Achieving Cost-Effective Power Grid Hardening through Transmission Network Topology Control

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***Abstract*—Vulnerability of power grid is a critical issue in power industry. In order to understand and reduce power grid vulnerability under threats, existing research often employs defender-attacker-defender (DAD) models to derive effective protection plans and evaluate grid performances under various contingencies. Transmission line switching (also known as topology control) is an effective operation to mitigate outages or attacks. In this paper, we include post-contingency transmission line switching operations into a defender-attacker-defender model. To solve this complicated tri-level formulation, we customize and implement an exact algorithm, i.e., nested column-and-constraint generation (NCCG) algorithm, to compute global optimal solutions. We perform a set of numerical experiments on IEEE RTS one-area system (1996), demonstrate the significant improvements from transmission line switching, and highlight that hardening plans derived from this new DAD model are very cost-effective compared to those obtained from traditional DAD model.**

***Index Terms*—power grid protection, transmission line switching, defender-attacker-defender model, column-and-constraint generation**

NOMENCLATURE

Indices, Sets and Parameters

$\mathbf{N}$	set of indices of buses
$\mathbf{J}$	set of indices of generators
$\mathbf{L}$	set of indices of transmission lines
j	generator index, $j \in \mathbf{J}$
l	transmission line index, $l \in \mathbf{L}$
n	bus index, $n \in \mathbf{N}$
$\mathbf{Jn}$	set of indices of generators connected to bus n
$o(l)$	origin bus of transmission line l
$d(l)$	destination bus of transmission line l
$\mathbb{Z}$	defender's protection decision set
$\mathbb{V}$	attacker's attack decision set
K	budget for attacker's desruption
R	budget for defender's protection
x_l	reactance at line l
D_n	demand at bus n
G_j	generation capacity of generator j
P_l	power flow capacity of transmission line l

Decision Variables

z_l	protection decision, 1 for protected, 0 otherwise
v_l	attack decision, 0 for attacked, 1 otherwise
w_l	transmission line switching, 0 for switched off, 1 otherwise
d_n	load shedding at node n
δ_n	phase angle at node n
g_j	generation level of generator$j$
p_l	power flow on line l
$\mathbf{z}$	vector of $z_l, \forall l \in L$, a protection plan
$\mathbf{v}$	vector of $v_l, \forall l \in L$, an attack plan
$\mathbf{w}$	vector of $w_l, \forall l \in L$, a transmission line switching plan
$\hat{*}$	fixed decision variable (or vector) of $*$

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I. INTRODUCTION

Power system vulnerability is a serious concern to power industry and the whole society. Terrorists and natural disasters' attacks on power grids are regarded as a national threat due to their massive damage. A recent report released by National Research Council emphasizes that a terrorist attack on U.S. power grids could be much more destructive than Hurricane Sandy because it could black out large segments of the country for months, cause hundreds of billions of economic damage, and lead to thousands of deaths [9]. Hence, protecting or hardening grid components against various disruptions is of a high national interest [1, 10]. Moreover, [14] clearly states that energy security efforts should start with hardening power grids.

To derive an effective plan that protects critical components with limited defensive resources, researchers often adopt game theory models to simulate the interaction between system operators and terrorists/natural disasters. Among them, defender-attacker-defender models are probably the most popular ones [6, 21, 5, 22, 2]. In those DAD models, system operators protect or harden grid components before any possible attack. Then, given the grid with some components non-attackable, attackers seek to destroy other critical components with the maximized load shedding. Once the attack happens, system operators respond to the disruptions by taking mitigation operations, for example, re-dispatching power flows, to minimize the load shedding. Because those models are challenging tri-level mixed integer programming problems, many research efforts focus on developing fast algorithms to solve practical instances. Examples include heuristic methods in [7, 5] and exact algorithms in [22, 2].

According to [22], it is critical to understand and model the defender's mitigation capability as it affects the attacker's attacking targets. Without a full description of the attacker's behaviors, the optimality of the protection plan will be forfeited. Nevertheless, to the best of our knowledge, all existing research on power grid defender-attacker-defender models neglect one important mitigation operation: *transmission line switching* (also known as topology control).

In fact, given that expanding transmission infrastructure is very costly and could pose many integration issues to existing power grids, the Federal Energy Regulatory Commission (FERC) orders call for improved economic operations of the electric transmission network. Especially, those orders promote the use of transmission line switching, which changes the transmission network topology, to improve utilization of existing transmission systems. Actually, transmission line switching has been included in real operations as a corrective mechanism to alleviate line overloads and voltage violations after contingencies [15, 20, 16, 8]. For example, PJM has incorporated the post-contingency transmission line switching actions into its Special Protection Schemes (SPSs) [18], which reflects a shift from preventive approach to a more economic corrective approach. The ISO New England studies the dispatch efficiency along with reliability requirements to determine the optimal periods to take off transmission lines for maintenance and earns a saving of 72.5 million dollars in 2008 [18]. Based on these current industry practices, it is believed that transmission line switching should be deeply investigated and effectively utilized.

To analytically study transmission line switching operations, [13] considers the optimal transmission line switching problem by introducing transmission line switching decision into a DC optimal power flow model. According to [13], a saving of 25% in dispatch costs can be achieved from transmission line switching. Recent work [19] also reveals the significant benefits of transmission line switching in utilizing wind power. Transmission line switching is also introduced to power system vulnerability analysis as system operator's post-contingency operations [4, 11, 25].

Given that transmission line switching can greatly improve dispatch capability of a power system, in this paper, we consider integrating protection operations and transmission line switching decisions under defender-attacker-defender model. We expect to derive cost-effective protection plans that can better use limited defensive resources. However, the challenge from incorporating transmission line switching is not only reflected in how to model this operation in the defender-attacker-defender framework, but also in how to solve the new model. All existing algorithms for general defender-attacker-defender models depend on the strong duality of the inner most linear programming problem, which is not the case when binary transmission line switching decisions are included. To address this challenge, we adopt the nested column-and-constraint generation (NCCG) algorithm [24] that extends the basic column-and-constraint method [23] and is designed specifically to deal with tri-level problem with an inner most mixed integer problem.

The rest of this paper is organized as follows. In Section II, we give the formulation of the defender-attacker-defender model with transmission line switching as post-contingency operations (i.e., DAD-TLS formulation) and present some structural properties. Section III describes the customized nested column-and-constraint generation algorithm to solve DAD-TLS formulation. Section IV shows our computational results. In Section V, we analyze the benefits from introducing transmission line switching operations to hardening decisions. Section VI concludes with a discussion.

II. Problem formulation

In this section, we present a tri-level min-max-min formulation of a defender-attacker-defender model for power grid protection problem that includes transmission line switching as a mitigation strategy. Similar formulations without transmission line switching can be found in [21, 22, 12].

A. Modeling Protection, Attack, and Transmission Line Switching

Study in [13] initially formulates transmission line switching operation into a DC optimal power flow model. Transmission line switching is represented by a binary variable, which takes effect on the Kirchhoff's law constraints. Similarly, our study considers transmission line switching as a corrective post-contingency/attack operation and employs binary transmission line switching variables in the inner DC optimal power flow model. Specifically, each transmission line l is associated with a binary variable w_l that represents whether a line is included in the system ($w_l = 1$) or disconnected (switched off) ($w_l = 0$). Hence, the Kirchhoff's law constraints in the defender-attacker-defender model with transmission line switching can be formulated as:

$$p_l x_l = w_l(z_l + v_l - z_l v_l)[\delta_{o(l)} - \delta_{d(l)}],\ \forall l \in \mathbf{L} \qquad (1)$$

where $(z_l + v_l - z_l v_l)$ represents the logic of protection (z_l) and attack (v_l) decisions. If $z_l = 1$, line l is protected, then no attack on that line would be possible since $z_l + v_l - z_l v_l = z_l = 1$. Note that if a transmission line is out-of-service after an attack, it must be non-protected ($\hat{z}_l = 0$) and attacked ($\hat{v}_l = 0$),which means $z_l + v_l - z_l v_l = 0$, and power flow p_l is zero. If a transmission line is not attacked ($\hat{v}_l = 1$), (1) reduces to $p_l x_l = w_l[\delta_{o(l)} - \delta_{d(l)}]$. Thus, transmission variable w_l will take effect on transmission line.

B. Defender-Attacker-Defender with Transmission Line Switching

Tri-level defender-attacker-defender model involves three agents acting sequentially. The top level corresponds to the defender's decision on allocating defensive resources to protect transmission lines throughout a power grid. The middle level decisions are controlled by an attacker, who seeks to maximize the load shedding of the power system by disconnecting a set of transmission lines that are unprotected. Then, after the disruption by the attacker is observed, the system operator reacts to that disruption by solving an optimal power flow problem with transmission line switching as network topology control to minimize the load shedding. Hence, the inner most problem is a binary mixed integer program. It differs from those in [7, 21, 5, 12, 3, 22] where the inner most level is simply a linear program for linear DC optimal power flow.

The mixed integer nonlinear programming (MINLP) formulation of the defender-attacker-defender model with transmission line switching (DAD-TLS) is:

$$\min_{\mathbf{z}\in\mathbb{Z}} \max_{\mathbf{v}\in\mathbb{V}} \min_{\{w_l,p_l,g_j,d_n,\delta_n\}} \sum_{n\in\mathbf{N}} d_n \tag{2}$$

$$st. \quad \sum_{l\in\mathbf{L}} z_l \leq R \tag{3}$$

$$\sum_{l\in\mathbf{L}} (1-v_l) \leq K \tag{4}$$

$$p_l x_l = w_l(z_l + v_l - z_l v_l)[\delta_{o(l)} - \delta_{d(l)}], \ \forall l \in \mathbf{L} \tag{5}$$

$$\sum_{j\in\mathbf{Jn}} g_j - \sum_{l|o(l)=n} p_l + \sum_{l|d(l)=n} p_l + d_n = D_n, \quad \forall n \in \mathbf{N} \tag{6}$$

$$-P_l \leq p_l \leq P_l, \ \ \forall l \in \mathbf{L} \tag{7}$$

$$0 \leq g_j \leq G_j, \ \ \forall j \in \mathbf{J} \tag{8}$$

$$0 \leq d_n \leq D_n, \ \ \forall n \in \mathbf{N} \tag{9}$$

$$v_l, z_l, w_l \in \{0,1\}, \ \ \forall l \in \mathbf{L} \tag{10}$$

where $\mathbb{Z} = \{\sum_{l\in\mathbf{L}} z_l \leq R, z_l \in \{0,1\}, \forall l \in \mathbf{L}\}$ is defender's protection decision set, and $\mathbb{V} = \{\sum_{l\in\mathbf{L}}(1-v_l) \leq K, v_l \in \{0,1\}, \forall l \in \mathbf{L}\}$ is attacker's attack decision set. R is the cardinality budget for the defender, which means that the defender could protect up to R transmission lines. Similarly, K is the cardinality budget for the attacker so that the attacker can remove up to K transmission lines. Note that it is consistent with N-K reliability consideration.

Constraints (5) capture the active DC power flows on a power grid following the Kirchhoff's Law with protection, attack, and transmission line switching decision variables. Constraints (6) preserve power flow balance at bus n. Constraints (7) simply state that the power flow on line l will be restricted within $[-P_l, P_l]$. Constraints (8) bound the power generation of each generator by zero and its capacity. Constraints (9) guarantee that the load shedding at load bus n does not exceed its nominal demand and is always nonnegative.

Note that, because of binary transmission line switching variables, the inner most optimal power flow problem with transmission line switching, becomes a mixed integer program. All existing exact algorithms in solving defender-attacker-defender models, e.g., the implicit enumeration in [12], the column-and-constraint generation in [22, 23], are no longer applicable as they depends on duality theory of linear program.

C. Structural Properties

In this subsection, we present some insights and properties of DAD-TLS formulation presented in (2-10). Let $f_{(R,K)}$ represent the optimal value for a given hardening budget R and a given attack budget K. The next result follows easily by analyzing the relaxation relationship between different Rs (and Ks).

Theorem II.1. *$f_{(R,K)}$ is non-increasing in R and non-decreasing in K.*

Indeed, it is easy to see that it is only necessary to consider cases where $R + K \leq |L|$. Otherwise, we can reduce K to $(|L| - R)$, given that extra attack efforts are useless.

Let $f_{(R,K)}(\hat{\mathbf{z}})$ be the optimal value for a given protection plan $\hat{\mathbf{z}}$ and $\hat{\mathbf{v}}$ be its corresponding optimal attack plan. By an abuse of notation, we also use $\hat{\mathbf{z}}$ (and $\hat{\mathbf{v}}$, respectively) to represent the set of associated transmission lines subject to protection (and attack, respectively).

Theorem II.2. (Intersection Theorem)

1) *$\hat{\mathbf{z}} \cap \hat{\mathbf{v}} = \varnothing$, i.e., an optimal attack plan does not involve any transmission lines under protection;*
2) *For a protection plan $\mathbf{z}^0$, if $\mathbf{z}^0 \cap \hat{\mathbf{v}} = \varnothing$, we have $f_{(R,K)}(\mathbf{z}^0) \geq f_{(R,K)}(\hat{\mathbf{z}})$. Therefore, unless $\hat{\mathbf{z}}$ is an optimal protection plan to DAD-TLS problem, it is necessary to have $\mathbf{z}^0 \cap \hat{\mathbf{v}} \neq \varnothing$ if $\mathbf{z}^0$ is an improved protection plan over $\hat{\mathbf{z}}$.*

Proof: Note that the first statement follows easily. Also, it is sufficient to prove the first part of the second statement. So, we have

$$\begin{aligned} & f_{(R,K)}(\mathbf{z}^0) \\ = \ & \max_{\mathbf{v}\in\mathbb{V}} \Big\{ \min \sum_{n\in\mathbf{N}} \mathbf{d}_n : (4) - (10), \mathbf{z} = \mathbf{z}^0 \Big\} \\ \geq \ & \Big\{ \min \sum_{n\in\mathbf{N}} \mathbf{d}_n : (5) - (10), \mathbf{z} = \mathbf{z}^0, \mathbf{v} = \hat{\mathbf{v}} \Big\} \end{aligned}$$

Because $\hat{\mathbf{z}} \cap \hat{\mathbf{v}} = \mathbf{z}^0 \cap \hat{\mathbf{v}} = \varnothing$, it is easy to see that

$$\begin{aligned} & \Big\{ \min \sum_{n\in\mathbf{N}} \mathbf{d}_n : (5) - (10), \mathbf{z} = \mathbf{z}^0, \mathbf{v} = \hat{\mathbf{v}} \Big\} \\ = \ & \Big\{ \min \sum_{n\in\mathbf{N}} \mathbf{d}_n : (5) - (10), \mathbf{z} = \hat{\mathbf{z}}, \mathbf{v} = \hat{\mathbf{v}} \Big\} \\ = \ & f_{(R,K)}(\hat{\mathbf{z}}) \end{aligned}$$

Therefore, we have $f_{(R,K)}(\mathbf{z}^0) \geq f_{(R,K)}(\hat{\mathbf{z}})$. ■

III. Solution Methodology

In this section, we customize and implement the nested column-and-constraint generation algorithm in [24] to solve DAD-TLS. In particular, by extending the inner most mix integer optimal power flow problem (with transmission line switching) into a bi-level program, the middle and inner levels' problem becomes a tri-level problem where the inner most one is a linear program. Then, using strong duality, we can further convert that tri-level formulation into a max-min-max format.

Specifically, for a given protection plan $\hat{\mathbf{z}}$, we have

$$\begin{aligned} & \max_{\mathbf{v}\in\mathbb{V}} \min_{\{w_l,p_l,g_j,d_n,\delta_n\}} \sum_{n\in\mathbf{N}} d_n \\ & = \max_{\mathbf{v}\in\mathbb{V}} \min_{w_l} \min_{\{p_l,g_j,d_n,\delta_n\}} \sum_{n\in\mathbf{N}} d_n \\ & = \max_{\mathbf{v}\in\mathbb{V}} \min_{w_l} \max_{\{\pi^1,\ldots,\pi^6\}} \beta \end{aligned}$$

where β is the dual objective function, (π^1, π^2) are dual variables for constraints (5) and (6), (π^3, π^4) are dual variables for constraints (7), π^5 is dual variable for constraints (8) and π^6 for (9). Given this max-min-max problem, we can solve it by the column-and-constraint generation algorithm to derive optimal attack plan $\mathbf{v}^*$. Then, with $\mathbf{v}^*$, we can again use the column-and-constraint generation algorithm to solve the complete defender-attacker-defender model in (2-10). Obviously, the column-and-constraint method is used in two levels, for which the whole procedure is called nested column-and-constraint method. To distinguish the master problems and sub problems of those two levels, we denote the master problem of outer level as NCCG master problem, the inner level master problem as CCG master problem, and the inner level sub problem as CCG sub problem. Next, to help the understanding of the whole algorithm, we present CCG master and sub problems. NCCG master and sub problems can be formulated in a similar way. Readers are also referred to [22] for detailed formulations that actually are NCCG master and sub problems.

Given a protection plan $\hat{\mathbf{z}}$ (a solution from NCCG master problem), and a set of transmission line switching plans $\hat{\mathbb{W}} = \{\hat{\mathbf{w}}^s, s = 1, ..., t\}$ (solutions from CCG sub problems, t is the current iteration number of column-and-constraint generation loop), CCG master problem can be formulated as follows.

$$\max_{\mathbf{v}\in\mathbb{V}} \quad \beta \tag{11}$$

$$s.t.\ \ \beta \le \sum_n D_n({\pi_n^2}^s - {\pi_n^6}^s) - \sum_j G_j {\pi_j^3}^s - \sum_l P_l({\pi_l^4}^s + {\pi_l^5}^s), \ \ \forall s = 1, ..., t \tag{12}$$

$$\sum_{l\in\mathbf{L}}(1 - v_l) \le K \tag{13}$$

$$x_l {\pi_l^1}^s - {\pi^2}^s_{n,o(l)=n} + {\pi^2}^s_{n,d(l)=n} + {\pi_l^4}^s - {\pi_l^5}^s = 0, \quad \forall l \in \mathbf{L}, s = 1, ..., t \tag{14}$$

$${\pi^2}^s_{n,j\in Jn} - {\pi_j^3}^s \le 0, \forall j \in \mathbf{J}, s = 1, ..., t \tag{15}$$

$${\pi_n^2}^s - {\pi_n^6}^s \le 1, \ \ \forall n \in \mathbf{N}, s = 1, ..., t \tag{16}$$

$$-\sum_{l,o(l)=n} {\pi_l^1}^s {\hat{w}_l}^s(\hat{z}_l + v_l - \hat{z}_l v_l) + \sum_{l,d(l)=n} {\pi_l^1}^s {\hat{w}_l}^s(\hat{z}_l + v_l - \hat{z}_l v_l) = 0, \quad \forall n \in \mathbf{N}, s = 1, ..., t \tag{17}$$

$${\pi_l^1}^s free, {\pi_l^4}^s \ge 0, {\pi_l^5}^s \ge 0, \ \ \forall l \in \mathbf{L}, s = 1, ..., t \tag{18}$$

$${\pi_n^2}^s free, {\pi_n^6}^s \ge 0, \ \ \forall n \in \mathbf{N}, s = 1, ..., t \tag{19}$$

$${\pi_j^3}^s \ge 0, \ \ \forall j \in \mathbf{J}, s = 1, ..., t \tag{20}$$

$$v_l \in \{0, 1\}, \ \ \forall l \in \mathbf{L}. \tag{21}$$

Note that nonlinear constraints in (17) cab be easily linearized using big-M method.

Given a protection plan $\hat{\mathbf{z}}$ (a solution from NCCG master problem), and an attack plan $\hat{\mathbf{v}}$ (a solution from CCG master problem), CCG sub problem can be formulated as follows,

$$\min_{\mathbf{w}} \sum_{n\in\mathbf{N}} d_n \tag{22}$$

$$st.\ \ p_l x_l = (\hat{z}_l + \hat{v}_l - \hat{z}_l \hat{v}_l) w_l [\delta_{o(l)} - \delta_{d(l)}], \ \ \forall l \tag{23}$$

$$\sum_{j\in\mathbf{Jn}} g_j - \sum_{l|o(l)=n} p_l + \sum_{l|d(l)=n} p_l + d_n = D_n, \forall n \in \mathbf{N} \tag{24}$$

$$-P_l \le p_l \le P_l, \ \ \forall l \in \mathbf{L} \tag{25}$$

$$0 \le g_j \le G_j, \ \ \forall j \in \mathbf{J} \tag{26}$$

$$0 \le d_n \le D_n, \ \ \forall n \in \mathbf{N} \tag{27}$$

$$w_l \in \{0, 1\}, \ \ \forall l \in \mathbf{L}. \tag{28}$$

Again, nonlinear constraints in (23) are linearizable using big-M method.

Next, we present the implementation steps of nested column-and-constraint generation algorithm. $\hat{\mathbb{V}}$ is a subset of attack plans $\hat{\mathbb{V}} = \{\hat{\mathbf{v}}^1, \dots, \hat{\mathbf{v}}^k\} \subseteq \mathbb{V}$ for NCCG master problem, $\hat{\mathbb{W}}$ is a subset of transmission switching plans $\hat{\mathbb{W}} = \{\hat{\mathbf{w}}^1, \dots, \hat{\mathbf{w}}^s\} \subseteq \mathbb{W}$ for CCG master problem. The optimality tolerance gap of algorithm is ϵ.

Algorithm 1 Nested Column-and-Constraint Generation for DAD-TLS

Initialization: set $LB \leftarrow -\infty$, $UB \leftarrow \infty$, $\hat{\mathbb{V}} \leftarrow \varnothing$, iteration index $k \leftarrow 1$
while gap $\ge \epsilon$ **do**
  solve NCCG master problem, update LB with optimal value $objMP$, update protection plan $\hat{\mathbf{z}}$, and gap
  solve NCCG sub problem with protection plan $\hat{\mathbf{z}}$ by the **Subroutine** below, obtain objective value $objSP$ and attack plan $\mathbf{v}^*$, $UB \leftarrow min\{UB, objSP\}$, update gap and $k \leftarrow k + 1$
  add $\mathbf{v}^*$ to $\hat{\mathbb{V}}$, create dispatch variables $(\mathbf{p}^k, \mathbf{g}^k, \mathbf{d}^k, \boldsymbol{\delta}^k)$, and add these variables (columns) with corresponding constraints to NCCG master problem
end while
return $\mathbf{z}^* \leftarrow \hat{\mathbf{z}}$

Subroutine: Solving NCCG sub problem

Initialization: set $LB_{in} \leftarrow -\infty$, $UB_{in} \leftarrow \infty$, $\hat{\mathbb{W}} \leftarrow \varnothing$, and inner iteration index $s \leftarrow 1$
while $gap^{'} \ge \epsilon$ **do**
  solve CCG master problem , update UB_{in} with optimal objective value $objMP_{in}$, obtain attack plan $\hat{\mathbf{v}}$, $gap^{'}$
  solve CCG sub problem with attack plan $\hat{\mathbf{v}}$, obtain optimal value $objSP_{in}$ and an optimal transmission line switching plan $\hat{\mathbf{w}}^s$, update $LB_{in} \leftarrow min\{LB_{in}, objSP_{in}\}$, $s \leftarrow s + 1$, $gap^{'}$
  add $\hat{\mathbf{w}}^s$ to $\hat{\mathbb{W}}$, create dual variables $(\pi^{1s}, \dots, \pi^{6s})$, and add these variables(columns) and their corresponding constraints to CCG master problem
end while
return $\mathbf{v}^* \leftarrow \hat{\mathbf{v}}$

A flowchart of the complete implementation of nested column-and-constraint generation algorithm is presented in Figure 1. As NCCG master problem, CCG master and sub problems, after linearization, are linear mixed integer programs, they can be readily solved by professional mixed integer programming solvers.

Remarks: We note that results in Intersection Theorem can help to reduce the solution space and therefore the computational complexity. Specifically, we include the following constraints in CCG master problem (for a given protection

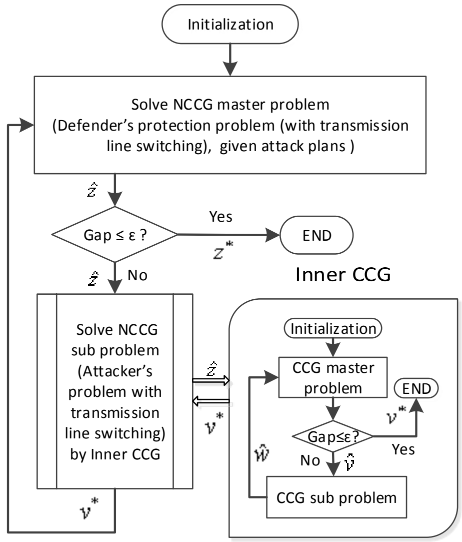

Fig. 1: flow chart of NCCG algorithm

plan $\hat{\mathbf{z}}$)

$$v_l = 1,\ \forall\, l \in \mathbf{L} \text{ s.t. } \hat{z}_l = 1;$$

and the following constraints in NCCG master problem (for attack plans in $\hat{\mathbb{V}}$)

$$\sum_{l:\hat{v}_l=0} z_l \geq 1,\ \forall\, \hat{\mathbf{v}}\ \in \hat{\mathbb{V}}.$$

IV. Computational Studies

In this section, we conduct computational experiments on a well-known IEEE Reliability Test System (RTS) one-area (1996) [17]. This system consists of 24 buses, 38 lines, 32 generators and 17 loads as illustrated in Fig 2. Data and parameters are adopted from [11]. The algorithm is implemented in C++ with CPLEX 12.5 on top of an Intel dual core 3.00GHz, 4GB memory PC. Tolerance gap ϵ is 0.1%.

A. Computational Results

We first present the results with hardening budget R ranging from 0 to 4 and attack budget K ranging from 0 to 6. Load shedding of optimal solutions are reported in Table I and their computational times are listed in Table II. N-K represent the worst-case contingency with K transmission lines under attack, specially, N-0 means no contingency in the grid.

TABLE I: load shedding (in 100MW) from DAD-TLS

N-K	$R=0$	$R=1$	$R=2$	$R=3$	$R=4$
N-0	0.00	0.00	0.00	0.00	0.00
N-1	1.31	1.29	0.79	0.73	0.69
N-2	2.79	2.79	2.59	2.29	1.66
N-3	4.29	3.90	3.38	3.16	2.46
N-4	5.38	5.16	4.46	3.96	3.05
N-5	6.88	5.96	5.01	4.48	3.77
N-6	7.75	6.48	5.77	5.27	4.42

As expected, we observe in Table I that more protection budget R (or more attack budget K, respectively) leads to less (or more, respectively) load shedding. Nevertheless, it can be seen that neither protection nor attack displays a linear behavior with respect to R or K in reducing or increasing load shedding, which indicates the complexity of physical laws and structures of power grid. On the one hand, an empirical understanding is that a hardening plan with $R = 2$, if implemented in an optimal way, often leads to significant load shedding reductions under different attack budgets. On the other hand, a similar understanding is that the worst N-2 contingencies could be very destructive.

TABLE II: computational time (in second) for DAD-TLS

N-K	$R=0$	$R=1$	$R=2$	$R=3$	$R=4$
N-0	1.2	1.1	1.2	0.9	1.0
N-1	5.3	7.7	9.1	18.6	36.9
N-2	13.5	15.1	10.5	18.6	67.3
N-3	13.9	54.7	112	183	270
N-4	9.8	93.1	204	788	9268
N-5	4.6	328	20147	12365	42605
N-6	21.1	5116	71989	411123	111039

In Table II, we observe that for some instances the computation time could be very long, especially for those considering N-5 and N-6 contingencies, which can be explained by the combinatorial nature of DAD-TLS model. Because we are dealing with power grid long-term planning problem, which does not need to work in a real time fashion, such computational time could be addressed by adopting more powerful computing facilities with sufficient computational budgets. Another strategy is, according to Theorem II.1, we can adopt optimal values from cases with N-K-1 and R-1 as strong bounds to facilitate the computation of those with N-K and R. Certainly, we will also explore advanced enhancement methods to improve the computational performance.

Indeed, due to the security or management issues in practice, we may not be able to switch off arbitrary transmission lines in the grid. Next, we study a few variants of DAD-TLS model where switching operations are restricted in different ways. For those variants, computational burdens are drastically reduced. To simplify our exposition, we refer to the original DAD-TLS with the full switching capability as DAD-OTS (optimal transmission switching).

B. A Few Variants of DAD-TLS

1) Budget for transmission line switching: In practice, it is not feasible to switch a large number of transmission lines when the power system is under attack. Hence, a power grid operator can put a budget on the number of switched transmission lines, which can be translated into the following constraint to bound the total number of switching-offs in the inner most minimization problem,

$$\sum_{l\in L}(1-w_l) \leq \text{PLS}$$

where PLS denotes the cardinality bound. The computational time and load shedding results are listed in Table III and Table IV with PLS $= 4$. Comparing the computational times in Table IV with those in Table II, we note that the computational time is greatly relieved by including a budget constraint on the total number of switched transmission lines. In the meanwhile, comparing results in Table III with those in Table I, only 6 out of 35 cases incur slightly higher load shedding. Those results suggest that we can achieve a trade-off between the load shedding reduction and the computational time by assigning PLS to an appropriate value.

TABLE III: load shedding (in 100MW) from DAD-TLS with PLS=4

N-K	$R=0$	$R=1$	$R=2$	$R=3$	$R=4$
N-0	0.00	0.00	0.00	0.00	0.00
N-1	1.35	1.29	1.05	0.73	0.69
N-2	2.79	2.79	2.64	2.29	1.79
N-3	4.29	3.90	3.46	3.16	2.46
N-4	5.38	5.16	4.46	3.96	3.21
N-5	6.88	5.96	5.01	4.48	3.77
N-6	7.75	6.48	5.77	5.27	4.42

TABLE IV: computational time (in second) from DAD-TLS with PLS=4

N-K	$R=0$	$R=1$	$R=2$	$R=3$	$R=4$
N-0	1.2	0.7	1.0	1.0	0.8
N-1	2.4	3.7	6.7	12.3	28.1
N-2	2.9	3.2	5.1	22.3	124
N-3	9.5	20.5	20.2	103	139
N-4	9.4	14.3	60	170	5637
N-5	5.3	18.2	87.6	529	7297
N-6	7.7	29.5	158	484	971

2) Candidate switchable lines: In this part, we investigate one situation where only a proper subset of transmission lines are switchable. To achieve system stabilization, we may not want to switch off transmission lines that carry significant amount of flows. So, we study one line switching strategy where only those with the least amount of power flows (measured when a power grid is in normal operating conditions: no attack/contingency) is switchable. In our experiment, we only allow the 10 least power flow lines to be switchable and computational results are reported in Table V and Table VI.

TABLE V: load shedding (in 100MW) of DAD-TLS with 10 least power flow lines switchable

N-K	$R=0$	$R=1$	$R=2$	$R=3$	$R=4$
N-0	1.10	1.10	1.10	1.10	1.10
N-1	1.97	1.90	1.90	1.72	1.71
N-2	3.07	2.79	2.78	2.77	2.51
N-3	4.29	4.11	3.66	3.38	3.16
N-4	5.38	5.16	4.46	4.02	3.96
N-5	6.88	5.96	5.01	4.73	4.66
N-6	7.75	6.48	5.75	5.53	5.25

Comparing results in Table V and Table VI with those reported in previous tables, we note that the computational time is further drastically reduced. However, the performance in reducing load shedding is not satisfactory, especially for cases with $K \leq 3$. Such result indicates that only considering those transmission lines with the least flows as switchable is not very effective in mitigating contingencies. Other lines, which may carry a significant amount of power flow, could be more effective in post-contingency operations.

TABLE VI: computational time (in second) of DAD-TLS with 10 least power flow lines switchable

N-K	$R=0$	$R=1$	$R=2$	$R=3$	$R=4$
N-0	0.4	0.3	0.3	0.2	0.3
N-1	0.6	1.2	1.4	3.2	3.3
N-2	1.1	2.4	2.5	3.6	12.6
N-3	2.1	3.2	4.3	12.6	30.0
N-4	3.5	4.0	8.2	47.5	49.4
N-5	4.1	8.2	35.0	55.5	65.1
N-6	6.0	11.3	40.0	146	192

A similar study is performed on a set of randomly selected transmission lines as switchable candidates. In Table VII and VIII, load shedding and computational times of such candidate set with 10 randomly selected switchable candidate lines are reported. Similar to those in Table V, the performance in reducing load shedding is not satisfactory, which again confirms the challenge and the importance of selecting switchable candidates to mitigate contingencies.

TABLE VII: load shedding (in 100MW) from DAD-TLS with random 10 lines switchable

N-K	$R=0$	$R=1$	$R=2$	$R=3$	$R=4$
N-0	0.50	0.50	0.50	0.50	0.50
N-1	1.91	1.59	1.46	1.26	1.22
N-2	2.99	2.86	2.85	2.85	2.05
N-3	4.29	4.18	3.72	3.16	2.81
N-4	5.44	5.16	4.46	3.96	3.46
N-5	6.88	5.96	5.11	4.48	4.08
N-6	7.75	6.48	5.81	5.27	4.77

TABLE VIII: computational time (in second) from DAD-TLS with random 10 lines switchable

N-K	$R=0$	$R=1$	$R=2$	$R=3$	$R=4$
N-0	0.7	0.3	0.4	0.5	0.8
N-1	2.3	1.6	1.3	3.3	4.0
N-2	3.2	4.0	4.4	5.7	86.1
N-3	2.2	5.5	9.6	20.2	46.3
N-4	11.6	3.5	12.1	38.5	45.7
N-5	2.9	11.0	65.6	48.2	184.3
N-6	7.2	36.3	39.7	303	519.5

V. Benefit Analysis of Transmission Line Switching in Hardening

To investigate the benefits of incorporating transmission network topology control through optimal transmission line switching into power grid hardening problem, in this section, we make a comparison of the hardening plans derived from defender-attacker-defender model with optimal transmission line switching, i.e., DAD-OTS, and those obtained from traditional DAD model (without switching) [22].

A. Performance of Hardening Plans from Transmission Line Switching

We first demonstrate that with transmission line switching, an optimal protection plan (and the associated most destructive attack plan) derived from DAD-OTS model could be very different from that obtained from the classical DAD model. As shown in Figure 2 where $R = 2$ and $K = 2$, an optimal protection plan from DAD-OTS model consists of line 10-12 and line 12-23, while an optimal protection plan from DAD model consists of line 12-13 and line 20-23.

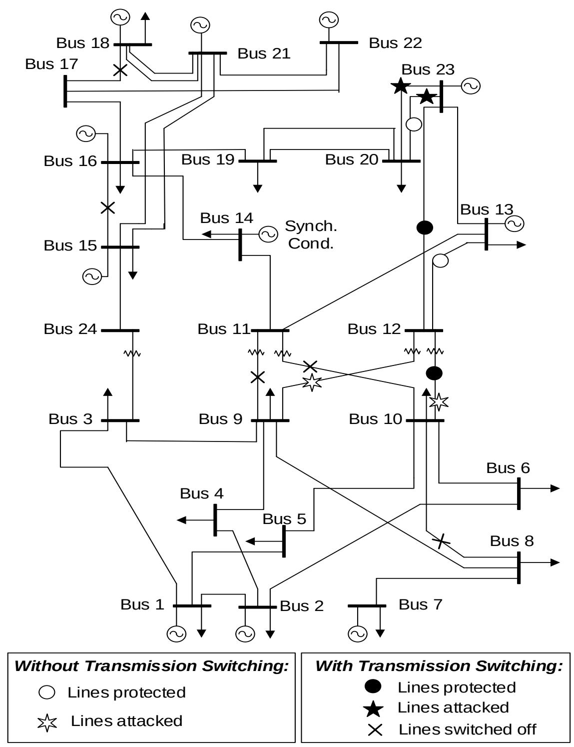

Fig. 2: DAD-OTS and DAD solutions for $R = 2$ and $K = 2$

To have a complete benchmark, we present in Table IX the load shedding of DAD model under all R and K combinations, which are derived in [22]. Comparing it with Table I, we can confirm that load shedding with transmission line switching is always less than those without transmission line switching. More straightforward comparisons can be found in Figure 3 and Figure 4, which present load shedding (averaged over different hardening budgets) from DAD-OTS and DAD models under different N-K contingencies and the relative load shedding reduction (in percentage) brought by transmission line switching.

As illustrated in Figures 3-4, transmission line switching has a very positive boosting effect on power grid hardening plans. In particular, for worst N-K contingencies with $K \leq 3$, more than 15% load shedding reduction can be easily achieved, comparing to hardening plans generated from DAD model. Nevertheless, such effect reduces with respect to K. It can be explained by the fact that, with K getting larger, there is less switching freedom left among the survived transmission lines in the grid. Hence, the benefit of transmission line switching becomes smaller.

B. Cost-Effectiveness Analysis

With our developed computing methods for DAD-TLS and DAD models, we can investigate the minimum hardening budget (i.e., the least number of transmission lines for hardening) to achieve a desired level of load satisfaction under various N-K criteria, which therefore provides a basic cost-effectiveness analysis tool for hardening. Next, we present a demonstration by considering the hardening budget under the worst N-3 contingency.

TABLE IX: load shedding (in 100MW) from DAD

N-K	$R=0$	$R=1$	$R=2$	$R=3$	$R=4$
N-0	1.43	1.43	1.43	1.43	1.43
N-1	2.30	2.06	2.04	2.04	2.02
N-2	3.97	3.27	3.11	2.91	2.91
N-3	4.84	4.47	3.98	3.59	3.50
N-4	5.70	5.36	4.46	4.37	4.25
N-5	7.06	5.96	5.29	5.02	4.80
N-6	7.95	6.67	6.06	5.69	5.47

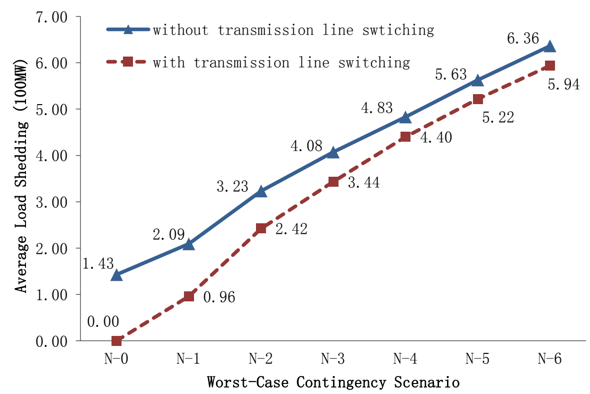

Fig. 3: load shedding in different hardening plans

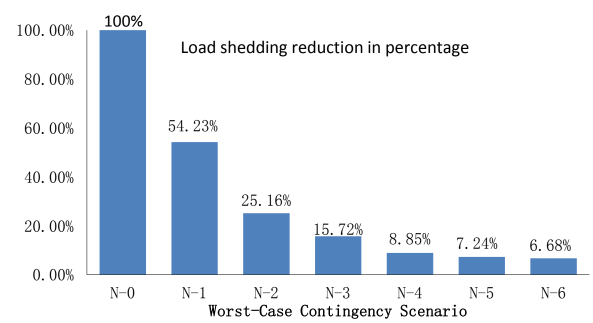

Fig. 4: load shedding reduction through transmission line switching in percentage

Figure 5 presents the numerical results between different load satisfaction requirements and protection budgets in DAD-OTS and DAD models. It is straightforward to realize that to have a higher proportion of load to be satisfied, we need larger protection budgets. However, hardening plans derived from DAD-OTS and DAD demand for drastically different economic investments. If we require that at least 85% total load must be met, 3 transmission lines should be hardened in optimal DAD solution, while protecting 2 lines is sufficient in optimal DAD-OTS solution. Such difference becomes more noticeable when the load satisfaction gets more stringent. For example, if at least 90% total load must be met, at least 14 lines should be hardened in optimal DAD solution while only 5 lines need to be protected in optimal DAD-OTS solution. Given the fact that practical transmission line hardening, e.g., placing lines underground, is very expensive, we can conclude that by modeling and implementing transmission line switching as a post-contingency operation, cost-effective protection plans can be derived that significantly outperform those obtained without considering this switching operation.

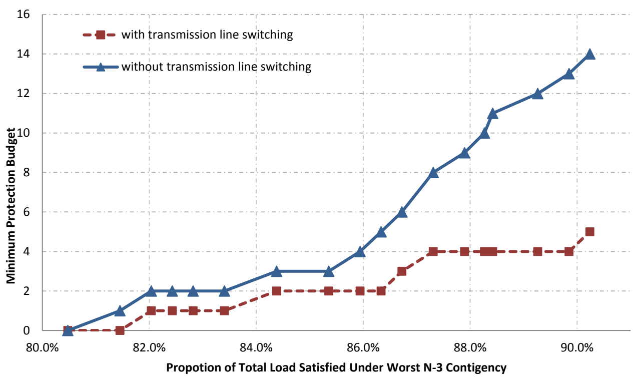

Fig. 5: hardening budget vs. load satisfaction

VI. Conclusion

This paper studies to incorporate transmission line switching operations into the traditional defender-attacker-defender model. For this challenging tri-level DAD-TLS formulation, we customize and implement nested column-and-constraint generation method to derive optimal solutions. A set of numerical experiments are performed on IEEE one-area RTS-96 system. Results verify the benefits of incorporating transmission line switching as a post-contingency operation into DAD model. In particular, it shows that resulting hardening plans from DAD-TLS could be different from those from DAD and they lead to very cost-effective hardening enhancement.

Future work will attempt to improve computational efficiency to address large-scale grids. In particular, we would like to explore parallel implementation and make use of grid structural properties for better computational performance.

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