

Real Options: A Survey

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Abstract

This survey paper provides an overview of real options, in particular the connection with financial options, valuation methods (analytical methods vs numerical methods based on simulation, lattice approximations to stochastic processes and finite-difference methods) and a wide array of application areas, from R&D to operations management to renewable energy project selection.

1 Real Options vs Financial Options

Managers today must not only take decisions under high uncertainty in fast-changing environments, but think ahead to the decisions they might want to take in the future, such as expanding a plant, contracting capacity or purchasing real estate that had been leased, and ensure now that they will have the ability to take those decisions later. Those opportunities are known in the literature as *real options* due to their similarities with financial options on stocks as well as their application to tangible (real) assets. They have emerged as a significant tool in the decision-maker's toolkit; however, they are in many ways more complex than their financial counterparts. Indeed, while the specific decision that the option will allow if exercised is usually embedded in the definition of the option and is thus fairly straightforward to articulate, it is in general difficult to value real options accurately, for instance because – in contrast with stocks – the underlying asset is rarely traded on public stock exchanges. This makes it challenging to create a replicating portfolio, the price of which would be equal to the option's fair value under the no-arbitrage principle. Another source of difficulty is that, while financial options are limited to the realm of finance, real options find applications in a vast array of domains, where the

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stochastic process driving the behavior of the underlying asset may be difficult to characterize, energy and licensing.

Outline. The purpose of this review paper is to describe the state of knowledge regarding real options. The remainder of this section focuses on the analogy between financial options and real options. Section 2 summarizes valuation tools for uncertain projects besides real options: discounted cash flow analysis, sensitivity analysis, decision-tree analysis and contingent-claim analysis. Section 3 describes analytical and numerical valuation methods. Section 4 reviews common applications of real options, in particular R&D projects, operations management, market competition, information technology, healthcare technology and energy. Finally, Section 5 describes some very recent research efforts and contains concluding remarks.

1.1 Financial options

Financial options, in their simplest form, give their owner the right, but not the obligation, to buy or sell an underlying asset at a pre-specified price on or before a pre-specified expiration date. Their pricing was pioneered by Black and Scholes (1973), who compute the fair value of a European call option (right to buy an underlying asset at a pre-specified price on a pre-specified date) when the underlying asset can take only two possible values by creating a portfolio of stocks and bonds that achieves the same payoff in those two states of nature; the option price is then the price of the replicating portfolio.

Cornuejols and Tütüncü (2007) describe how to compute the fair value of a European call option in a one-period, two-states framework. The following notation will be used.

- S_0 : Current stock price,
- R : One-period interest rate,
- K : Strike price of the call option,
- u : Multiplicative factor of the stock price (becoming $u S_0$)
in the case of a upward movement in each period,
- d : Multiplicative factor of the stock price (becoming $d S_0$)
in the case of a downward movement in each period,
- C_0 : Current option price (to be determined),
- C_u^1 : Option payoff in up state,
- C_d^1 : Option payoff in down state,
- x : the number of shares of stock used to create the replicating portfolio (decision variable),
- y : amount in dollars invested in the bond.

Assuming that $d < R < u$ so that no arbitrage exists, the portfolio of stock and bond that replicates the payoff of the option satisfies the following two equations in x and y :

$$\begin{aligned} S_0 u \cdot x + R y &= C_u^1 \\ S_0 d \cdot x + R y &= C_d^1 \end{aligned}$$

This leads to:

$$\begin{aligned} x &= \frac{1}{S_0} \frac{C_u^1 - C_d^1}{u - d} \\ y &= \frac{1}{R} \left(\frac{u C_d^1 - d C_u^1}{u - d} \right). \end{aligned}$$

The fair value of the call option is then $x S_0 + y$ or:

$$C_0 = \frac{1}{R} (p_u C_u^1 + p_d C_d^1) \tag{1}$$

with $p_u = (R - d)/(u - d)$ and $p_d = (u - R)/(u - d)$. Note that $p_u > 0$, $p_d > 0$ and $p_u + p_d = 1$ under the no-arbitrage assumption. Hence, p_u and p_d are interpreted as probabilities and called risk-neutral probabilities.

Eq. (1) states that the option's fair price is the present value of the expected value of the option's payoffs, where the expected value is computed using the risk-neutral probabilities. A key feature of this formula is that it does not depend on the actual probabilities of the stock's going up or down.

The time horizon is then divided in a lattice or binomial tree where each time step is small enough to apply the previous results, and the price of the European call option is computed by proceeding backwards from the last time period, using a system of recursive equations. As the time step goes toward 0, the optimal price of a European call option with expiration time T and strike price (pre-specified purchase price at time T) K , denoted $C(K, T)$, reaches the following limit value, known as the Black-Scholes pricing formula:

$$C(K, T) = S_0 N(d_1) - K e^{-rT} N(d_2), \tag{2}$$

with:

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left(\ln(S/K) + \left(r + \frac{\sigma^2}{2} T \right) \right) \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

where we use the following additional notation:

The closed-form nature of this formula and the ability to quantify the impact of its various parameters on the option price easily have made it a widely-used tool in finance; however, it

S_0 : current stock price
 r : the risk-free rate
 T : time to expiration
 σ : volatility of the underlying asset.

only applies to European options. (A pricing formula similar to Eq. (2) exists in the case of put options.)

More complex options, such as American options – where the decision maker can exercise an option at *any* time until the expiration date – or Asian options – path-dependent options that use as payoff the *average* value taken by the stock price over a past period of time – are priced using either lattice models of stock prices (proceeding backwards from the last time period and using a system of recursive equations), numerical methods based on finite differences of partial differential equations, or simulation (Hull, 2014). We explore numerical methods in greater depth in Section 3.

1.2 The link between real options and financial options

Real options have been the focus of significant research interest in the financial economics literature since they were first introduced by Myers (1977), who addresses a gap in modern finance theory regarding corporate debt policy by arguing that firms' growth opportunities can be viewed as call options. Specifically, Myers (1977) suggests that corporate borrowing is inversely related to the proportion of market value accounted for by real options and introduces the analogy between call options and corporate investment opportunities.

Trigeorgis (1993) provides an extensive classification of real options. In particular:

- An **option to defer** provides the flexibility of delaying the start of an investment (e.g., opening a new plant) depending on available information (e.g., new product demand).
- An **option to abandon** gives the opportunity to abandon a project if the market conditions deteriorate.
- An **option to alter operating scale** lets managers adapt the scale of production depending on changes in factors affecting the profitability of the project.
- An **option to switch** enables the management to modify the output mix of a facility (product flexibility) when the price or demand of its products changes. Alternatively, the same outputs can be produced using different types of inputs (process flexibility).

- **Growth options** can be interpreted as inter-project compound options. (An early investment or outlay can be regarded as a prerequisite for subsequent investment opportunities in complementary products.)
- A **multiple-interactions option** is a combination of options enhancing upward potential and options protecting against downside risk, e.g., opening a new plant while also decreasing capacity at an existing one.
- A **time-to-build option** can be regarded as a combination of several options to abandon with consecutive exercise times. At each single decision point, the manager has an opportunity to abandon the project based on market conditions or investors' interest.

The analogy between financial options and real options is further investigated in Trigeorgis (1996b). A simple correspondence between the parameters of call options and real options is provided in Table 1.

Table 1: Analogy between a call option on a stock and a real option on a project (Trigeorgis, 1996b)

Call Option on stock	Real option on project
Current value of stock	Gross PV of expected cash flows
Exercise price	Investment cost
Time to expiration	Time until opportunity vanishes
Stock value uncertainty	Project value uncertainty
Risk free interest rate	Risk free interest rate

An issue in pricing real options is that there is usually no tradable asset to play the role of the stock in Eq. (2). Hence, a portfolio that exhibits high correlation in risk and payoff with the real option is often used instead (Trigeorgis, 1996b). Copeland and Antikarov (2001) and Rogers (2002) describe methods to price and implement real options for corporate finance managers.

Investment decisions are subject to multiple sources of uncertainty such as project life time, interest rate, currency rate, market share, oil prices, etc. Each source of uncertainty can be the focus of a real option if the decision maker seeks to ensure flexibility and postpone irreversible decisions until more information has been revealed. Busby and Pitts (1997) interview finance officers about the occurrence of different types of flexibility in their capital expenditure projects. Their findings are summarized in Table 2.

Table 2: Frequency of occurrence of types of flexibility in capital investments (Busby and Pitts, 1997)

Frequency (%)	Postponement	Abandonment	Rescaling	Growth	Technical Change
0-20	21	49	30	14	43
21-40	16	28	23	21	29
41-60	16	9	16	12	12
61-80	16	9	16	28	10
81-100	30	5	14	26	7

1.3 Real options and strategy

Luehrman (1998a) argues that strategy can be viewed as a portfolio of real options. This is because any strategy usually involves a *sequence* of decisions, some taken immediately while others are deferred until more information becomes available. Luehrman (1998a), extending the work by Luehrman (1998b), which maps the characteristics of a capital project to the five variables determining the value of a financial call option on a stock, recommends the use of a two-dimensional “option space” plotting value-to-cost against volatility, in order to improve decision making regarding the sequence and timing of a portfolio of strategic investments.

The adoption of real options as a decision-making technique, however, has sometimes been fraught with difficulties. van Putten and MacMillan (2004) point out that real options have been adopted only slowly by managers due to the risk of overestimating the value of uncertain projects. They state: “This reluctance stems at least in part from a suspicion that it is risky to apply valuation tools that have been developed for well-defined financial options to complex business projects.” They suggest that managers use both real options and discounted cash flows as project valuation methods. They also recommend to not only consider the risks associated with a project’s revenues but also its costs, and to incorporate the “benefit of failure” in the analysis, i.e., the value of what a failed investment could bring to the decision maker. They define Total Project Value as the sum of three terms: Net Present Value, Adjusted Option Value and Abandonment Value.

Brosch (2008) formulates the real options portfolio selection problem as a stochastic mixed integer problem with a dynamic budget and path dependency constraints, while the objective function is the expected value of the optimal real options exercise policy. The model accounts for managerial flexibility, inter-project and intra-project options interactions. It is solved using a simultaneous forward- and backward-looking procedure, which introduces path dependency and backward recursion; however, due to the complexity of the problem, a closed form solution cannot be obtained. Finally, Miller (2010) combines real options theory with Bayesian decision

making.

2 Benchmarking approaches

In this section, we describe traditional capital budgeting methods that can be used to complement real options valuation techniques: discounted cash flow analysis, sensitivity analysis, decision tree analysis and contingent claim analysis.

2.1 Discounted Cash Flow Analysis

Net present value (NPV), payback period and internal rate of return are some of the traditional measures that use discounted cash flow (DCF) analysis to evaluate the project's profitability.

The NPV of a project over T time periods with a discount factor r and a stochastic cash flow (C_t), whose expected value is ($E[C_t]$), is calculated as:

$$NPV = \sum_{t=0}^T \frac{E[C_t]}{(1+r)^t}.$$

The manager should go ahead with the project if $NPV \geq 0$ and stop if $NPV < 0$.

The *payback period* is the period it takes to recoup a project's initial investment, given its subsequent cash flows. The *internal rate of return* is the discount factor that brings the NPV of the project to 0. The NPV metric is widely regarded as being the most accurate among these measurements (Trigeorgis, 1996b).

A drawback to DCF methods is that they disregard the effect of managerial control during the lifetime of the project: they assume that managers do not revise their decisions regarding the project. In fact, the market is dynamic and subject to multiple sources of uncertainty. Therefore, managers do update their decisions according to information revealed up to that point, so that they can defer, extend, abandon or contract the project during its life time (Trigeorgis, 1996b).

In addition, the NPV approach calculates the project value on an "expected cash flow" basis by assuming that the cash flow structure of the project is static. Trigeorgis (1996b) asserts that this assumption may lead to an unrealistic project valuation especially when the probability distribution of the project returns is asymmetric.

Uncertainty, which results from sources such as effective tax rate, inflation rate, and the project's life time, can be captured by defining a risk-adjusted discount factor as in the Capital Asset Pricing Model (CAPM). Then, the value of the project equals to the sum of the expected value of the future net cash flows discounted by the risk adjusted rate (Fama, 1996). In the

CAPM model, the risk-adjusted risk factor r' is defined as:

$$r' = r_f + \beta(r_m - r_f),$$

where r_f is the risk free interest rate, r_m is the expected market return, and $\beta = \text{Cov}(r, r_m)/\text{Var}(r_m)$ is the beta of the project.

The NPV approach assumes that the beta of the project stays the same during the project's life time; however, in practice the beta of the project can change over time. Moreover, the NPV approach considers neither market competition nor the interaction between different projects. For instance, a competitor's reaction to a project implementation might affect not only the cash flow structure of that project but also that of the other ongoing projects.

2.2 Sensitivity Analysis

A project's NPV depends on forecasted values of factors such as the project's life time, the cash flow structure, the risk free rate, the market rate, and more. Sensitivity analysis, or "what if" analysis, aims at identifying the key primary variables and determining their impact on NPV when varying each variable at a time; however, sensitivity analysis may not give realistic insights if those variables are interdependent. Traditional simulation techniques including Monte Carlo simulation can be applied to determine the probability distribution of the project's NPV by repeatedly sampling from the probability distributions of the key variables affecting the Net Cash Flows for each period, but accurately reflecting interdependencies between primary variables through their probability distributions is fraught with difficulties and conclusions obtained in this approach may be questionable (Trigeorgis, 1996b).

2.3 Decision-Tree Analysis

Decision-tree analysis (DTA) represents a project as a sequence of decisions and possible realizations of chance events with known probability distributions in a tree structure during the life time of the project (Freund and Bertsimas, 2004). It can also incorporate the interdependency between decisions taken at different time points and the effect of different realizations of chance events on the project's cash flow structure. This eases management's task of visualizing the project's inherent options and price them into the project's value. However, as the number of decisions or chance event realizations increases, the number of possible paths in the decision tree increases geometrically, which turns the decision tree in the analysis into a "decision bush" that quickly becomes intractable; in addition, DTA uses the same discount factor throughout the

life time of the project and neglects the dynamic nature of the project's riskiness (Trigeorgis, 1996b). Updating the discount factor based on available information at each time period could help overcome this problem but this idea is hard to implement in practice (Schulmerich, 2010).

2.4 Contingent-Claim Analysis

Contingent claims analysis (CCA) has its origin in the pricing of financial options, and motivates the derivation of Eq. (1). In the words of Schulmerich (2010), CCA “*seeks to replicate the payoff structure of the project and its real options via financial transactions in order to determine the NPV of the project... With DTA the basic idea is to calculate with real probabilities and a constant, risk-adjusted interest rate as the discount rate. With CCA the basic idea is to transform the real probabilities into risk-adjusted probabilities such that the algorithm can use a constant, risk-free interest rate that is independent of the project's risk structure.*”

As such, CCA is a fundamental technique to value real options rather than an alternative model to quantify the value of a risky project. The use of CCA reminds the decision-maker of the key assumptions underlying option pricing theory: a continuously tradable underlying asset, no arbitrage, and a Geometric Brownian motion as the stochastic process driving the asset price. Therefore, CCA seems better suited to financial options than to real options, but is used to value real options due to the lack of a clearly superior, alternative method and the simplicity of the Black-Scholes pricing formula (1).

Dixit and Pindyck (1994) provide further discussions on real options, their advantages compared to NPV and valuation methods. The landmark Dixit-Pindyck model they describe refers to a contingent-claims valuation method for real options based on solving a partial differential equation, invoking Ito's lemma and finding boundary conditions. The decision maker must be able to span the risk using existing assets in order to construct a riskless portfolio. If this condition is not satisfied, the manager may use dynamic programming with an exogenously specified discount rate instead. The authors then advocate decomposing the value of the project at time t as the sum of two terms: (i) the operating profit over $(t, t + dt)$ and (ii) the continuation value beyond $t + dt$. Invoking Ito's lemma then leads to the derivation of a new differential equation that can be solved easily and allows for in-depth analysis of the structure of the option value, both in the cases with and without spanning assets.

The simplest version of the Dixit-Pindyck framework considers a firm that has the monopoly right to invest in a project that will produce a given output flow in perpetuity, at an investment cost of I . The question is to decide when to incur the cost of I when the exogenous price of the output P follows a Geometric Brownian Motion with drift α and volatility σ . Let ρ be the

risk-adjusted discount rate. It can be shown that the expected present value V of the project when the current price P is $V = P/(\rho - \alpha)$, and thus also follows a Geometric Brownian Motion with the same drift α and volatility σ . Dixit and Pindyck (1994) shows that it is optimal to invest when the expected value of the project reaches (or exceeds) the threshold value:

$$V^* = \frac{\beta_1}{\beta_1 - 1} I,$$

with β_1 the larger root of the quadratic equation:

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - \rho = 0.$$

Note that the option value multiple $\frac{\beta_1}{\beta_1 - 1}$ is always greater than 1, so that there is value in waiting even if the current expected value of the project exceeds the investment cost I .

Henderson (2004) investigates how to value real options without a perfect spanning asset, with a focus on an option to invest; the author finds that the value of the option to invest and the trigger level are both lowered when the spanning asset is less than perfect, and investment should take place earlier.

3 Real Options Valuation Methods

Real options valuation methods can be classified in two main groups: analytical and numerical (Schulmerich, 2010).

3.1 Analytical Models

Eq. (2) is the simplest example of an analytical model, obtained in the case of a European call option; however, analytical approaches are sometimes possible for other options. In this approach, the problem of determining when to exercise the option can be modeled as an optimal stopping-time problem, solved using recursive backward equations called Bellman's equations, usually with a finite-time rather than infinite-time horizon (Dixit and Pindyck, 1994). The option's fair value is then the value of the stopping-time problem at time 0. Analytical methods are also discussed in Trigeorgis (1996b). Examples of options that can be analyzed in this framework are the option to defer, option to abandon and option to switch, below.

Option to defer: McDonald and Siegel (1986) model the gross project value $(V_t)_{t \geq 0}$ by a diffusion process given via the stochastic differential equation (SDE):

$$dV_t = \alpha V_t dt + \sigma V_t dB_t, \quad t \geq 0, \alpha \in R^+, \sigma \in R^+,$$

where α is the instantaneous drift on the project and σ is the instantaneous volatility. Paddock et al. (1988) value the option to defer for the project, which has a payout rate D , with the SDE:

$$dV_t = (\alpha - D)V_t dt + \sigma V_t dB_t, \quad t \geq 0, \alpha \in R^+, \sigma \in R^+$$

Option to abandon: McDonald and Siegel (1985) model the unit output price's diffusion process $(P_t)_{t \geq 0}$ as:

$$dP_t = \alpha P_t dt + \sigma P_t dB_t, \quad t \geq 0, \alpha \in R^+, \sigma \in R^+.$$

while Myers and Majd (1990) consider the following process:

$$dP_t = \alpha(D - P_t)dt + \sigma P_t dB_t, \quad t \geq 0, \alpha \in R^+, \sigma \in R^+,$$

where D is the instantaneous cash payout or dividend.

Option to switch: Margrabe (1978) values an option to exchange one risky asset for another with the same diffusion process for each asset's price. In contrast with the Black-Scholes model (2), this model assumes neither the existence of a risk-free asset nor that of traded bonds. We will use the following notation:

- S_{i0} : current stock price of asset i , $i = 1, 2$,
- q_i : (constant) dividend yields of asset i , $i = 1, 2$,
- σ_i : (constant) volatility of asset i , $i = 1, 2$,
- ρ : (constant) correlation between the assets,
- T : time to expiration.

Margrabe's formula values the exchange option at:

$$e^{-q_1 T} S_{10} N(d_1) - e^{-q_2 T} S_{20} N(d_2), \quad (3)$$

with:

$$\begin{aligned} d_1 &= \frac{\log(S_{10}/S_{20}) + (q_2 - q_1 + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \\ d_2 &= d_1 - \sigma\sqrt{T}, \\ \sigma &= \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}. \end{aligned}$$

Analytical methods can value a single real option; however, they cannot account for the interaction between several real options properly (Schulmerich, 2010). In general, option valuation problems can be solved using analytical methods based on partial differential equations if the state variable is an Ito process, but these methods will not work if the state variable is a function of other variables evolving as Ito processes. Therefore, analytical methods are not capable of valuing complex real options, which require the use of numerical methods.

3.2 Numerical Models

Common numerical methods used in valuing real options are described in Trigeorgis (1996b). They are of two main types: (i) approximations of the underlying stochastic process as a lattice model – usually a binomial tree – often used in conjunction with Monte-Carlo simulation to generate sample paths and value the option (Monte-Carlo simulation can also be applied more widely to stochastic processes without the lattice approximation), and (ii) approximations of the partial differential equations using finite-difference methods.

3.2.1 Monte-Carlo simulation and approximation of the underlying stochastic process

The Black-Scholes formula Eq. (2) recovers the limit value of a binomial tree model when the time step goes to zero, and thus bridges analytical and numerical methods. When such closed-form expressions are not available, the equations must be evaluated using numerical methods. Boyle (1977) pioneers the use of Monte-Carlo simulation to value options and discusses the use of variance reduction techniques such as control variates. Improvements in efficiency to the Monte-Carlo framework are discussed in Boyle et al. (1997). The Monte-Carlo simulation is computationally expensive due to the need to evaluate the value function at each time step, and thus motivates the development of a more efficient method called Least-Squares Monte Carlo Simulation Method (LSM), due to Longstaff and Schwartz (2001). LSM reduces the computational cost by using least-squares regression at each step. It is particularly cost-effective in obtaining option values for high-dimensional variables.

Models such as Cox et al. (1979), Hull and White (1988) and Trigeorgis (1991) approximate the underlying stochastic process as a binomial tree. Trigeorgis (1991) develops a log-transformed lattice approach with constant risk free interest rate and argues that this method is a consistent, stable, and efficient binomial tree method that can value complex investments with interacting real options. The author further makes the case that lattice approaches are superior to Monte Carlo simulation in terms of simplicity and flexibility in handling different stochastic processes,

options payoffs, early exercise of the other intermediate decisions (interaction), etc. In addition, they can handle real option packages and compound real options; however, Schulmerich (2010) points out that the lattice approaches value the option for only one underlying start value. Valuing the option with various initial conditions can be time consuming. The author modifies the binomial tree approach of Cox et al. (1979) and the log-transformed binomial tree approach of Trigeorgis (1991) in order to value real options in the presence of stochastic interest rates.

3.2.2 Approximation of the partial differential equations

Finite-difference and numerical integration methods are used to approximate partial differential equations (PDEs). The finite-difference approximation involves space/time discretization on a grid, leading to a finite system of recursive equations in the underlying state variable i and the time state j . Parkinson (1977) focuses on numerical integration, which may use for instance Taylor expansions of the derivatives. Brennan (1979) and Brennan and Schwartz (1978) explore implicit and explicit finite difference schemes; explicit methods evaluate the state of the system “now” based on the state of the system at earlier time periods, while implicit methods evaluate the state of the system “now” based on equations incorporating the state of the system at earlier time periods as well as “now”. Explicit finite-difference methods are analogous to lattice model approximations to the stochastic process solved backward from the expiration date T to time 0. Implicit methods use Gauss algorithm to solve the resulting linear equation system. Barone-Adesi and Whaley (1987) apply quadratic approximation schemes, which they argue are accurate and considerably more computationally efficient than finite-difference, binomial or compound-option pricing methods.

Note that the log-transformation in Trigeorgis (1991) is motivated by the improved numerical properties of the method; in particular, the coefficients in the finite-difference method no longer depend on the state of the underlying variables. Trigeorgis (1996b) provides a detailed review of partial differential approximation approaches. A comprehensive example is provided in Hull (2014).

4 Applications

Real options can be applied to a wide range of settings. R&D projects, operations management, market competition, commercial lease contracts, information technology, energy and revenue management are some of the areas where the real options approach is employed as a decision tool.

4.1 R&D Projects

Compound options

R&D projects have been the most common application area for real options, especially for biopharmaceutical projects. Pennings and Lint (1997) develop a stochastic jump amplitude model to better capture the option value in R&D, and apply their framework to a division of Philips Corporate Research. Panayi and Trigeorgis (1998) model R&D projects with multi-stage decisions as compound options, which are combinations of sequential investment options, to reflect the three main stages of the project: research, technical construction-development, and implementation-commercialization. Childs et al. (1998) use a real options approach to analyze interrelated projects, where a firm may invest in the development stage of two projects (in parallel or in sequence) and then will select only a single project to implement. The authors show the optimality of sequential development over parallel development when projects have highly correlated values and when they require a large commitment of capital for development, are short term in nature, and have relatively low volatility.

Further, Childs and Triantis (1999) examine dynamic R&D investment policies and the valuation of R&D programs using contingent claims analysis, in presence of the following factors: learning-by-doing, collateral learning between different projects in the program, interaction between project cash flows, periodic reevaluations of the program, different intensities of investment, capital rationing constraints, and competition. They make the following findings: (i) the optimal strategy is not very sensitive to misestimation of volatility for high volatility projects; (ii) the option to accelerate the lead project (if a project dominates the others) is often more valuable than the option to exchange projects when in the presence of a budget constraint; and (iii) competition from other firms leads to more parallel investment in the early development stages of projects, less parallel investment in the latter stages, and lower investment overall.

Bollen (1999) explicitly incorporates a product life cycle in the option valuation framework, and argues that standard techniques with a constant expected-growth rate are inaccurate for the valuation of capacity options in high-technology industries with regular introductions of newly improved products. Sarkar (2000) shows that in certain situations, an increase in uncertainty can actually increase the probability of investing, and thereby have a positive impact on investment. Pennings and Lint (2000) use the real options approach to find the optimal timing and region to roll out a new product with known unit cost and stochastic profit margin and demand following a correlated geometric Brownian motion considering the market competition. The cash flow value at each time unit is represented by a stochastic equation and a financial options valuation method is used to calculate the value of the real option giving the right to roll out a new product.

The authors provide a case study about Philips Electronics and conclude that the market and technology uncertainty impact the value of the phased roll-out strategy.

Jensen and Warren (2001) use a three-phase lifecycle for a compound options model to value research in the service sector, with an example drawn from the e-commerce area. Loch and Bode-Greuel (2001) argue that, because the major risks in R&D projects are typically project-specific and cannot be replicated in external markets, an asset with the same – or at least correlated – payoffs cannot be found to replicate the payoff of the project. In their opinion, decision trees are superior tools to represent managerial options for R&D projects and evaluate the value of these projects. The authors use their framework to evaluate compound growth options arising from research at a pharmaceutical company.

Bowman and Moskowitz (2001) caution against some of the assumptions underlying most standard option valuation models in a case study involving a R&D project at Merck, and advocate the creation of customized real option models in strategic analysis. Huchzermeier and Loch (2001) use a real options approach to address project management under risk and evaluate flexibility in R&D. The authors consider five types of operational uncertainty, namely, the market pay-off, budget, performance, market requirement, and schedule uncertainties. They then evaluate the impact of these operational uncertainties on the value of managerial flexibility in R&D projects using the real options approach in conjunction with stochastic dynamic programming. They observe that uncertainty may decrease the probability of real options being exercised, therefore, may decrease the value of flexibility. In addition, the value of flexibility increases with the level of uncertainty if the decision is made after uncertainty is resolved and before costs and revenue augment. Lint and Pennings (2001) model the product development process as a series of real options with reducing uncertainty over time and show how to assign any particular project within a 2 x 2 matrix of uncertainty versus R&D option value, using insights drawn from the product development process at Philips Electronics.

Neely and de Neufville (2001) suggest combining the best features of DCF and real options valuation. They distinguish between project risks, to which they recommend applying decision analysis tools, and market risks, for which real options are more appropriate, and call their technique *hybrid option valuation*. It consists in inserting chance events reflecting market conditions into the decision tree that already includes decision nodes about the investment strategy and chance events about technical difficulties. Next, the decision maker calculates the project value by discounting the project value (based on option-exercising decisions) at each node using the risk-adjusted discount rate. Finally, the authors check the effects of assumptions and parameters on the project value using sensitivity analysis.

Weeds (2002) considers investing in (competing) research projects with uncertain technological success and a stochastic value of the patent to be won, and analyzes how the fear of preemption in a winner-take-all patent system affects optimal decision-making in a non-cooperative equilibrium. Folta and Miller (2002) investigate decision-makers' optimal strategy when they may acquire additional equity in partner firms in research-intensive industries and face the threat of preemption by rivals. They test their assumptions using data from minority investments in the biotechnology industry. A key result is that greater uncertainty encourages commitment. MacMillan and McGrath (2002) treat R&D projects as one of three possible real options (positioning options, scouting options and stepping-stone options), depending on technical and market uncertainty, and explain how to design a portfolio of R&D projects that is consistent with a company's strategy.

Paxson (2003) provides seventeen articles written by various researchers on wide-ranging topics related to real R&D options including learning and incomplete information. Cassimon et al. (2004) model the process of developing a new drug in the pharmaceutical industry as a series of consecutive phases from R&D to commercialization, and model each phase as an option on executing the following phase, i.e., a compound option. The authors argue that, in the case of new drug applications, the R&D phase can best be described as a six-fold compound option and derive a closed-form solution for a n-fold compound option model. McGrath and Nerkar (2004) apply real options reasoning to R&D investment in the pharmaceutical industry and identify three factors that influence a biotech firm's propensity to invest in R&D.

Santiago and Vakili (2005) investigate whether the value of a R&D project increases as uncertainty increases, when the problem of determining the optimal managerial decisions is formulated using dynamic programming. They argue that when the source of variability is development uncertainty or market requirement uncertainty, it is not possible to make a general statement regarding the impact of increased uncertainty. If the source of variability is market payoff, increased variability will increase either the overall project value or the project option value.

Santiago and Bifano (2005) introduce multidimensional decision trees to assess the development of a new product under technical, market and cost factors. The model provides optimal managerial actions to be taken at each stage of the review process. Tsui (2005) applies real options to value an innovative R&D project in the automotive industry. First, uncertain demand is predicted using Monte Carlo simulation, then a linear optimization model is solved to obtain the optimal product portfolio for cases with and without the innovative product at each decision node. The difference between the profit amounts predicted in each case determines the decision

to exercise the option. The optimal exercise time is obtained through backward recursion.

Hartmann and Hassan (2006) survey pharmaceutical companies and the healthcare departments of financial services firms regarding the application of real options. They observe that pharmaceutical companies tend to apply real options in the clinical phases of R&D development, while financial services firms observe the highest values in the pre-clinical and early clinical phases. Wong (2007) shows that the critical value of a project that triggers the exercise of the real option exhibits a U-shape against the project's volatility. The positive investment-uncertainty is more likely for relatively safe projects and for high-growth ones. Zhang et al. (2007) explore the connection between the centrality of a firm's R&D organization structure and its propensity to form strategic alliances, and validate their ideas using 2,647 strategic alliances formed by 43 pharmaceutical companies over 1993-2002.

According to Wang and Hwang (2007), the traditional financial analysis approach underestimates the R&D project value, because it ignores the fact that long lead times of R&D projects decrease the credibility of the original data collected in order to determine the optimal portfolio of R&D projects. The authors name this type of information corruption "R&D uncertainty" and suggest that a fuzzy integer portfolio selection model can overcome this deficiency. The authors combine compound options pricing model introduced by Geske (1963) with the fuzzy set theory in order to calculate the R&D projects' value under R&D uncertainty. Next, they transform the fuzzy integer programming problem into a crisp mathematical model using a qualitative possibility theory. The new model is solved via an optimization technique.

Bekkum et al. (2009) analyze portfolios of R&D projects using the real options approach. The authors show that if the projects are positively correlated, diversification is an effective tool for reducing risk. On the other hand, strategies such as synergies and spillovers should be considered rather than diversification under negative correlation. In addition, they observe that if high-risk projects are considered in the portfolio, then the overall portfolio risk is less sensitive to correlation.

Martzoukos and Zacharias (2013) develop a real options framework to study a research joint venture where two firms have to decide on both the optimal level of coordination in R&D activities and the optimal level of effort and money spent on information acquisition activities considering the spillover effects. In other words, each firm holds an investment option and aims to maximize the profit potential through information acquisition or investing in R&D projects to improve the potential for cost reduction and revenue increase. The authors propose a game theoretic approach allowing firms to coordinate their R&D activities due to the spillover effect between firms' R&D actions. A two-stage closed-loop stochastic game is proposed to determine the optimal set of

decisions for the firms and the values of the embedded real options.

4.2 Operations Management

Joint ventures

Kogut (1991) models a joint venture as a real option to expand in order to respond to future technological and market developments, and justifies his interpretation by analyzing 92 manufacturing joint ventures. Kogut and Kulatilaka (1994) show that the global operations of a multinational with geographically dispersed subsidiaries can be interpreted as owning an option, the value of which depends on the real exchange rate. Folta (1998) argues that minority direct investments and joint ventures should be viewed as options to defer either internal development or acquisition of a target firm by considering a sample of 402 transactions in the biotechnology industry.

Reuer and Leiblein (2000), however, argue using real options theory that U.S. manufacturing firms with greater investment in international joint ventures do not generally obtain lower levels of downside risk. Kouvelis et al. (2001) study the effects of exchange rates on firms entering foreign markets, which can choose between exporting (EXP), joint ventures with local partners (JV) and wholly owned production facilities (WOS) in the foreign country. They identify a hysteresis phenomenon that characterizes switching behavior between strategies in the presence of switchover cost. Their numerical results suggest that a weak home currency favors the JV over the WOS, and the EXP mode over the WOS or JV.

International joint ventures (IJV) are further studied in Tong et al. (2008), who show that ownership structure, product-market focus and geographic location are important factors affecting the value of embedded growth options in the IJV framework. Graf and Kimms (2011) employ an option-based procedure regarding the capacity control problem for the strategic alliance of two airlines, where the main decision is the number of seats allocated to booking classes of each airline in the alliance.

Lukas et al. (2012) study mergers and acquisition deals involving contingent earn-outs in a game-theoretic real options approach. The authors consider a buyer and a target firm both of which are risk-neutral. The target firm's cash flows are assumed to follow a Geometric Brownian motion. Sunk transaction costs occur in the acquisition process; however, the buyer firm enjoys the possible synergies and future cash flows of the target firm later. Possible synergies are modeled as a positive, monotonously increasing, and a concave function. The problem of determining the optimal earnout and initial payment conditions and timing of the acquisition is solved by means of dynamic programming.

Exchange rate

Using contingent claims analysis, Kamrad and Siddique (2004) analyze and value supply contracts in the presence of exchange rate uncertainty, supplier-switching options, order-quantity flexibility, profit sharing, and supplier reaction options, and model how flexibility can benefit both the producer and the suppliers.

Lin and Wu (2004) consider an export-oriented manufacturer planning to the transfer production location from a domestic country to a foreign country. The exchange rate is assumed to follow a geometric Brownian motion. The problem of determining the optimal labor and raw material allocation decisions along with the production shift decision (American type options) is formulated as a stochastic control problem and dynamic programming and Lagrange multipliers approaches are used to solve the problem.

Burnetas and Ritchken (2005) investigate supply chain options, when the retailer has the right to reorder (or to return) items at a fixed price. The authors show how the introduction of option contracts affects the wholesale price and the retail price. They also derive conditions under which the manufacturer prefers to use options, in which case the retailer may be worse off if the uncertainty is sufficiently high.

Nembhard et al. (2005) investigate the effect of the time lag between the time when the real option is decided to be exercised and the time that the decision is implemented on the on the outcome of the switching (supplier, production plant etc.) decisions in a supply chain under exchange rate uncertainty. The exchange rate between the home country currency and that of a foreign country is assumed to follow a geometric Brownian motion. The authors formulate the problem of valuing the switching option under exchange rate uncertainty as a stochastic dynamic problem where the recursive value function is optimized at each stage in order to maximize the profit by selecting an option for a given state variable (exchange rate) value and the option selected in the previous stage. The option valuation process is handled via modeling exchange rate movements by two alternative approaches; namely, a multi-nominal lattice approach and a Monte-Carlo simulation. The authors observe that the option value decreases as the time lag increases. In addition, the proposed Monte Carlo simulation method provides closer approximations to the true option value, and handles the valuation process for the cases with large number of variables more efficiently, than the lattice approach. Fujita (2007) formulates an international trade model considering stochastic exchange rates using the real option approach, and measures the effect of foreign exchange rates on the exporting country. The author observes that higher uncertainty on foreign exchange rates leads to higher growth rate and variance of the welfare of the exporting country.

Supply chain options, lease contracts and transportation planning

Trigeorgis (1996a) evaluates lease contracts with operational options such as option to buy, cancel, and renew using contingent claim analysis. The author suggests a CCA-based numerical analysis for leasing contracts with multiple interacting options.

Cortazar et al. (1998) consider a copper production plant that has to obey an environmental regulation schedule limiting the disposal amount. The environmental impact of the production facilities can be lessened by investing in R&D projects and new technologies, which are assumed to be irreversible investments and to increase the operational costs; otherwise, the production amount should be kept in low levels to match the regulations. The authors use the real options approach to determine the optimal output price level at which the investment option on environmental technologies is exercised. A Geometric Brownian motion is used to formulate the output (copper) and input (copper concentrate) prices. The authors propose a model on continuous environmental investments at each point in time where the decision variables are the environmental investment schedule and the output levels. The original problem does not have an analytical solution; however, it can be solved by numerical methods. However, if the input (concentrate) price is assumed to be a fraction of the output (copper) price, then only one uncertainty source of price remains in the model and the problem can be solved analytically. The authors conclude that the environmental regulations might cause production plants under emission restrictions to decrease their output levels instead of investing in environmental technologies when the output price volatility is high.

Dangl (1999) applies the real options approach for a strategic investment problem of a firm where the optimal timing and capacity of an irreversible investment have to be determined under demand uncertainty. The optimal timing and size of the capacity extension decision are obtained via a stochastic dynamic programming approach.

Schwartz and Smith (2000) use a two-factor stochastic commodity price model that reflects the mean-reversion in the short-run prices and the uncertainty in the equilibrium price level to which prices converge in the long-run. The changes in the equilibrium price level is formulated according to geometric Brownian motion with drift expressing the expectations of the consumption of the existing supply, improvements in production technologies, new commodity reserve discoveries, inflation, and political and regulatory effects. Mean-reverting Ornstein-Uhlenbeck process is used to model the short term deviations (the difference between the spot and the equilibrium prices) which revert to zero. These deviations result from some short-term changes in demand, supply, or price dynamics. Kalman Filtering, an iterative procedure for estimating unobserved state variables based on observations whose values are affected by these state vari-

ables, is employed. The authors use the proposed stochastic commodity pricing approach in a real options valuation problem where the decision maker has a right to build an oil production plant which starts producing oil after a determined time lag. The problem to determine the value of the investment and the optimal exercise strategy is solved by a discrete-time, infinite-horizon dynamic programming where at each period the decision maker either exercises the option to develop the production plant or postpones the decision till next period. They observe that the proposed method provides closer commodity price estimations and, therefore, real options valuations than the benchmark models.

Martzoukos and Trigeorgis (2002) propose an asset valuation approach where the underlying asset follows a mixed jump-diffusion process with multiple jumps, each of which is assumed to be independent of each other and to have a log-normally distributed jump-size and a Poisson-distributed inter-arrival time. The authors provide a general valuation framework and an analytical solution for European-type real options and a Markov-chain solution approach for valuing both American- and European-type real options. The authors argue that the proposed asset valuation with multiple jumps method leads to more realistic option values than the prevailing methods for both financial and real options.

Bellalah (2002) provides a valuation method for lease contracts in the real options framework under incomplete information. The author presents a term structure of lease rates under incomplete information and a framework for the equilibrium lease rate. The incomplete information modeling is inspired by Grenadier (1995), who a unified framework for pricing a variety of leasing contracts. The structure of the model is analogous to models of the term structure of interest rates in finance. Bellalah (2002) computes the equilibrium rents on leases with options to renew and options to cancel.

Bengtsson and Olhager (2002) use a real options approach to value the product-mix flexibility in presence of uncertain demand, correlation between products, and relative demand distribution within the product-mix. The problem is to maximize the total contribution margin of the production subject to production capacity and demand constraints. The authors model the demand for each single product through a mean reverting stochastic differential equation and use a Monte Carlo simulation method to solve the problem, which helps them estimate the value of the option using the pay-off values over all simulation runs. In addition, they address the need for incorporating an equilibrium model such as the in-temporal capital asset pricing method (ICAPM) while using the traditional option pricing method.

Berling and Rosling (2005) consider the systematic risk of the stochastic demand and purchase price and analyze their effect on the inventory policies in a real options framework. A stochastic

Wiener process is used to model the stochastic factors such as demand and price, and two inventory models (a single-period newsboy model and an infinite horizon model with a fixed set-up cost) are employed. The authors observe that the systematic purchase-price risk has a significant effect on the inventory policies (re-order point and order quantity), whereas that of the systematic risk of stochastic demand is negligible.

Clark and Easaw (2007) study the problem determining the optimal access price to enter a natural monopolistic network under cash flow uncertainty. The price of the commodity and the demand evolve according to a Geometric Brownian motion. The entrant firm has an option to postpone entrance where entrance into the network corresponds to undertaking the entire investment. The value of the option to invest is then calculated.

Cortazar et al. (2008) investigate a computer-simulation based least squares estimation method (LSM) that incorporates a three factor stochastic process to model commodity prices in order to efficiently and effectively value American-type real options on coal mine investments. The authors conclude that the simulation based real options valuation methods are promising tools that provide a higher degree of freedom to use rigorous models than classical methods, without the concern of obtaining analytical solutions.

Shibata (2008) studies the effect of uncertainty on real options valuation using an extension of Bernardo and Chowdhry (2002). While standard real options pricing models consider only profit uncertainty, Shibata's model accounts for three uncertainty sources: profit, information and estimation uncertainty. Information and estimation uncertainty results from incomplete information. The main motivation for the paper is the fact that the cumulative profit of the initial action at time t can be observed; however, the current realized value of the underlying state variable is not determined with certainty. The author investigates the effect of the three sources of uncertainty on the value of the real option.

Secomandi (2010) investigates the optimal inventory-trading policy (in terms of in-terms of inventory availability and prevailing commodity price), under both space and capacity constraints. The author considers an exogenous Markov process to model the evolution of the commodity spot price. The decision maker has control over both operational and inventory trading decisions corresponding to capacity injection/withdrawal. The author argues that decoupling these two types of decisions is generally difficult and proposes an optimal trading policy at each iteration, where the operational decisions and capacity injection/withdrawal decisions depend on both the spot price and the inventory level. He shows the value of such an inter-dependence structure using real data from the natural gas industry.

Chow and Regan (2011) quantify the value of flexibility for deferral and design strategies in

investments made in a network in the presence of non-stationary uncertainty. They propose a model to determine the value of a network investment deferral option (NIDO) with the ability to redesign the network. They show that the option premium can be decomposed into a basic deferral premium and a flexible network design premium, representing the cost of committing to a transportation planning choice. The model is tested on the Sioux Falls, SD network. The optimal option exercising time is obtained by solving a dynamic programming problem with the network design subproblem approached using the least-squares Monte-Carlo simulation method (LSM). LSM decreases the computational burden that would otherwise be incurred by the traditional Monte-Carlo simulation method to value a network-based objective function.

Löffler et al. (2012) examine the vendor selection process with several key variables including the timing of the contracting, transfer payments, and set-up, switching and abandonment decisions in an asymmetric information setting when a new supplier enters the market. The information asymmetry arises from the fact that the new entrant has imperfect information about its costs, whereas the incumbent supplier has perfect information about its own costs. The buyer selects one of these two suppliers to form a supply chain. The authors focus on the impact of the asymmetric information on the timing of contracting with the new entrant firm and on the buyer's set of actions.

Oh and Özer (2013) study forecast information sharing and decision making under uncertainty with multiple decision makers having asymmetric information. Specifically, the authors focus on the problem of a supplier extracting credible forecast information from a manufacturer to plan its capacity investment decision. The supplier has an option to defer the capacity investment decision and obtain more information from the manufacturer, which will decrease the degree of uncertainty that the investment decision is subject to. On the other hand, waiting for further information leads to a tighter deadline for the capacity expansion project, which increases the cost of the project. Specifically, the supplier decides on the timing of the capacity expansion, on the size of the capacity expansion, and whether to associate with the manufacturer for information sharing (at a cost). The authors represent the degree of demand uncertainty and that of information asymmetry with parameters whose value changes over time and propose a model for the dynamic evolutions of asymmetric forecasts. In addition, the value of the option for the capacity expansion is valued based on the proposed forecasting approach.

4.3 Market Competition

Theoretical results

Recently, researchers have begun to combine market competition and real options. Savva and Scholtes (2002) study partnership contracts under uncertainty with cooperative options (exercised jointly, seeking to maximize the total contract value), non-cooperative options (exercised unilaterally) or coalitions (where the option is exercised to maximize the option holders' payoffs). The authors use standard contingent claims analysis under a complete markets assumption and dynamic programming in the case of heterogenous risk aversion. They conclude that non-cooperative options can be powerful bargaining tools but can also destroy partners' incentive to participate in the contract.

Murto et al. (2004) focus on the valuation of the investment projects (with the purpose of adjusting production cost and capacity) in a oligopoly market for a homogeneous commodity. The authors aim to determine the optimal timing of the granular investment project considering the oligopolistic competition and price uncertainty. The price uncertainty arises from exogenous uncertainty and new capacity investments' impact. Market demand evolves stochastically and the firms move sequentially. The authors first obtain a unique Markov-perfect Nash equilibrium, then a Monte Carlo simulation is run to generate demand realizations over time, which will be used to determine the values of the firms as a result of their investment decisions. Smit and Trigeorgis (2006) use real options to investigate corporate investment opportunities under uncertainty, and in particular whether it is optimal to compete independently or to collaborate via strategic alliances. The authors apply their framework to consumer electronics and telecom, and provide insights into the optimal strategic decisions.

Kong and Kwok (2007) investigate strategic investment games between two firms that compete for optimal entry in a project that generates uncertain revenue flows, when both the sunk cost of investment and the revenue flows of the two competing firms are asymmetric. The authors provide a complete characterization of pre-emptive, dominant and simultaneous equilibriums. Ferreira et al. (2009) provide a toolkit for strategic investment decisions in a competitive environment based on options games. The key methodological tool lies in overlaying the binomial trees of real options analysis with game theory payoff matrices that capture competitive interactions. These options games help the decision maker better evaluate the trade-off between flexibility and strategic commitment. While they are most obviously suited to companies in capital-intensive, oligopolistic markets in the presence of high demand volatility, the framework can be applied to a wide range of settings in practice. Thijssen (2010) considers preemption in a real option game with a first-mover advantage and player-specific uncertainty. Payoffs are driven

by a player-specific stochastic state variable. The author shows that there exists an equilibrium with qualitatively different properties from those in standard real options games.

Siddiqui and Takashima (2012) study games of lumpy capacity expansion projects under output price uncertainty with different settings including monopolistic and duopolistic markets. Sequential decision making for capacity expansion offers the manager the right to defer exercising the investment option until it is in his best interest to do so, based on market competition and the output price. In addition, the exogenous shock to demand is assumed to evolve according to a Geometric Brownian motion. The sequential capacity expansion decisions are determined by a dynamic programming problem in the case of a monopolistic market. In the duopolistic market case, a dynamic sequential game approach is used to determine the optimal timing for the capacity expansion decision. The authors provide insights into the effect of the uncertainty on the value of flexibility for both cases.

Real estate

Grenadier (1995) develops a unified framework for pricing a wide variety of leasing contracts using a real-options approach to endogenously derive the entire term structure of lease rates. The model is flexible enough to determine the equilibrium lease rates for forward leases, adjustable rate leases, leases with options to cancel or renew and leases with payments contingent on asset usage. Childs et al. (1996) examine the effect of mixed-use and redevelopment options on property values. They investigate how the ability to mix uses and redevelop over time affects the timing of initial land development and highlight the impact of the marginal revenue (especially whether it is constant or decreasing to scale) on the shape of the development boundary. They find that mixing uses can significantly increase property value when the correlation between payouts from different property types is low or when redevelopment costs are low, and provide policy implications regarding multiple-use zoning.

Buetow and Joseph (1998) model the market price of a real-estate asset and its rental rate both via a Geometric Brownian Motion and a mean reverting process. Using the no-arbitrage assumption and a riskless-hedge portfolio, the authors obtain a system of stochastic differential equations, whose solution they approximate using the finite-difference method, to derive the values of the option to renew the lease at a rent indexed to the Consumer Price Index (CPI) and the option to purchase the leased space at a price indexed to CPI. Grenadier (2002) presents a continuous time model to price real estate leases with competitive interactions. Cunningham (2006) investigates the hypothesis that greater price uncertainty should delay the timing of development and raise land prices. He finds that a one-standard-deviation increase in uncertainty

lowers the likelihood of development by 11 percent and raises vacant land prices by 1.6 percent in a case study using land prices in King County, WA.

Bulan et al. (2006) study the relationship between uncertainty and investment delays, with a focus on the impact of competition on this relationship, using a sample of 1,214 condominium developments in Vancouver, Canada. They find that a one-standard deviation increase in the return volatility reduces the probability of investment by 13 percent. Increases in (idiosyncratic or systematic) risk lead developers to delay new real estate investments, but increases in the number of potential competitors counterbalances the relationship between idiosyncratic risk and development. Schwartz and Torous (2007) test Grenadier's real estate lease pricing model and argue that greater competition among local developers is associated with more building starts.

4.4 Other Applications

4.4.1 Information Technology

McGrath (1997) investigates real options to initiate technology positioning investments with a focus on the relationship between boundary conditions and uncertainty (in demand, adoption rate, access to markets, expropriation, imitation, access to infrastructure, and more), especially in hyper-competitive environments. Benaroch and Kauffman (1999) apply the Black-Scholes option pricing model for the exercise time of a deployment option of POS (point-of-service) debit services by Yankee 24. Benaroch and Kauffman (2000) consider a real options framework to expand an electronic banking network; they show that traditional methods would have provided erroneous recommendations. Taudes et al. (2000) apply real options to a software platform upgrade involving two releases of the SAP software. Schwartz and Zozaya-Gorostiza (2003) describe how to value information technology investment projects (either development or acquisition projects) using real options, when investment benefits are represented as a stream of stochastic cash flows. Kauffman and Li (2005) consider a firm deciding whether to adopt one of two incompatible and competing technologies and provide a continuous-time stochastic model to help determine the optimal timing for managerial adoption.

4.4.2 Healthcare Technology

Palmer and Smith (2000) use the real options approach to assess an irreversible healthcare technology investment decision with options to defer. The authors also address applicability of the real options approach at the microlevel of the individual patient treatment in which

uncertainty and reversibility are observed. In addition, they show results of sensitivity analysis for cost-per-QALY (Quality-Adjusted Life-Years) for 7 medical procedures.

Levaggi and Moretto (2008) analyze a hospital's optimal investment decision in a new health-care technology, which allows the hospital to increase the quality level of the care provided at the cost of an irreversible investment. The authors show how the technology investment is best incentivized using a long-term contract where the number of treatments reimbursed depends on the level of investment made when the technology is new.

Özgul et al. (2009) build a model to value a real-world hospital information system (HIS) project with compound options. The authors define HIS as a customized and upgraded enterprise resource planning (ERP) system, which supports strategic service offering, resource and supply chain planning, collaborative care support, patient management, enterprise management, and support capabilities. A binomial lattice model is applied for the pricing of real options. Sensitivity analysis supports the claim that the method is robust against uncertainty in parameters and interaction between options.

4.4.3 Energy

Tseng and Barz (2002) use real options to value power plants with unit commitment constraints over a short-term period. They show that failing to consider physical constraints such as minimum uptime and downtime may significantly overvalue a power plant. Thompson et al. (2004) use real options theory to derive nonlinear partial-integro-differential equations for the valuation and optimal operations of hydroelectric and thermal power generators in deregulated electricity markets. The electricity price is subject to uncertainty because of the competition in the deregulated market and of the dynamic nature of the demand and production cost. Other sources of uncertainty include the water inflow, power function, cost of fuel, lead time in power generation, control response time lags, and output rates. The authors model the price and cost using mean-reverting stochastic differential equations with jumps. They solve the problem using numerical methods and determine the optimal operational strategies along with the expected cash flow. They further argue that their framework achieves high levels of computational speed and accuracy while incorporating a wide range of spot price dynamics and operational characteristics.

Rothwell (2006) uses real options to evaluate new nuclear power plants, specifically, the building of an advanced boiling water reactor in Texas, and determines the risk premium associated with net revenue uncertainty. Tseng and Lin (2007) consider the real option to commit or de-commit a generating unit at a power plant. The decision to exercise the real option depends on the fuel (used by the power plant to produce electricity) and electricity prices. The authors

assume that the fuel and electricity prices follow correlated geometric mean reverting processes and propose a lattice framework to represent the price movements. The option valuation problem is formulated as a stochastic dynamic programming problem. The real-options method in Bockman et al. (2008) provides, for small hydropower projects, an electricity price threshold below which it is never optimal to initiate the project and above which investment is made according to an optimal-size function, and is illustrated using three Norwegian examples. Yang et al. (2008) present an analysis undertaken by the International Energy Agency to study the effects of uncertainty in government climate policy on private investors' decision-making in the power sector, with case studies in gas, coal and nuclear power investment.

Lee and Shih (2010) evaluate quantitatively the policy value provided by developing renewable energy in the face of uncertain fossil fuel prices and renewable-energy-policy-related factors. They discuss how policy planning uncertainty including managerial flexibility influences renewable energy development. Madlener and Stoverink (2011) value a coal-fired power plant investment project in presence of market liberalization. Cheng et al. (2011) use compound real options for cleaner energy development projects that incorporate the lead time for power plant investments and demand uncertainty.

4.4.4 Revenue Management

The dynamic nature of the customer demand, raw material and commodity prices, and exchange rates and shifts in market competition in supply chain systems result in the need to adapt to changes and the flexibility in the decision making process.

Tsai and Hung (2009) address demand uncertainty in Internet retailing and propose a dynamic pricing method integrating the real options (RO) approach with goal programming (GO) and the analytic hierarchy process or AHP (a technique developed to structure and analyze complex decisions) for the revenue management problem of Internet auctions. RO is used to determine upper and lower bounds of the value of each auction commodity; AHP is employed to calculate the increment and decrement volumes for each commodity based on criteria such as demand growth, market share, life cycle, competitive power, and long term return/volatility ratio. Timely quota increment and decrement values are calculated based on AHP weights and updated as new information is obtained. A goal programming approach is used to minimize the penalties resulting from the under- and over-achievements of the targeted goals while satisfying the available budget, limited capacity, and AHP process-related constraints where the decision variables are the quotas of the auction commodities, increment and decrement of the

initial quotas, and deviation variables denoting under and over achievement of the targeted goals on the revenue. The authors observe that a firm can increase the profitability of its Internet auction practices by following the recommendations obtained from the proposed method, since it incorporates risk information.

4.4.5 Licensing

Miller and Bertus (2005) study the applicability of real options to license valuation in the aerospace maintenance, repair, and overhaul industry. They argue that, in the example they consider, 20% of licensing opportunities (those that are similar to existing product lines) can be valued using discounted cash flow and the remaining 80% are good candidates for real options analysis: 20% in a traditional licensing scenario that should be valued using the European futures option framework, 30% as licenses depending on each other and thus modeled as compound options, 10% as opportunities with investment cost uncertainty that should be valued using Margrabe's formula (see Margrabe (1978) and Eq. (3) below) and, for the 10% of riskiest licensing opportunities to evaluate new parts, a perpetual call option model.

Ziedonis (2007) applies the real options framework to technology licensing, where firms can use options contracts when considering whether to acquire rights to commercialize university technologies. He finds that firms are more likely to purchase option contracts for more uncertain technologies and that firms that are better able to evaluate an external technology are less likely to purchase options before licensing.

5 Recent Research Efforts

In this last section, we conclude by mentioning a few recent papers on real options that illustrate current research efforts in this field. Most of these efforts are specific to a given research team and its expertise; however, there seems to be an important stream of developing research that applies real options to renewable energy investment problems.

5.1 Research in management, finance, game theory

McCarter et al. (2011) introduce the concept of collective real options, which manage social uncertainty by developing trust between alliance partners and producing relational small wins. Alvarez and Dixit (2014) provide a real-options perspective on the future of the Euro; specifically, they use a multi-country real options model (an n-dimensional optimal stopping problem with country-specific shocks and "convergence" of member economies) in computing the option value

of breaking up the Eurozone and find a non-negligible but small option value. Ignoring this option value leads to a one time loss of about 4% annual GDP for each country.

Grenadier and Malenko (2011) study games in which the decision to exercise an option is a signal of private information to outsiders, which distorts the timing of exercise. The authors connect the direction of distortion with the change in the decision-maker's utility due to outsiders' belief about the payoff from exercise. They apply their model to four corporate finance settings: investment under managerial myopia, venture capital grandstanding, investment under cash flow diversion and product market competition. Grullon et al. (2012) provide evidence that the positive relation between firm-level stock returns and firm-level return volatility is due to firms' real options. Finally, Bensoussan et al. (2014) examine irreversible investment decisions in duopoly games with a variable economic climate. The central modeling feature of the paper is to integrate timing flexibility, competition and changes in the economic environment in the form of a cash flow process with regime switching. The authors formulate the problem as a stopping-time game under Stackelberg leader-follower competition, and obtain the regime-dependent optimal policies for both the leader and the follower.

5.2 Research in energy

Zhu and Fan (2011) investigate a carbon capture and storage (CCS) investment evaluation model incorporating the following uncertainties: existing thermal power generating cost, carbon price, thermal power with CCS generating cost and investment in CCS technology deployment. They solve their model using the Least Squares Monte Carlo (LSM) method. They conclude based on their analysis that the current investment risk of CCS is high. Martinez-Cesena and Mutale (2011) consider real options in the context of renewable energy generation projects in the case of flexibility in design. They illustrate the potential of their approach on hydropower projects. Krogh-Boomsma et al. (2012) analyze investment timing and capacity choice for renewable energy projects under support schemes such as feed-in tariffs and renewable energy certificate trading. In a Nordic case study based on wind power, the authors find that the feed-in tariff encourages earlier investment but renewable energy certificate trading creates incentives for larger projects once investment has been undertaken.

5.3 Concluding remarks

Real options continue to be a vibrant research area bridging finance and management. While their pricing is often complex and fraught with difficulties, they present a unique opportunity

for the decision maker to incorporate and value managerial flexibility in a business environment characterized by always increasing amounts of uncertainty and fast-paced change.

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