, $\mathbf{z}^T = [z_{1211}, z_{1212}, \ldots, z_{12\,m\,m}, z_{1311}, \ldots, z_{n,n-1,m,m}]$. Let $\mathbf{a}^T = [\mathbf{y}^T \mid \mathbf{z}^T]$.

a) Let $\mathbf{b}^T_{y_{ik}} = e(y_{ik})$, $\quad \mathbf{b}^T \in \mathbb{B}^{|\mathbf{a}^T|}$, $\quad \forall i, k$

b) Let $\mathbf{c}^T_{z_{ijkl}} = e(y_{ik}) + e(y_{jl}) + e(z_{ijkl}), \quad \mathbf{c}^T \in \mathbb{B}^{|\mathbf{a}^T|}, \quad \forall i, j, k, l : j \neq i$

where $e(i)$ is the unit vector having i-th entry equal to 1 and 0, elsewhere .

Note that $\mathbf{b}^T_{y_{ik}}$ and $\mathbf{c}^T_{z_{ijkl}}, \forall i, j, k, l$ are all feasible solutions of R-CROSS-DOCK.

Let $A = \begin{bmatrix} \mathbf{b}^T \\ \mathbf{c}^T \end{bmatrix}$ be an square matrix of $|N|(|N|-1)|M||M| + |N||M|$.

It can be easily shown that the rows of A can be re-ordered to represent a lower triangular matrix where all the diagonal elements are 1.

The matrix A is non-singular and all rows of A form a set of $|N|(|N|-1)|M||M|+|N||M|$ linearly independent vectors.

Therefore, R-CROSS-DOCK polytope in which none of the constraints (16) and (18)) imply equality, is a full-dimensional polytope. □

Example 1. *let $|M| = 2$ and $N = 3$ be the total number of docks and trucks, respectively.*

$$\begin{aligned} \mathbf{a}^T = [&y_{11}, y_{12}, y_{21}, y_{22}, y_{31}, y_{32} \mid z_{1211}, z_{1212}, z_{1221}, z_{1222}, z_{1311}, z_{1312}, z_{1321}, z_{1322}, z_{2111}, z_{2112}, \\ &z_{2121}, z_{2122}, z_{2311}, z_{2312}, z_{2321}, z_{2322}, z_{3111}, z_{3112}, z_{3121}, z_{3122}, z_{3211}, z_{3212}, z_{3221}, z_{3222}]. \end{aligned} \tag{27}$$

The following lines compose a square matrix $A_{30\times30}$ where $30 = |N|(|N|-1)|M||M| + |N||M|$.

$$\begin{aligned}
\mathbf{b}^T_{y_{11}} &= [\mathbf{1},0,0,0,0,0 \mid 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{b}^T_{y_{12}} &= [0,\mathbf{1},0,0,0,0 \mid 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{b}^T_{y_{21}} &= [0,0,\mathbf{1},0,0,0 \mid 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{b}^T_{y_{22}} &= [0,0,0,\mathbf{1},0,0 \mid 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{b}^T_{y_{31}} &= [0,0,0,0,\mathbf{1},0 \mid 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{b}^T_{y_{32}} &= [0,0,0,0,0,\mathbf{1} \mid 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{1211}} &= [\mathbf{1},0,\mathbf{1},0,0,0 \mid \mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{1212}} &= [\mathbf{1},0,0,\mathbf{1},0,0 \mid 0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{1221}} &= [0,\mathbf{1},\mathbf{1},0,0,0 \mid 0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{1222}} &= [0,\mathbf{1},0,\mathbf{1},0,0 \mid 0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{1311}} &= [\mathbf{1},0,0,0,\mathbf{1},0 \mid 0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{1312}} &= [\mathbf{1},0,0,0,0,\mathbf{1} \mid 0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{1321}} &= [0,\mathbf{1},0,0,\mathbf{1},0 \mid 0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{1322}} &= [0,\mathbf{1},0,0,0,\mathbf{1} \mid 0,0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{2111}} &= [\mathbf{1},0,\mathbf{1},0,0,0 \mid 0,0,0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{2112}} &= [0,\mathbf{1},\mathbf{1},0,0,0 \mid 0,0,0,0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{2121}} &= [\mathbf{1},0,0,\mathbf{1},0,0 \mid 0,0,0,0,0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{2122}} &= [0,\mathbf{1},0,\mathbf{1},0,0 \mid 0,0,0,0,0,0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{2311}} &= [0,0,\mathbf{1},0,\mathbf{1},0 \mid 0,0,0,0,0,0,0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{2312}} &= [0,0,\mathbf{1},0,0,\mathbf{1} \mid 0,0,0,0,0,0,0,0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{2321}} &= [0,0,0,\mathbf{1},\mathbf{1},0 \mid 0,0,0,0,0,0,0,0,0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{2322}} &= [0,0,0,\mathbf{1},0,\mathbf{1} \mid 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0,0]. \\
\mathbf{c}^T_{x_{3111}} &= [\mathbf{1},0,0,0,\mathbf{1},0 \mid 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\mathbf{1},0,0,0,0,0,0,0].
\end{aligned}$$

$$\mathbf{c}^T_{x_{3112}} = [0, \mathbf{1}, 0, 0, \mathbf{1}, 0 \mid 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \mathbf{1}, 0, 0, 0, 0, 0, 0].$$
$$\mathbf{c}^T_{x_{3121}} = [\mathbf{1}, 0, 0, 0, 0, \mathbf{1} \mid 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \mathbf{1}, 0, 0, 0, 0, 0].$$
$$\mathbf{c}^T_{x_{3122}} = [0, \mathbf{1}, 0, 0, 0, \mathbf{1} \mid 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \mathbf{1}, 0, 0, 0, 0].$$
$$\mathbf{c}^T_{x_{3211}} = [0, 0, \mathbf{1}, 0, \mathbf{1}, 0 \mid 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \mathbf{1}, 0, 0, 0].$$
$$\mathbf{c}^T_{x_{3212}} = [0, 0, 0, \mathbf{1}, \mathbf{1}, 0 \mid 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \mathbf{1}, 0, 0].$$
$$\mathbf{c}^T_{x_{3221}} = [0, 0, \mathbf{1}, 0, 0, \mathbf{1} \mid 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \mathbf{1}, 0].$$
$$\mathbf{c}^T_{x_{3222}} = [0, 0, 0, \mathbf{1}, 0, \mathbf{1} \mid 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \mathbf{1}].$$

$det(A) = 1$ (lower triangular) and the rows are linearly independent (notice also that the origin is a feasible solution of the model). Therefore, the dimension of this polytope is 30.

Lemma 2. *Let $C \geq f_{ij}, \forall i, j \neq i$. Let also those equality implying constraints (16) and (18) exist in A and matrix A' being obtained from matrix A by removing the rows in $\mathbf{c}^T$ corresponding to every equality constraint in (16) and (18). $rank(A') = dim(\text{R-CROSS-DOCK})$ is the dimension of resulting polytope.* □

Theorem 3 (Some Facets of R-CROSS-DOCK).

1) *The valid inequalities (20) and (21) are face-defining.*
2) *The valid inequalities (23) and (24) are face-defining.*
3) *The valid inequalities (25) and (26) are face-defining.*

Proof of Theorem 3. 1) For a given i, k, all the rows of A' after removing the one corresponding to $\mathbf{b}^T_{y_{ik}} = e(y_{ik})$ are linearly independent and satisfy (20), (23) and (25) with equality. Hence $rank(A') - 1$ linearly independent solutions satisfying these inequalities with equality.

2) For a given j, l, all the rows of A after removing the one corresponding to $\mathbf{b}^T_{y_{jl}} = e(y_{jl})$ are linearly independent and satisfy (21), (24) and (26) with equality. Hence $rank(A') - 1$ linearly independent solutions satisfying these inequalities with equality.

Theorem 4. *The valid inequalities (13) and (14) are face-defining.*

Proof of Theorem 4. The proof is the same as in Theorem 3.

6. Numerical results

The data are generated in the same way as in (Miao et al., 2009). The instances are publicly available[1].

Our computational experiments are conducted on an Intel(R)Xeon(R), 2 processors of 2.40 GHz and 2.39 GHz with 12GB RAM.

In total we have 85 instances ranging from $|M| = 3, |N| = 10$ up to $|M| = 8, |N| = 40$. A time limit of 1500 seconds has been set in CPLEX 12.4 for solving these instances. However, in some cases, where resolution of the LP at particular node of branch-and-bound(-and-cut) algorithm takes longer time, CPLEX does not manage to terminate exactly on 1500 seconds and it terminates upon termination of the LP resolution. This might cause going beyond the 1500 seconds for some instances.

Table A.1 through Table A.6 report the computational results for the following cases:

A.1 reports the numerical results using the revised model without any additional valid inequality (i.e. R-CROSS-DOCK).

[1] www.LGI2A.univ-artois.fr/~gelareh/downloads/cross_dock/data.rar

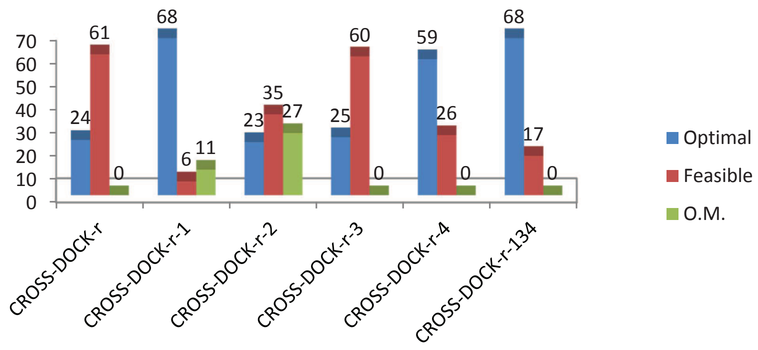

Figure 1: Statistics on resolution of instances using variants of CROSS-DOCKS-r.

- A.2 is concerned with the revised model and constraints (20)- (21) (i.e. R-CROSS-DOCK-1).
- A.3 reports numerical results of the revised model and (22) (i.e. R-CROSS-DOCK-2).
- A.4 considers the revised model and constraints (23)- (24) (i.e. R-CROSS-DOCK-3).
- A.5 is concerned with the revised model and constraints (25)- (26) (i.e. R-CROSS-DOCK-4).
- A.6 summarizes the numerical results of the revised model and all the facet-defining inequalities (20), (21), (23), (24), (25) and (26)(i.e. R-CROSS-DOCK-134).

Figure 1 summarizes the statistics of termination of CPLEX on instances of different models.

As shown in this figure, in the revised model, only 24 instances were solved to optimality within the given time limit while in all other cases where facet-defining inequalities were added, the number of optimally solve instances is more than twice the revised model (the model without any additional constraints).

While the number of instances where CPLEX terminated to feasibility status (hitting the time limit with feasible solution) without proof of optimality almost always decreases by introducing new additional valid inequalities, however, R-CROSS-DOCK-2 and R-CROSS-DOCK-3 (which contains facet-defining inequalities) cause CPLEX to run into memory issue with some instances (our-of-memory (O.M.)).

The general conclusion is that R-CROSS-DOCK-134 is an absolutely superior model and for every solution, the quality is known.

Figure 2 reports that the average solution qualities for those instances for which CPLEX terminated with optimality or feasibility. Clearly, R-CROSS-DOCK-1, R-CROSS-DOCK-4 and R-CROSS-DOCK-134 are superior.

When also Figure 1 is taken into account, R-CROSS-DOCK-134 is absolutely superior.

The average computational time (again for those terminated to feasibility or optimality) is also reported in Figure 3.

In the first glance, it seems that R-CROSS-DOCK-1 is superior, but we should recall that the number of optimally solved instances in R-CROSS-DOCK-134 is more than in R-CROSS-DOCK-1.

Figure 4 depicts a comparison between the average number of branch-and-bound nodes for instances solved by each model.

A comparison between Figure 4 and Figure 3 reveals that many more nodes being processed within much less computational time in R-CROSS-DOCK-134 and R-CROSS-DOCK-1. That means the LP resolution becomes significantly easier after adding the corresponding constraints.

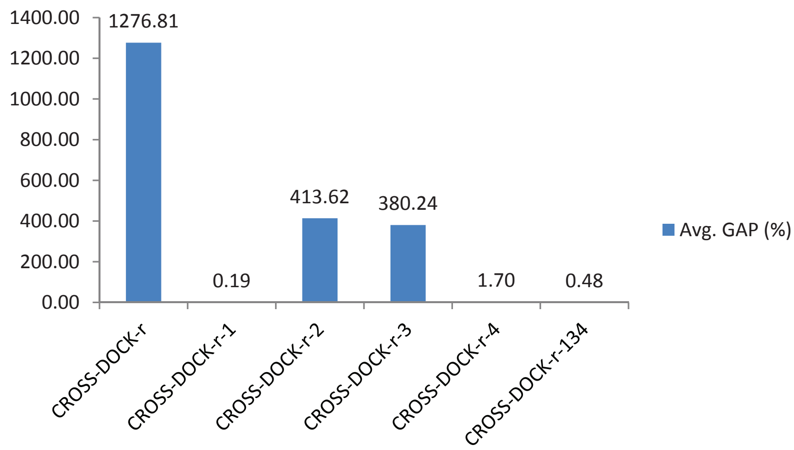

Figure 2: Average solution quality within the time limit for variants of CROSS-DOCKS-r with feasible or optimal termination.

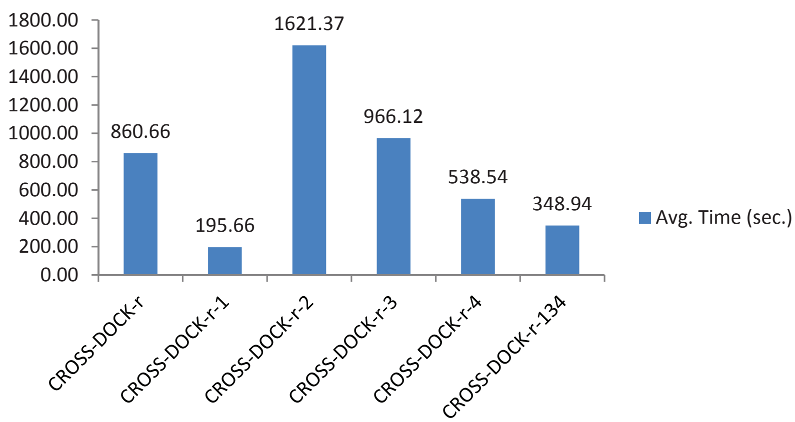

Figure 3: Average CPU time for variants of CROSS-DOCKS-r with feasible or optimal termination.

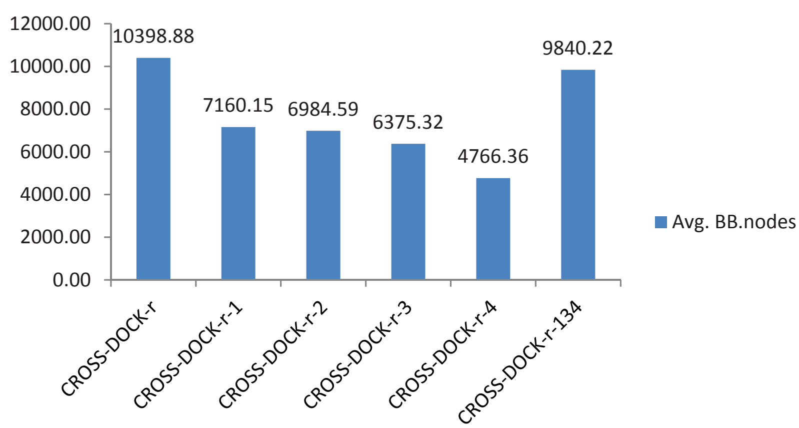

Figure 4: Average number of branch-and-bound nodes for variants of CROSS-DOCKS-r with feasible or optimal termination.

The root node quality (the LP bound) for all the models is depicted in Figure 5.

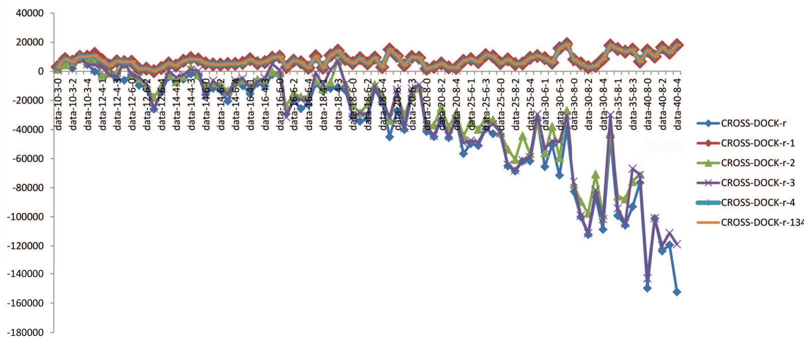

Figure 5: The root node values for variants of CROSS-DOCKS-r with feasible or optimal termination.

In Figure 5, the results for CROSS-DOCK-r-1, CROSS-DOCK-r-4 and CROSS-DOCK-r-4 coincide.

Clearly, CROSS-DOCK-r-1, CROSS-DOCK-r-4 and CROSS-DOCK-r-4 report much better LP relaxation polytopes than the others. This can justify why many more nodes can be processed in much less computational time as depicted in Figure 4.

Extensive computational result for each model is reported in Table A.1 through Table A.6. All the model used the same set of instances. In each table, results are reported in two blocks (left and right). In each block, the table is organized as follows. The first column reports the name of instance. The second column reports the LP relaxation values. In the third column, for those instances where optimality is proven or terminated to feasibility, we report the objective value. Those instances for which CPLEX failed to report any solution, we indicated it by '–'. The CPLEX status upon termination is reported in the fourth column (Status). (O.M.) in this column indicates that CPLEX has run out of memory and no solution to the problem has been reported. Total number of branch-and-bound processed by CPLEX are reported in the fifth column (nbNode). In some case, the number of nodes reported by CPLEX was incorrect (e.g. negative integer numbers) where we needed to consult our log file to determine the minimum number of processed nodes. The computational time reported by CPLEX is brought in the next column (CPUTime) and the solution qualities are also reported in the last column (MIPRelativeGap).

7. Summary, conclusion and outlook to future work

We have shown that the formulation proposed earlier for the cross dock problem needed to be corrected due to the fact that it eliminated some (perhaps optimal) feasible solutions. We then proposed a correction and several classes of valid inequalities. We also showed that the valid inequalities significantly contribute in accelerating resolution and performance of CPLEX. Moreover, after determining the dimension of polytope, we showed that some of those valid inequalities are, in fact, the facet-defining inequalities.

One can use some separation algorithms to separate the introduced facet-defining inequalities on the fly instead of adding all of them to the model. They can also be introduced as lazy constraint which is currently in use within most of the general-purpose MIP solvers. This might help in avoiding to start with a huge LP to be solved at every node of branch-and-bound and improve the efficiency.

Our future work will be on identifying more facets of the polytope and developing more efficient branch-and-bound-and-cut methods. Metaheuristics and other exact methods such as decompositions also deserve attention.

Acknowledgement

The authors would like to thank Professor Andrea Lodi for distributing the code for optimizing on Chvàtal-Gomory closure.

References

Balas, E., 1997. Recognizing facet defining inequalities. Acta Mathematica Vietnamica 22 (1), 7–28.

Fischetti, M., Lodi, A., 2007. Optimizing over the first chvtal closure. Mathematical Programming 110, 3–20.

Lim, A., Ma, H., Miao, Z., 2006. Truck dock assignment problem with operational time constraint within crossdocks. In: Ali, M., Dapoigny, R. (Eds.), Advances in Applied Artificial Intelligence. Vol. 4031 of Lecture Notes in Computer Science. Springer Berlin Heidelberg, pp. 262–271.

Miao, Z., Lim, A., Ma, H., 2009. Truck dock assignment problem with operational time constraint within crossdocks. European Journal of Operational Research 192 (1), 105 – 115.

Appendix A. Numerical experiments.

Table A.1: Numerical results: R-CROSS-DOCK

Instance	R.Obj.	Obj.	Status	nbNode	CPUTime	MIPRelativeGap	Instance	R.Obj.	Obj.	Status	nbNode	CPUTime	MIPRelativeGap
data-10-3-0	1525.2	3045	Optimal	109	2.75	0	data-18-6-3	-12385.4	9557	Feasible	9088	501.67	115.63
data-10-3-1	8410	8410	Optimal	0	2.15	0	data-18-6-4	-21116.6	2995	Feasible	>12062	493.98	363.42
data-10-3-2	2077.33	6545	Optimal	83	3.81	0	data-20-6-0	-45026.1	17635	Feasible	5696	1013.4	206.51
data-10-3-3	8266	10004	Optimal	148	2.53	0	data-20-6-1	-27178	10829	Feasible	>11243	551	140.85
data-10-3-4	4710.28	9985	Optimal	212	3.12	0	data-20-6-2	-40021.8	4393	Feasible	>21097	1089.4	534.78
data-12-4-0	-91.3711	13413	Optimal	962	16.22	0	data-20-6-3	-17052.9	10302	Feasible	>11548	763.02	171.46
data-12-4-1	-3252.84	7747	Optimal	4862	29.34	0	data-20-6-4	-11491.2	10776	Feasible	>16107	712.58	125.79
data-12-4-2	-494	4032	Optimal	2531	22.28	0	data-20-8-0	-41343.1	2088	Feasible	12565	1044.4	1082.83
data-12-4-3	-5101	8562	Optimal	1752	58.05	0	data-20-8-1	-44834.4	3454	Feasible	>9749	1129.45	697.51
data-12-4-4	-6125.44	6353	Optimal	943	15.65	0	data-20-8-2	-33807.9	5731	Feasible	8691	1603.52	372.3
data-12-6-0	-2665.94	8641	Feasible	>14788	178.42	90.92	data-20-8-3	-45576.4	3178	Feasible	11063	1265.26	1126.03
data-12-6-1	-9672.03	1723	Feasible	>21438	218.14	113.48	data-20-8-4	-32882.2	2751	Feasible	12006	914.32	951.15
data-12-6-2	-10842.7	1989	Feasible	17561	307.29	179.68	data-25-6-0	-56541.8	7478	Feasible	14425	1026.77	550.73
data-12-6-3	-25956.2	259	Feasible	>19225	210.74	2557.23	data-25-6-1	-49634.3	9313	Feasible	>12462	1399.92	382.18
data-12-6-4	-14301.2	2401	Feasible	18558	252.05	281.57	data-25-6-2	-50954.8	6744	Feasible	11020	1678.2	471.89
data-14-4-0	-3855.88	5627	Optimal	5870	46.22	0	data-25-6-3	-39582	12228	Feasible	>12196	1309.72	220.48
data-14-4-1	-7530.79	3932	Optimal	36027	58.66	0	data-25-6-4	-42844.2	10708	Feasible	12342	1656.7	319.53
data-14-4-2	-5476.53	7298	Optimal	6156	47.21	0	data-25-8-0	-43234	6778	Feasible	4157	1743.98	602.21
data-14-4-3	-1737.13	9570	Optimal	7639	26.08	0	data-25-8-1	-65019.6	8853	Feasible	5618	1630.35	575.7
data-14-4-4	-3095.92	8762	Optimal	8096	43.63	0	data-25-8-2	-68505.9	6293	Feasible	4955	1643.44	1042.47
data-14-6-0	-15955.4	5796	Feasible	>17008	271.43	136.8	data-25-8-3	-61732.7	5885	Feasible	4730	1709.85	882.87
data-14-6-1	-11643.1	6376	Feasible	>21645	393.37	144.03	data-25-8-4	-61583.5	10523	Feasible	3534	1736.35	582.93
data-14-6-2	-12784.2	5084	Feasible	11579	331.6	160.13	data-30-6-0	-33592.6	11801	Feasible	4314	1876.72	275.96
data-14-6-3	-20456.7	5518	Feasible	>19202	414.7	190.78	data-30-6-1	-65503.7	10756	Feasible	3587	1768.65	569.08
data-14-6-4	-8176.49	5768	Feasible	>18238	244.88	127.06	data-30-6-2	-50624.6	6377	Feasible	3559	1837.05	670.07
data-16-4-0	-9883.65	5793	Optimal	50057	190.34	0	data-30-6-3	-71442.2	17479	Feasible	3517	1784.15	401.92
data-16-4-1	-15545.2	9050	Optimal	43445	136.63	0	data-30-6-4	-39128.7	19482	Feasible	5699	1747.26	168.35
data-16-4-2	-8202.56	5579	Optimal	7356	29.66	0	data-30-8-0	-82584.9	9152	Feasible	900	1808.68	931.32
data-16-4-3	-10093.7	6649	Optimal	9688	61.96	0	data-30-8-1	-100149	5922	Feasible	3278	1692.55	1594.08
data-16-4-4	-2229.71	9320	Optimal	3376	36.36	0	data-30-8-2	-112415	3618	Feasible	2306	1695.73	3005.9
data-16-6-0	-2921.02	11177	Feasible	7168	359.49	69.08	data-30-8-3	-86273.8	4996	Feasible	888	1856.99	1730.83
data-16-6-1	-29401.4	3015	Feasible	>20729	438.39	450.02	data-30-8-4	-108864	10177	Feasible	1883	1839.61	1062.64
data-16-6-2	-19841.8	7813	Feasible	7567	493.45	155.09	data-35-8-0	-45586.2	19806	Feasible	638	2008.28	312.65
data-16-6-3	-25605.2	6395	Feasible	>24344	330.61	256.28	data-35-8-1	-99417.9	17618	Feasible	948	1966.07	642.66
data-16-6-4	-22884.6	2618	Feasible	>12845	483.24	382.68	data-35-8-2	-105972	14741	Feasible	2191	1730.11	758.48
data-18-4-0	-8329.27	10376	Optimal	60100	195.92	0	data-35-8-3	-93231.7	18489	Feasible	531	1922.06	567.23
data-18-4-1	-13765.8	2585	Feasible	>32357	225.58	125.62	data-35-8-4	-76753.6	7820	Feasible	1180	2033.07	990.69
data-18-4-2	-11757.3	12451	Optimal	20263	113.46	0	data-40-8-0	-149270	17772	Feasible	195	2048.17	906.6
data-18-4-3	-10909.3	15609	Optimal	4370	45.23	0	data-40-8-1	-101318	15237	Feasible	272	2191.41	746.51
data-18-4-4	-12985.8	9467	Optimal	26330	120.67	0	data-40-8-2	-123759	21896	Feasible	319	1915.9	645.58
data-18-6-0	-30809.4	5875	Feasible	7293	531.48	339.92	data-40-8-3	-119571	16153	Feasible	443	1703.33	804
data-18-6-1	-34361.2	10046	Feasible	>12877	886.37	232.07	data-40-8-4	-151861	22167	Feasible	46	2278.57	758.53
data-18-6-2	-32524.2	6000	Feasible	>16060	955.16	318.15							

Table A.2: Numerical results: R-CROSS-DOCK and (20) and (21)

Instance	R.Obj.	Obj.	Status	nbNode	CPUTime	MIPRelativeGap
data-10-3-0	3045	3045	Optimal	0	1.12	0
data-10-3-1	8410	8410	Optimal	0	1.23	0
data-10-3-2	6545	6545	Optimal	0	1.06	0
data-10-3-3	10004	10004	Optimal	0	1.08	0
data-10-3-4	9985	9985	Optimal	0	1.11	0
data-12-4-0	12004.4	13413	Optimal	162	3.54	0
data-12-4-1	7747	7747	Optimal	0	2.34	0
data-12-4-2	4008	4032	Optimal	45	3.42	0
data-12-4-3	6923.08	8562	Optimal	108	4.4	0
data-12-4-4	6353	6353	Optimal	0	1.84	0
data-12-6-0	6592.39	8635	Optimal	4543	10.89	0
data-12-6-1	1633.79	1718	Optimal	373	8.05	0
data-12-6-2	1855.04	1983	Optimal	311	7.35	0
data-12-6-3	146.84	255	Optimal	690	11.59	0
data-12-6-4	2313.88	2397	Optimal	77	8.38	0
data-14-4-0	5418.12	5627	Optimal	75	5.26	0
data-14-4-1	3932	3932	Optimal	0	3.54	0
data-14-4-2	7298	7298	Optimal	0	2.5	0
data-14-4-3	9087.25	9570	Optimal	168	4.57	0
data-14-4-4	8130.4	8762	Optimal	105	6.41	0
data-14-6-0	5525.22	5790	Optimal	357	14.12	0
data-14-6-1	4739.49	6355	Optimal	2974	17.78	0
data-14-6-2	4659.49	5076	Optimal	642	18.36	0
data-14-6-3	5064.27	5505	Optimal	507	14.41	0
data-14-6-4	5091.67	5755	Optimal	1097	10.58	0
data-16-4-0	5778.5	5793	Optimal	65	9.62	0
data-16-4-1	8379.56	9050	Optimal	89	8.71	0
data-16-4-2	5561	5579	Optimal	39	4.51	0
data-16-4-3	6381.4	6649	Optimal	21	5.02	0
data-16-4-4	9320	9320	Optimal	0	2.93	0
data-16-6-0	10019.9	11153	Optimal	570	19.67	0
data-16-6-1	2839.54	3003	Optimal	207	17.13	0
data-16-6-2	7328.04	7799	Optimal	11020	31.61	0
data-16-6-3	6232.82	6366	Optimal	1939	17.19	0
data-16-6-4	2431.88	2602	Optimal	573	19.73	0
data-18-4-0	10375	10376	Optimal	0	7.24	0.01
data-18-4-1	2457.88	2585	Optimal	106	8.16	0
data-18-4-2	10971	12451	Optimal	112	10.78	0
data-18-4-3	14155.9	15609	Optimal	0	10.72	0
data-18-4-4	8474.01	9467	Optimal	408	6.9	0
data-18-6-0	5627.64	5800	Optimal	129	23.93	0
data-18-6-1	9044.94	10009	Optimal	1964	34.15	0
data-18-6-2	5438.94	5920	Optimal	762	24.73	0
data-18-6-3	9394.56	9552	Optimal	191	17.74	0
data-18-6-4	2870.95	2979	Optimal	1257	22.32	0
data-20-6-0	14951	17491	Optimal	3475	66.94	0
data-20-6-1	10611.9	10820	Optimal	1140	35.94	0
data-20-6-2	4180.5	4365	Optimal	13075	68.38	0
data-20-6-3	9879.25	10254	Optimal	427	30.3	0
data-20-6-4	9351.83	10773	Optimal	929	47.58	0
data-20-8-0	1805.57	2007	Optimal	695	71.2	0
data-20-8-1	3049.92	3347	Optimal	7227	154.58	0
data-20-8-2	5427.44	5693	Optimal	6477	122.83	0
data-20-8-3	2689.31	3027	Optimal	6962	129.25	0
data-20-8-4	2327.43	2640	Optimal	44531	322.64	0
data-25-6-0	7209.28	7436	Optimal	1642	44.59	0
data-25-6-1	8345.12	9202	Optimal	1169	60.25	0
data-25-6-2	6411.62	6699	Optimal	5470	81.54	0
data-25-6-3	11182	12122	Optimal	623	68.64	0
data-25-6-4	10288.4	10633	Optimal	622	48.92	0
data-25-8-0	5636.36	6575	Feasible	52173	1554.53	2.49
data-25-8-1	8247.15	8632	Optimal	12937	259.76	0
data-25-8-2	5001.81	5993	Optimal	81502	1435.44	0
data-25-8-3	5381.14	5716	Optimal	6780	149.31	0
data-25-8-4	9212.44	–	O.M.	–	–	–
data-30-6-0	10520.7	11497	Optimal	21653	185.16	0
data-30-6-1	9051.08	–	O.M.	–	–	–
data-30-6-2	5933.22	6234	Optimal	5923	196.41	0
data-30-6-3	15788.9	17281	Optimal	25928	278.35	0
data-30-6-4	18737.4	19365	Optimal	1983	89.69	0
data-30-8-0	8164.89	8860	Feasible	56436	1589.39	0.9
data-30-8-1	5358.27	5717	Feasible	37812	1602.24	2.37
data-30-8-2	2717.33	3163	Feasible	39629	1597.9	3.92
data-30-8-3	3597.62	4275	Optimal	3507	443.03	0
data-30-8-4	8396.84	–	O.M.	–	–	–
data-35-8-0	17759.4	–	O.M.	–	–	–
data-35-8-1	15304.3	–	O.M.	–	–	–
data-35-8-2	13428.8	–	O.M.	–	–	–
data-35-8-3	14931.1	15811	Feasible	37860	1637.34	1.4
data-35-8-4	6547.83	7028	Feasible	19578	1639.06	3.25
data-40-8-0	14313.2	–	O.M.	–	–	–
data-40-8-1	10044.2	–	O.M.	–	–	–
data-40-8-2	16408.4	–	O.M.	–	–	–
data-40-8-3	12727.4	–	O.M.	–	–	–
data-40-8-4	17956.9	–	O.M.	–	–	–

Table A.3: Numerical results: R-CROSS-DOCK and (22)

Instance	R.Obj.	Obj.	Status	nbNode	CPUTime	MIPRelativeGap	Instance	R.Obj.	Obj.	Status	nbNode	CPUTime	MIPRelativeGap
data-10-3-0	1436.5	3045	Optimal	102	3.35	0	data-18-6-3	-8956.91	–	O.M.	–	–	–
data-10-3-1	4381.8	8410	Optimal	33	9.69	0	data-18-6-4	-16114.5	–	O.M.	–	–	–
data-10-3-2	6545	6545	Optimal	0	7.82	0	data-20-6-0	-33739.8	17661	Feasible	9	2628.45	287.62
data-10-3-3	10004	10004	Optimal	0	4.96	0	data-20-6-1	-16107	–	O.M.	–	–	–
data-10-3-4	7522	9985	Optimal	141	17.93	0	data-20-6-2	-29973	4406	Feasible	5877	1900.59	539.12
data-12-4-0	7485.92	13413	Optimal	842	105.57	0	data-20-6-3	-16875.9	–	O.M.	–	–	–
data-12-4-1	-2761.5	7747	Optimal	3583	165.78	0	data-20-6-4	-11608.5	10819	Feasible	3840	1980.4	128.36
data-12-4-2	-1428.4	4032	Optimal	2334	44.7	0	data-20-8-0	-37523	2117	Feasible	971	2127.78	1746.18
data-12-4-3	-645.94	8562	Optimal	1369	156.42	0	data-20-8-1	-36845	4363	Feasible	0	3020.41	904.45
data-12-4-4	6353	6353	Optimal	0	68.89	0	data-20-8-2	-24765.9	5798	Feasible	0	2507.53	480.34
data-12-6-0	-2745.45	8635	Optimal	146904	1300.46	0	data-20-8-3	-38470.5	3781	Feasible	14	2420.04	1086.32
data-12-6-1	-4987.63	–	O.M.	–	–	–	data-20-8-4	-28749.4	2748	Feasible	209	2232.06	1089.56
data-12-6-2	-10117.5	–	O.M.	–	–	–	data-25-6-0	-44410.2	7501	Feasible	1502	2168.74	661.85
data-12-6-3	-17929.8	255	Feasible	38453	1890.84	2104.72	data-25-6-1	-34670.3	9529	Feasible	929	2559.82	451.43
data-12-6-4	-10318.1	–	O.M.	–	–	–	data-25-6-2	-39933.4	–	O.M.	–	–	–
data-14-4-0	-2643.46	5627	Optimal	5870	119.53	0	data-25-6-3	-32473.9	12606	Feasible	597	2422.49	327.49
data-14-4-1	-7243.36	3932	Optimal	27664	131.28	0	data-25-6-4	-33294.9	11761	Feasible	103	2461.63	373.14
data-14-4-2	-3574.85	7298	Optimal	5163	211.74	0	data-25-8-0	-42255.2	6808	Feasible	252	2023.97	689.09
data-14-4-3	2574.31	9570	Optimal	7020	102.9	0	data-25-8-1	-53587.4	11537	Feasible	0	3040.27	533.28
data-14-4-4	-2529.2	8762	Optimal	7802	257.96	0	data-25-8-2	-60630.9	7726	Feasible	0	3043.83	866.82
data-14-6-0	-9758.07	–	O.M.	–	–	–	data-25-8-3	-44293.6	–	O.M.	–	–	–
data-14-6-1	-7945.81	–	O.M.	–	–	–	data-25-8-4	-56529.3	11369	Feasible	0	2622.69	576.55
data-14-6-2	-9063.13	–	O.M.	–	–	–	data-30-6-0	-34704.5	12551	Feasible	2276	2010.62	319.39
data-14-6-3	-13512.1	–	O.M.	–	–	–	data-30-6-1	-56135.1	–	O.M.	–	–	–
data-14-6-4	-7202.48	–	O.M.	–	–	–	data-30-6-2	-37854.5	–	O.M.	–	–	–
data-16-4-0	-6166.79	–	O.M.	–	–	–	data-30-6-3	-59334.9	19632	Feasible	2	2572.11	393.4
data-16-4-1	-10279.7	–	O.M.	–	–	–	data-30-6-4	-26943.6	20025	Feasible	100	2806.44	222.74
data-16-4-2	-5595.29	5579	Optimal	6058	77.7	0	data-30-8-0	-77674.4	11065	Feasible	0	3070.35	787.03
data-16-4-3	-5751.18	6649	Optimal	7184	240.47	0	data-30-8-1	-89620.9	9317	Feasible	0	3071.69	1039.15
data-16-4-4	-302.817	9320	Optimal	3585	106.53	0	data-30-8-2	-97848.6	4966	Feasible	0	3072.08	2019.25
data-16-6-0	-1093.49	–	O.M.	–	–	–	data-30-8-3	-70846.3	5795	Feasible	1	3071.52	1305.45
data-16-6-1	-23764.6	3040	Feasible	6632	1984.41	538.18	data-30-8-4	-98977.4	14066	Feasible	0	3073.66	796.78
data-16-6-2	-15168	7816	Feasible	13028	1730.99	150.89	data-35-8-0	-41898.5	21959	Feasible	0	3113.98	284.07
data-16-6-3	-17557.4	–	O.M.	–	–	–	data-35-8-1	-85970.7	18702	Feasible	0	3113	545.5
data-16-6-4	-17491.2	2623	Feasible	8376	2008.56	527.42	data-35-8-2	-87981.3	18671	Feasible	0	3083.02	560.79
data-18-4-0	-6564.4	–	O.M.	–	–	–	data-35-8-3	-76009	18753	Feasible	0	3112	492.58
data-18-4-1	-12337.7	2585	Optimal	52346	681.99	0	data-35-8-4	-70496.4	16688	Feasible	0	3112.45	509.95
data-18-4-2	-7425.16	12451	Optimal	18490	511.34	0	data-40-8-0	–	–	O.M.	–	–	–
data-18-4-3	7843.5	15609	Optimal	2689	393.5	0	data-40-8-1	–	–	O.M.	–	–	–
data-18-4-4	-8298.97	9467	Optimal	9366	366.9	0	data-40-8-2	–	–	O.M.	–	–	–
data-18-6-0	-23440.7	5853	Feasible	7206	1821.19	379.04	data-40-8-3	–	–	O.M.	–	–	–
data-18-6-1	-28532.6	–	O.M.	–	–	–	data-40-8-4	–	–	O.M.	–	–	–
data-18-6-2	-22650.1	6051	Feasible	6184	2072.26	272.21							

Table A.4: Numerical results: R-CROSS-DOCK and (23) and (24)

Instance	R.Obj.	Obj.	Status	nbNode	CPUTime	MIPRelativeGap
data-10-3-0	1605.83	3045	Optimal	81	3.29	0
data-10-3-1	8410	8410	Optimal	0	2.59	0
data-10-3-2	6545	6545	Optimal	0	4.23	0
data-10-3-3	10004	10004	Optimal	0	1.79	0
data-10-3-4	4811.67	9985	Optimal	83	4.79	0
data-12-4-0	4423	13413	Optimal	672	34.87	0
data-12-4-1	2810.76	7747	Optimal	0	80.28	0
data-12-4-2	-2476.44	4032	Optimal	1525	11.47	0
data-12-4-3	-3522.77	8562	Optimal	1617	25.97	0
data-12-4-4	6353	6353	Optimal	0	38.09	0
data-12-6-0	-2375.72	8638	Feasible	>23406	224.34	25.04
data-12-6-1	-4259.34	1718	Feasible	>20297	288.12	87.78
data-12-6-2	-8465.21	1988	Feasible	>24020	416.57	65.12
data-12-6-3	-25545.2	255	Feasible	7826	306.04	2503.07
data-12-6-4	-13953.1	2398	Feasible	8409	247.57	119.92
data-14-4-0	228.434	5627	Optimal	285	107.98	0
data-14-4-1	-4108.89	3932	Optimal	8192	41.17	0
data-14-4-2	-2243.85	7298	Optimal	871	90.67	0
data-14-4-3	1155.49	9570	Optimal	584	49.59	0
data-14-4-4	347.348	8762	Optimal	1369	102.52	0
data-14-6-0	-16223.1	5790	Feasible	>16066	446.48	100.5
data-14-6-1	-6352.76	6376	Feasible	>22848	435.01	116.99
data-14-6-2	-11517.5	5076	Feasible	>7959	446.71	91.45
data-14-6-3	-15980.1	5516	Feasible	>18959	428.39	153.16
data-14-6-4	-6129.22	5755	Feasible	>31203	321.56	55.16
data-16-4-0	-4867.85	5793	Optimal	4901	102.41	0
data-16-4-1	-11387.2	9050	Optimal	2653	124.91	0
data-16-4-2	-7151.86	5579	Optimal	3798	33.76	0
data-16-4-3	-4656.35	6649	Optimal	1538	106.38	0
data-16-4-4	5070.08	9320	Optimal	71	42.29	0
data-16-6-0	775.336	11163	Feasible	17979	412.65	49.6
data-16-6-1	-31430.5	3004	Feasible	>23509	634.53	436.73
data-16-6-2	-19499.4	7815	Feasible	>21739	710.9	141.95
data-16-6-3	-19422.6	6392	Feasible	7230	733.53	192.24
DATA-16-6-4	-20825	2624	Feasible	>10950	546.22	291.72
data-18-4-0	-912.574	10376	Optimal	1127	123.6	0
data-18-4-1	-9513.03	2585	Optimal	21975	246.62	0
data-18-4-2	918.213	12451	Optimal	4429	395.28	0
data-18-4-3	6925.32	15609	Optimal	698	159.43	0
data-18-4-4	-8900.03	9467	Optimal	7979	232.99	0
data-18-6-0	-31333.7	5864	Feasible	>12389	679.04	279.19
data-18-6-1	-28528.6	10030	Feasible	4796	926.35	219.64
data-18-6-2	-30209	5983	Feasible	>9530	1054.58	237.14
data-18-6-3	-10160.5	9557	Feasible	>18068	1062.04	73.54
data-18-6-4	-20102.8	2993	Feasible	>21804	654.66	316.99
data-20-6-0	-31649.6	17569	Feasible	10121	1774.31	163.05
data-20-6-1	-12585.8	10851	Feasible	0	1024.8	170.94
data-20-6-2	-38976.2	4386	Feasible	>22529	1510.56	476.33
data-20-6-3	-11895	10310	Feasible	5292	871.84	130.1
data-20-6-4	-9199.54	10956	Feasible	>10771	878.08	118.74
data-20-8-0	-37330.5	2099	Feasible	6016	909.33	867.23
data-20-8-1	-44285.6	3435	Feasible	6559	1622.49	757.91
data-20-8-2	-30997.2	5740	Feasible	6775	1636.25	252.63
data-20-8-3	-44540.2	3205	Feasible	5917	1657.57	1006.62
data-20-8-4	-32602.7	2722	Feasible	10398	1612.57	964.79
data-25-6-0	-49145.3	7470	Feasible	5453	1311.22	371.41
data-25-6-1	-47497.8	9263	Feasible	11346	1694.83	364.69
data-25-6-2	-48642.8	7422	Feasible	0	886.15	519.96
data-25-6-3	-39488.5	12227	Feasible	11088	1635.97	211.6
data-25-6-4	-36292.3	10744	Feasible	7773	1731.24	277.52
data-25-8-0	-42042	6754	Feasible	3557	1620.51	607.74
data-25-8-1	-64225.6	8695	Feasible	2851	1659.41	656.6
data-25-8-2	-67966.2	6350	Feasible	3523	1484.71	1033.57
data-25-8-3	-61127.9	5928	Feasible	3272	1698.23	898.04
data-25-8-4	-59024.3	10512	Feasible	1610	1800.19	536.09
data-30-6-0	-29431.3	11751	Feasible	0	909.14	319.39
data-30-6-1	-53009.4	11564	Feasible	0	1273.33	522.87
data-30-6-2	-47789.3	6529	Feasible	3408	1534.96	515.74
data-30-6-3	-48324.5	17367	Feasible	3175	1916.25	310.76
data-30-6-4	-30447.8	20158	Feasible	0	1908.89	128.5
data-30-8-0	-75957.7	9740	Feasible	656	2058.32	808.81
data-30-8-1	-99514.4	5992	Feasible	1961	1737.42	1597.79
data-30-8-2	-111501	3480	Feasible	1681	1753.03	3123.49
data-30-8-3	-81702.8	5428	Feasible	154	1967.89	1491.88
data-30-8-4	-101767	10216	Feasible	842	2074.14	1018.82
data-35-8-0	-30051.2	20700	Feasible	0	2308.02	224.58
data-35-8-1	-94390.9	18249	Feasible	2	2068.43	560.29
data-35-8-2	-104741	14723	Feasible	1408	1778.36	760.12
data-35-8-3	-67024.2	18638	Feasible	0	2948.62	421.01
data-35-8-4	-71326.8	7242	Feasible	306	2278.21	990.04
data-40-8-0	-142497	19223	Feasible	4	2376.35	803.85
data-40-8-1	-101111	14002	Feasible	14	2161.89	786.73
data-40-8-2	-119960	20015	Feasible	5	2103.16	675.2
data-40-8-3	-111334	16423	Feasible	0	1826.35	755.46
data-40-8-4	-119117	22596	Feasible	0	2975.34	562.38

Table A.5: Numerical results: R-CROSS-DOCK and (25) and (26)

Instance	R.Obj.	Obj.	Status	nbNode	CPUTime	MIPRelativeGap	Instance	R.Obj.	Obj.	Status	nbNode	CPUTime	MIPRelativeGap
data-10-3-0	3045	3045	Optimal	0	1.06	0	data-18-6-3	9365.92	9552	Optimal	209	45.33	0
data-10-3-1	8410	8410	Optimal	0	1.22	0	data-18-6-4	2812.63	2979	Optimal	3563	72.18	0
data-10-3-2	6545	6545	Optimal	0	1.29	0	data-20-6-0	14913.5	17491	Optimal	3268	165.67	0
data-10-3-3	10004	10004	Optimal	0	1.5	0	data-20-6-1	10371.6	10820	Optimal	3738	99.44	0
data-10-3-4	9985	9985	Optimal	0	1.56	0	data-20-6-2	4141.63	4365	Optimal	27428	328.3	0
data-12-4-0	10787.5	13413	Optimal	157	6.77	0	data-20-6-3	9547.89	10254	Optimal	648	47.91	0
data-12-4-1	6867.45	7747	Optimal	55	6.41	0	data-20-6-4	9304.74	10773	Optimal	699	76.38	0.01
data-12-4-2	3946	4032	Optimal	189	4.67	0	data-20-8-0	1757.47	2007	Optimal	18068	773.94	0
data-12-4-3	6976.22	8562	Optimal	105	4.85	0	data-20-8-1	2984.23	3353	Feasible	16466	1556.47	3.7
data-12-4-4	6353	6353	Optimal	0	3.2	0	data-20-8-2	5417.11	5693	Optimal	41577	1144.22	0
data-12-6-0	6562.42	8635	Optimal	6203	36.07	0	data-20-8-3	2644.84	3039	Feasible	16723	1554.18	6.47
data-12-6-1	1611.36	1718	Optimal	785	23.73	0	data-20-8-4	2195.31	2668	Feasible	>12824	1136.47	12.42
data-12-6-2	1824.88	1983	Optimal	518	30.23	0	data-25-6-0	7158.16	7436	Optimal	3769	124.88	0
data-12-6-3	171.938	255	Optimal	1160	91.14	0	data-25-6-1	8294.76	9202	Optimal	1589	153.85	0
data-12-6-4	2272.98	2397	Optimal	96	21.08	0	data-25-6-2	6215.47	6699	Optimal	14179	558.19	0
data-14-4-0	5436.66	5627	Optimal	51	8.75	0	data-25-6-3	11383.4	12122	Optimal	2797	153.82	0
data-14-4-1	3832.72	3932	Optimal	79	10.84	0	data-25-6-4	10206.8	10633	Optimal	1840	169.95	0
data-14-4-2	6888.7	7298	Optimal	30	6.22	0	data-25-8-0	5493.71	6579	Feasible	7736	1563.01	7.62
data-14-4-3	8506.57	9570	Optimal	308	10.56	0	data-25-8-1	8236	8632	Feasible	23927	1558.51	0.58
data-14-4-4	7571.53	8762	Optimal	164	12.57	0	data-25-8-2	4935.18	6027	Feasible	9314	1565.75	6.38
data-14-6-0	5476.69	5790	Optimal	588	35.13	0	data-25-8-3	5327.13	5743	Feasible	11093	1571.68	3.52
data-14-6-1	4709.53	6355	Optimal	3818	41.78	0	data-25-8-4	9171.1	10306	Feasible	8840	1560.63	4.96
data-14-6-2	4604.34	5076	Optimal	1976	38.11	0	data-30-6-0	10789.2	11497	Feasible	>23649	807.23	1.35
data-14-6-3	5046.51	5505	Optimal	1859	41.36	0	data-30-6-1	8673.2	10589	Feasible	>31954	1267.82	3.4
data-14-6-4	5073.11	5755	Optimal	1975	25.48	0	data-30-6-2	5790.51	6234	Feasible	>17379	705.56	2.8
data-16-4-0	5176	5793	Optimal	149	18.53	0	data-30-6-3	15721	17281	Optimal	16937	695.16	0
data-16-4-1	8104.91	9050	Optimal	203	21.86	0	data-30-6-4	18733.7	19365	Optimal	3305	273.64	0
data-16-4-2	5481.02	5579	Optimal	85	12.14	0	data-30-8-0	8272.78	8891	Feasible	5204	1601.35	4.53
data-16-4-3	6306.22	6649	Optimal	45	9.22	0	data-30-8-1	5264.29	5722	Feasible	5966	1629.6	5.19
data-16-4-4	9320	9320	Optimal	0	4.07	0	data-30-8-2	2695.21	3186	Feasible	3626	1620.66	10.73
data-16-6-0	9988.67	11153	Optimal	1144	40.11	0	data-30-8-3	3527.66	4296	Feasible	2148	1652.41	11.03
data-16-6-1	2846.25	3003	Optimal	344	56.44	0	data-30-8-4	8325.36	9979	Feasible	3592	1598.06	11.9
data-16-6-2	7661.82	7799	Optimal	1073	40.45	0	data-35-8-0	17768.6	18970	Feasible	5320	1644.27	2.6
data-16-6-3	6165.27	6366	Optimal	5898	88.81	0	data-35-8-1	15242.8	15970	Feasible	2150	1666.59	3.13
data-16-6-4	2368.2	2602	Optimal	1512	44.27	0	data-35-8-2	13391.4	14279	Feasible	2548	1686.62	5.74
data-18-4-0	9873.1	10376	Optimal	162	18.33	0	data-35-8-3	14900.6	15930	Feasible	3905	1639.96	5.09
data-18-4-1	2364.25	2585	Optimal	490	15.94	0	data-35-8-4	6493.71	7038	Feasible	3141	1671.24	5.1
data-18-4-2	10952.8	12451	Optimal	114	14.88	0	data-40-8-0	14285.4	14982	Feasible	1369	1728.37	4.49
data-18-4-3	14093.2	15609	Optimal	105	12.54	0	data-40-8-1	9995.14	10601	Feasible	1652	1656.28	4.46
data-18-4-4	8465.07	9467	Optimal	941	9.81	0	data-40-8-2	16340	18133	Feasible	1267	1730.57	9.86
data-18-6-0	5520.45	5800	Optimal	649	67.05	0	data-40-8-3	12620.2	13430	Feasible	1208	1746.99	5.36
data-18-6-1	8830.84	10009	Optimal	1776	88.77	0	data-40-8-4	17883.8	18260	Feasible	1680	1686.01	1.71
data-18-6-2	5273.58	5920	Optimal	2040	57.67	0							

Table A.6: Numerical results: R-CROSS-DOCK and (20), (21),(22), (23),(24), (25), (26)

Instance	R.Obj.	Obj.	Status	nbNode	CPUTime	MIPRelativeGap
data-10-3-0	3045	3045	Optimal	0	1.03	0
data-10-3-1	8410	8410	Optimal	0	1.06	0
data-10-3-2	6545	6545	Optimal	0	1.03	0
data-10-3-3	10004	10004	Optimal	0	1.08	0
data-10-3-4	9985	9985	Optimal	0	1.14	0
data-12-4-0	10810.8	13413	Optimal	112	5.19	0
data-12-4-1	7333.2	7747	Optimal	63	3.67	0
data-12-4-2	3977.5	4032	Optimal	47	3.29	0
data-12-4-3	6920.23	8562	Optimal	86	5.87	0
data-12-4-4	6353	6353	Optimal	0	1.87	0
data-12-6-0	6592.01	8635	Optimal	5599	11.7	0
data-12-6-1	1638.85	1718	Optimal	612	9.87	0
data-12-6-2	1855.23	1983	Optimal	463	11.47	0
data-12-6-3	153.617	255	Optimal	488	14.66	0
data-12-6-4	2320.82	2397	Optimal	34	9.91	0
data-14-4-0	5529.68	5627	Optimal	52	5.93	0
data-14-4-1	3932	3932	Optimal	0	3.7	0
data-14-4-2	7298	7298	Optimal	0	2.76	0
data-14-4-3	8346.56	9570	Optimal	111	5.52	0
data-14-4-4	8075.33	8762	Optimal	96	8.03	0
data-14-6-0	5523.72	5790	Optimal	360	12.25	0
data-14-6-1	4740.99	6355	Optimal	3717	17.16	0
data-14-6-2	4597.04	5076	Optimal	712	15.9	0
data-14-6-3	5058.67	5505	Optimal	1383	15.88	0
data-14-6-4	5178.28	5755	Optimal	1101	11.09	0
data-16-4-0	5756.67	5793	Optimal	56	9.55	0
data-16-4-1	8254.45	9050	Optimal	35	12.78	0
data-16-4-2	5579	5579	Optimal	0	4.48	0
data-16-4-3	6649	6649	Optimal	0	5.66	0
data-16-4-4	9320	9320	Optimal	0	3.2	0
data-16-6-0	10005.3	11153	Optimal	1125	26.79	0
data-16-6-1	2855.75	3003	Optimal	95	19.8	0
data-16-6-2	7373.14	7799	Optimal	9873	41.93	0
data-16-6-3	6228.04	6366	Optimal	1777	21.64	0
data-16-6-4	2444.53	2602	Optimal	425	20.14	0
data-18-4-0	10376	10376	Optimal	0	7.21	0
data-18-4-1	2585	2585	Optimal	0	9.69	0
data-18-4-2	11065.8	12451	Optimal	93	11.08	0
data-18-4-3	14123.9	15609	Optimal	0	9.28	0
data-18-4-4	8470.37	9467	Optimal	743	7.97	0
data-18-6-0	5624.35	5800	Optimal	142	26.38	0
data-18-6-1	8932.03	10009	Optimal	1505	39.81	0
data-18-6-2	5344.77	5920	Optimal	443	26.85	0
data-18-6-3	9397.27	9552	Optimal	133	18.56	0
data-18-6-4	2840.17	2979	Optimal	1623	24.13	0
data-20-6-0	14948.9	17491	Optimal	2565	74.66	0
data-20-6-1	10635.1	10820	Optimal	950	48.86	0
data-20-6-2	4183.86	4365	Optimal	8970	51.59	0
data-20-6-3	9818.66	10254	Optimal	261	30.53	0
data-20-6-4	9353.62	10773	Optimal	885	43.48	0
data-20-8-0	1806.04	2007	Optimal	1622	77.7	0
data-20-8-1	3045.71	3347	Optimal	8306	183.5	0
data-20-8-2	5427.41	5693	Optimal	6866	98.73	0
data-20-8-3	2714.99	3027	Optimal	7083	161.1	0
data-20-8-4	2295.18	2640	Optimal	40728	569.39	0
data-25-6-0	7210.98	7436	Optimal	11435	98.73	0
data-25-6-1	8717.23	9202	Optimal	1128	65.02	0
data-25-6-2	6434.29	6699	Optimal	3335	75.43	0
data-25-6-3	11269	12122	Optimal	921	79.31	0
data-25-6-4	10278.2	10633	Optimal	967	51.14	0
data-25-8-0	5552.23	6575	Feasible	>42130	1567.28	0.68
data-25-8-1	8251.42	8632	Optimal	14790	308.45	0
data-25-8-2	5002.77	5999	Feasible	63280	1558.22	1.88
data-25-8-3	5382.92	5716	Optimal	4443	169.12	0
data-25-8-4	9209.29	10276	Optimal	42629	1080.85	0
data-30-6-0	10700.1	11497	Optimal	24193	212.78	0
data-30-6-1	8854.64	10583	Optimal	159737	1269.49	0.01
data-30-6-2	6020.6	6234	Optimal	4643	164.35	0
data-30-6-3	15792	17281	Feasible	28267	334.19	0
data-30-6-4	18708.8	19365	Optimal	2619	104.58	0.01
data-30-8-0	8257.85	8861	Feasible	>27374	1051.91	1.82
data-30-8-1	5360.64	5721	Feasible	37894	1610.93	2.77
data-30-8-2	2720.72	3160	Feasible	38551	1604.58	3.6
data-30-8-3	3746.83	4275	Optimal	1724	371.16	0
data-30-8-4	8384.5	9939	Feasible	>19365	1071.02	7.41
data-35-8-0	17764.2	18990	Feasible	>27899	1177.65	0.77
data-35-8-1	15300.9	15895	Feasible	29538	1643.25	1.49
data-35-8-2	13431.4	14266	Feasible	>21837	1540.23	2.83
data-35-8-3	14930.4	15802	Feasible	30294	1234.98	1.99
data-35-8-4	6543.67	7024	Feasible	21751	1462.31	3.49
data-40-8-0	14315.8	14954	Feasible	>10540	1488.98	3.86
data-40-8-1	10048.8	10595	Feasible	16118	1697.42	2.04
data-40-8-2	16428	18113	Feasible	7671	1594.16	2.14
data-40-8-3	12724.3	13408	Feasible	>11585	1417.27	2.86
data-40-8-4	17954.2	18263	Feasible	18421	1687.67	0.98